Wide field imaging

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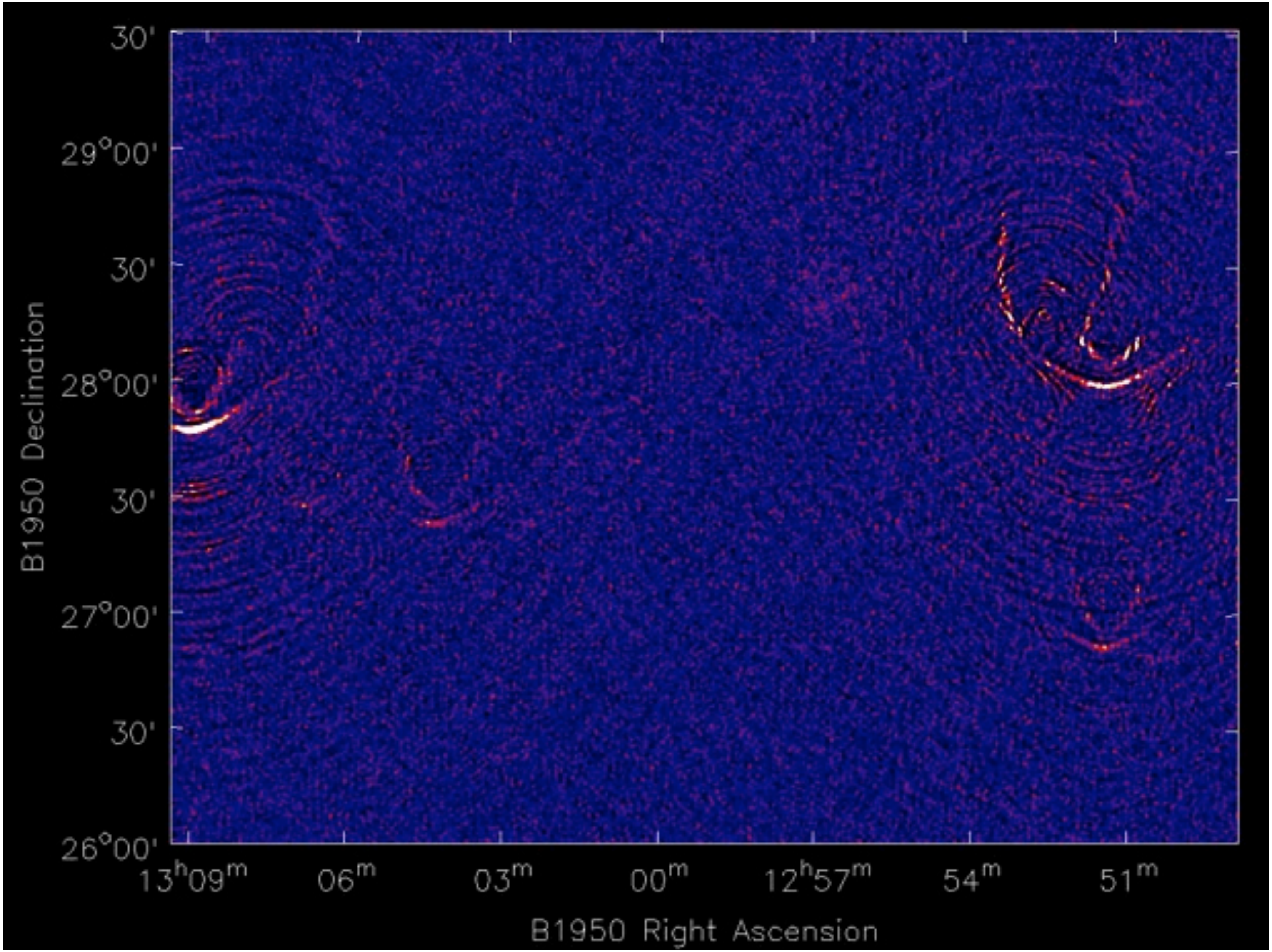
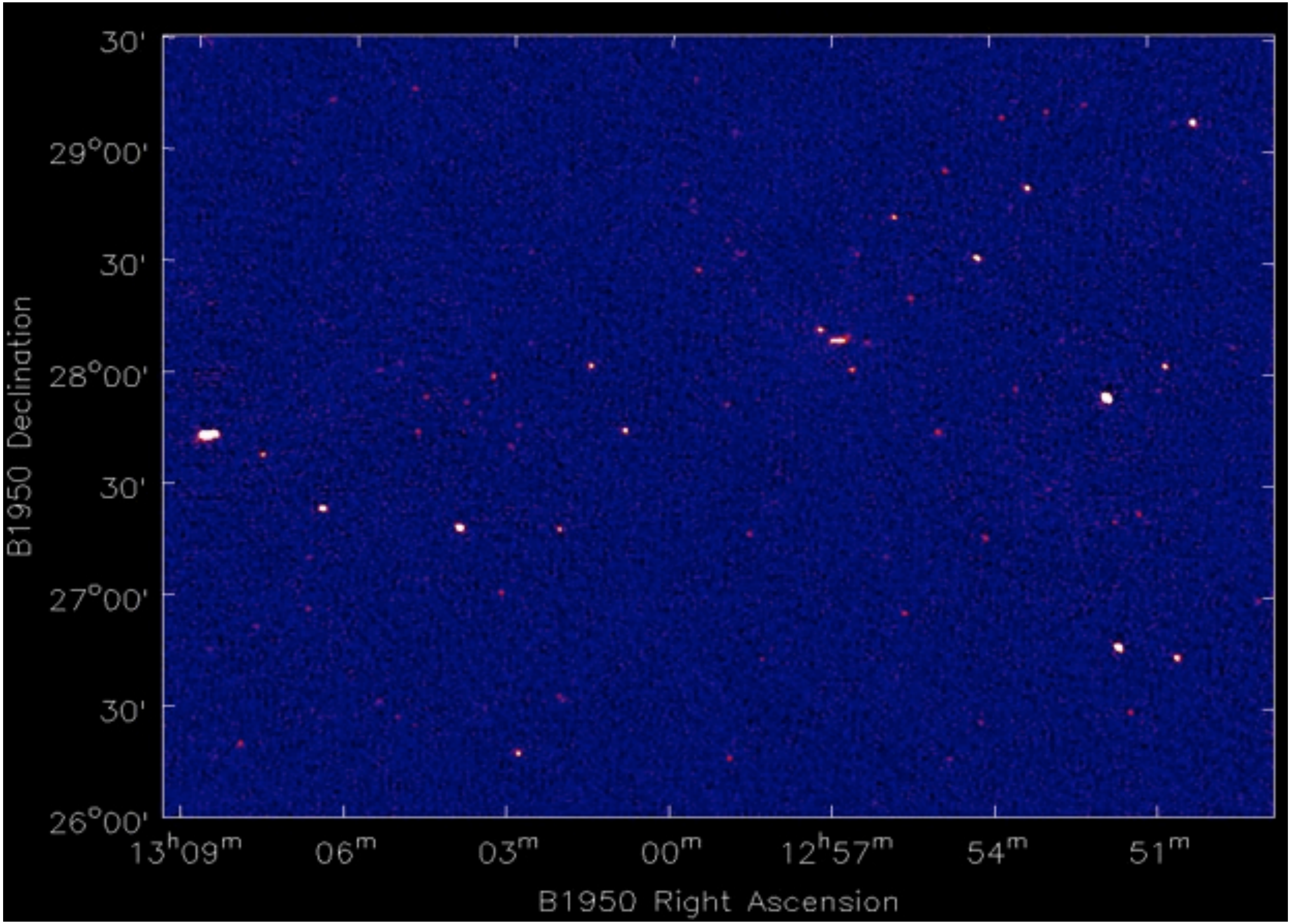
Standard 2D processing



Faceted processing



Radio telescope imaging

Spatial coherence of electric field (visibility) is Fourier transform of sky brightness

