PRINCIPLES OF INTERFEROMETERS II

R Subrahmanyan

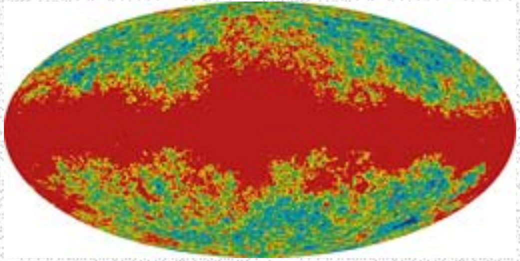
NRAO, Socorro

RRI, Bangalore

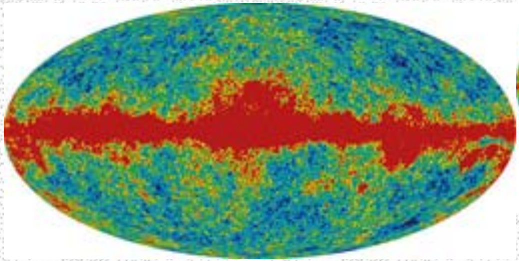
Contents of this talk

- What does a single radio antenna placed on the surface of the Earth measure?
- What do arrays of antennas distributed on the ground measure?
- How sensitive are the measurements by radio antennas and interferometers?

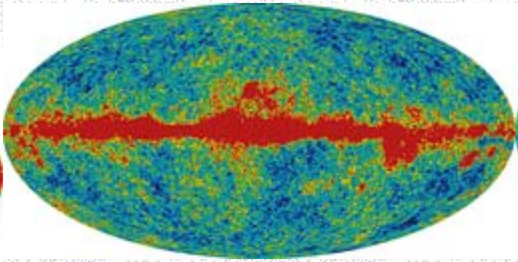
23 GHz



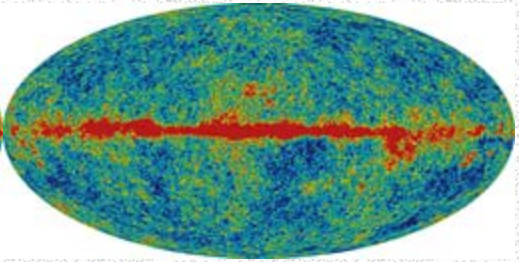
33 GHz



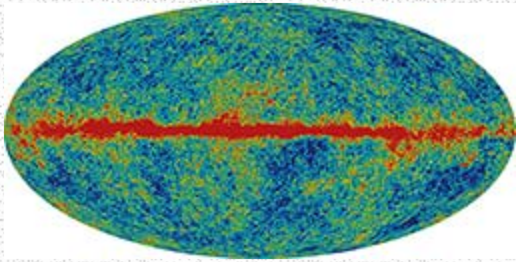
41 GHz



61 GHz



94 GHz



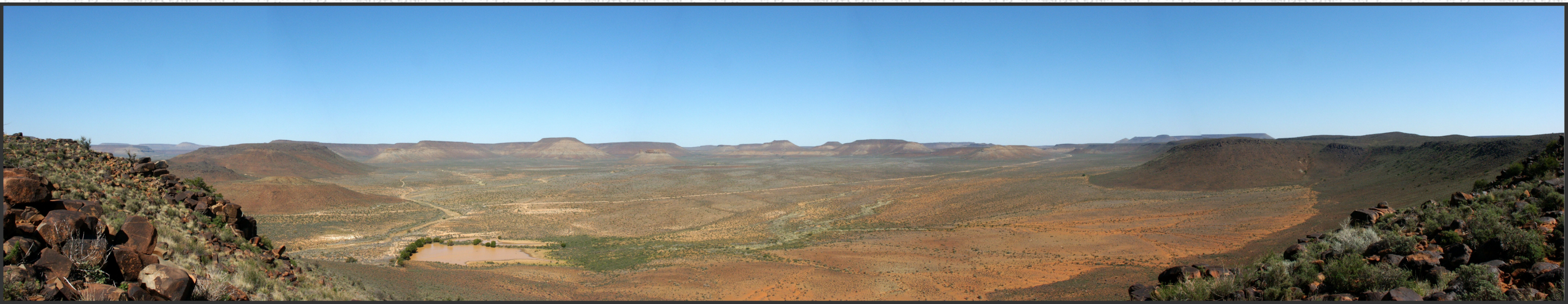
Any antenna in the ground receives electromagnetic radiation from the sky above

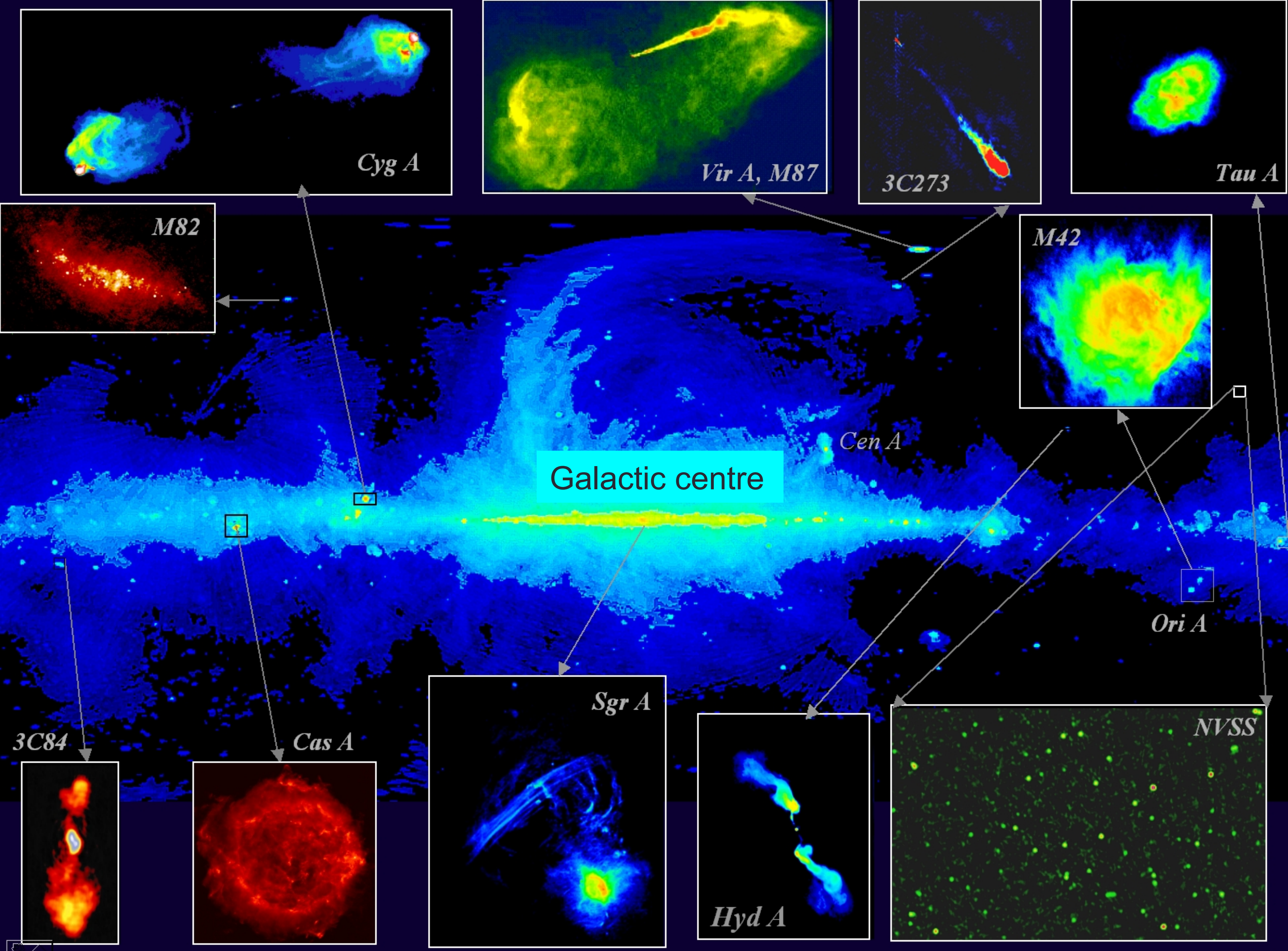
And the ground below

Changing

- Prominence of galactic plane
- Intensity of sky brightness

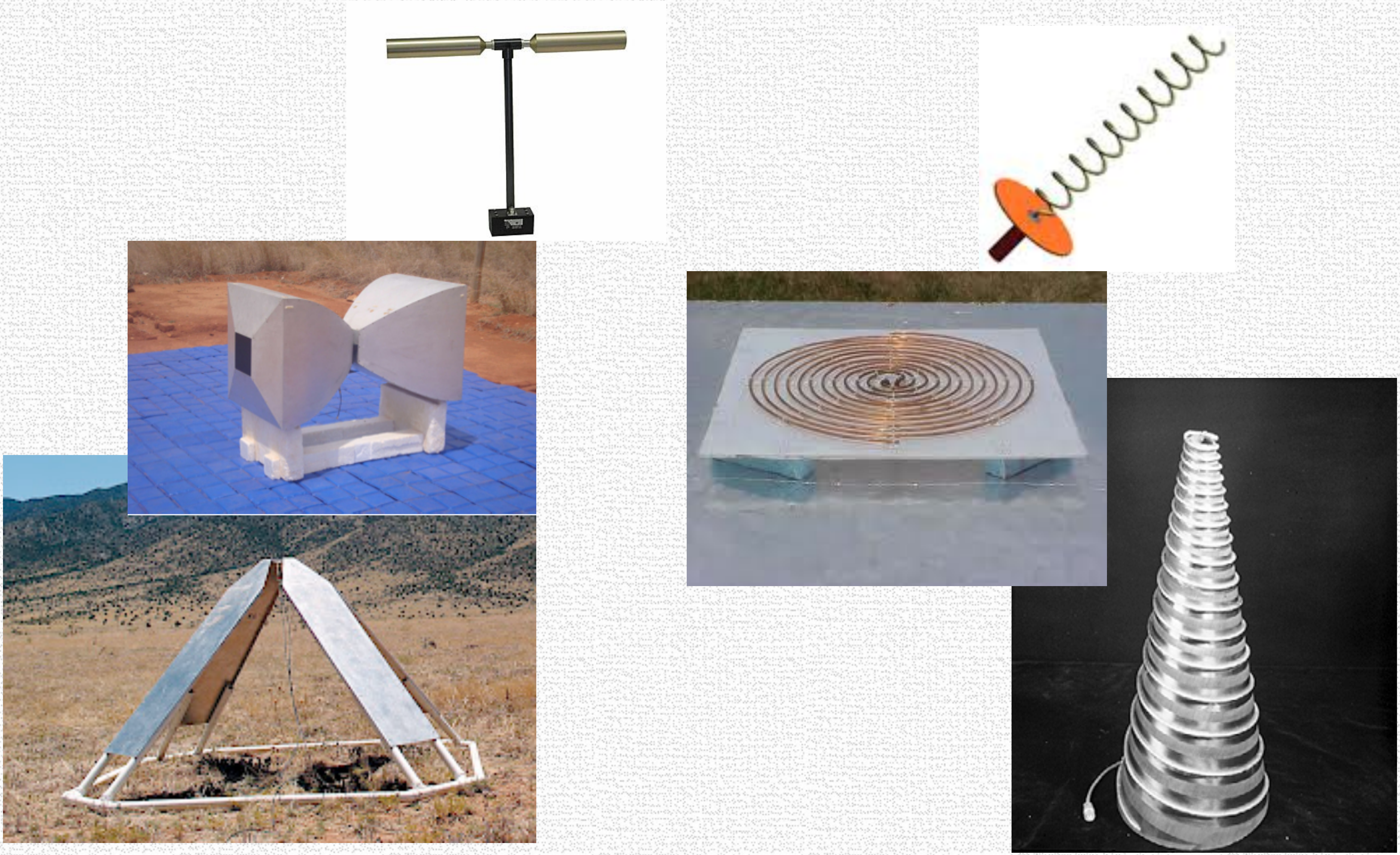
- Prominence of the ground



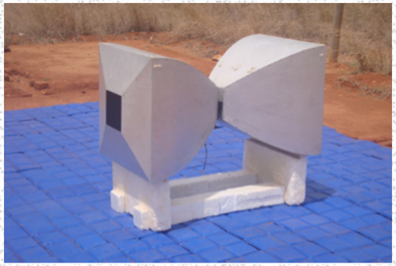


Galactic centre

Radio antennas come in different shapes and sizes



A radio antenna is a ‘sensor’ of the EM field at its location in space



- The electric field is created by electromagnetic radiation arriving at that location in space from the sky and ground.

- The field is caused by a large number of independent atomic processes in the sky and ground

- The electric field is not a constant in time.

  It is time varying.

  It is not a sine wave.

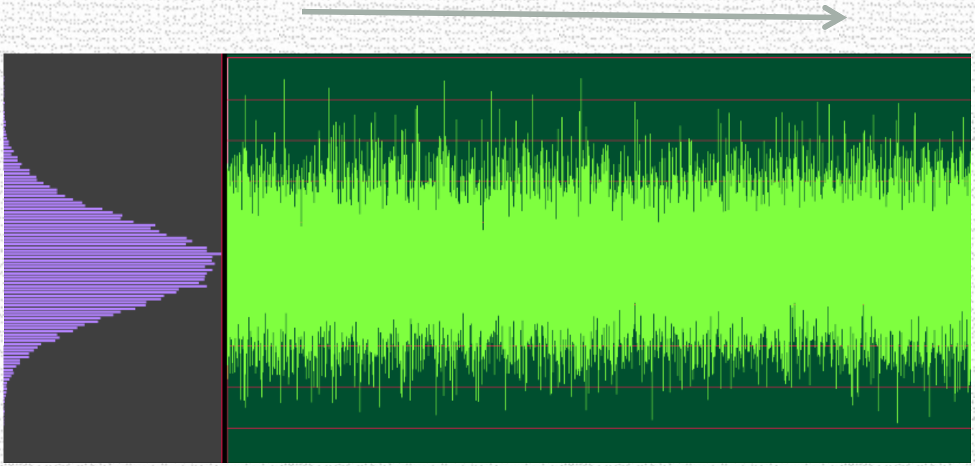
  The field is a random variate.

