HIGH DYNAMIC RANGE IMAGING & SELF CALIBRATION

R Subrahmanyan
NRAO, Socorro
RRI, Bangalore

Synthetic images from a Fourier Synthesis telescope

Radio frequency images of the sky show

- ‘Point’ sources
- Somewhat resolved sources
- Double radio sources
- Diffuse radio sources
- Complex radio sources

Even a ‘perfect’ synthesis image does have ‘noise’ arising from measurement uncertainty

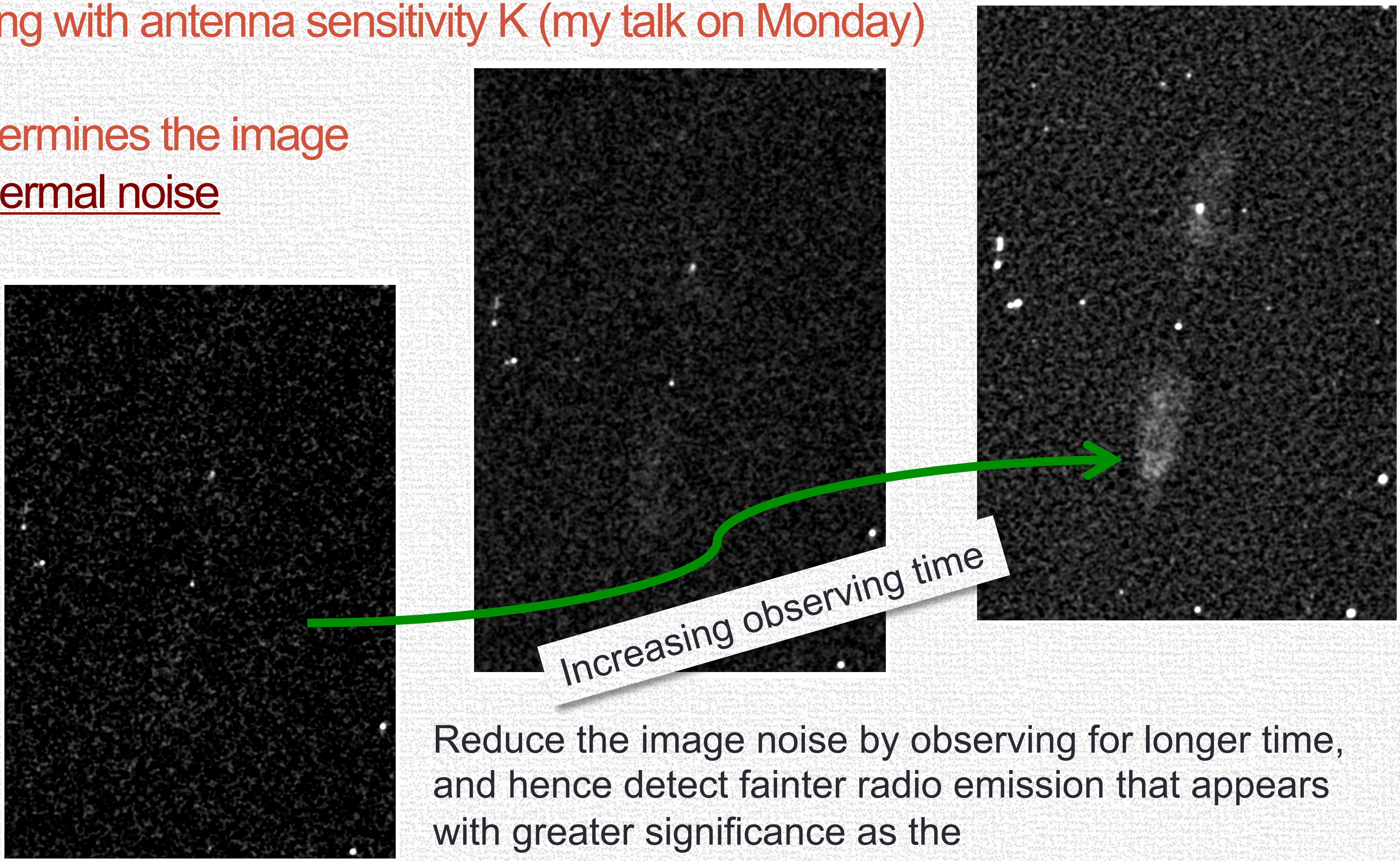
The image noise arises from :

- Emission from the background sky – galactic emission, the unresolved uniform extragalactic emission including CMB, sources in the field
- Atmosphere emission
- Ground emission
- Antenna emission
- Additive noise from amplifiers, attenuators, digitizers
- Finite data

System Temperature, the number of independent samples that go to synthesize the image, their relative weights (Urvashi 's talk on Tuesday), along with antenna sensitivity K (my talk on Monday)

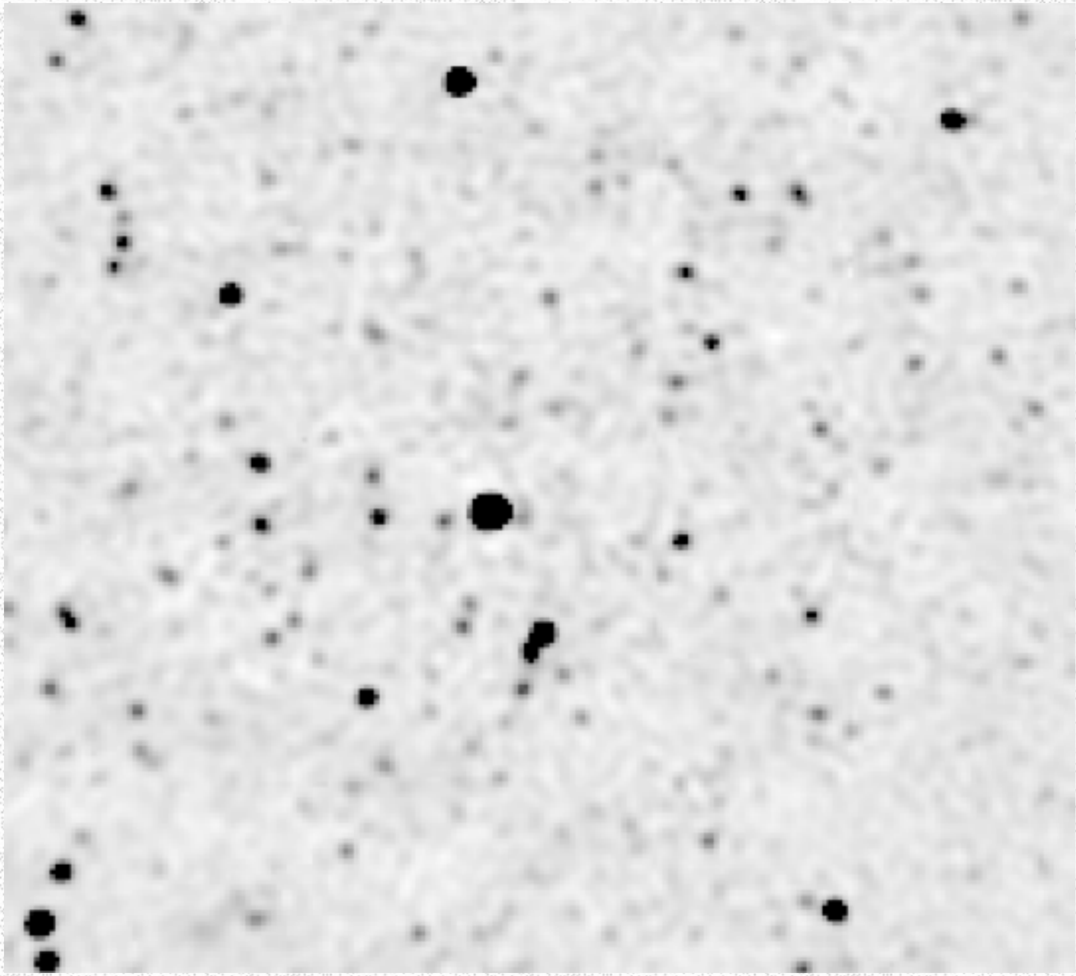
determines the image

Thermal noise



Reduce the image noise by observing for longer time, and hence detect fainter radio emission that appears with greater significance as the image noise = uncertainty continually reduces.