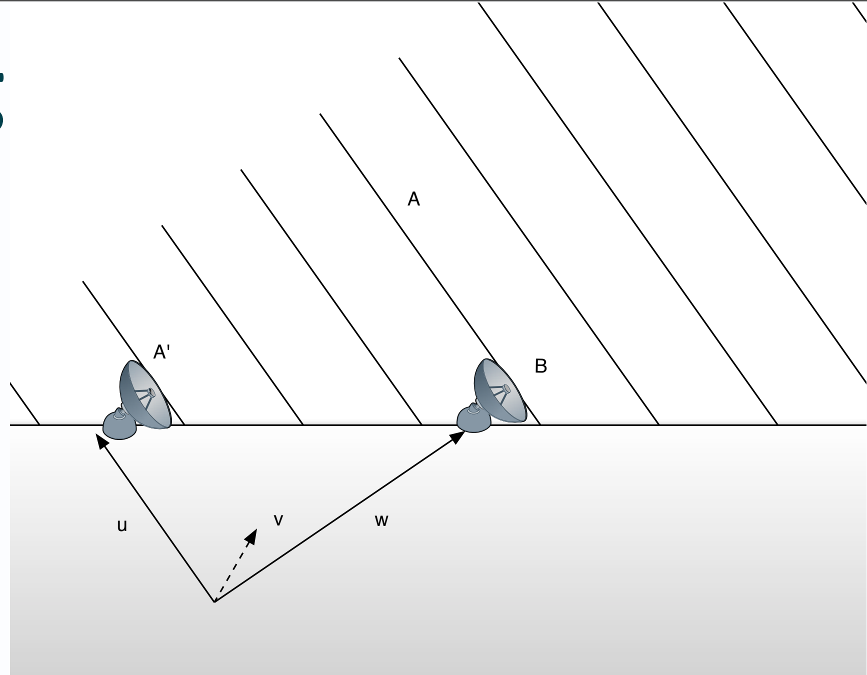
$$V_{A'B} = \left\langle E_{A'} E_B^* \right\rangle_t$$
$$= e^{-2\pi j w} \int I(l,m) e^{-2\pi j (ul + vm)} dl dm$$

Measure for many values of the Fourier components u,v

Invert Fourier relationship to get image of sky brightness

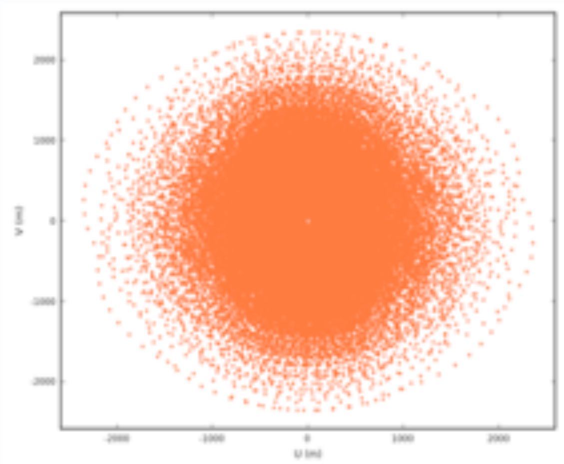
Typical problems

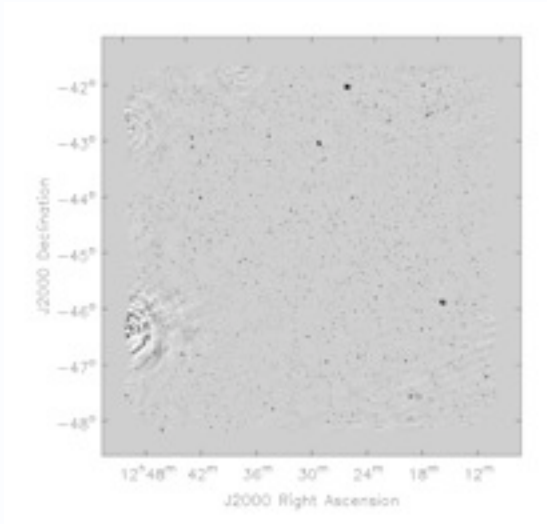
- Incomplete u,v, sampling
- Calibration



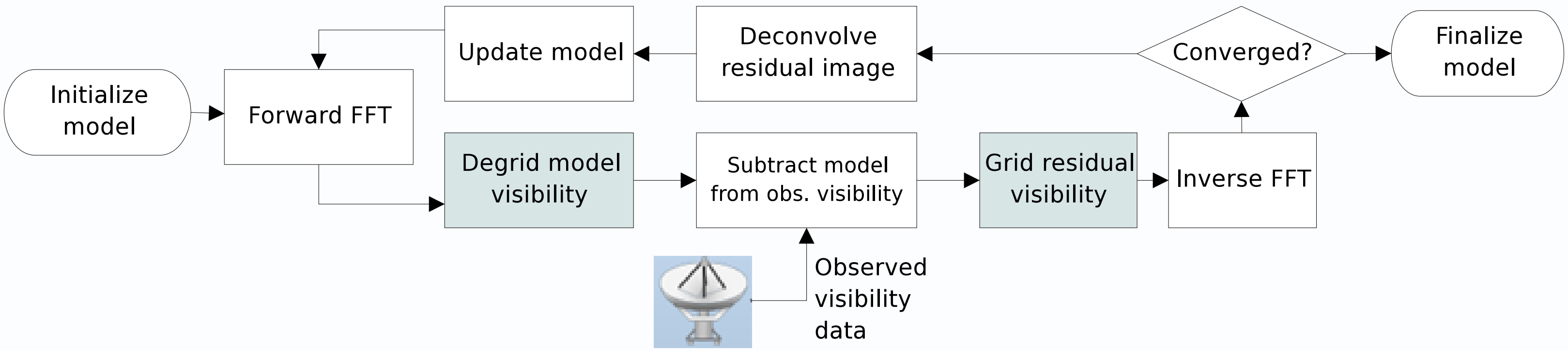
Iterative imaging

Iterate to find model of sky that fits the data





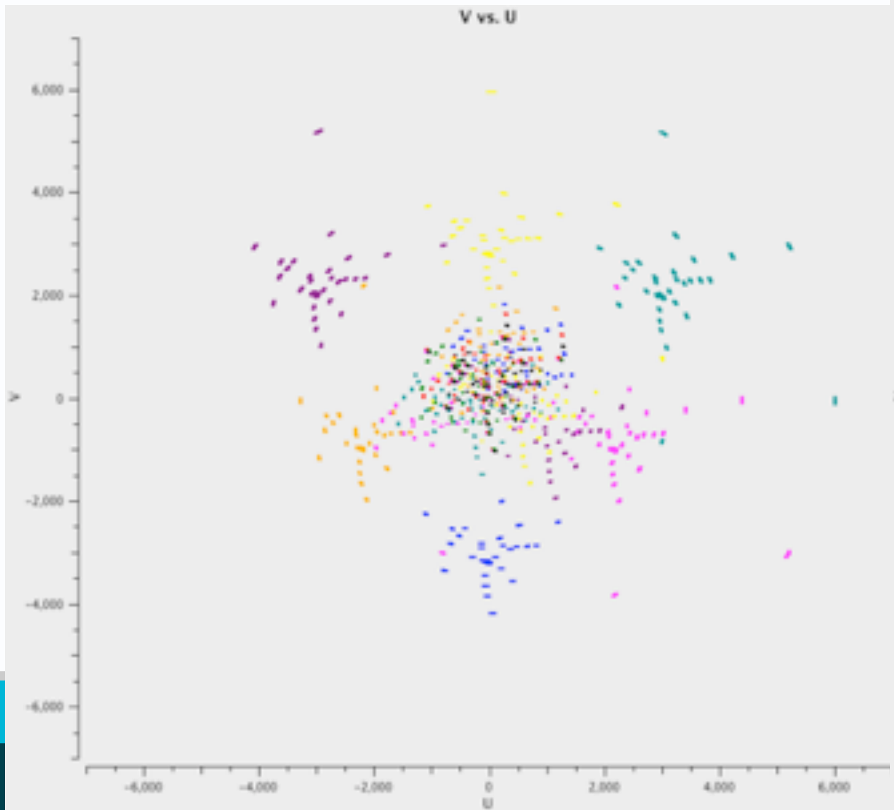
Multiple transforms between data and image space