- The field has a Normal (or Gaussian) probability distribution.

- Aside from RFI and ETI.

A radio antenna is a ‘sensor’ of the EM field at its location in space

- A radio antenna has metal conductors in which the fluctuating electric field causes time-varying currents.
- A radio antenna converts the fluctuating electric field into a fluctuating voltage on a transmission line = cable.



Histogram of the voltage from the antenna

Voltage from antenna vs Time

Radio antennas may be arrayed and the voltages summed to yield the summed field over an aperture.

These are called aperture arrays.

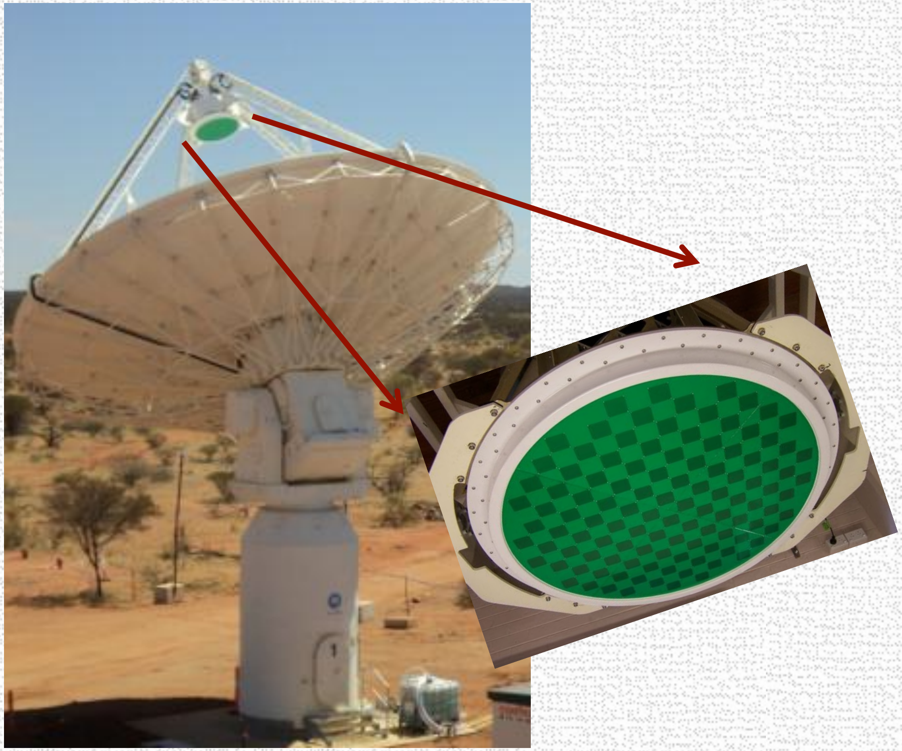
MWA tile

LOFAR low and high band stations

Apertures arrays may be phased to yield the EM field on the ground arising from selectively different patches of the sky.

Parabolic reflectors sum the field over the aperture of the dish.

Larger the aperture and higher the frequency – better will be the cancellation of fields owing to off-axis sources on the sky.

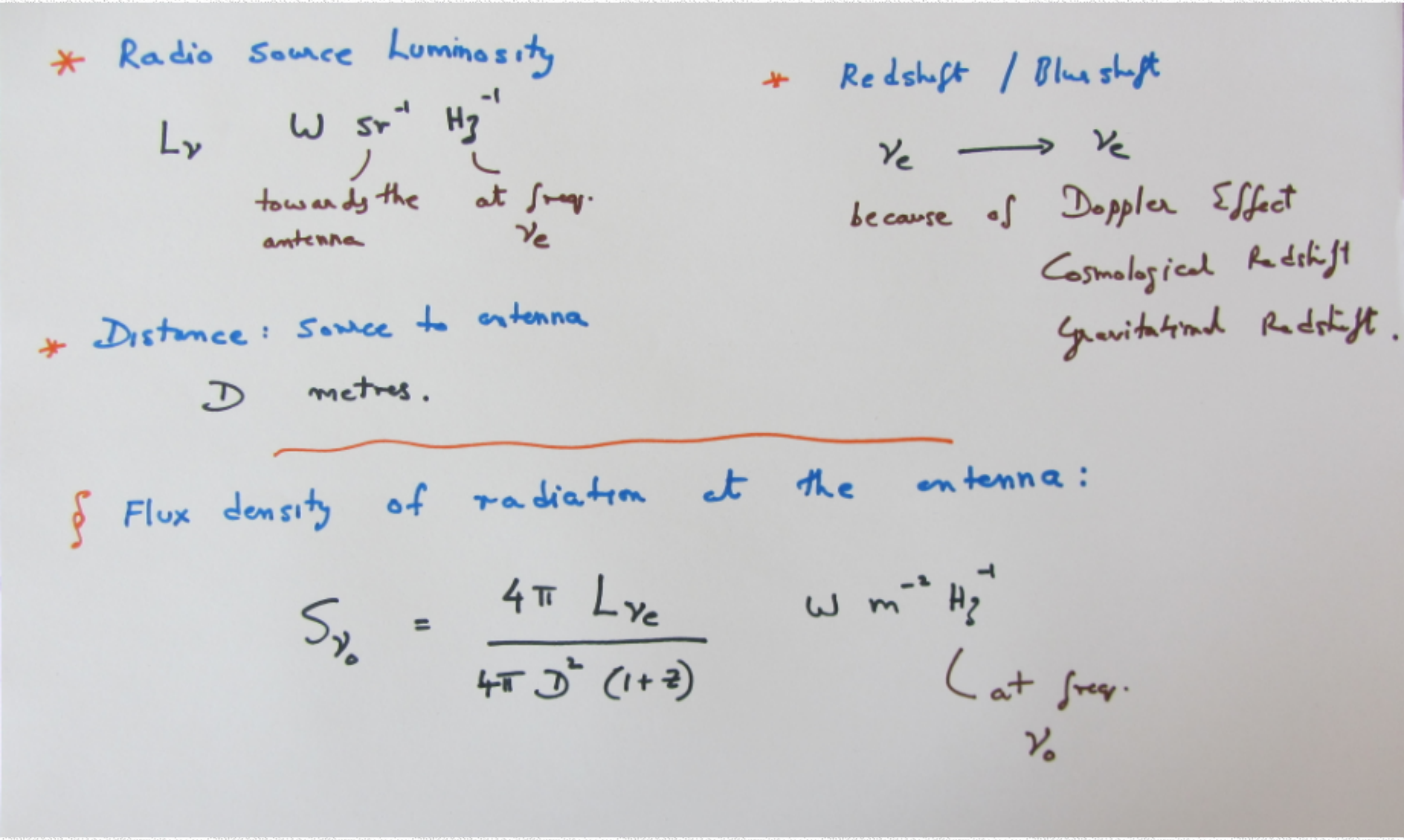


A 2D set of EM field sensors at the focal plane of a parabolic dish

Focal Plane Array

measures the summed EM fields from sky regions that are offset in a 2D raster.

Every radio source on the sky results in radiation density on the ground



- Radio source Luminosity

L_ν W sr^{-1} Hz^{-1}

towards the antenna

at freq. ν_e

- Distance : source to antenna

D metres.

- Redshift / Blueshift

$\nu_e \longrightarrow \nu_e$

because of Doppler Effect

Cosmological Redshift

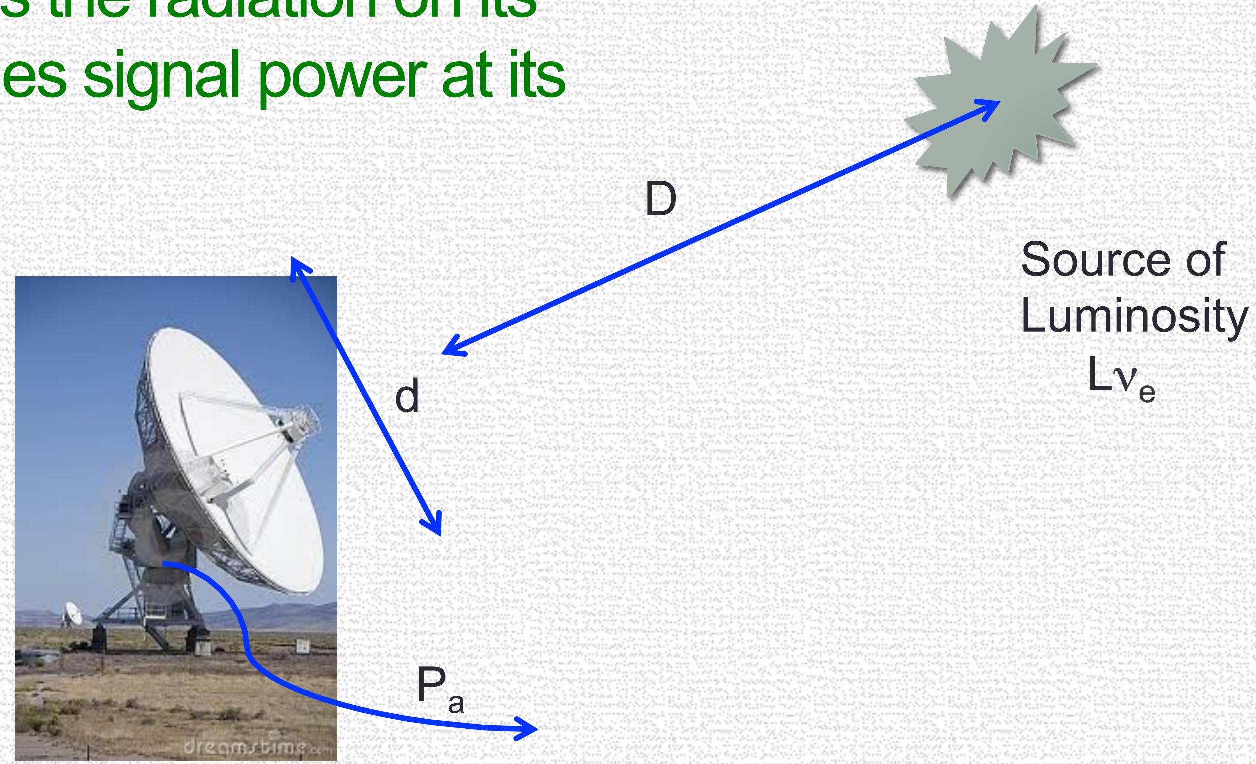
Gravitational Redshift.

§ Flux density of radiation at the antenna :

$$S_{\nu_o} = \frac{4\pi L_{\nu_e}}{4\pi D^2 (1+z)} \quad \text{W m}^{-2}\,\text{Hz}^{-1}$$

at freq. ν_o

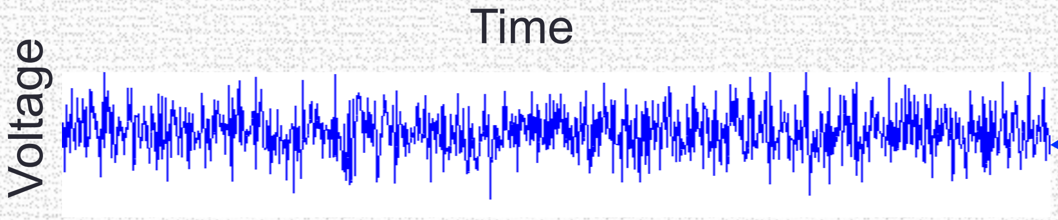
The antenna senses the radiation on its aperture and provides signal power at its terminals

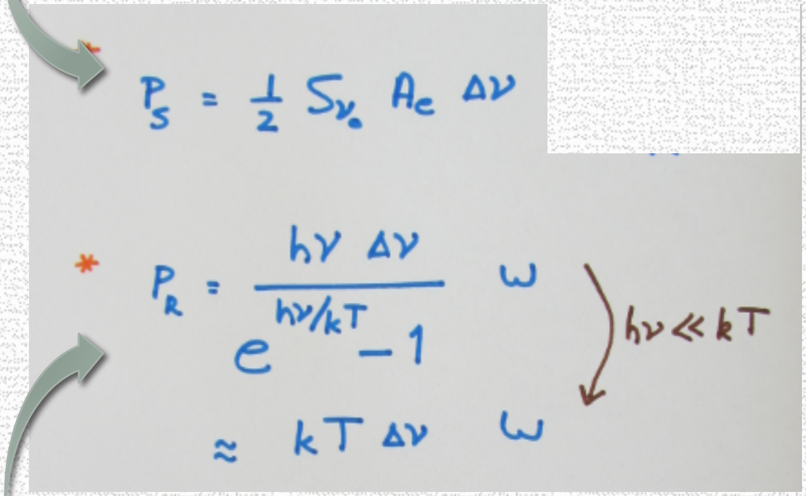


* Signal power from the source

$$P_s = (f_p\, S_{\nu_0}) \times \left(A_e = \eta_a A = \eta_a \pi d^2/4\right) \times \Delta\nu \quad W$$

a single probe picks up one polarization : $f_p = \frac{1}{2}$ if src is randomly polarized.