Sensitivity in images may be limited by confusion noise



Pictor-A field made using 32T MWA [F Briggs, ANU]

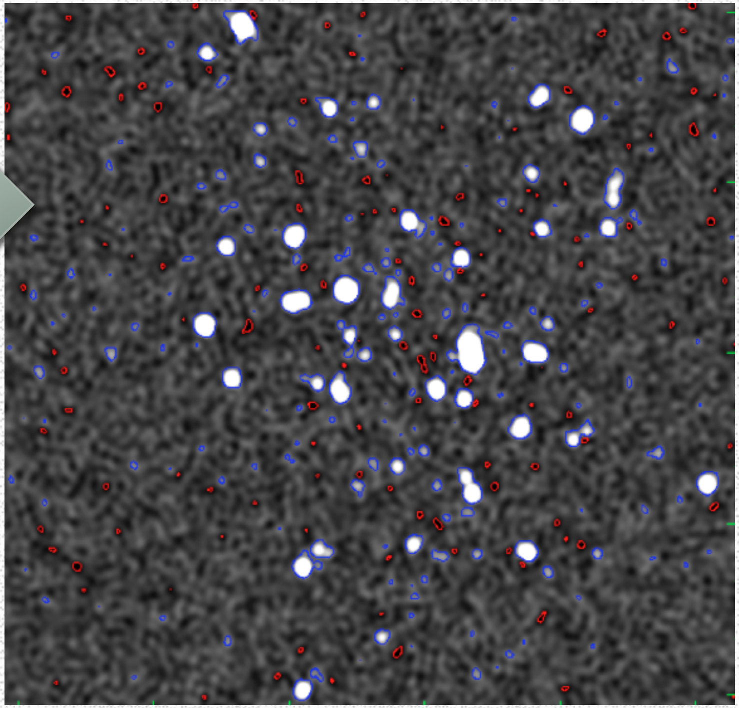
When the thermal image noise is reduced until a source appears in every image resolution element.

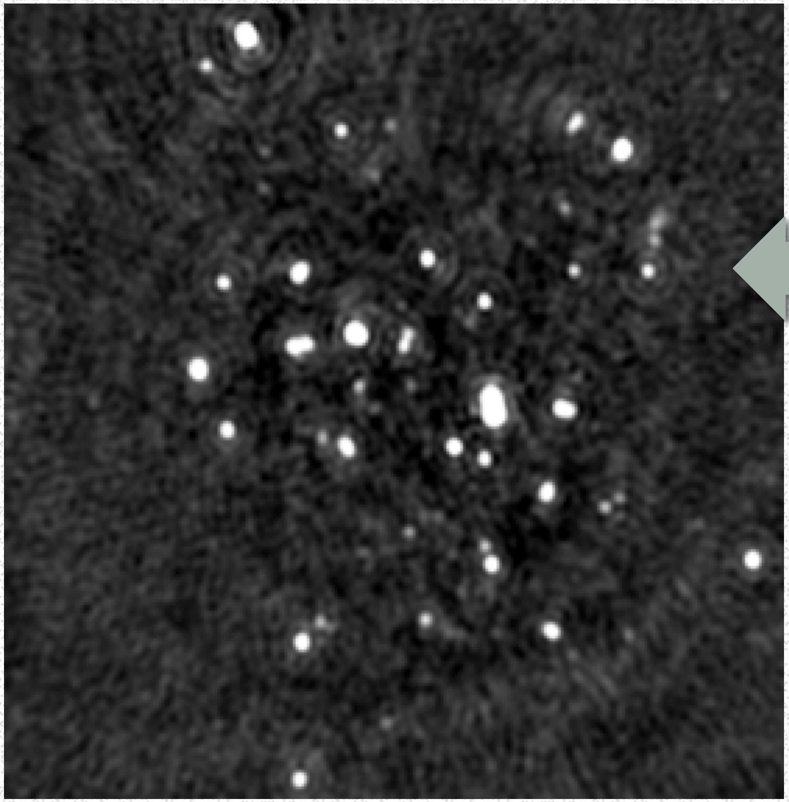
Rms image noise due to sources in every image resolution element dominates the rms image noise arising from System temperature or the measurement uncertainty.

Confusion noise may be classical confusion (due to sources in every beam) or sidelobe confusion.

Dynamic range:

In a good quality image:
DR is the ratio of
peak image intensity to
rms image noise.



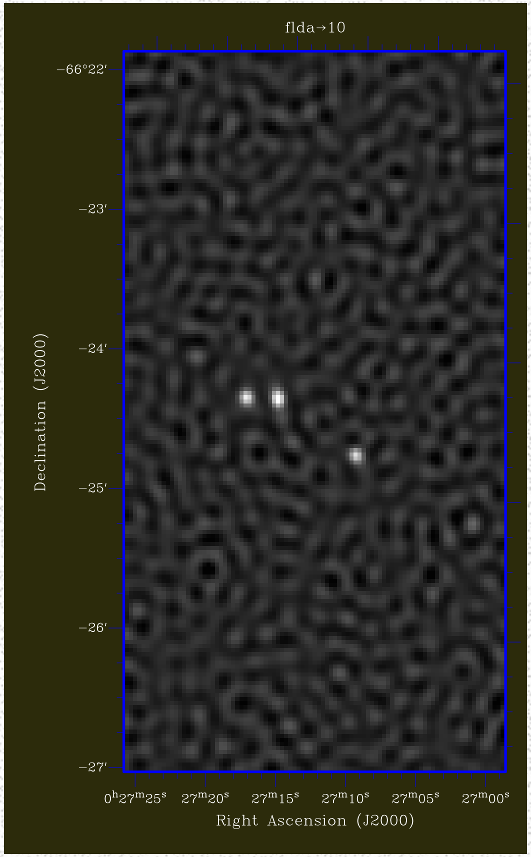


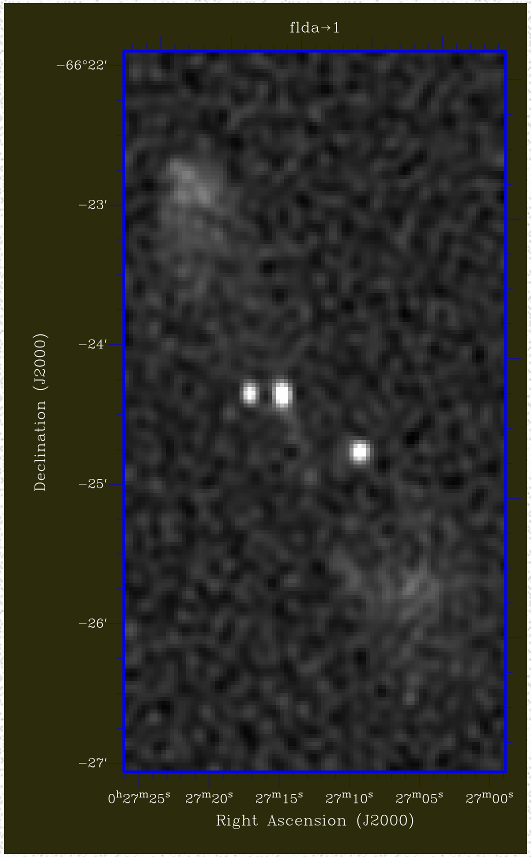
In the real world:
the synthesized image has errors, which may be several times the thermal noise in the image away from sources.