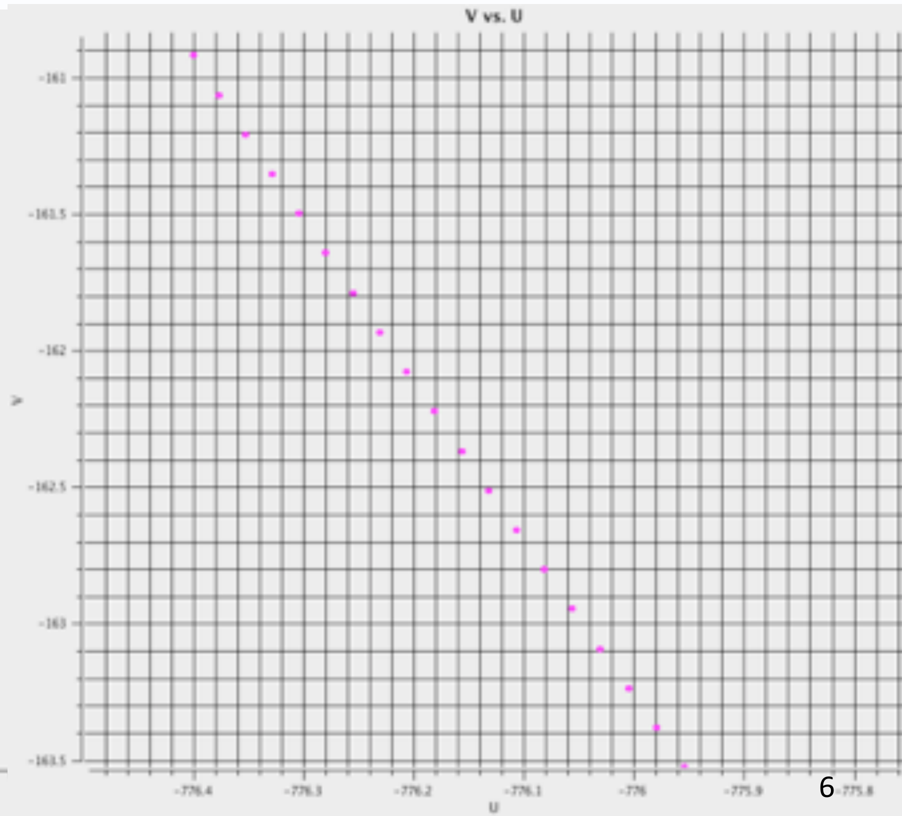
Gridding/degridding visibility data

- For FFT, need to place visibility samples on a grid
- Moving to nearest neighbour causes aliasing in image plane
- Use prolate spheroidal wave function for anti-aliasing
- Can also include physical effects as well

ASKAP snapshot Fourier plane coverage

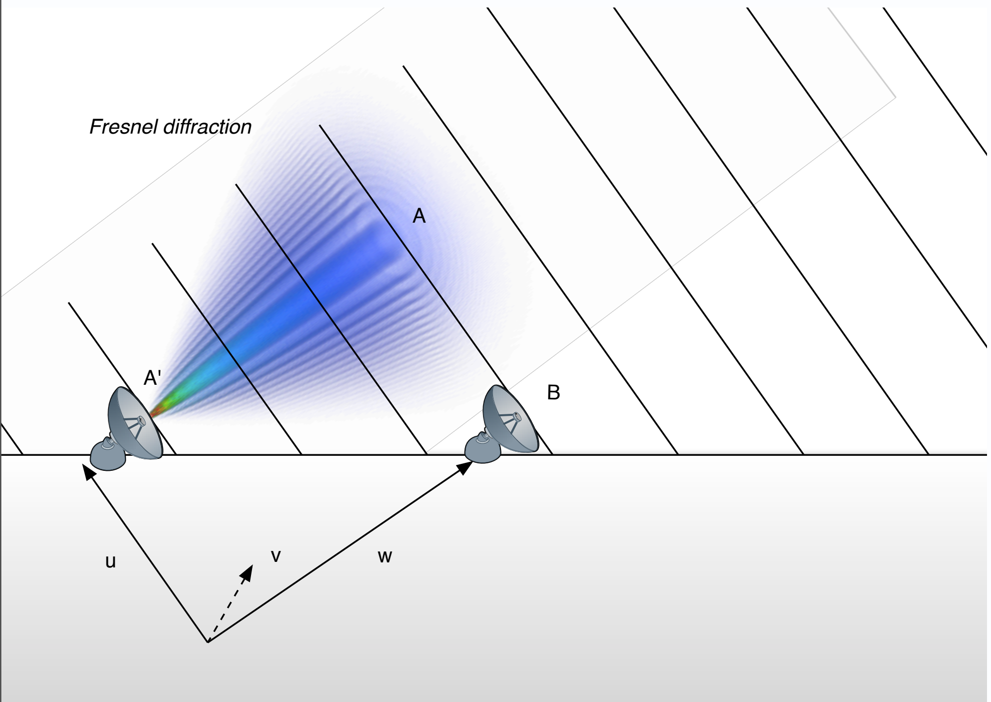


Zoom of track of one pair of antennas superimposed on Fourier plane grid



Wide field imaging

- Each antenna sees radiation from a cone of directions
- Each ray in the cone requires a different delay correction
  - For small field of view, the delay correction is linear
  - For wide field of view, there is a quadratic term
- Fresnel propagation from AB plane to A'B plane