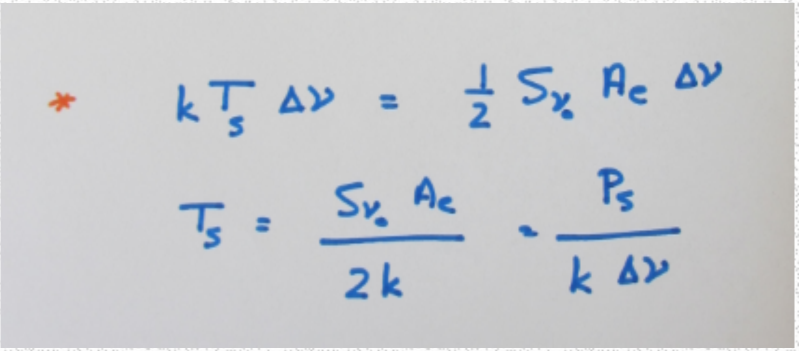


$$* \quad P_s = \frac{1}{2} S_{\nu_0} A_e \Delta\nu$$

$$* \quad P_R = \frac{h\nu \, \Delta\nu}{e^{h\nu/kT} - 1} \text{ W} \quad \Big\rangle \, h\nu \ll kT$$

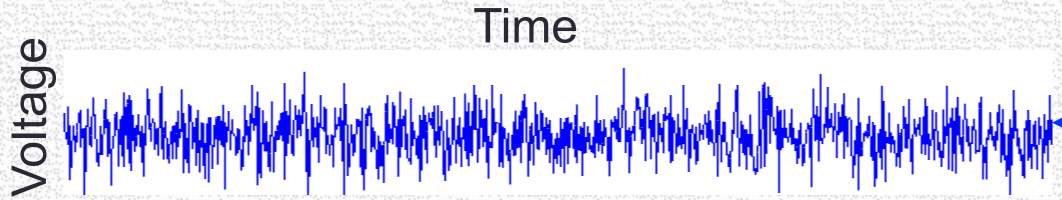
$$\approx kT \, \Delta\nu \text{ W}$$

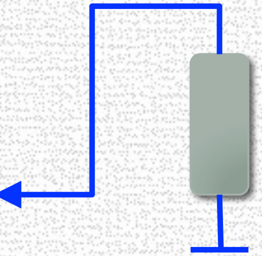
When T is adjusted so that the power from the resistor equals the power from the antenna



$$* \quad kT_s \, \Delta\nu = \frac{1}{2} S_{\nu_0} A_e \, \Delta\nu$$

$$T_s = \frac{S_{\nu_0} A_e}{2k} = \frac{P_s}{k \, \Delta\nu}$$

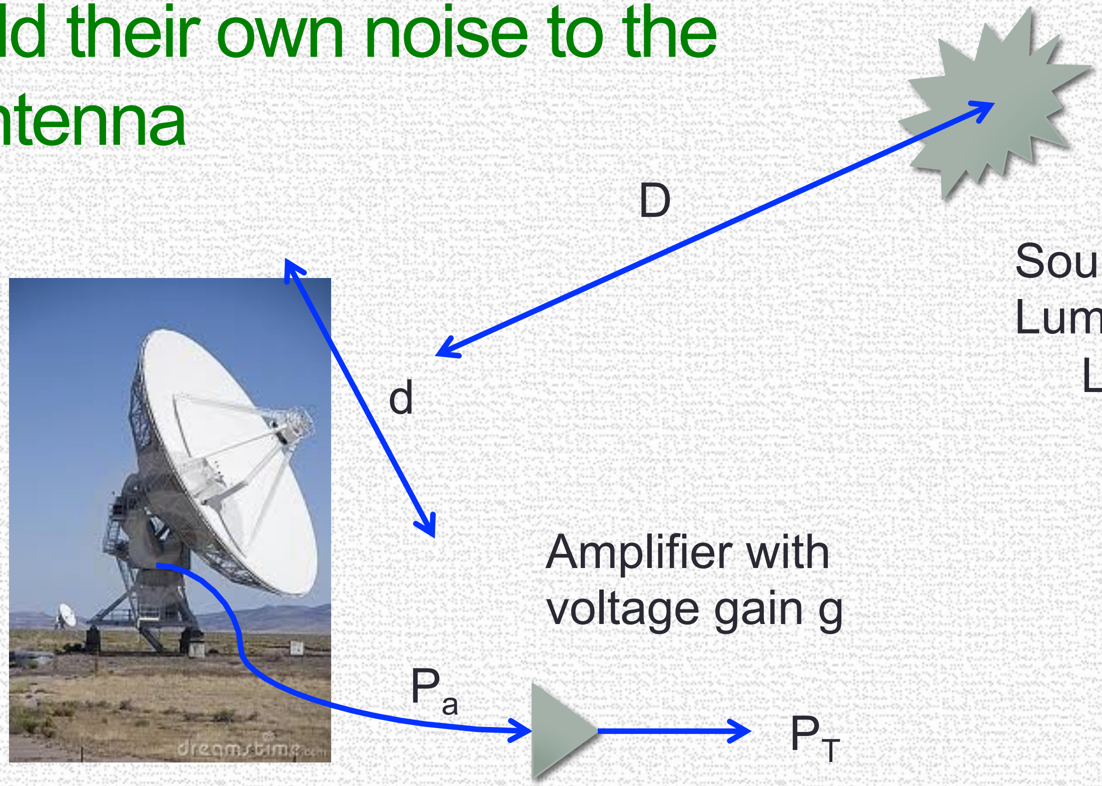




Resistance R at temperature T

Radio astronomers use Temperature as a unit of power from the antenna!

Amplifiers also add their own noise to the signal from the antenna



D

d

Source of Luminosity $L\nu_e$

Amplifier with voltage gain g

P_a

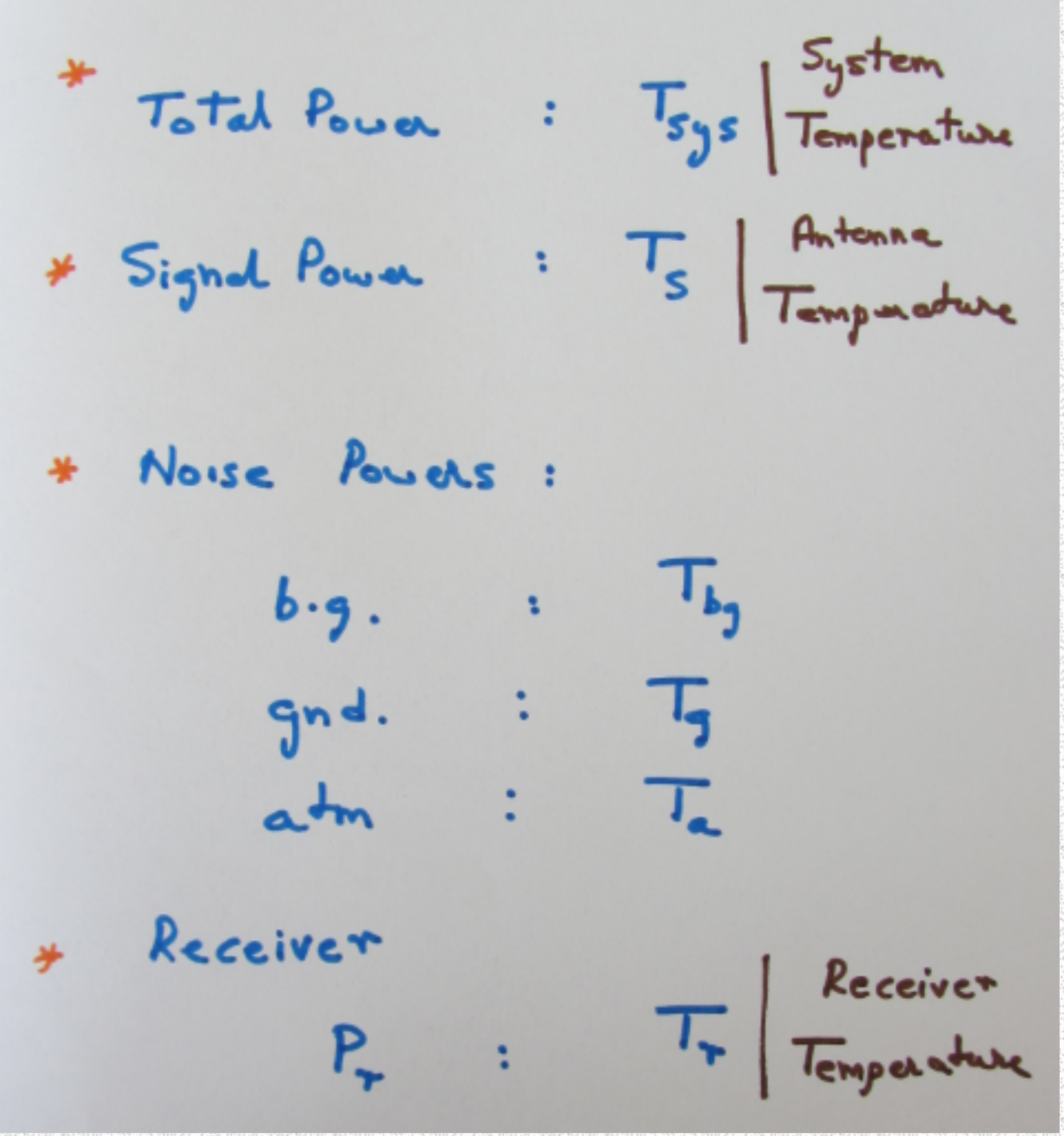
P_T

* Following the amplifier :

$$P_s' = g^2 P_s = \frac{1}{2} g^2 S_{\nu_0} A_e \Delta\nu \qquad \omega$$

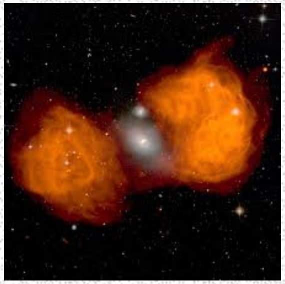
$$P_T = P_s' + \underbrace{P_{atm}}_{\text{atmosphere}} + \underbrace{P_{bg}}_{\text{b.g. sources}} + \underbrace{P_g}_{\text{ground}} + \underbrace{P_r}_{\text{amplifier}}$$

And all the power components in the cable from the antenna is given units of Temperature (K) corresponding to the equivalent temperature of a resistor that replaces the antenna



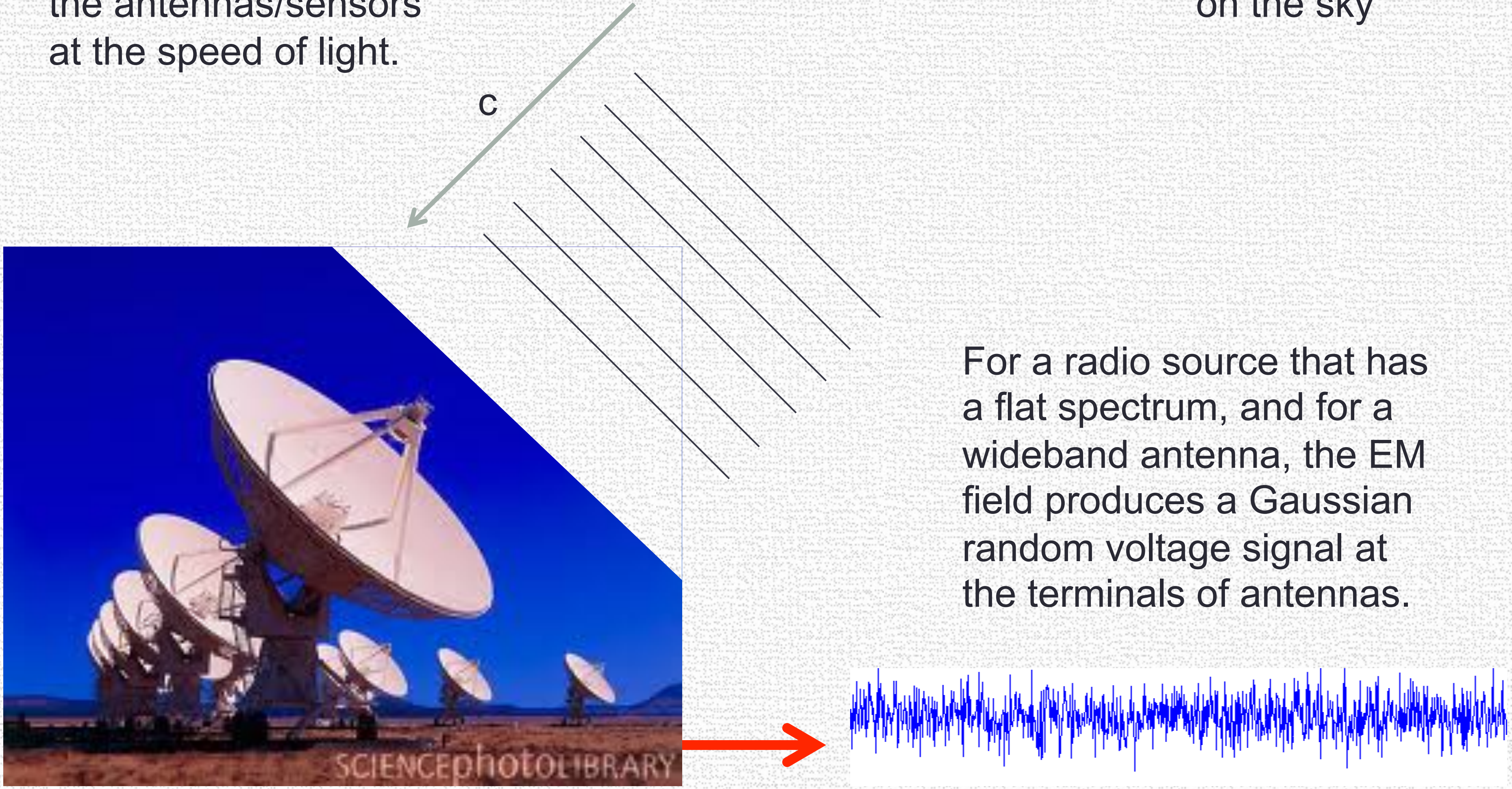
- Total Power : T_{sys} | System Temperature
- Signal Power : T_s | Antenna Temperature
- Noise Powers :
  - b.g. : T_{bg}
  - gnd. : T_g
  - atm : T_a
- Receiver
  - P_r : T_r | Receiver Temperature

The radio sources in the sky result in a radiative electromagnetic field on the ground



Radio source
on the sky

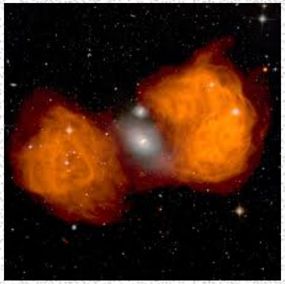
That rush past
the antennas/sensors
at the speed of light.