The DR is the ratio of peak image intensity to brightest spurious feature in the image

High dynamic range

does NOT mean that the image you have is a faithful reproduction of the sky!

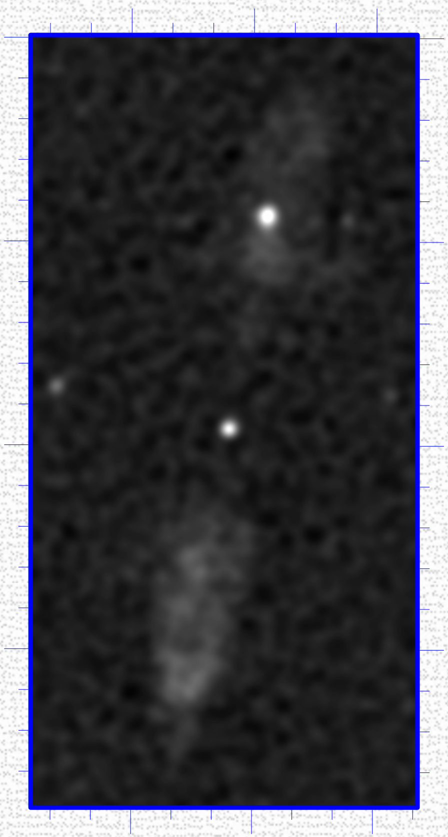




Both images have similar dynamic range

But the image on the left does not have adequate short uv-spacings/baselines and so has little information on extended structure!

High dynamic range images require to have adequate visibility coverage to image the range of spatial scales in the sources and be



High fidelity images!

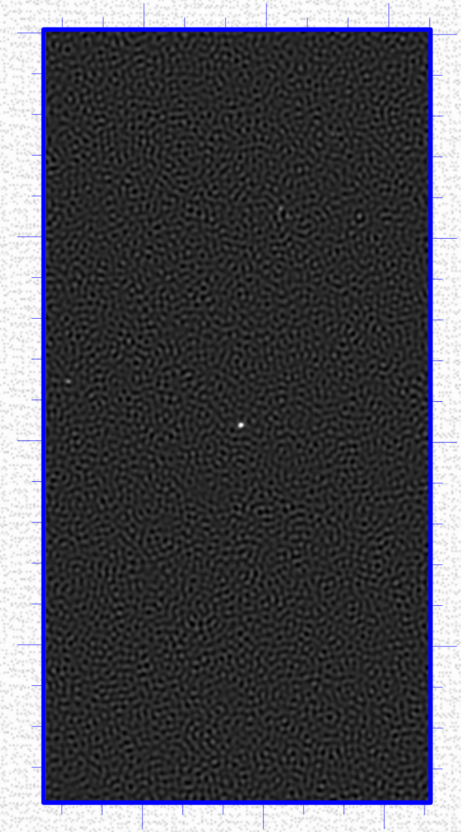
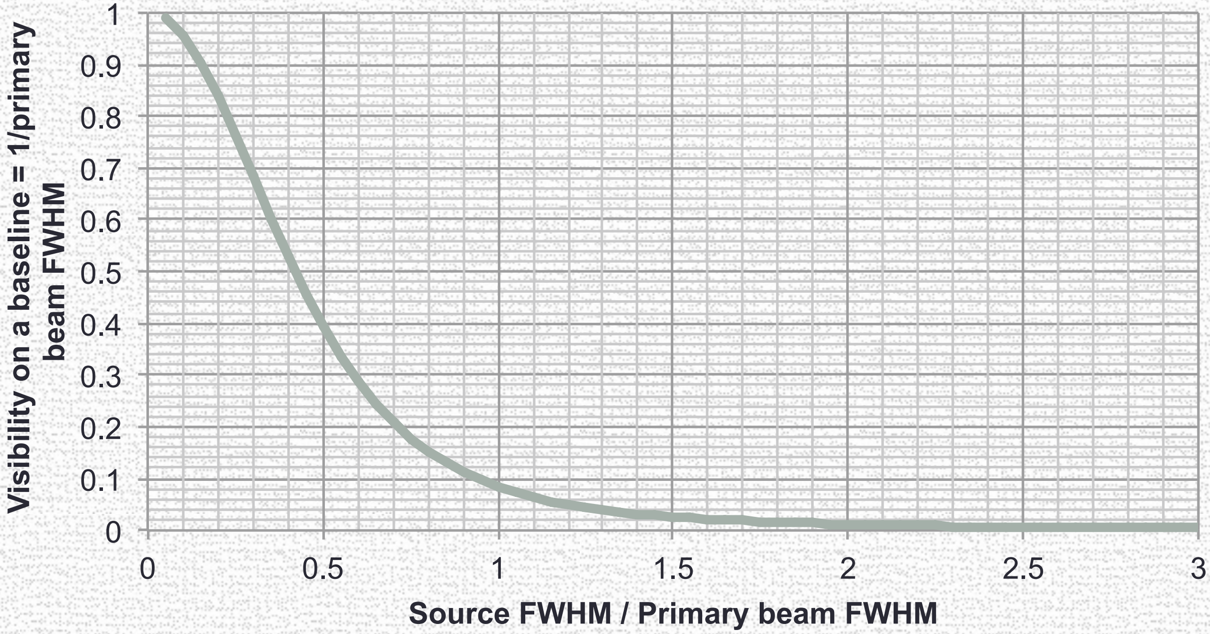


Image fidelity: defined in terms of the difference between synthesized image and true image

High dynamic range does not always imply high fidelity!

To image extended emission components of sources we need to include short spacing interferometers

However:



Even suppose we had a short spacing:

with baseline length = diameter of antenna element

Interferometer visibility drops off if source FWHM >> primary beam FWHM

Unfortunately the antenna optics averages the EM field over the antenna aperture

Minimum baseline
= diameter of the dish

- Information on scales smaller than the shortest spacing = dish diameter exists in short spacing interferometers!