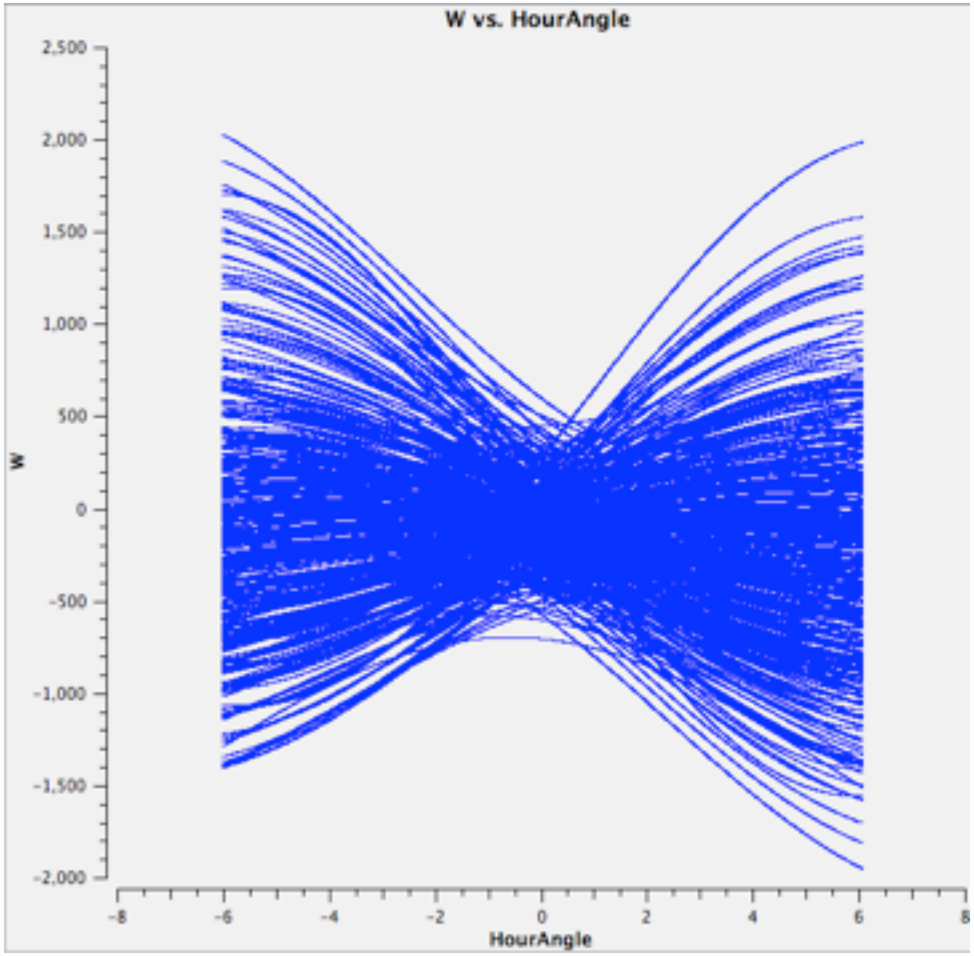


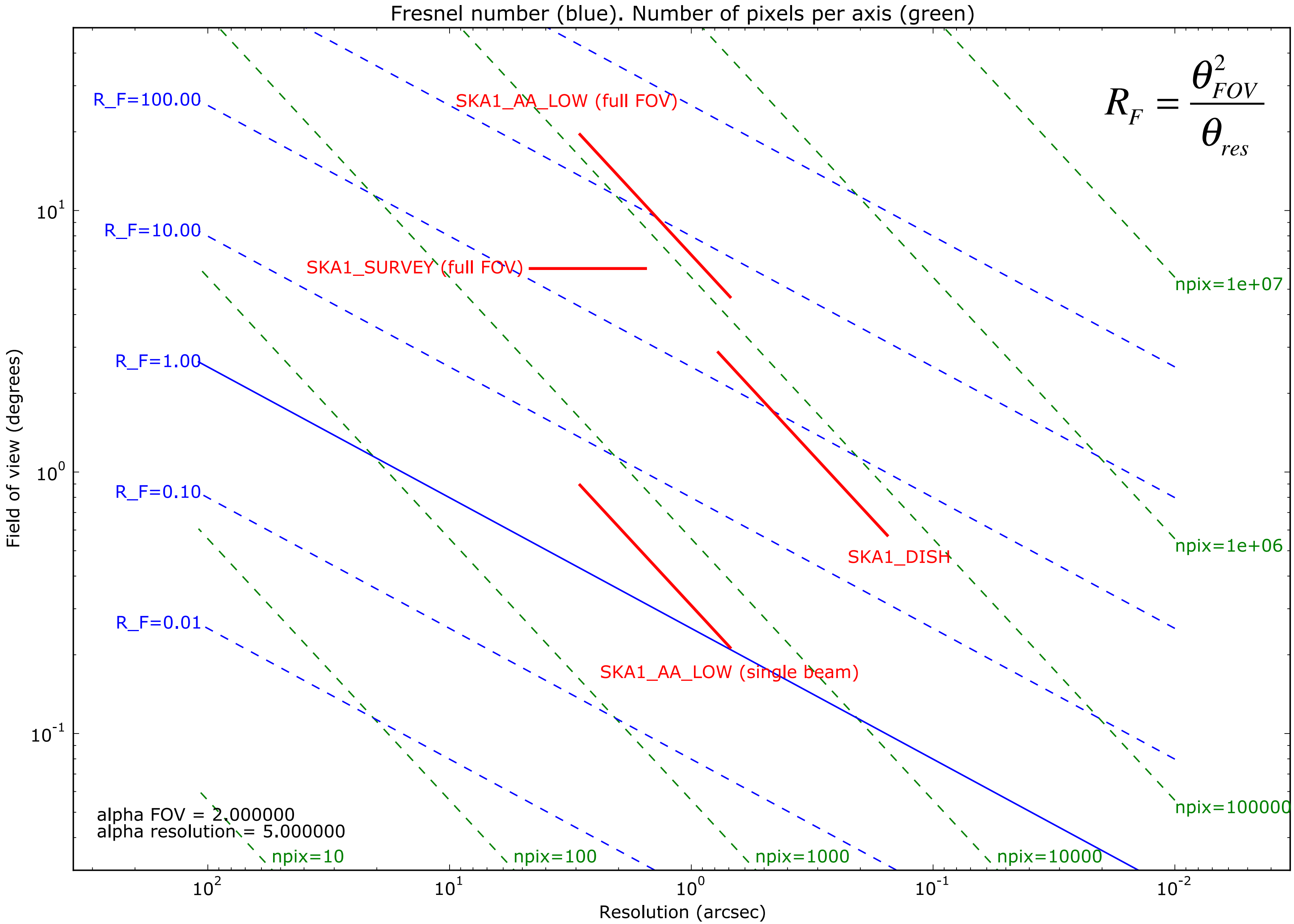
$$V(u,v,w) = \int \left[\frac{I(l,m) e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}} \right] e^{j2\pi(ul+vm)} dldm$$

$$\phi_w = 2\pi w\left(\sqrt{1-l^2-m^2}-1\right)$$

$$\sim \pi w\left(l^2+m^2\right) \sim \frac{\theta_{FOV}^2}{\theta_{res}}$$

w versus hour angle for an ASKAP 2km observation



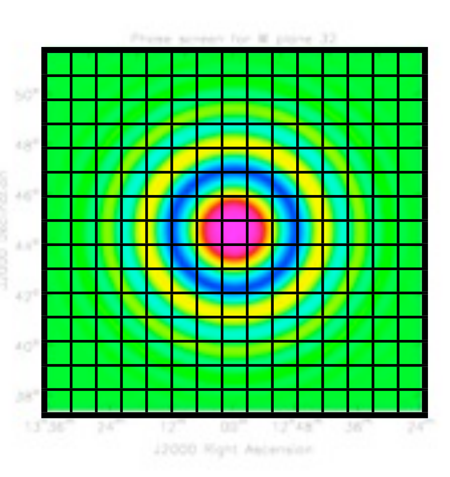


SKA1_SURVEY baseline = 20km, others = 200km

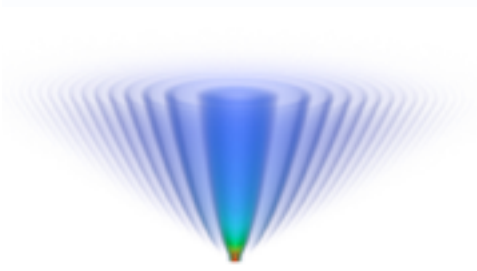
Wide field imaging algorithms

Quadratic phase term added to Fourier Transform

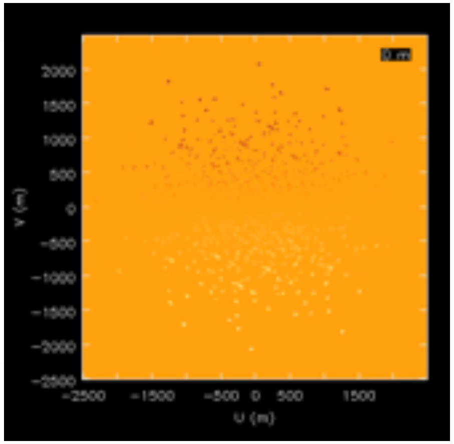
$$V(u,v,w) = \int \left[\frac{I(l,m)e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}} \right] e^{j2\pi(ul+vm)}\,dl\,dm$$



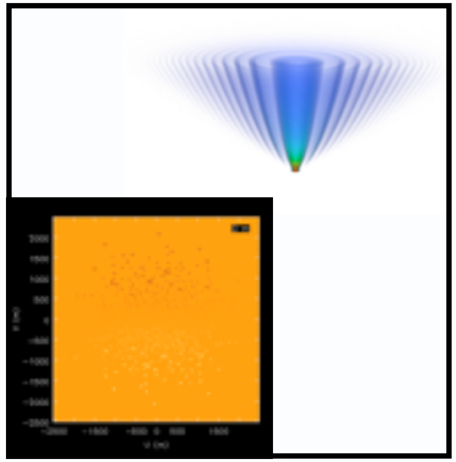
faceting



w projection



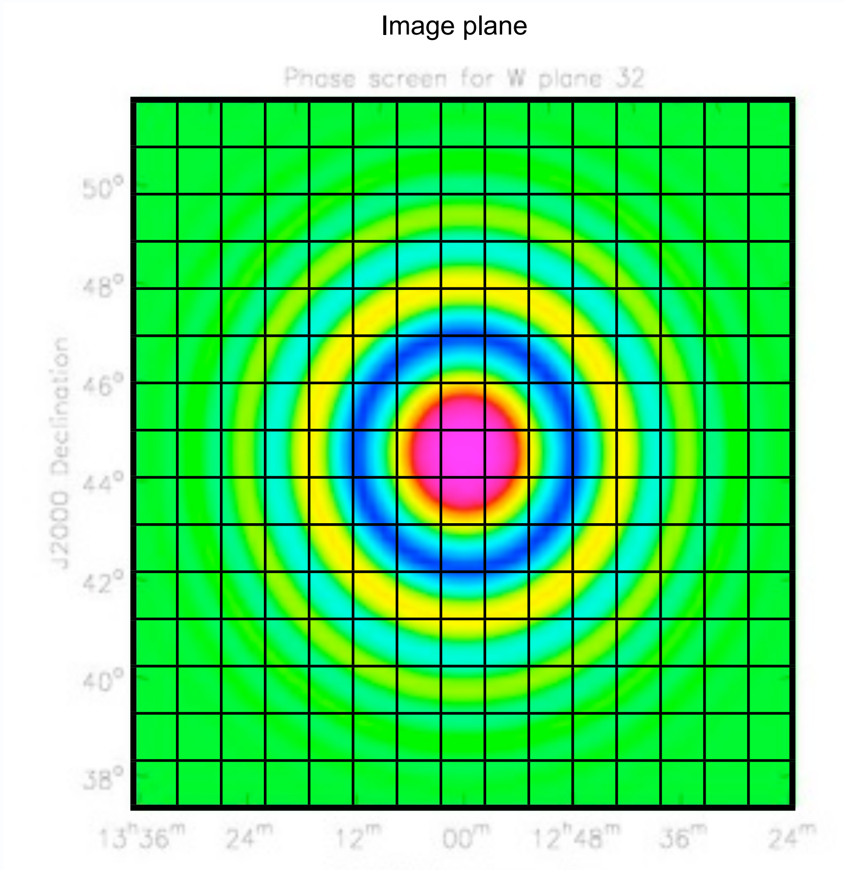
snapshots



w snapshots

Algorithm 1: faceting

$$e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)} \approx \sum_k e^{j2\pi w\left(\sqrt{1-l_k^{\,2}-m_k^{\,2}}-1\right)} \Pi\left(\frac{l-l_k}{\Delta l}\right)\Pi\left(\frac{m-m_k}{\Delta m}\right)$$