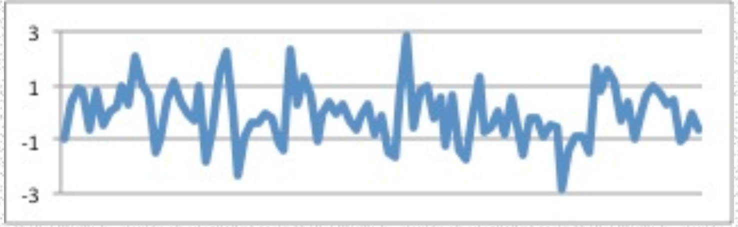
For a radio source that has a flat spectrum, and for a wideband antenna, the EM field produces a Gaussian random voltage signal at the terminals of antennas.

Principle of interferometers

Extended radio source of angular size θ radians

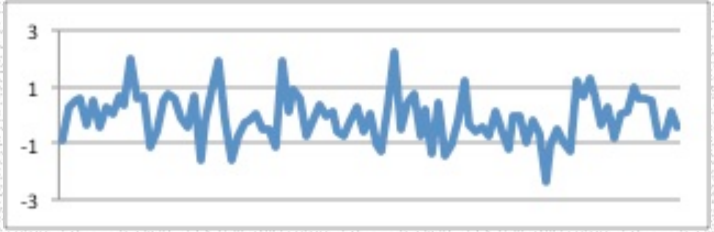


Spacing $< \lambda/\theta$



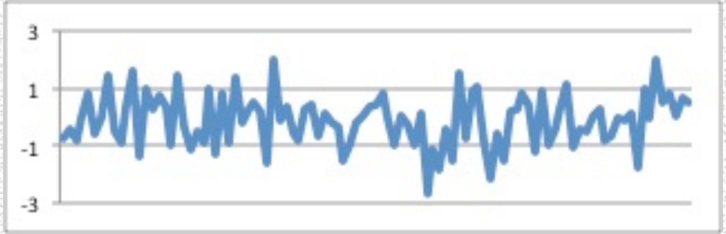
Spacing $> \lambda/\theta$

Voltage

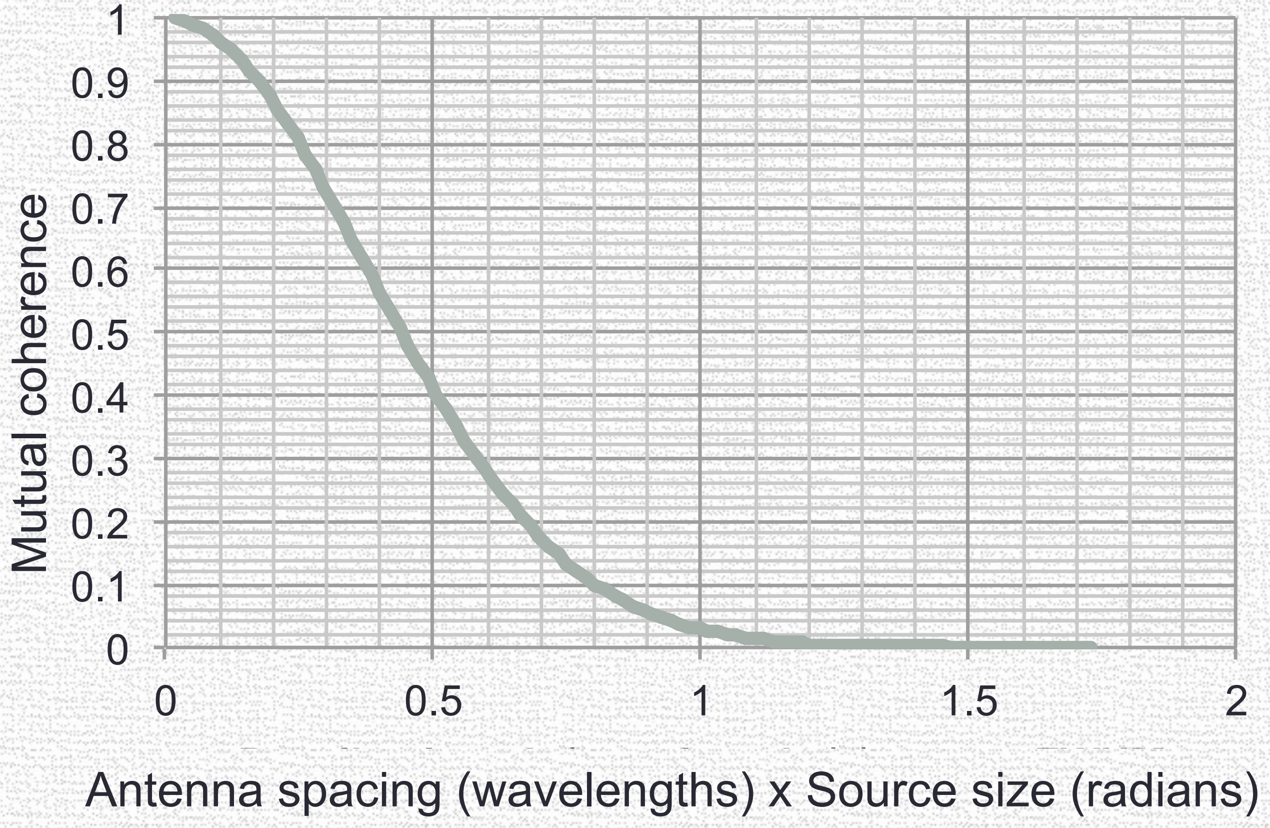


Time

Mutual coherence of the EM fields

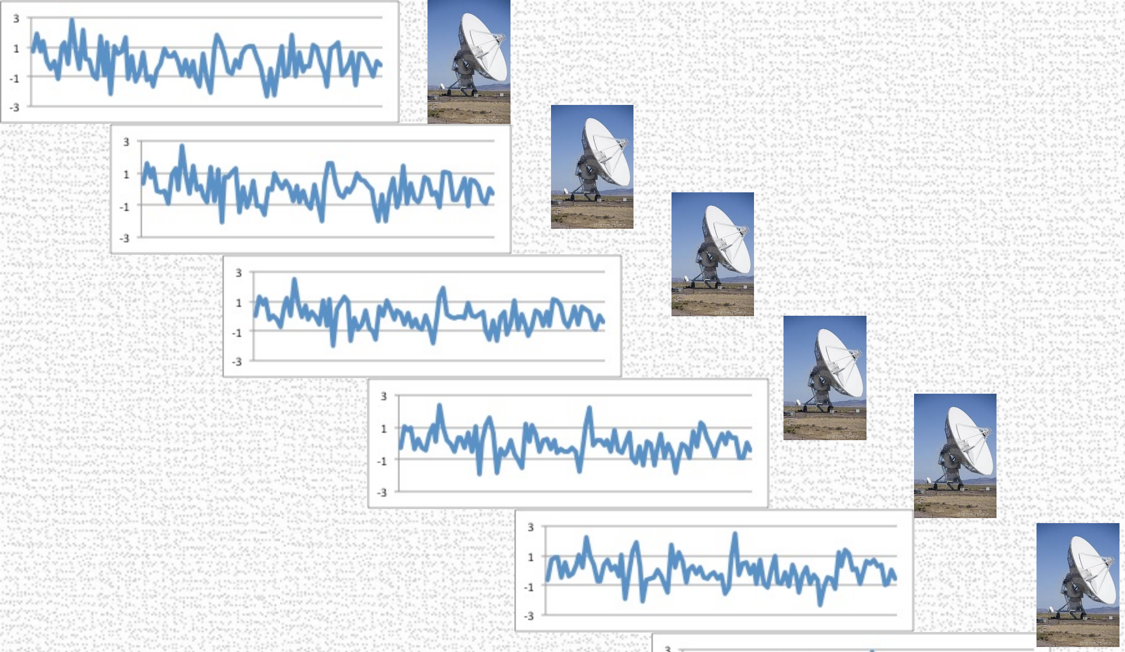


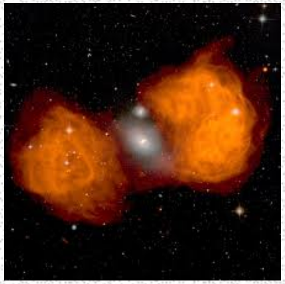
Mutual coherence between voltages from antenna elements drops off with separation



Mutual coherence drops off as baseline length exceeds inverse of source size

Mutual coherence drops off as Source size exceeds inverse of antenna spacing

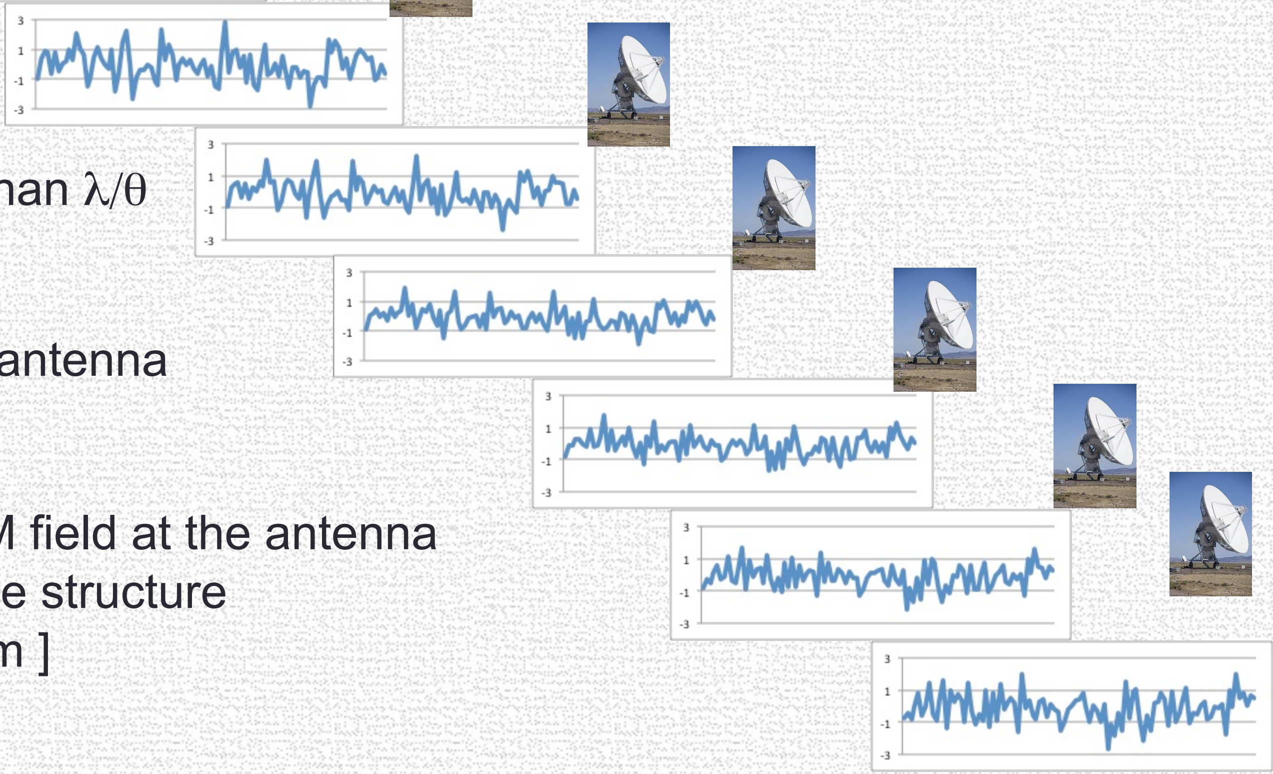




Voltage waveforms
are similar for antenna
pairs that are spaced less than λ/θ

Voltage waveforms are
progressively dissimilar for antenna
separations exceeding λ/θ

Mutual coherence of the EM field at the antenna
locations depends on source structure
[Cittert von Zernike theorem]



Principle of synthesis imaging using radio interferometers

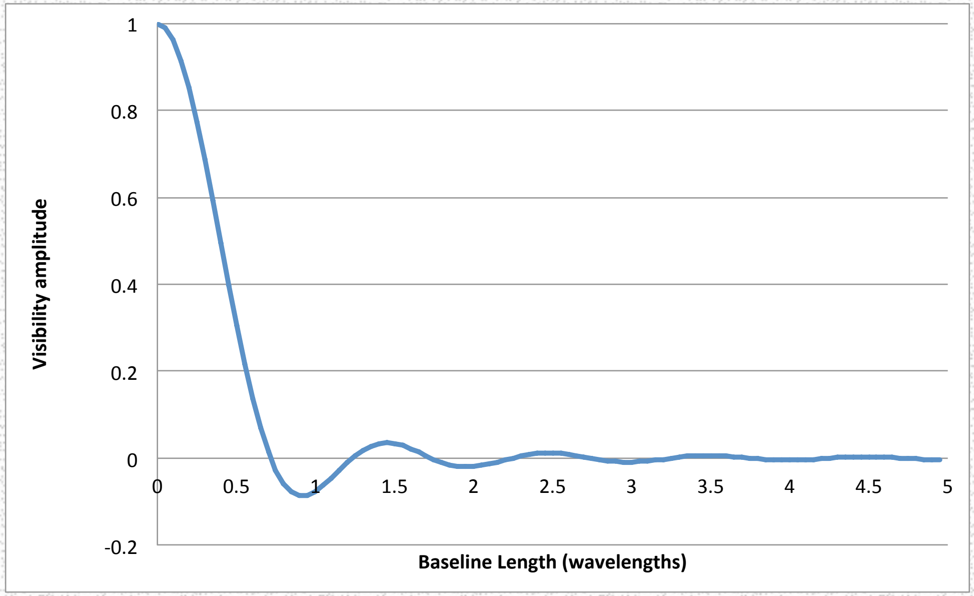
Structure in brightness distribution on the sky →
mutual coherence in the EM field on the ground