- MOSAICING can decompose the visibilities on any baseline

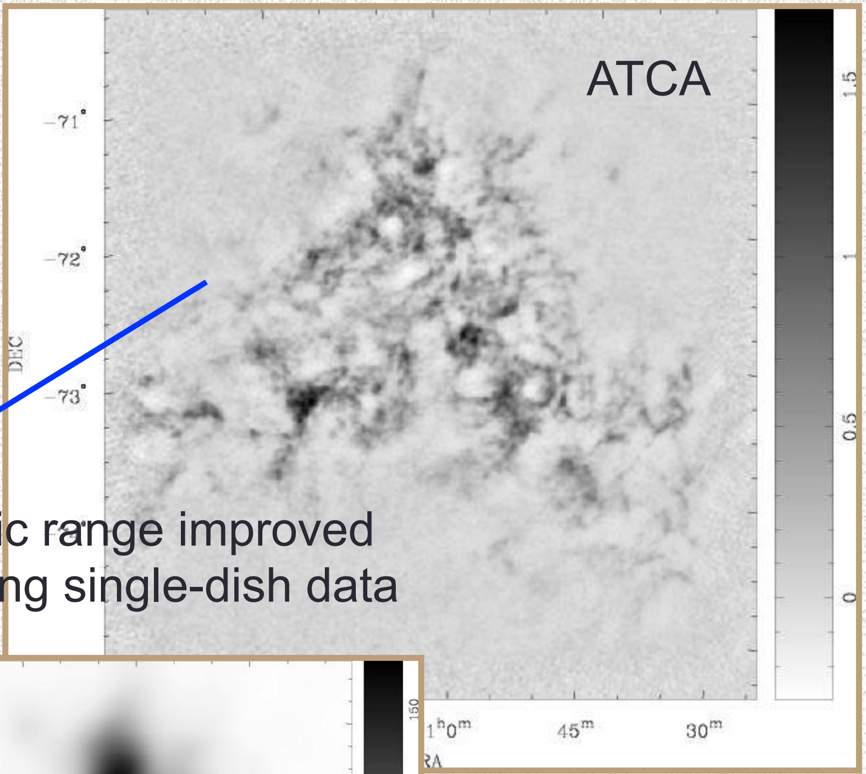
→ wide fields can be imaged
using arrays of large aperture antennas

Range of baseline data actually present in the measurement set

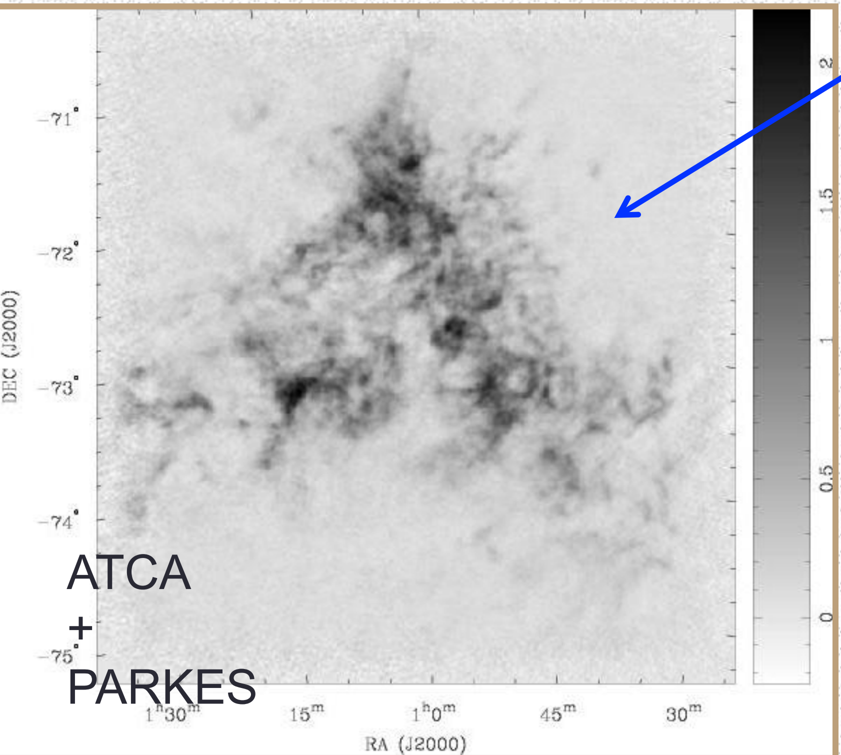
[Mosaicing talk Thursday by Lister]

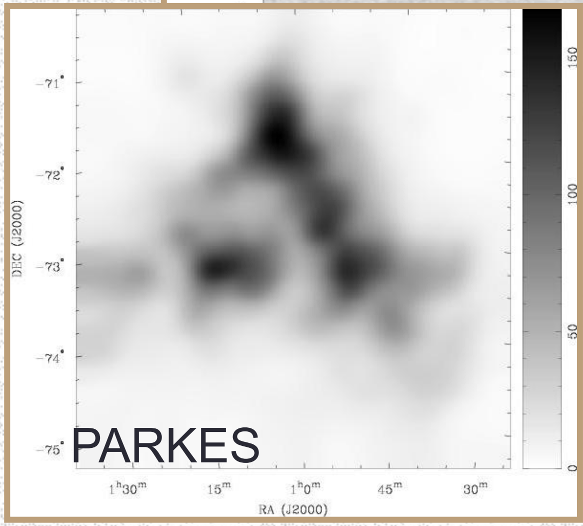
Sometimes the shortest baselines of the interferometer may not have sufficient information on the large scale structure

This may be corrected by adding the missing short spacing information from single-dish raster-scan observations



Dynamic range improved by adding single-dish data



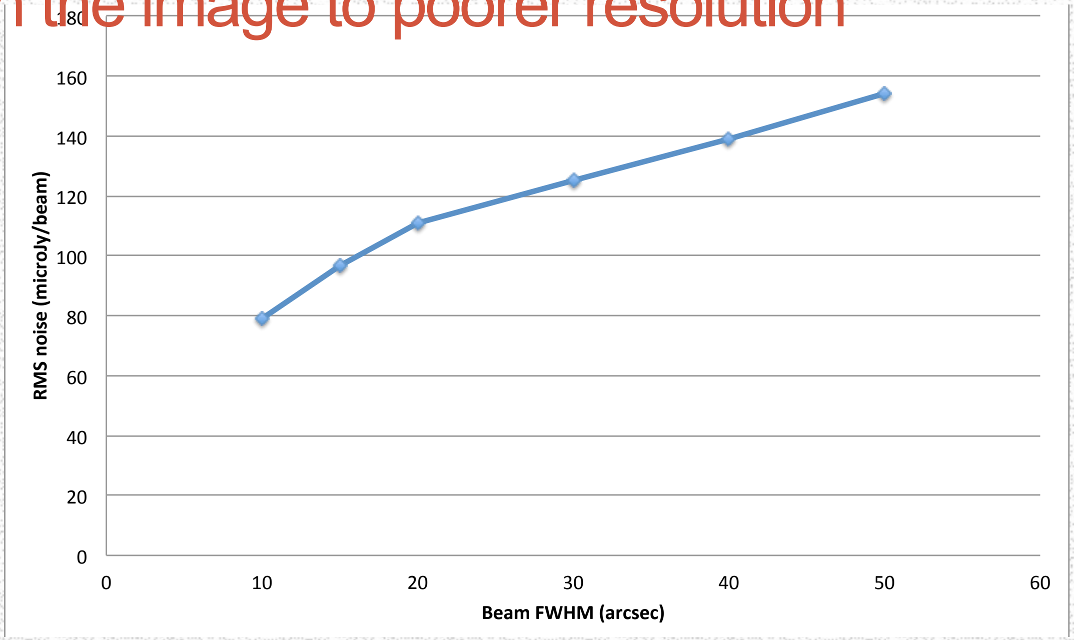


SMC

[Stanimirovic et al. 1999]

Dynamic range:
changes when we smooth the image to poorer resolution

Rms thermal noise & peak intensity both depend on synthesized beam



- In Fourier synthesis imaging, the image RMS noise (in Jy/beam units) <u>increases</u> with smoothing! The trend depends on the visibility coverage and relative weighting of the data during the imaging process.
- The peak intensity (in Jy/beam units) increases with smoothing for extended emission.
- The surface brightness sensitivity generally improves with smoothing; however, sources blend and images get confused.