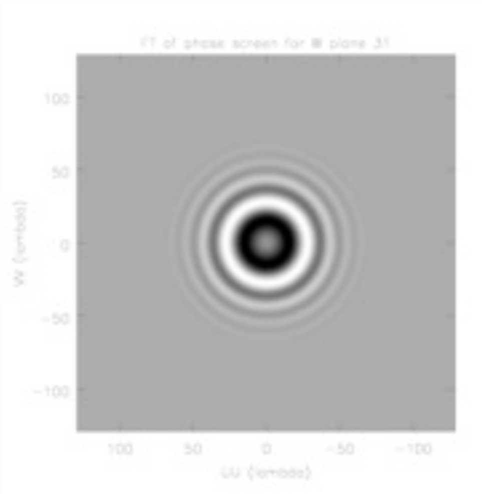
- Conceptually straightforward
- Easy to code
- Facet methods are equivalent to representing phase screen by regular grid of sinusoids
  - Quite poor approximation!
- Asymptotic limit is the Direct Fourier Sum

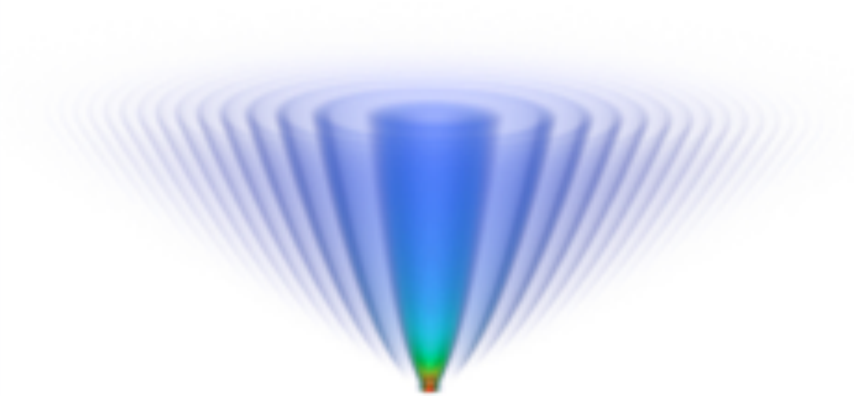
Algorithm 2: w projection

$$V(u,v,w) = \int \left[\frac{I(l,m) e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}} \right] e^{j2\pi(ul+vm)} dl dm$$

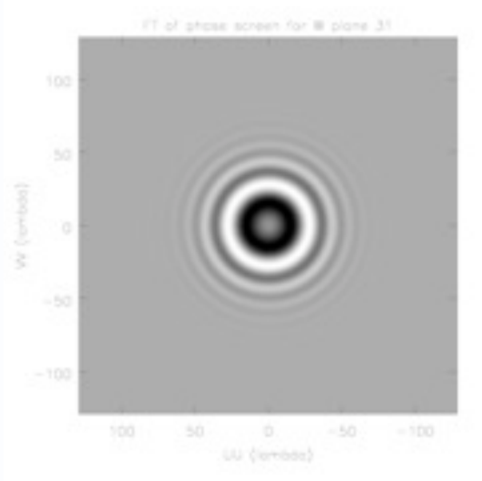
$$G(u,v,w) = \int \left[\frac{e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}} \right] e^{j2\pi(ul+vm)} dl dm$$

$$V(u,v,w) = G(u,v,w) \otimes V(u,v,w=0)$$



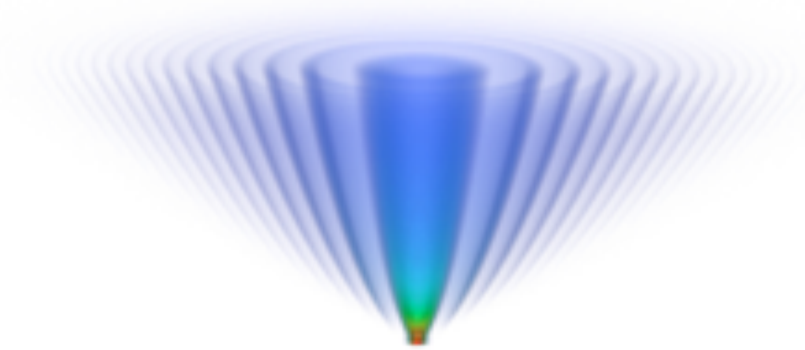


Algorithm 2: w projection



1. Initialise output 2D grid in u,v space
2. Project visibility sample onto the grid using the appropriate plane of the w-dependent convolution function
3. Repeat for all data
4. 2D FFT to obtain output image

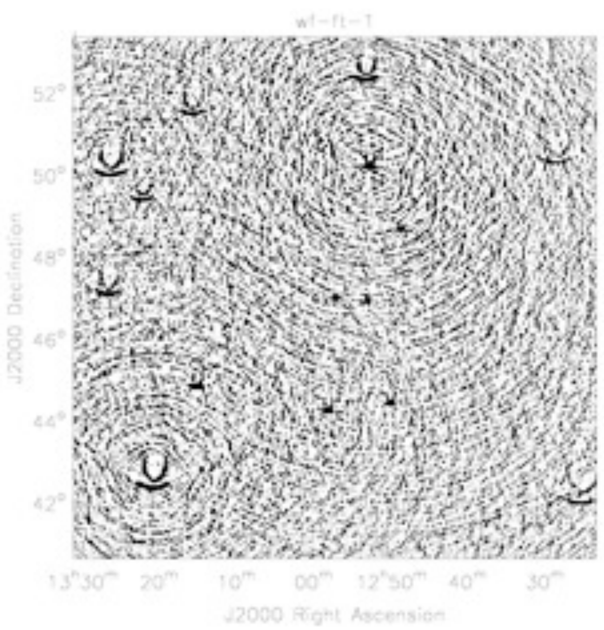
$$T_{wprojection} \sim N_{vis} \mu_{wproj} R_F^2$$



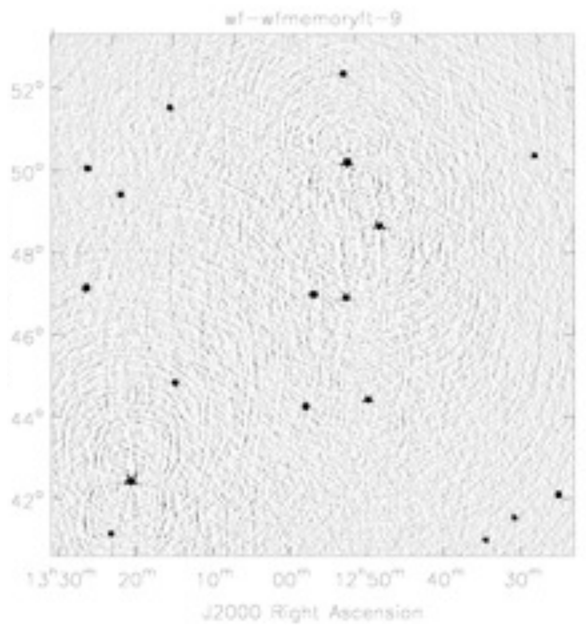
Comparison of 2D, faceted, w projection algorithms