Mutual coherence between fields →
correlation properties of voltage waveforms sensed by antennas

Synthesis Imaging:

1. Measuring the cross-correlation between voltage waveforms sensed by pairs of antennas with a variety of separations = OBSERVING
2. To derive the mutual coherence properties of the EM field on the ground = CALIBRATION
3. Which may be inverted to solve for brightness distribution on the sky = IMAGING

Key advantage of using interferometers to measure the radio sky:



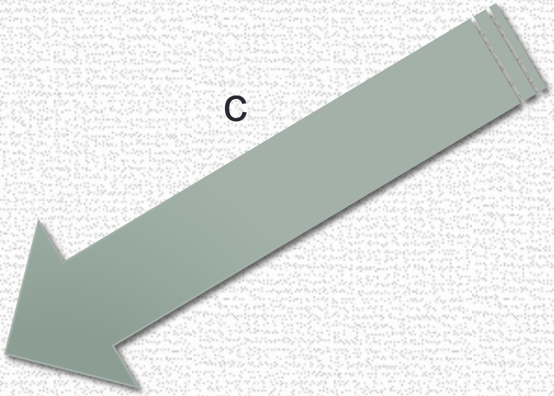
Interferometers see discrete sources and are blind to the uniform background

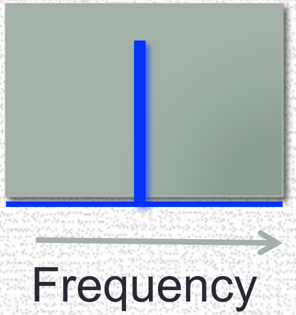
Interferometer response to mean sky brightness falls off on baselines exceeding a wavelength!

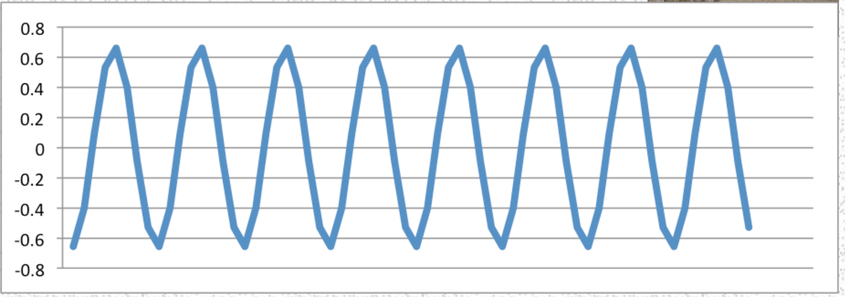
Interferometers are also blind to receiver noise, atmosphere radiation, ground radiation!

Time variation in the measured electric field

If the source in monochromatic
Emission is at a single frequency

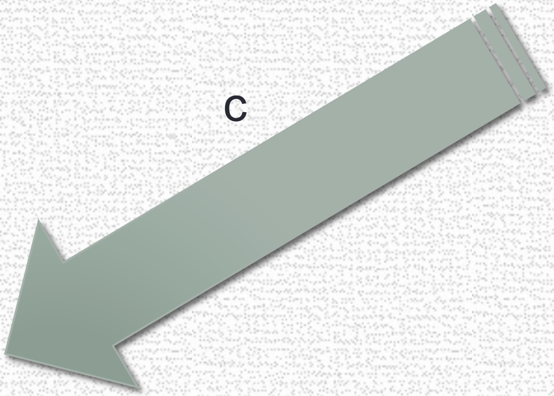


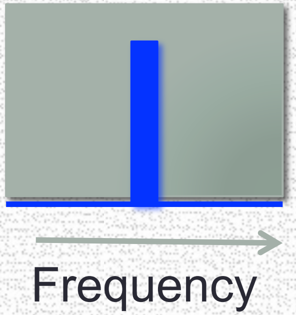


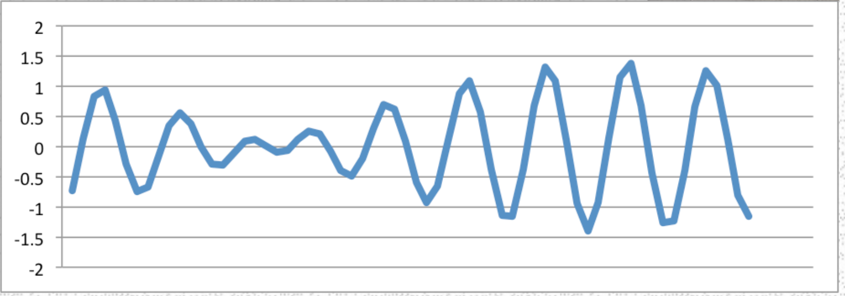


The measured electric field is
a sine wave over time with
constant amplitude.

If the source emission is over a narrow band

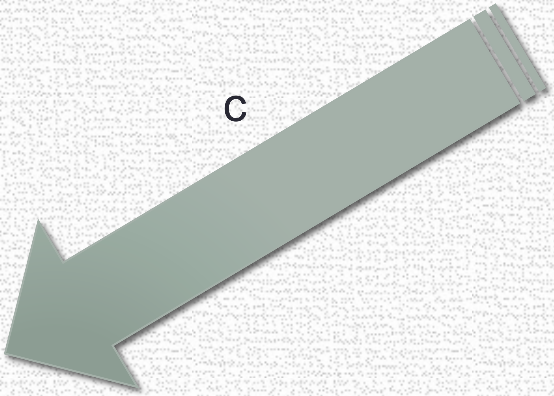


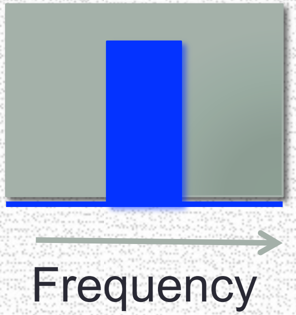


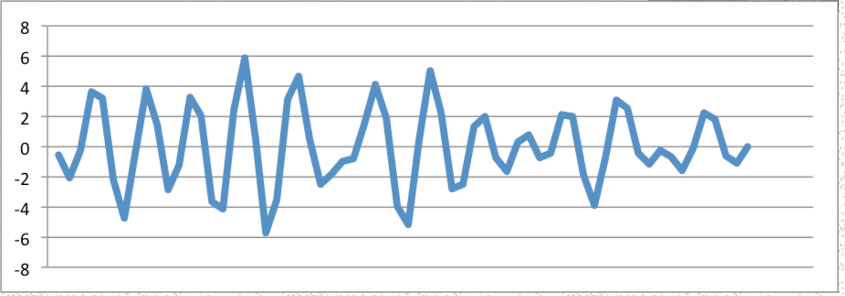


The measured electric field is a sine wave over time with slowly varying amplitude.

If the source emission is over a wide band



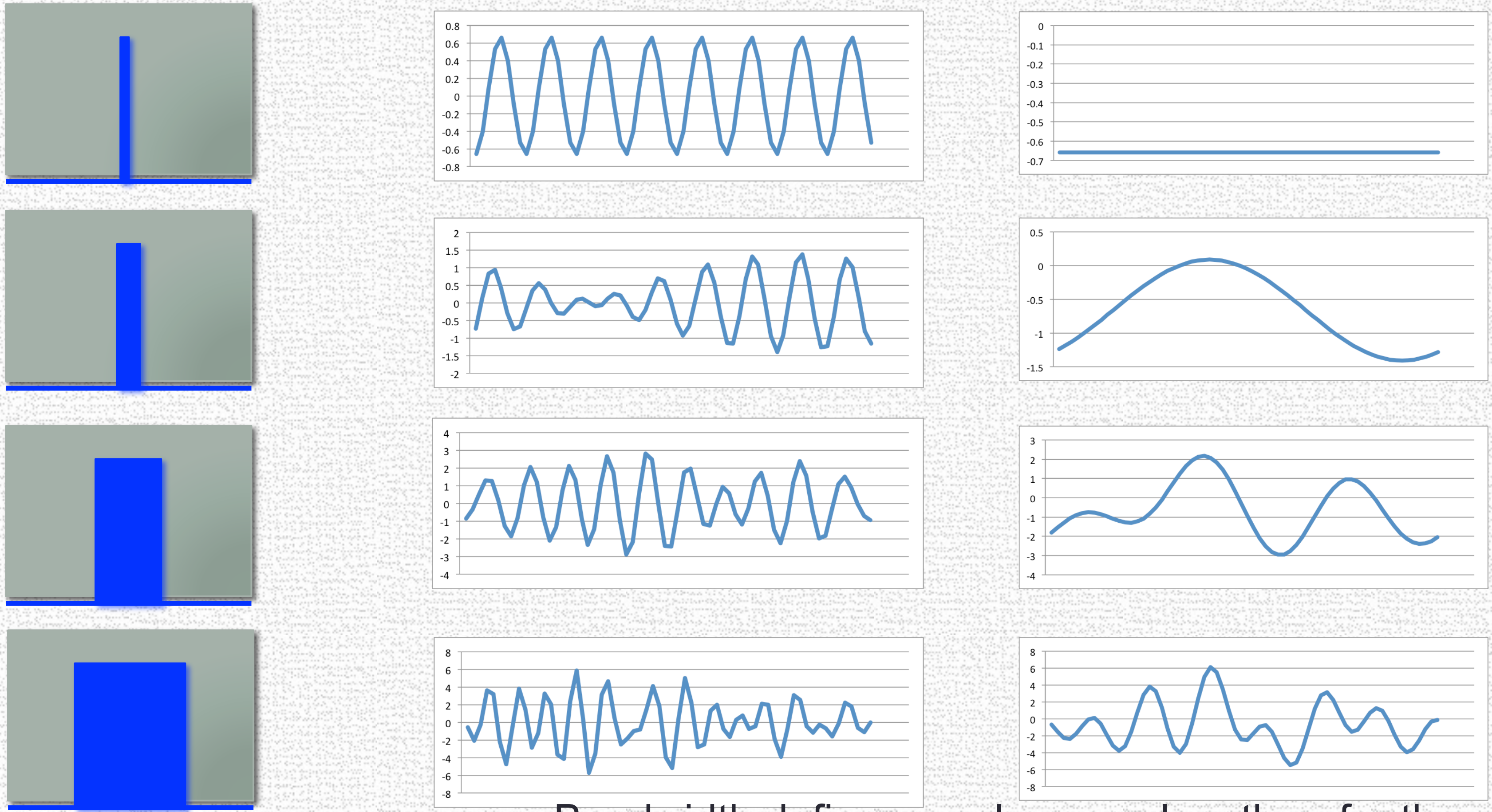




The measured electric field is a sine wave over time with rapidly varying amplitude.

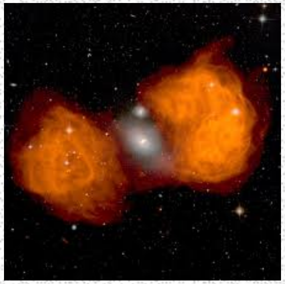
Wider the band, more quickly does the amplitude of the received field vary with time

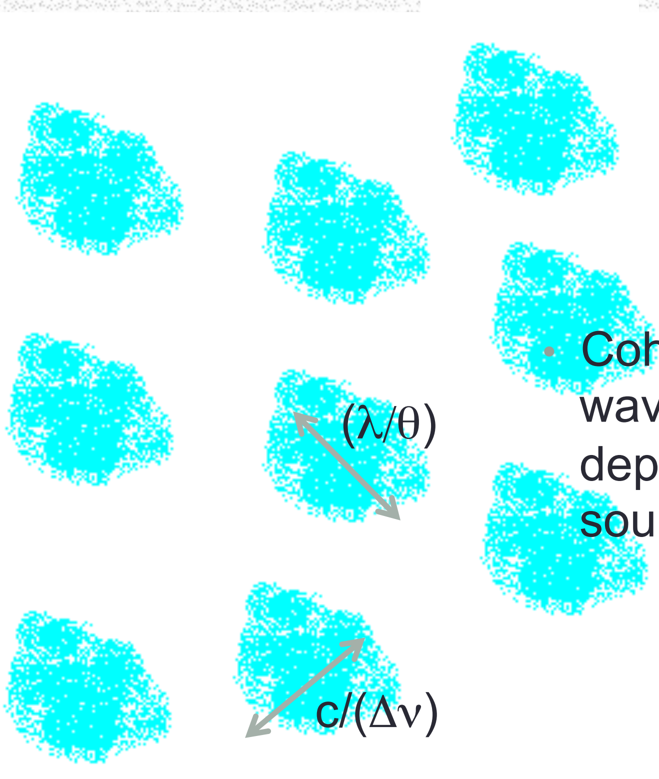
Emission Spectrum | Received waveform | Received field amplitude



Bandwidth defines a coherence length $c\tau$ for the amplitude of the received field amplitude

Depending on the source angular size and bandwidth: the EM field on the ground has a spatial-temporal coherence

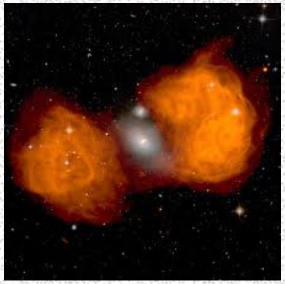


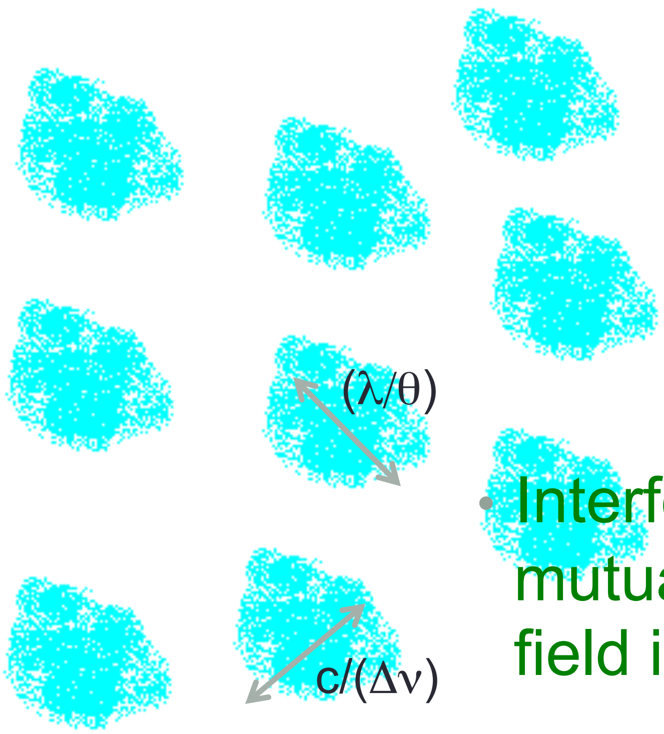


- Coherence scale is (λ/θ) parallel to the wave front from the source, and depends on the angular size of the source.