High dynamic range images made with good visibility coverage

may be smoothed to improve surface brightness sensitivity and reveal low surface brightness emission

Images are either

‘thermal noise limited’, or

‘confusion noise limited’, or

‘dynamic range limited’.

Why do we have image errors that limit the dynamic range?

1. Because the visibilities have additive or multiplicative errors

2. Visibilities recorded are not truly the expected coherence for the pair of antennas and for the real sky.

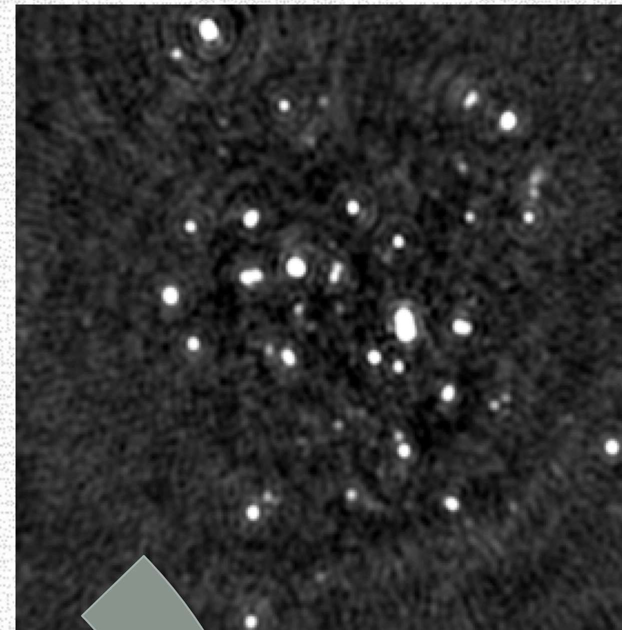
3. Almost all of these may not be errors but simply a reflection of our ignorance of the system – the antenna and receiver characteristics.

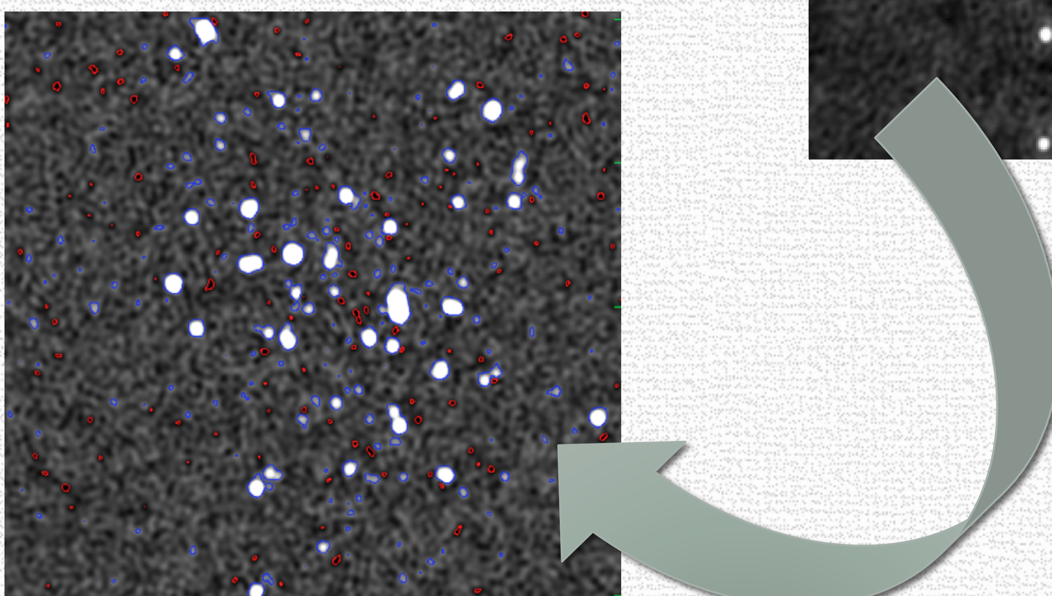
4. Because of errors in the image reconstruction process because of insufficient information to create an image of the field of view

In most cases we receive a measurement set that has errors.
These are errors beyond thermal noise.

Calibration may be imperfect.

How do we make
high dynamic range images
using such measurements?





Principle of high dynamic range imaging:

Known unknowns are ok;
but can't have unknown unknowns

- We have a measurement set, and

- We have a measurement equation that relates the sky, model parameters for the atmosphere, and model parameters for the telescope, to the measurement.

- If the measurement set is sufficiently large, the inversion problem of solving for the sky and known unknowns given the measurement set may be overdetermined.

Needs to be over determined because there is measurement noise

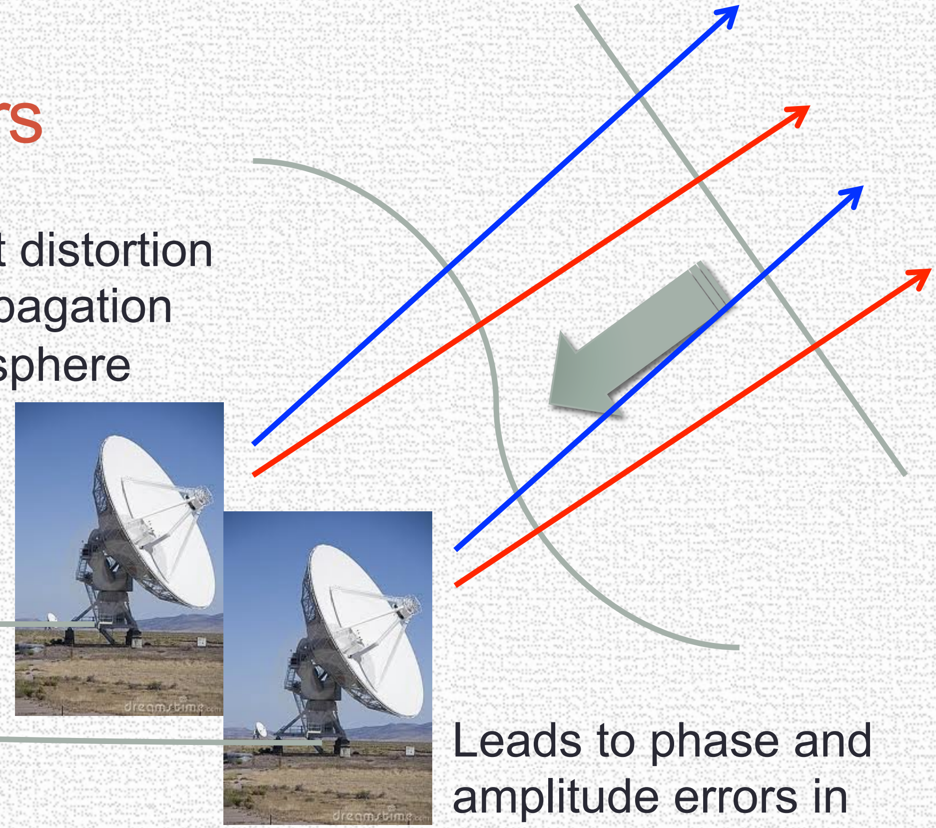
- We may make a joint solution for the sky model and parameters for the atmosphere and parameters describing the telescope, given the measurement set and measurement equation.