- Simulation of VLA 74 MHz long integration

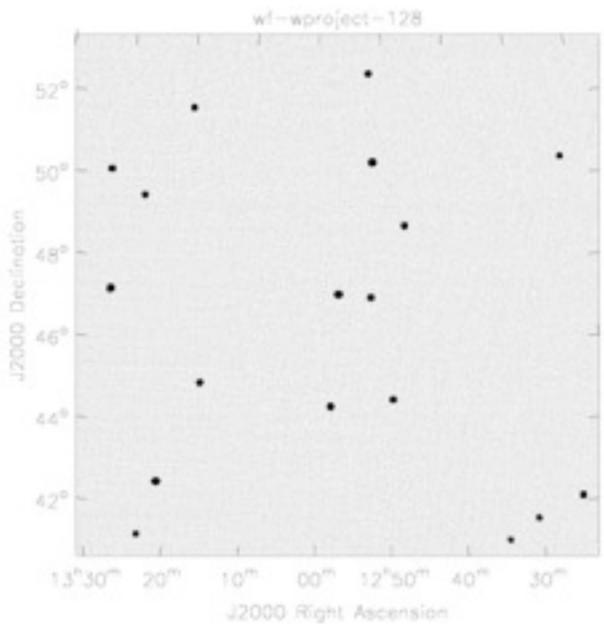
Single Fourier transform



Faceted Fourier transforms

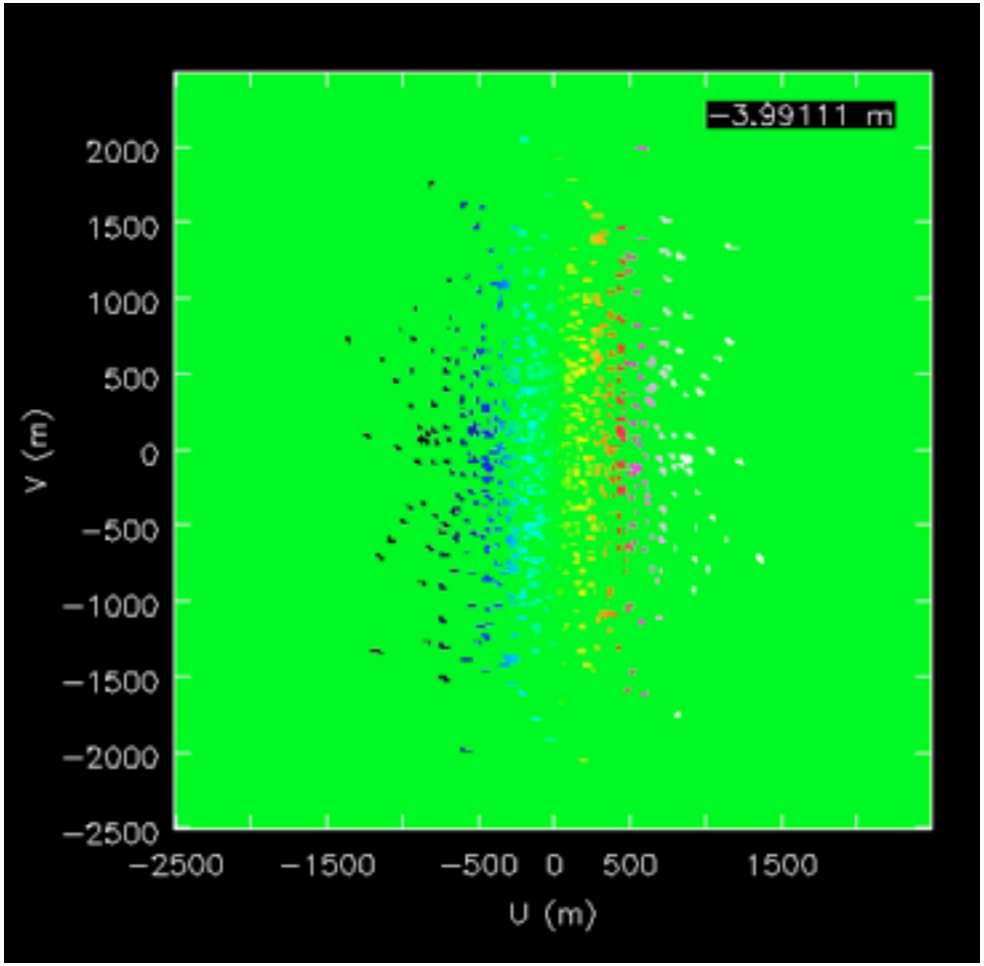


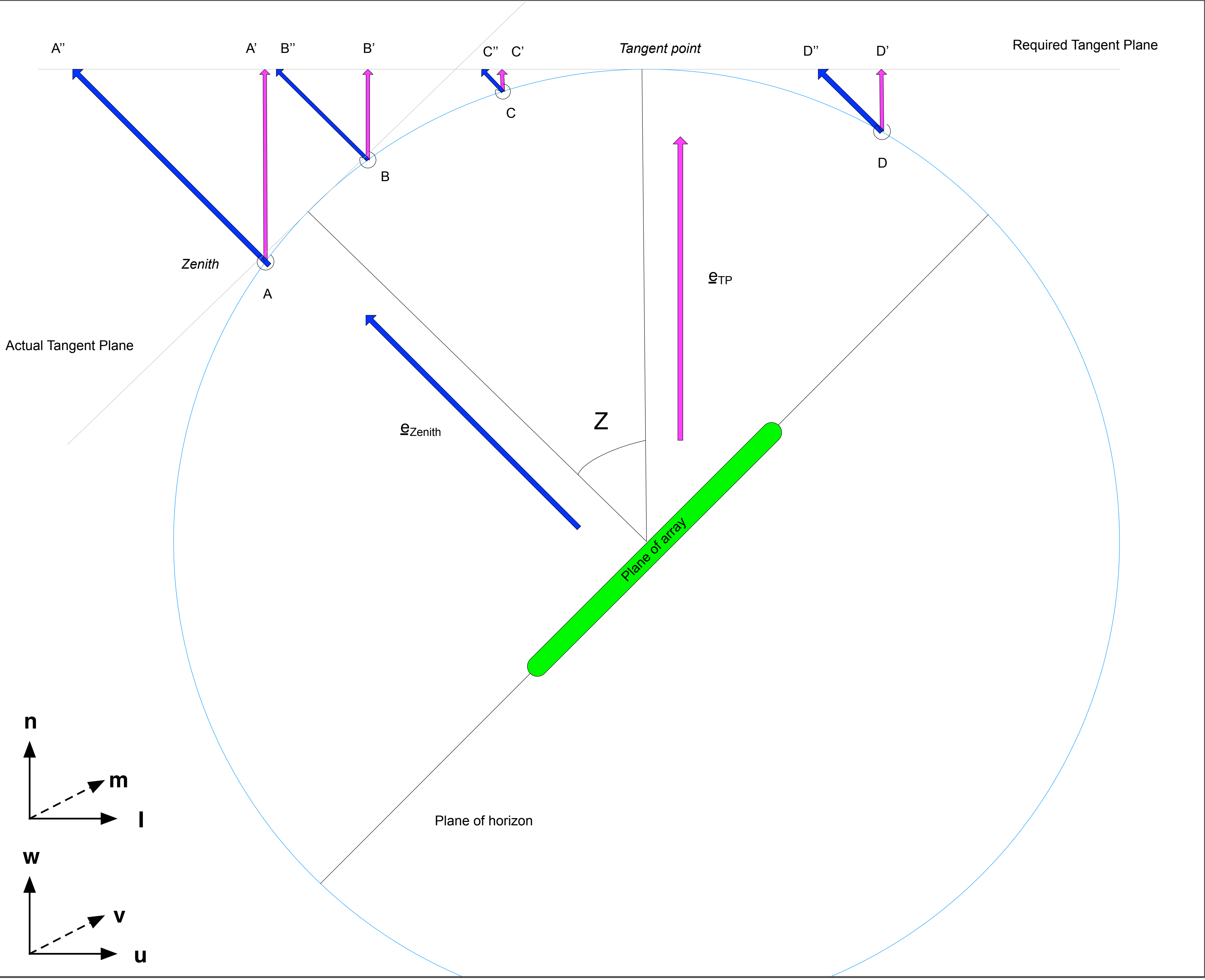
W projection



Small arrays are instantaneously flat

- Rotation of flat uv plane with time
- Starting with source rising in the east
- Stopping with source setting in the west




A''
A'
B''
B'
C''
C'
Tangent point
D''
D'
Required Tangent Plane
C
B
D
Zenith
A
Actual Tangent Plane
$\underline{e}_{TP}$
$\underline{e}_{Zenith}$
Z
Plane of array
n
m
l
w
v
u
Plane of horizon

Algorithm 3: snapshots

- Array is often approximately planar and rotates slowly
- Since u,v,w sampled is planar then the transform is instantaneously 2D with a coordinate distortion