- Coherence scale is c/(Δν) along the line of sight to the source = perpendicular to the wave front, and depends on the bandwidth of the source emission.

Depending on the source angular size and bandwidth: the EM field on the ground has a spatial-temporal coherence





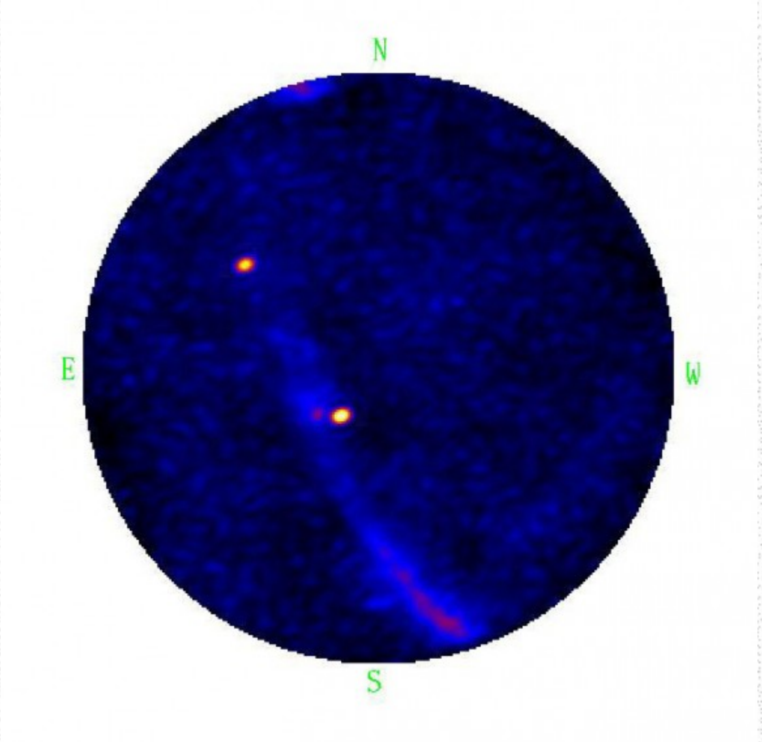
- Interferometers measure the mutual coherence of the EM field in space and time.

- Which is a measure of the structure of the emission in angular scale and frequency.

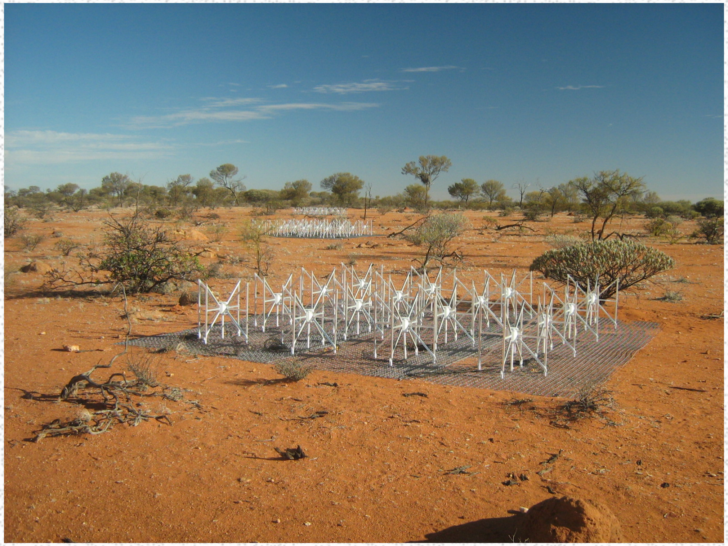
Wide-field interferometer arrays

Measure the coherence properties of the field on the ground corresponding to emission from the entire visible sky

And can image the whole sky at every instant!



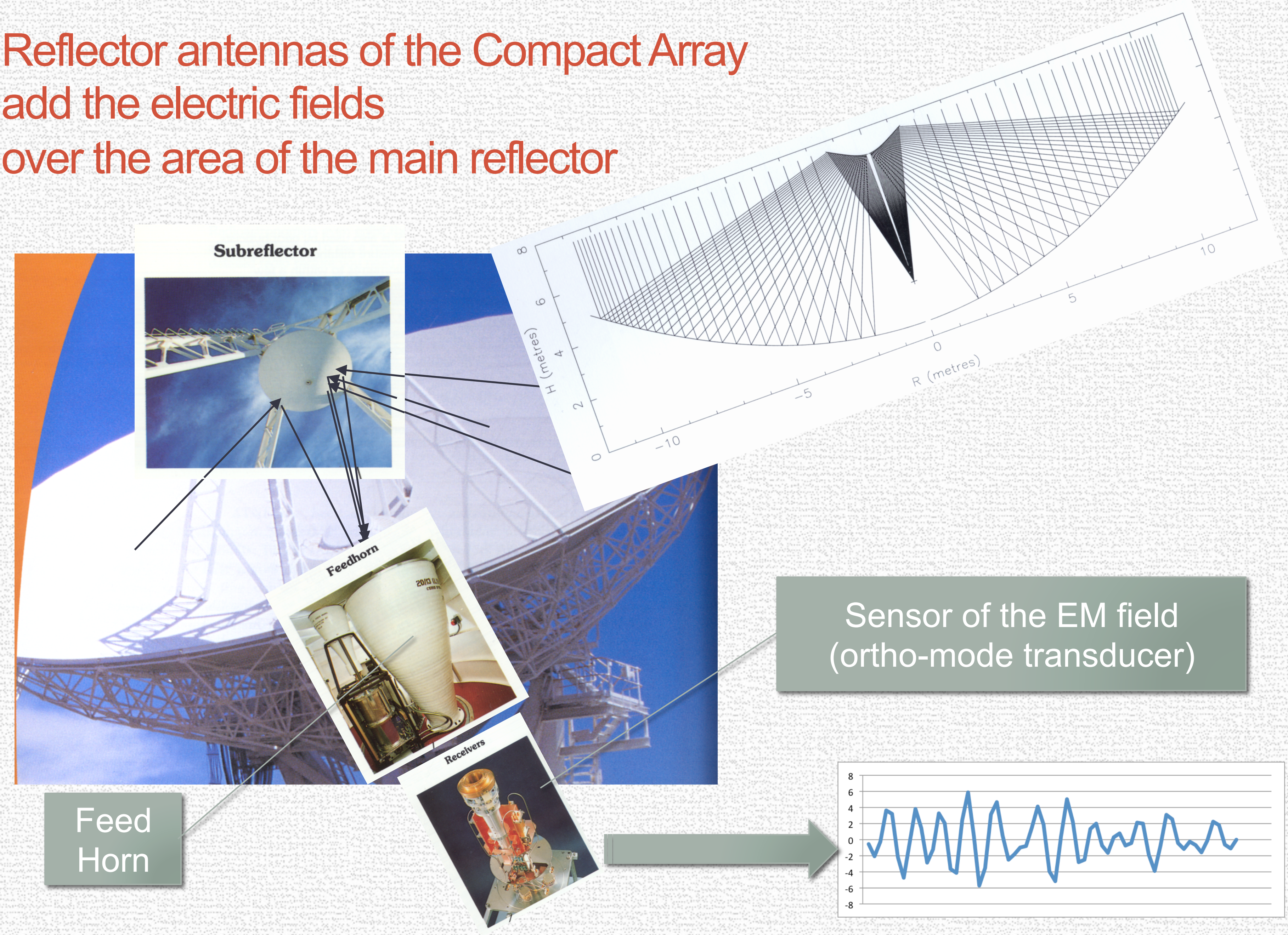
Interferometers made of Aperture Arrays and Reflector Arrays



measure the coherence between the EM fields averaged over two regions of space

=> antenna apertures

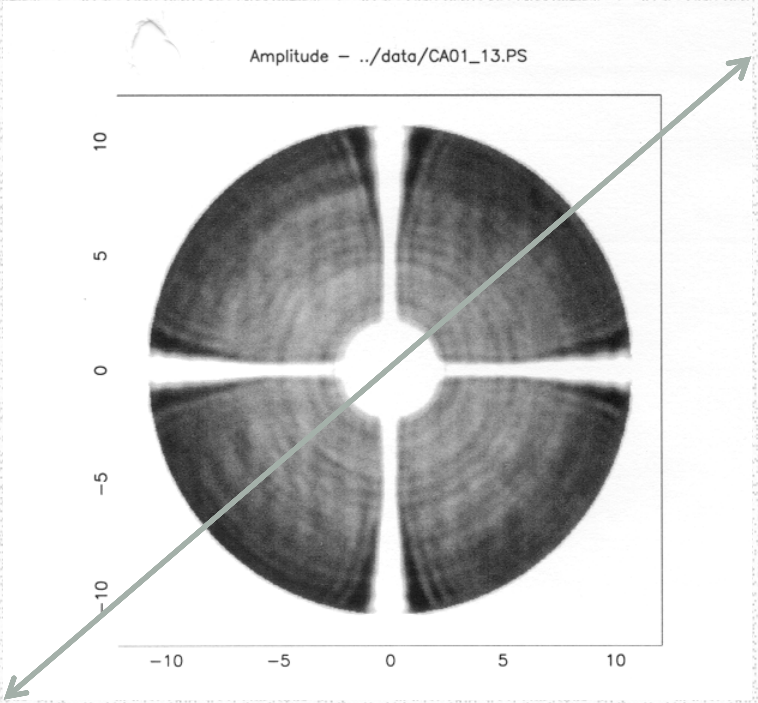
Reflector antennas of the Compact Array add the electric fields over the area of the main reflector

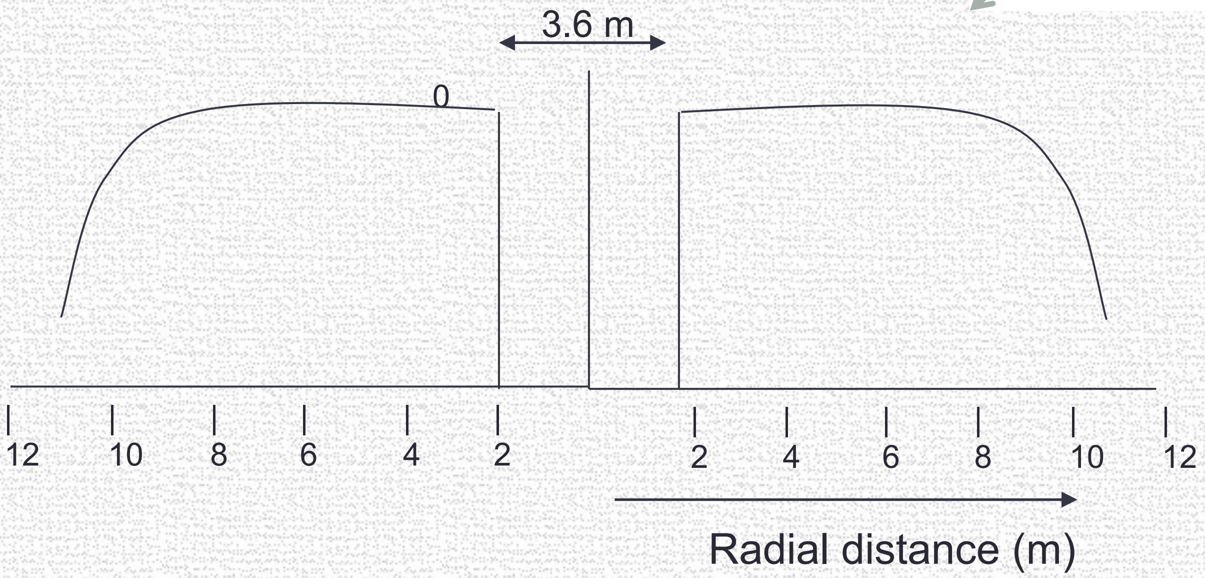


Sensor of the EM field (ortho-mode transducer)

Feed Horn

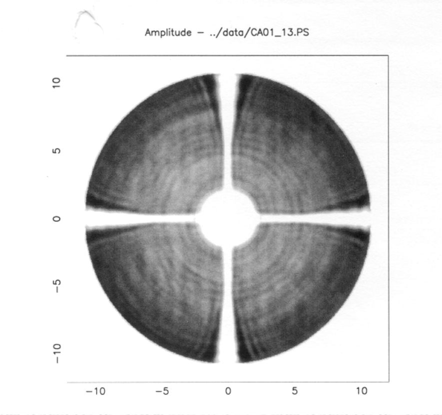
The OPTICS – shapes of the reflector surfaces and the design of the antenna element at the focus (FEED HORN) – decide the weighting of the addition in fields over the ground.