Origin of visibility errors 1: Signal path propagation errors

Wave-front distortion during propagation thro' atmosphere

Propagation in long transmission lines may introduce differential amplitude and phase errors

Correlator at a central site



Leads to phase and amplitude errors in signals arriving at the antennas

These are

- Antenna based errors
- Amplitude/phase calibration errors
- Multiplicative errors

System propagation errors may be corrected using noise calibration and round trip phase measurement

Observing a nearby calibrator helps remove most part of the propagation error; however, residual errors remain because of time varying gains and non-uniform atmosphere

Effect of visibility errors on image dynamic range

Consider an array with N antennas, N(N−1)/2 baselines, and imaging of instantaneous visibilities – snapshot imaging.

- If one baseline has phase error ϕ (in radians): $DR \approx N^2/\{\sqrt{2}\ \phi\}$
- If one baseline has amplitude error ε: $DR \approx N^2/\{\sqrt{2}\ \varepsilon\}$

10° phase is equivalent to 17% ampl

- If one antenna has a random phase error ϕ on all (N-1) baselines to that antenna:

  $DR \approx \sqrt{(N/2)} \times N/\phi$

- If all antennas have a random phase error on all baselines:

  $DR \approx N/\{\sqrt{2}\ \phi\}$

[Derivations in Rick Perley's lecture notes in ASP Conf Ser vol.180]

If N=6, a 10° phase error on all baselines: DR ≈ 20

If N=36, a 10° phase error on all baselines: DR ≈ 120

These numbers are for snapshot images.

Earth-rotation synthesis improves dynamic range if independent times have independent errors. Typically the number of independent time in a 12-hr observation may be 25-1000, which improves DR by factors between 5-30

With a few 10's of antennas the DR achieved in typically a few 1000 with standard transfer of calibration from a nearby calibrator source.

To reduce antenna based errors below 1° and improve DR to better than 10,000:
SELF-CALIBRATION

Closure Phase

Complex multiplicative error introduced by propagation: g1, which has phase $\phi 1$

Visibility measurement set:

Vm_{12} = g1 x g2* x Vt_{12}
Vm_{23} = g2 x g3* x Vt_{23}
Vm_{13} = g1 x g3* x Vt_{13}

has phase terms:

$$\phi m12 = \phi t12 + \phi 1 - \phi 2$$
$$\phi m23 = \phi t23 + \phi 2 - \phi 3$$
$$\phi m31 = \phi t31 + \phi 3 - \phi 1$$

g2, with phase $\phi 2$

These are errors that are present after calibration!

g3, with phase $\phi 3$

Closure phase:

Sum the measured phases over baselines between sets of three antennas

$$\phi m12 + \phi m23 + \phi m31 = \phi t12 + \phi t23 + \phi t31$$

The phase terms due to propagation errors drops out!

Closure phase is a ‘good’ quantity that is free of antenna based errors

Closure Amplitude

Complex multiplicative error introduced by propagation: g1

g2

Visibility measurement set:

$Vm_{12} = g1 \times g2^* \times Vt_{12}$
$Vm_{23} = g2 \times g3^* \times Vt_{23}$
$Vm_{14} = g1 \times g4^* \times Vt_{14}$
$Vm_{43} = g4 \times g3^* \times Vt_{43}$

These are errors that are present after calibration!

Closure amplitude
defined for a loop of visibility amplitudes
measured between any set of four antennas

g3

$$\frac{|Vm_{12}| \times |Vm_{43}|}{|Vm_{14}| \times |Vm_{23}|} = \frac{|Vt_{12}| \times |Vt_{43}|}{|Vt_{14}| \times |Vt_{23}|}$$

g4

The amplitude gain terms due to
propagation errors drops out!

Closure amplitude is a ‘good’ quantity that is once again free of antenna based errors

Self calibration

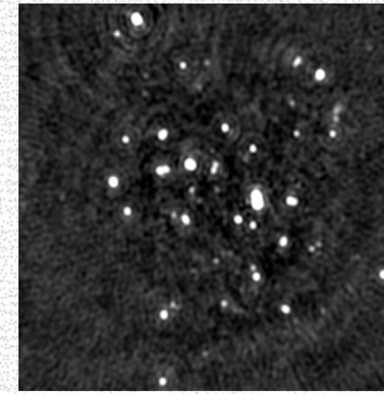
Visibility data may have antenna based errors owing to uncorrected gains of the propagation paths in arms of the interferometers.

Nevertheless, closure phases and closure amplitudes are free from such antenna-based calibration errors.

Principle of self-calibration: allow the complex antenna gains to be free parameters.

Observed visibilities

Fourier Transform

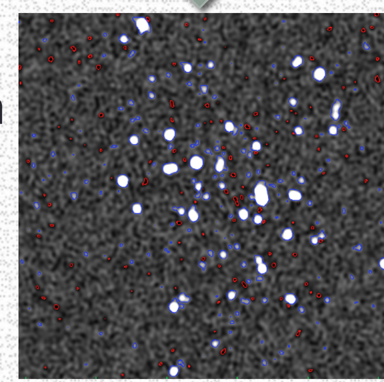


Allow antenna gains to change

Constrain the image to rid itself of errors; positive Stokes I image

Selfcalibrated visibilities

Fourier Transform



Self-calibration works if the problem is over-determined: if the ratio of number of constraints (closure quantities) to number of degrees of freedom (antenna gains) is large.

Iterative algorithm for self calibration:

Using calibrated/corrected visibilities Vc - compute a model $I_m(x,y)$ for the source based on constraints on what we think the sky ought to look like

This uses good deconvolution algorithms

Compute model visibilities Vm corresponding to the model image

Solve for complex antenna gains, which may be a function of time, that when applied to the measurement set minimizes:

$$S = \sum\sum w_{ij} \, |Vm_{ij} - g_i g_j^* Vc_{ij}|^2$$

Which is the difference between corrected visibilities and model visibilities.

General solution to self-calibration problem:

- Iterative solutions seem to work in most cases.
- A general method is simultaneous solution for
  - complex antenna gains and
  - image I(x,y) on the sky

  driven by the constraints that define our expectation for what the image ought to look like (positivity, maximizing image entropy…..)

Making self calibration work to improve the sky image:

Arrays with N elements have
Free antenna gains: (N-1) phases, N amplitudes.