$$w = au + bv$$

$$l' = l + a\left(\sqrt{1 - l^2 - m^2} - 1\right)$$

$$m' = m + b\left(\sqrt{1 - l^2 - m^2} - 1\right)$$

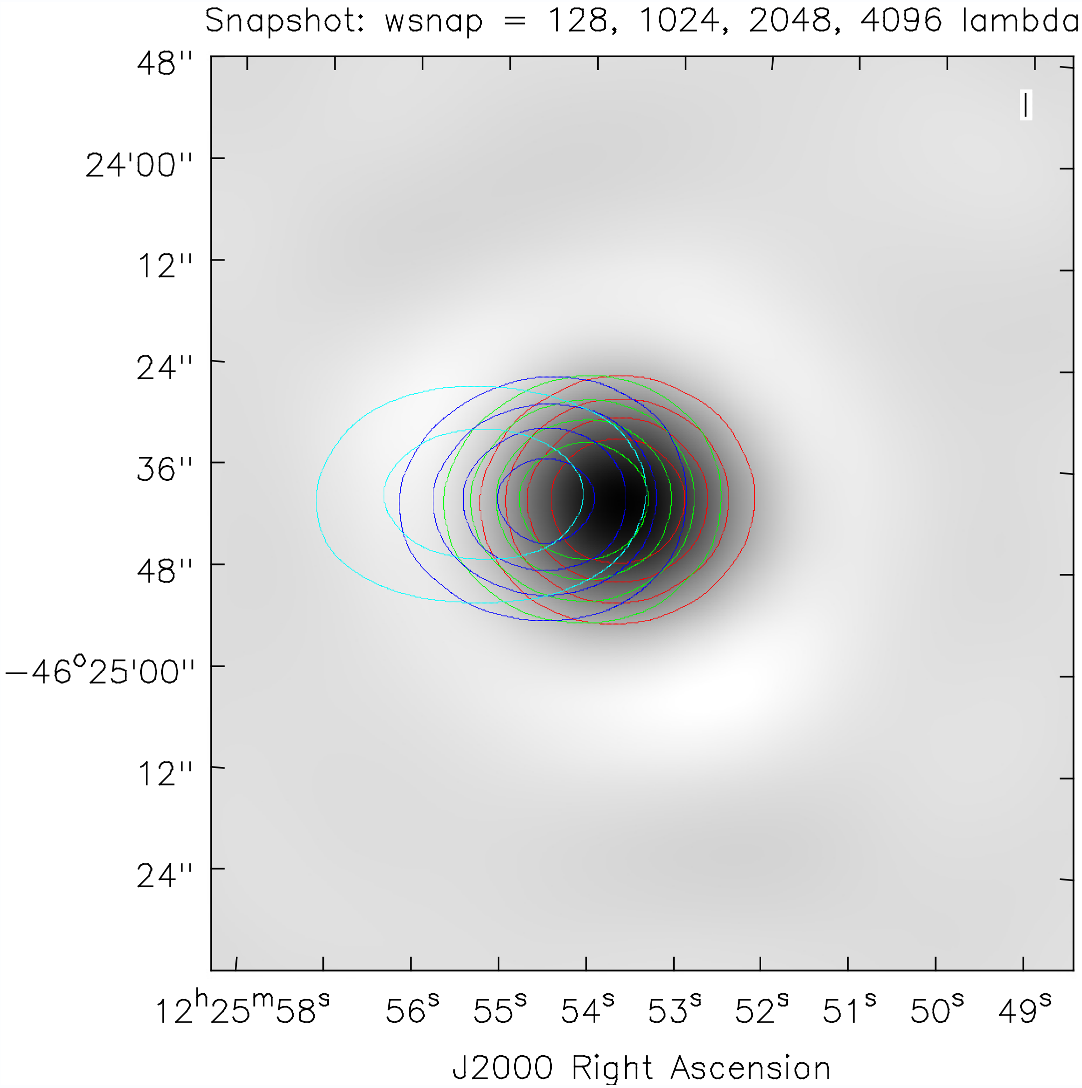
$$V(u,v,w) = \int \left[\frac{I(l',m')}{\sqrt{1 - l'^2 - m'^2}} \right] e^{j2\pi(ul'+vm')} dl dm$$

Algorithm 3: snapshots

1. Initialise output 2D image
2. Fit and remove best plane in u,v,w coordinates and remove
3. Grid visibility data onto this 2D plane
4. Has the plane rotated too much in u,v,w space?
   - Yes: FFT data, correct for image coordinate distortion and add to image
   - Yes: Fit and remove plane again
5. Repeat for all data

$$T_{snapshot} \sim N_{vis}\mu_{wproj}R_{aa}^2 + N_{pixel}^2\mu_{reproj}h_{obs}\frac{R_F}{\sqrt{8\Delta A}}$$