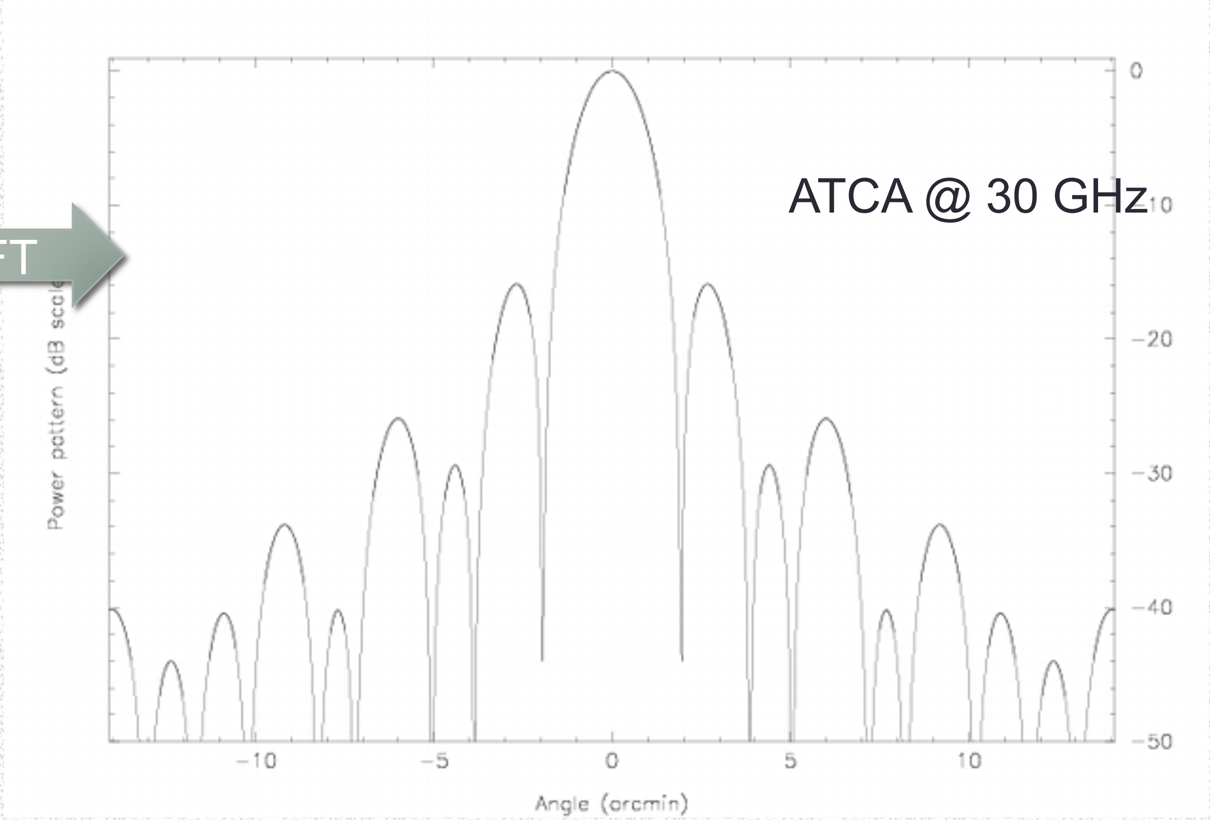


Fourier Transform of the weighting of the EM field over the aperture
→ PRIMARY BEAM

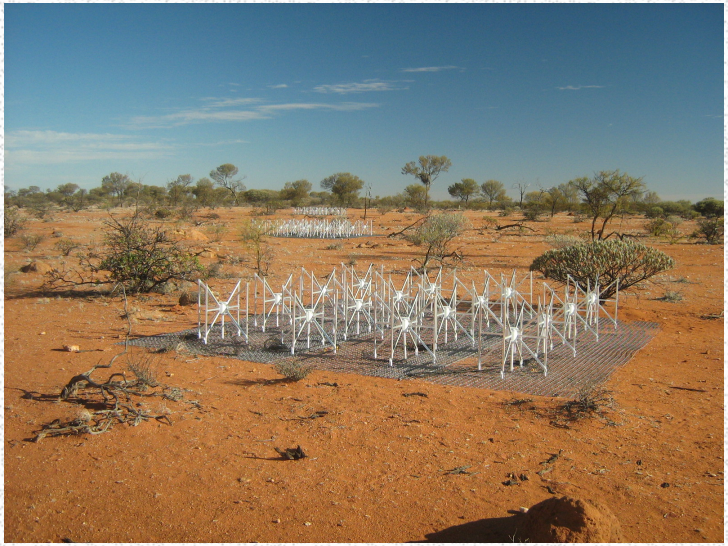
Weighting in the addition of field over the antenna surface
decides
the sensitivity over the sky



FT


ATCA @ 30 GHz

Interferometers made of Aperture Arrays and Reflector Arrays



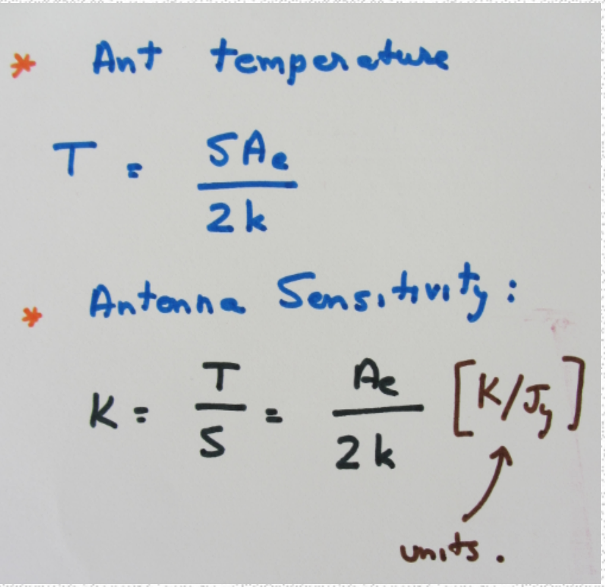
measure the coherence between the EM fields averaged over two regions of space = antenna apertures.

Because of the weighted averaging of fields on the extended apertures,

These interferometers measure the mutual coherence in fields arising exclusively from the region of the primary beam.

Single source of flux density S_T in the boresight of an antenna

S_T W/m²/Hz



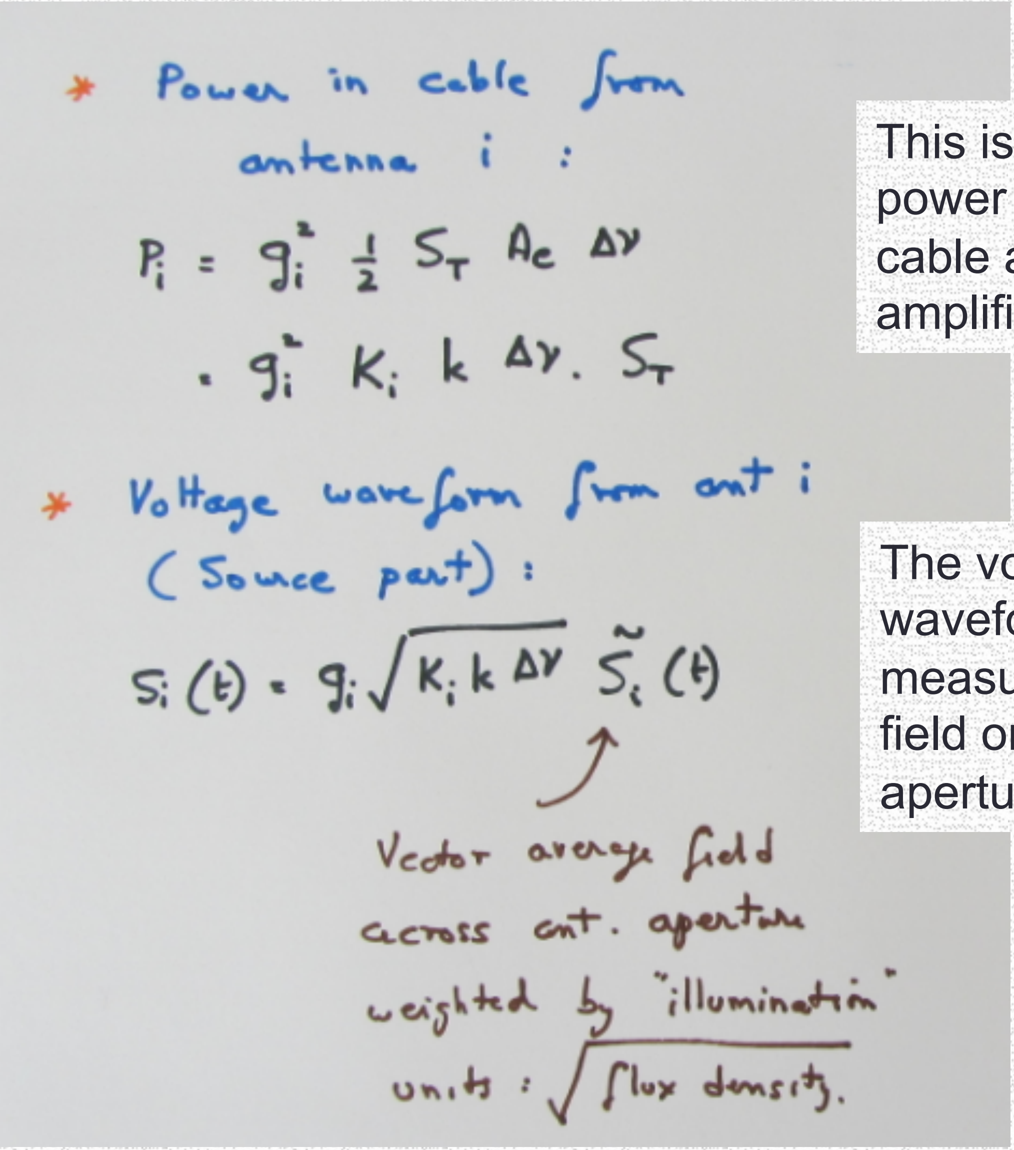
* Ant temperature

$$T = \frac{S A_e}{2k}$$

* Antenna Sensitivity:

$$K = \frac{T}{S} = \frac{A_e}{2k} \quad [K/Jy]$$

units.



* Power in cable from antenna i :

$$P_i = g_i^2 \frac{1}{2} S_T A_e \Delta\nu$$

$$= g_i^2 K_i k \Delta\nu \cdot S_T$$

* Voltage waveform from ant i (Source part):

$$s_i(t) = g_i \sqrt{K_i k \Delta\nu}\, \tilde{S}_i(t)$$

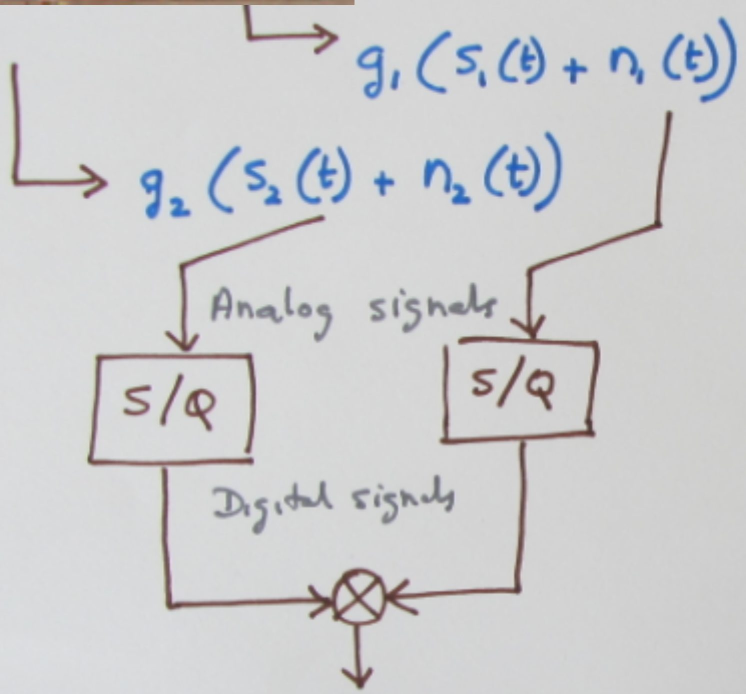
Vector average field across ant. aperture weighted by "illumination"

units: $\sqrt{\text{flux density}}$.