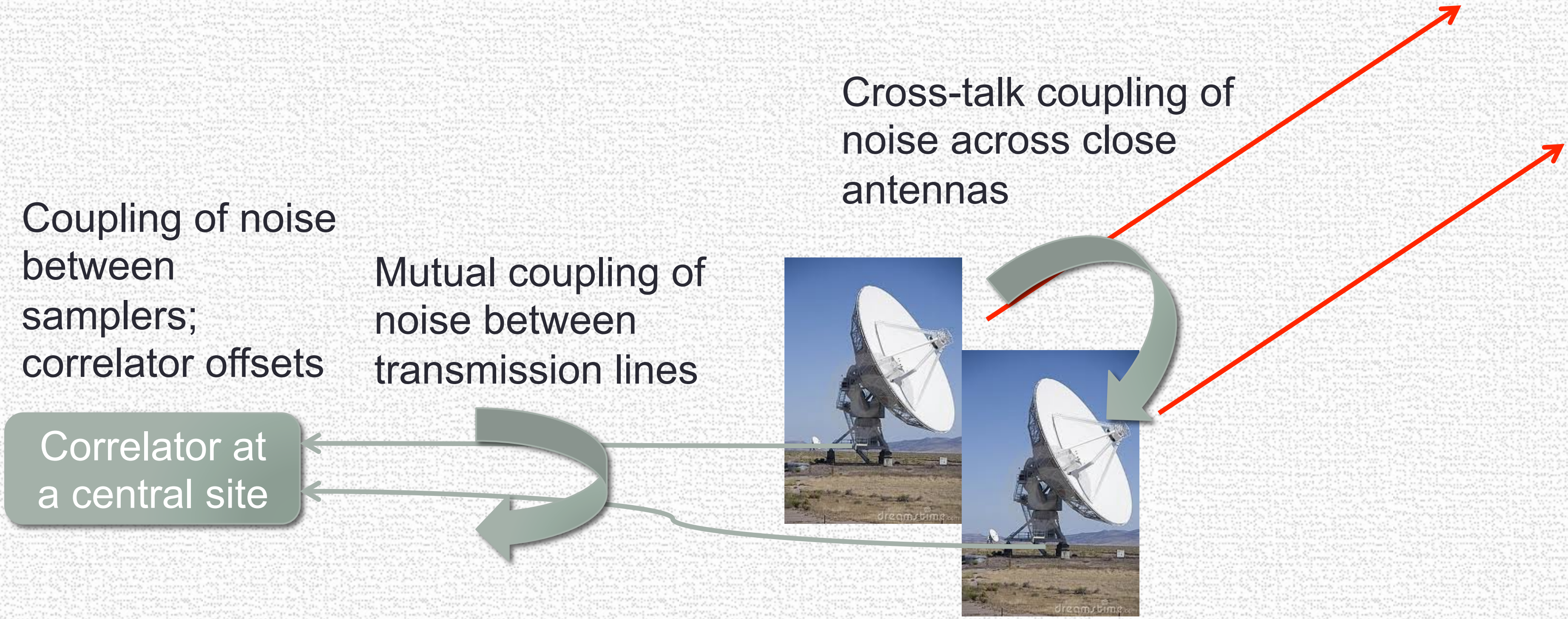
Ratio of closure constraints to free parameters steeply rises with number of antennas –

Selfcal works better when arrays have large N

Representation of the sky requires large numbers of components if field of view is large and synthesized beam is small and sensitivity is great.

Selfcal works better if number of good constraints from the data (closure quantities) well exceeds the number of free parameters (antenna gains and image components): we must have enough information to make a good image!

Origin of visibility errors 2: Closure errors



These are

- Not antenna based errors, but baseline based or closure errors
- Amplitude/phase errors
- Additive and multiplicative errors

Additive errors in the transmission paths and in the correlator may be canceled by phase switching the sky signal as it enters the signal path.

Closure errors that are multiplicative survive: Baseline-based error correction

- Baseline-based multiplicative errors are determined by
  - Observing strong unresolved sources,
  - Using a good model for the sky including other sources in the field,
  - Dividing the measured visibilities by the model visibilities,
  - Averaging for long durations (so that the complex visibility contributions from other sources in the field – inadequacies in the model – average out).

- The factors resulting from the above represent ‘constant’ baseline-based errors that are used as corrections for visibilities on the target field.

- Usually difficult to do this on short baselines – where the problem is often most obvious.

Origin of visibility errors 3: Direction-dependent (DD)

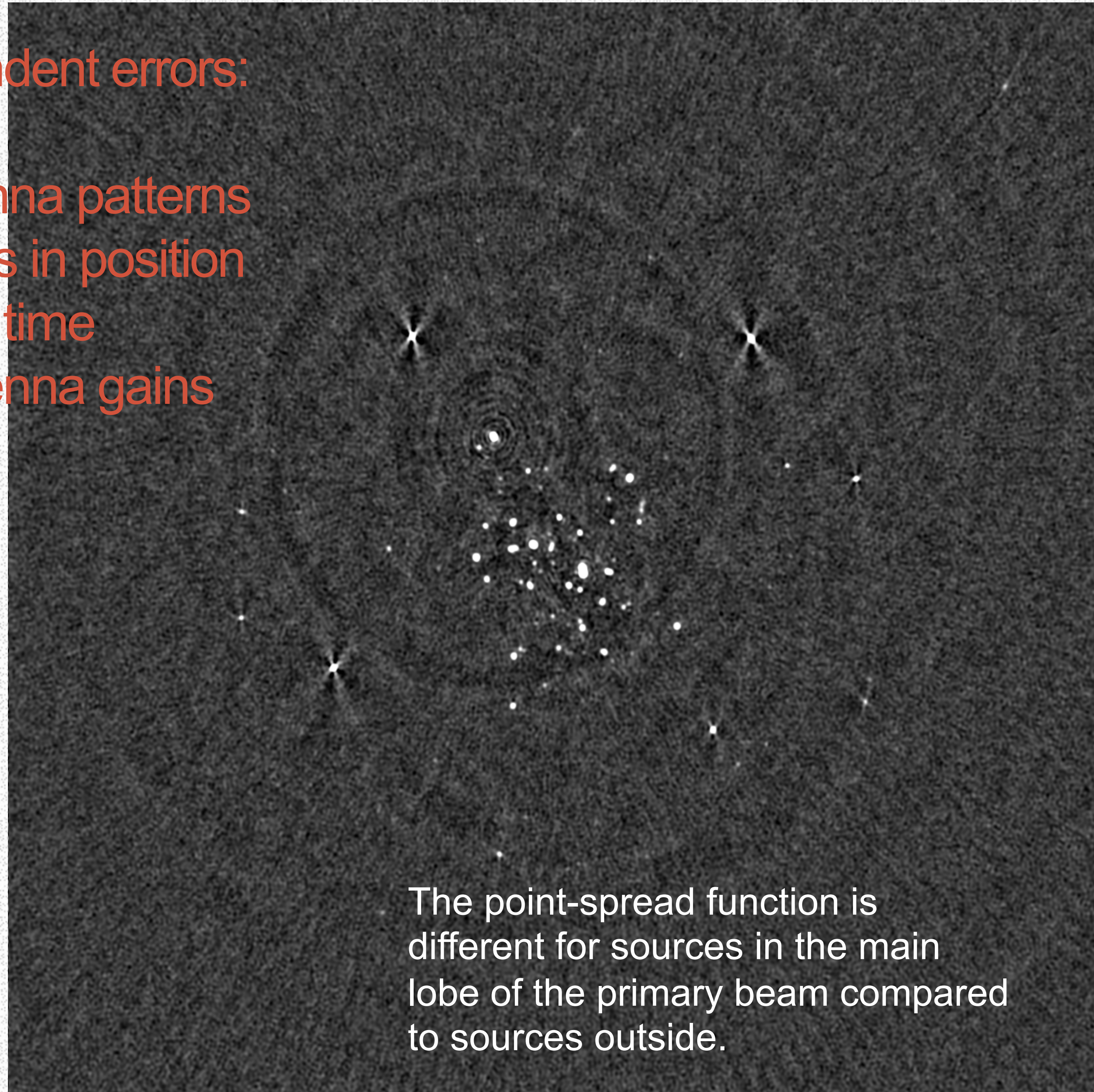
- Visibility errors that are dependent on sky position – column density structure in ionosphere, troposphere – these are antenna based multiplicative errors, which are sky-position dependent and also time variable.

- Visibility errors because antennas are not identical – this is an antenna-based direction dependent multiplicative error.

- Visibility errors because antennas are time-varying -
  - Antenna pointing errors that are different in different antennas and may also vary with time
  - Antennas with alt-az mounts have beams that rotate on the sky
  - Antennas built as aperture arrays have beams that change with phasing of the aperture and the projected area changes with sky direction.