snapshots: position and width errors

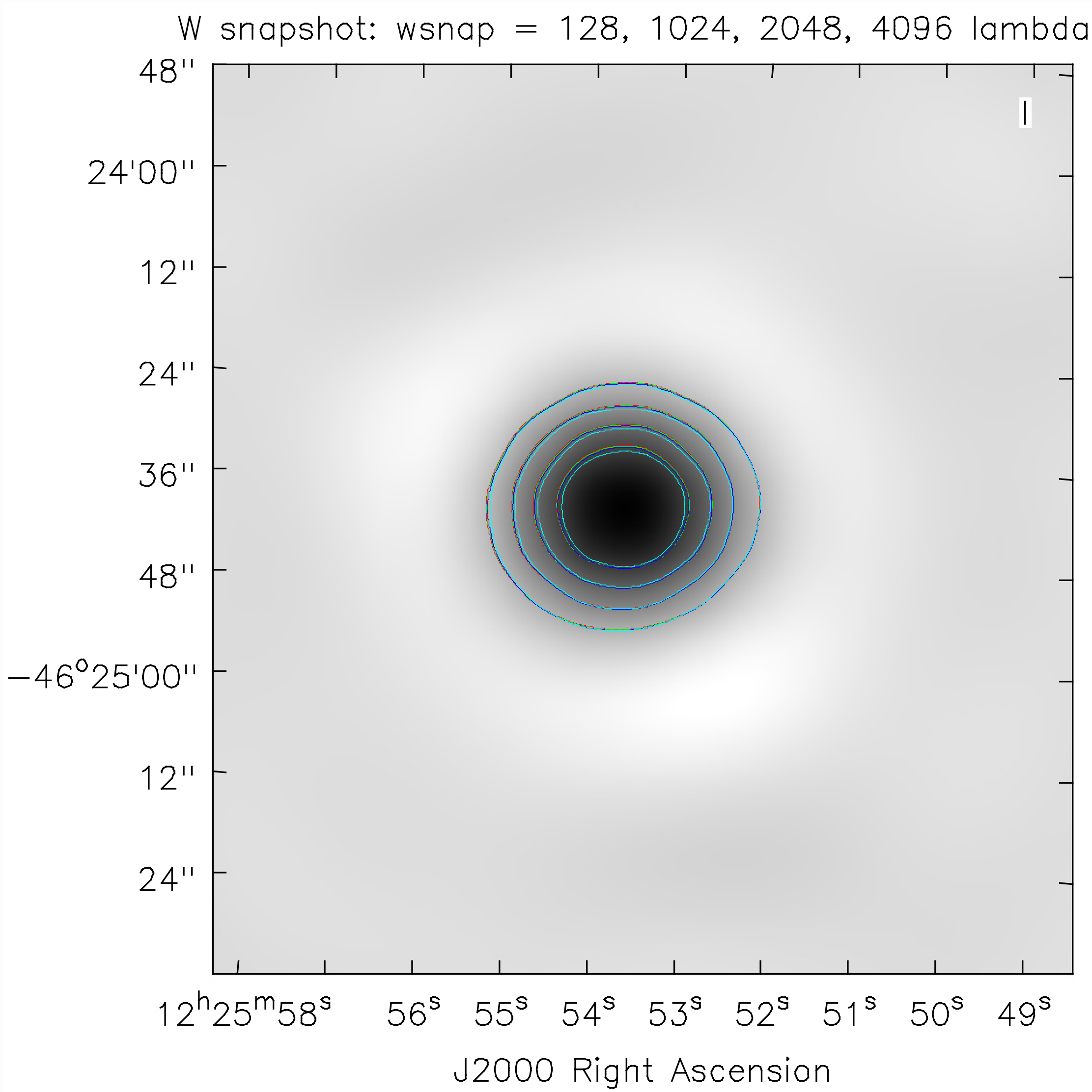


Algorithm 4: w snapshots

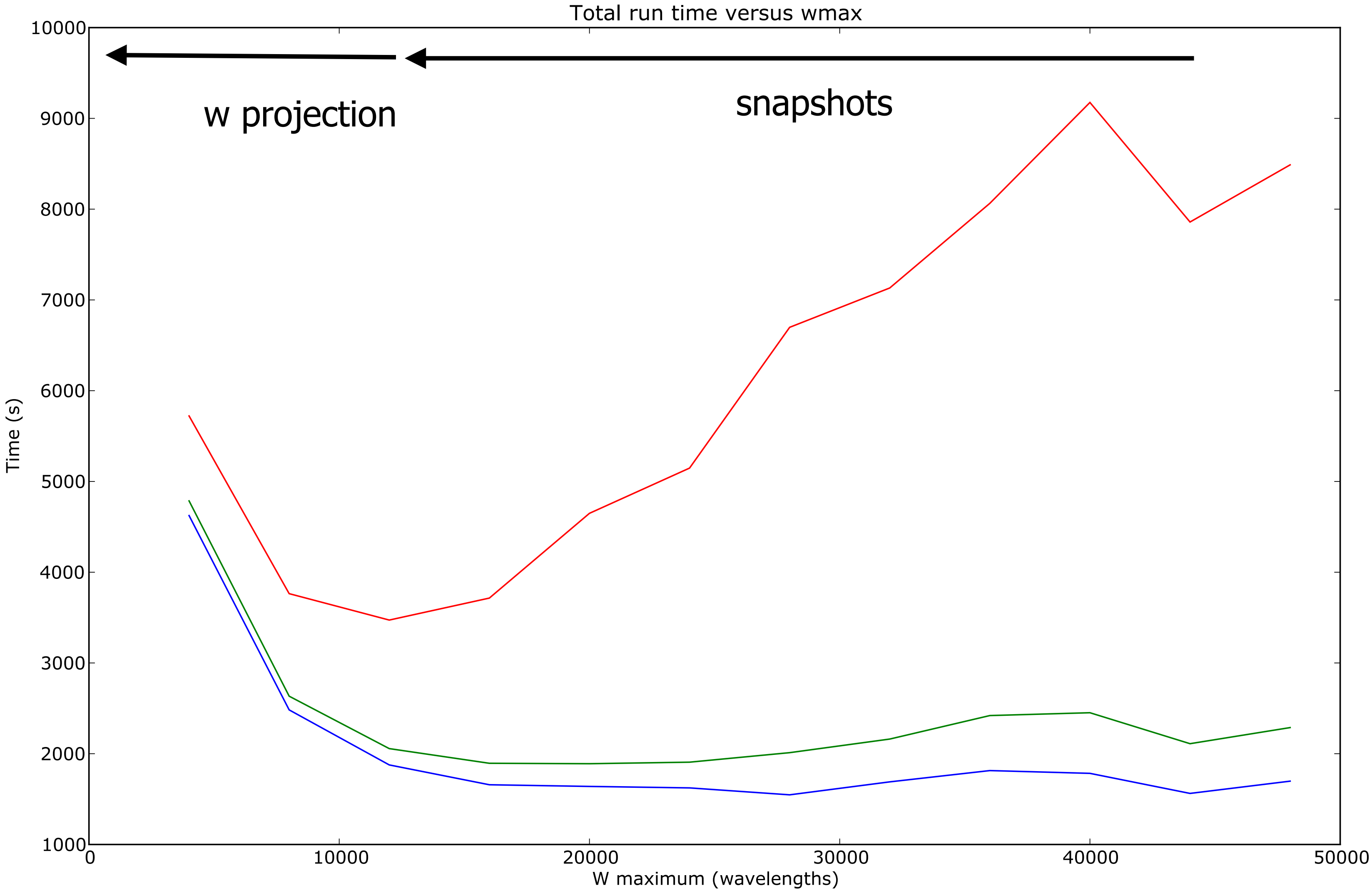
1. Initialise output 2D image
2. Fit and remove best 2D plane from u,v,w coordinates
3. W project visibility data onto this 2D plane
4. Has the 2D plane rotated too much in u,v,w space?
   - Yes: FFT data, correct for image coordinate distortion and add to 2D image
   - Yes: Fit and remove plane again
5. Repeat for all data

$$T_{wsnapshots} \sim N_{pixel}^2 \mu_{reproj} h_{obs} \left(\frac{w_{rms}}{w_{step}}\right) + N_{vis} \mu_{wproj} R_F^2 \left(\frac{w_{step}}{w_{rms}}\right)^2$$

w snapshots: position and width errors



Tuning of w snapshots



Optimal tuning of w snapshots

Scaling with number of visibilities and number of pixels

$$T_{wsnapshots,opt} \sim \left(\left(N_{pixel}^2 \mu_{reproj} h_{obs}\right)^2 N_{vis} \mu_{wproj} R_F^2\right)^{\frac{1}{3}}$$

Improvement over w projection and snapshots

$$\frac{T_{wsnapshots,opt}}{T_{wprojection}} \sim \left(\frac{N_{pixel}^2 \mu_{reproj} h_{obs}}{N_{vis} \mu_{wproj} R_F^2}\right)^{\frac{2}{3}}$$

$$\frac{T_{wsnapshots,opt}}{T_{snapshot}} \sim \frac{\sqrt{8\Delta A}}{R_F} \left(\frac{N_{vis} \mu_{wproj} R_F^2}{N_{pixel}^2 \mu_{reproj} h_{obs}}\right)^{\frac{1}{3}}$$

Scaling with Fresnel number

$$R_F = \frac{\theta_{FOV}^2}{\theta_{res}}$$

w projection	R_F^2	unbiased but slow
snapshots	R_F	biased but fast
w snapshots	$R_F^{2/3}$	unbiased and fastest

Algorithm 5: aw projection

Add primary beam *A(l,m)* to imaging equation

$$V(u,v,w)=\int\left[\frac{A(l,m)e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}}\right]I(l,m)e^{j2\pi(ul+vm)}dldm$$

$$V(u,v,w)=\tilde{G}(u,v,w)\otimes V(u,v)$$

$$\tilde{G}(u,v,w)=\int\frac{A(l,m)e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}}e^{j2\pi(ul+vm)}dldm$$

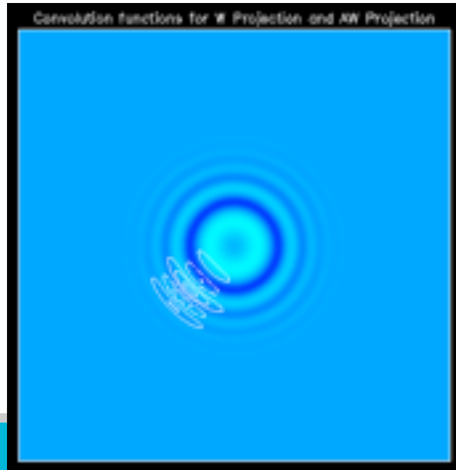
Algorithm 5: aw projection

Add primary beam *A(l,m)* to imaging equation

$$V(u,v,w) = \int \left[\frac{A(l,m) e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}} \right] I(l,m) e^{j2\pi(ul+vm)} dl dm$$

$$V(u,v,w) = \tilde{G}(u,v,w) \otimes V(u,v)$$

$$\tilde{G}(u,v,w) = \int \frac{A(l,m) e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}} e^{j2\pi(ul+vm)} dl dm$$



Algorithm 6: DDE projection

Add antenna-based direction-dependent effects to imaging equation

$$V_{i,j}(u,v,w) = \int \left[\frac{G_i(l,m)G_j^*(l,m)e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}} \right] I(l,m)e^{j2\pi(ul+vm)}dldm$$

$$V_{i,j}(u,v,w) = \tilde{G}_{i,j}(u,v,w) \otimes V(u,v)$$

$$\tilde{G}_{i,j}(u,v,w) = \int \frac{G_i(l,m)G_j^*(l,m)e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}}{\sqrt{1-l^2-m^2}} e^{j2\pi(ul+vm)}dldm$$

Computational load can be very large - not yet demonstrated

Common feature in all algorithms

- Reduce problem to 2D - then can apply 2D FFT
- Faceting
  - tiled collection of small fields of view
- W Projection
  - project onto w=0 plane using convolution function
- Snapshots
  - remove instantaneous flat plane, correct image geometry
- W snapshots
  - Two stages: (approximate) snapshots then (exact) w projection

Options

Options

- MIRIAD

Options

- MIRIAD
  - Does not handle wide field imaging at all

Options

- MIRIAD
  - Does not handle wide field imaging at all
- CASA

Options

- MIRIAD
  - Does not handle wide field imaging at all
- CASA
  - w projection

Options

- MIRIAD
  - Does not handle wide field imaging at all
- CASA
  - w projection
  - faceting

Options

- MIRIAD
  - Does not handle wide field imaging at all
- CASA
  - w projection
  - faceting
- ASKAPSoft

Options

- MIRIAD
  - Does not handle wide field imaging at all
- CASA
  - w projection
  - faceting
- ASKAPSoft
  - aw projection

Options

- MIRIAD
  - Does not handle wide field imaging at all
- CASA
  - w projection
  - faceting
- ASKAPSoft
  - aw projection
  - snapshots

Options

- MIRIAD
  - Does not handle wide field imaging at all
- CASA
  - w projection
  - faceting
- ASKAPSoft
  - aw projection
  - snapshots
  - aw snapshots

Options

- MIRIAD
  - Does not handle wide field imaging at all
- CASA
  - w projection
  - faceting
- ASKAPSoft
  - aw projection
  - snapshots
  - aw snapshots
  - aw stack

Summary

- Multiple algorithms available
- w projection best generally available
- w snapshots likely to take over soon

- Finding computationally feasible algorithms for SKA is still a research area

- Next frontier is solving for and correcting for antenna-based direction-dependent effects

Thank you

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