This is the power in the cable after the amplifier

The voltage waveform is a measure of the field on the aperture

Source of flux density S_T along the boresights of two antennas



$$P_{12} = g_1 g_2 (s_1 + n_1)(s_2 + n_2)$$

$$\langle P_{12} \rangle = g_1 g_2 \langle s_1 s_2 + \underbrace{s_1 n_2 + s_2 n_1 + n_1 n_2}_{\rightarrow 0} \rangle$$

$$= g_1 g_2 \langle s_1(t)\, s_2(t) \rangle$$

s_1, s_2 : voltage waveforms due to the source

n_1, n_2 : noise waveforms (amp, gnd, bg ...)

n_1, n_2 are independent gaussian random variates

$$\langle n_1(t)\, n_2(t) \rangle \rightarrow 0.$$

Noise waveforms are uncorrelated!

$$* \quad \langle P_{12}\rangle = g_1 g_2 \langle S_1(t)\, S_2(t)\rangle$$

In terms of the EM fields on the antenna surfaces

$$\langle P_{12}\rangle = g_1 g_2 \sqrt{K_1 K_2}\; k\,\Delta\nu \underbrace{\langle \tilde{S}_1(t)\, \tilde{S}_2(t)\rangle}$$

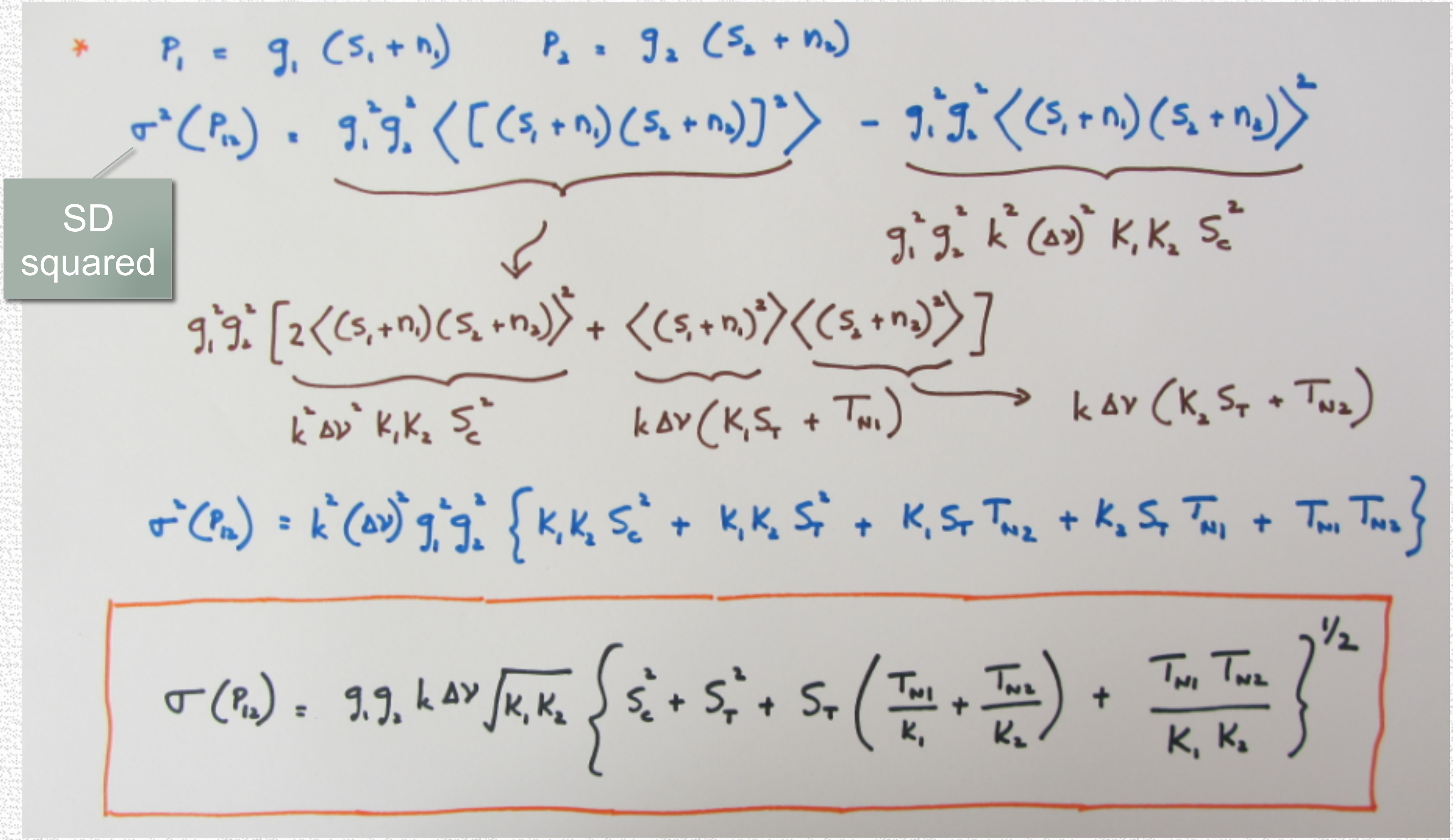
$$= g_1 g_2 \sqrt{K_1 K_2}\; k\,\Delta\nu \quad S_c$$

Partial coherence between the fields at the two antennas

$$|S_c| \le S_T$$

Product of the voltage waveforms is a measure of the mutual coherence of the fields on the antenna surfaces

Uncertainty in P_{12}



$$* \quad P_1 = g_1(s_1+n_1) \qquad P_2 = g_2(s_2+n_2)$$

$$\sigma^2(P_{12}) = \underbrace{g_1^2 g_2^2 \langle [(s_1+n_1)(s_2+n_2)]^2 \rangle}_{\downarrow} - \underbrace{g_1^2 g_2^2 \langle (s_1+n_1)(s_2+n_2) \rangle^2}_{g_1^2 g_2^2 k^2 (\Delta\nu)^2 K_1 K_2 S_c^2}$$

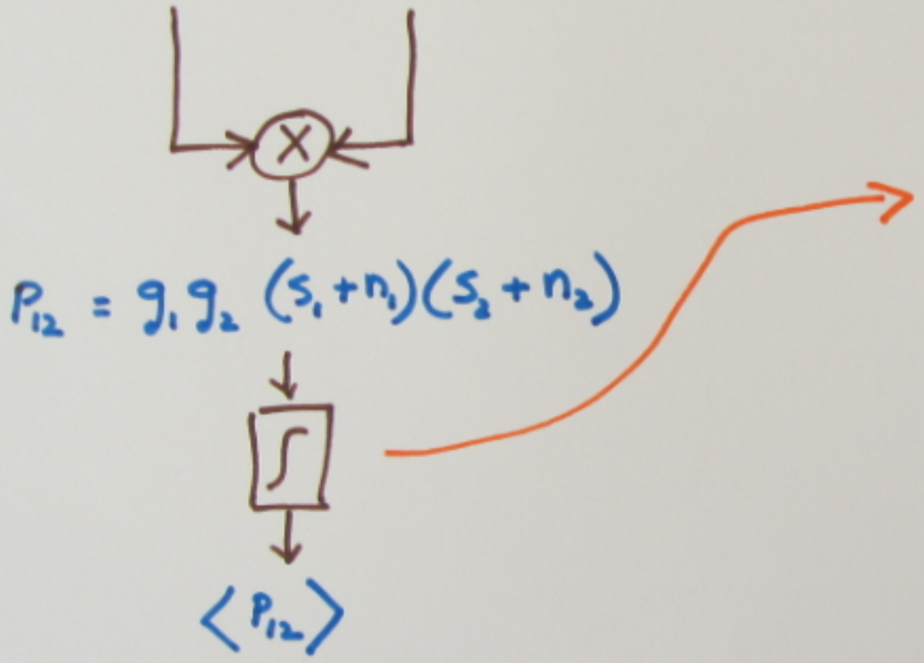
$$g_1^2 g_2^2 \Big[2 \underbrace{\langle (s_1+n_1)(s_2+n_2) \rangle^2}_{k^2 \Delta\nu^2 K_1 K_2 S_c^2} + \underbrace{\langle (s_1+n_1)^2 \rangle}_{k\Delta\nu(K_1 S_T + T_{N1})} \underbrace{\langle (s_2+n_2)^2 \rangle}_{\longrightarrow\, k\Delta\nu(K_2 S_T + T_{N2})} \Big]$$

$$\sigma^2(P_{12}) = k^2 (\Delta\nu)^2 g_1^2 g_2^2 \left\{ K_1 K_2 S_c^2 + K_1 K_2 S_T^2 + K_1 S_T T_{N2} + K_2 S_T T_{N1} + T_{N1} T_{N2} \right\}$$

$$\boxed{\sigma(P_{12}) = g_1 g_2 k \Delta\nu \sqrt{K_1 K_2} \left\{ S_c^2 + S_T^2 + S_T \left(\frac{T_{N1}}{K_1} + \frac{T_{N2}}{K_2} \right) + \frac{T_{N1} T_{N2}}{K_1 K_2} \right\}^{1/2}}$$

SD squared

The correlator = multiplier is followed by an integrator that averages the coherence for a time during which this quantity does not change



Measurement uncertainty in the integrated product

Sampling rate = $2\Delta\nu$

Integration time = τ

# of samples = $2\Delta\nu\tau$

$$\boxed{\sigma(\langle P_{12}\rangle) = \frac{\sigma(P_{12})}{\sqrt{2\Delta\nu\tau}}}$$

The integrated product is calibrated for the gains of the interferometer arms and antenna sensitivity

The integrated output is divided by

$$g_1 g_2 k \Delta\nu \sqrt{K_1 K_2}$$

to convert to Jy units

$$\langle P_{12} \rangle \Big|_{cal} = S_c$$

$$\sigma(\langle P_{12} \rangle) \Big|_{cal} = \frac{\left\{ S_c^2 + S_T^2 + S_T \left(\frac{T_{N1}}{K_1} + \frac{T_{N2}}{K_2} \right) + \frac{T_{N1} T_{N2}}{K_1 K_2} \right\}^{1/2}}{\eta_c \sqrt{2 \Delta\nu Z}}$$

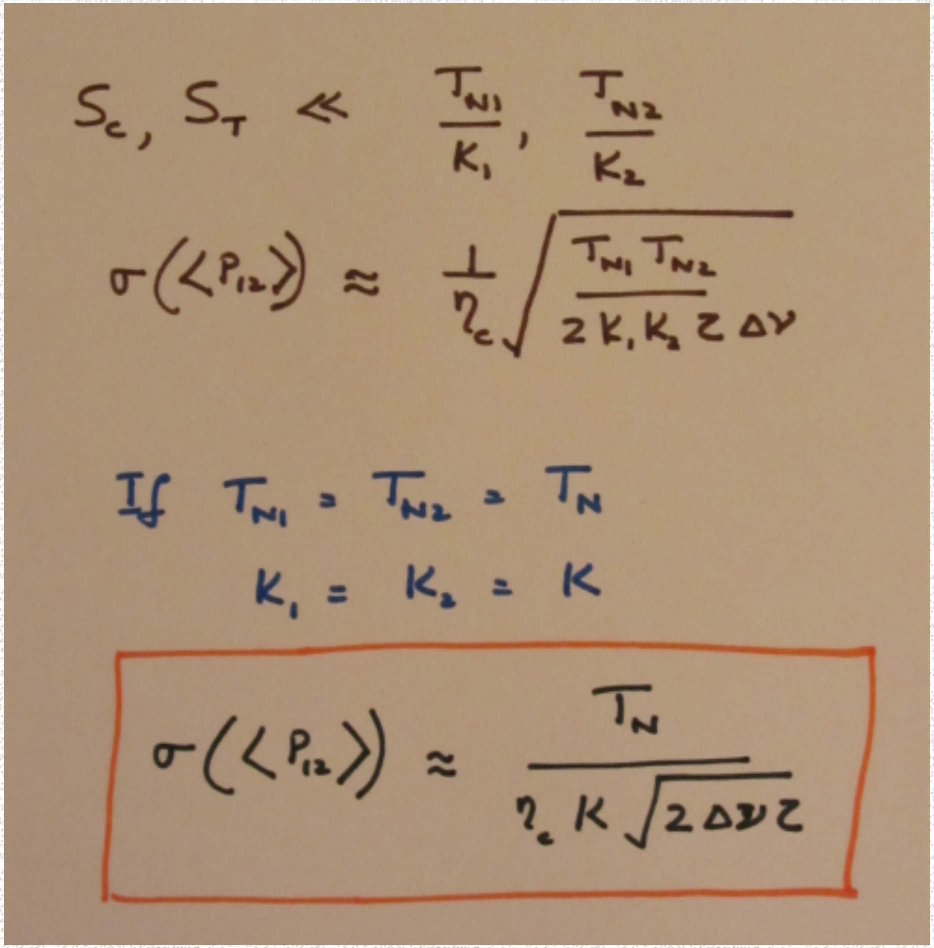
This is the measurement uncertainty in the real and imaginary parts of the complex product

Most often:

power due to the source

is very much less than

power due to the receiver noise + b.g. sky + atm + gnd



$$S_c, S_T \ll \frac{T_{N1}}{K_1}, \frac{T_{N2}}{K_2}$$

$$\sigma(\langle P_{12}\rangle) \approx \frac{1}{\eta_c}\sqrt{\frac{T_{N1}T_{N2}}{2K_1K_2\tau\Delta\nu}}$$

If $T_{N1} = T_{N2} = T_N$

$K_1 = K_2 = K$

$$\boxed{\sigma(\langle P_{12}\rangle) \approx \frac{T_N}{\eta_c K\sqrt{2\Delta\nu\tau}}}$$

Sensitivity of an interferometer
improves with
lower receiver+atm+gnd noise,
better antenna and sampler efficiency,
wider bandwidth and observing time

Last Slide

Assume 22 m antennas with 60% efficiency:

[A 1 Jy source gives 4000 times less power compared to your hand covering the antenna!]

Assume 3K background and 25 K receiver noise

[Only if the antenna is pointed at a 350 Jy source will the power from the source equal the noise power from the background sky and receiver]

Modern Synthesis telescopes image sources with 10 μJy flux density and operate with GHz bandwidths. Assume $\Delta\nu$ = 4 GHz and the operating frequency 10 GHz.

[The digital signals from the antenna are sampled every 0.125 ns, and a source photon arrives at the antenna only once every million samples]

Every photon from the source is buried in 35 million photons from the receiver and background sky: modern interferometers + software can make images of these sources!