Direction dependent errors:

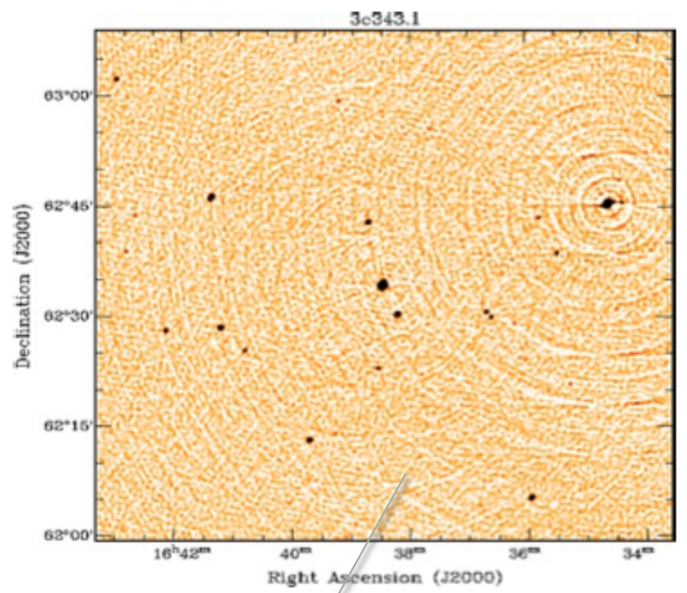
Changing antenna patterns over time results in position dependent and time dependent antenna gains



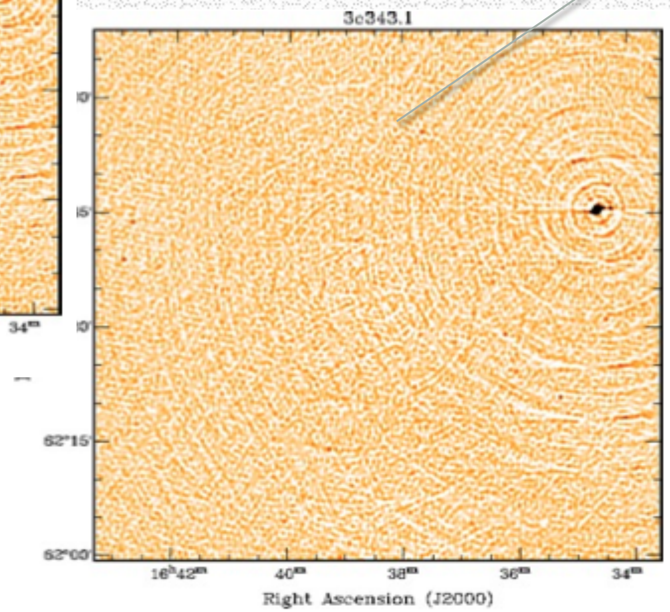
The point-spread function is different for sources in the main lobe of the primary beam compared to sources outside.

‘Peeling’: as a technique to subtract strong off-axis sources when DD antenna gains vary with time

[Example from Tom Oosterloo, ASTRON]



After subtracting sources close to the field centre



After selfcalibration on the off-axis source

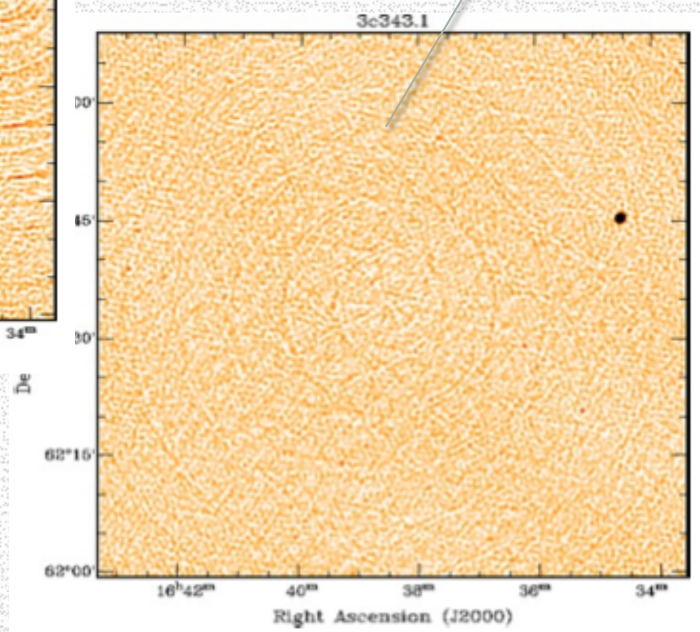


Image after selfcalibrating full field

Peeling……continued

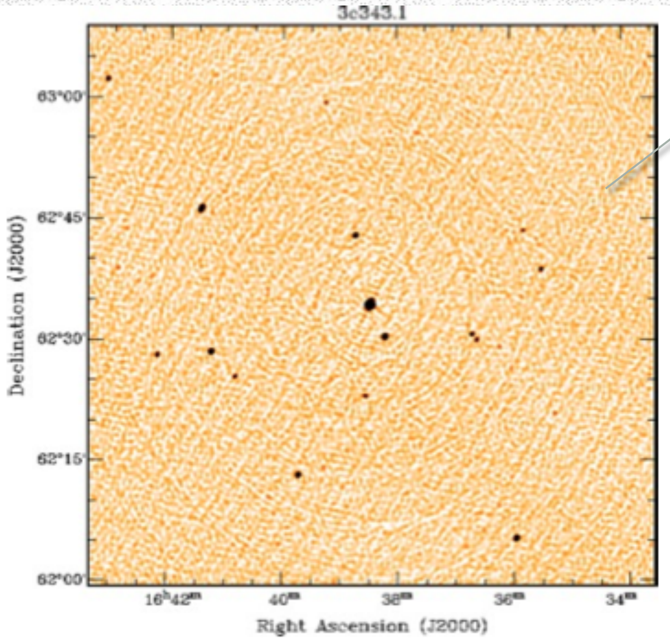
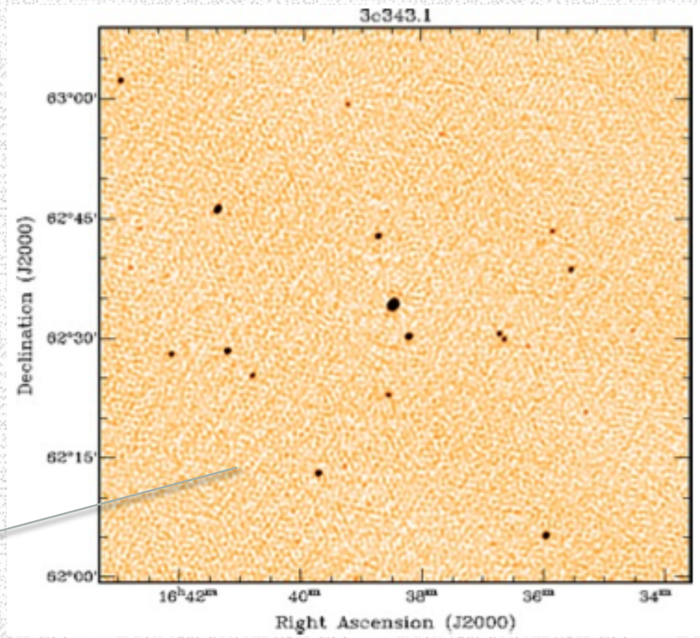


Image after subtracting the off-axis source and undoing corrections based on the off-axis source



Final image after selfcalibration based on sources close to boresight

[Example from Tom Oosterloo, ASTRON]

Example of application of peeling – imaging M81 field

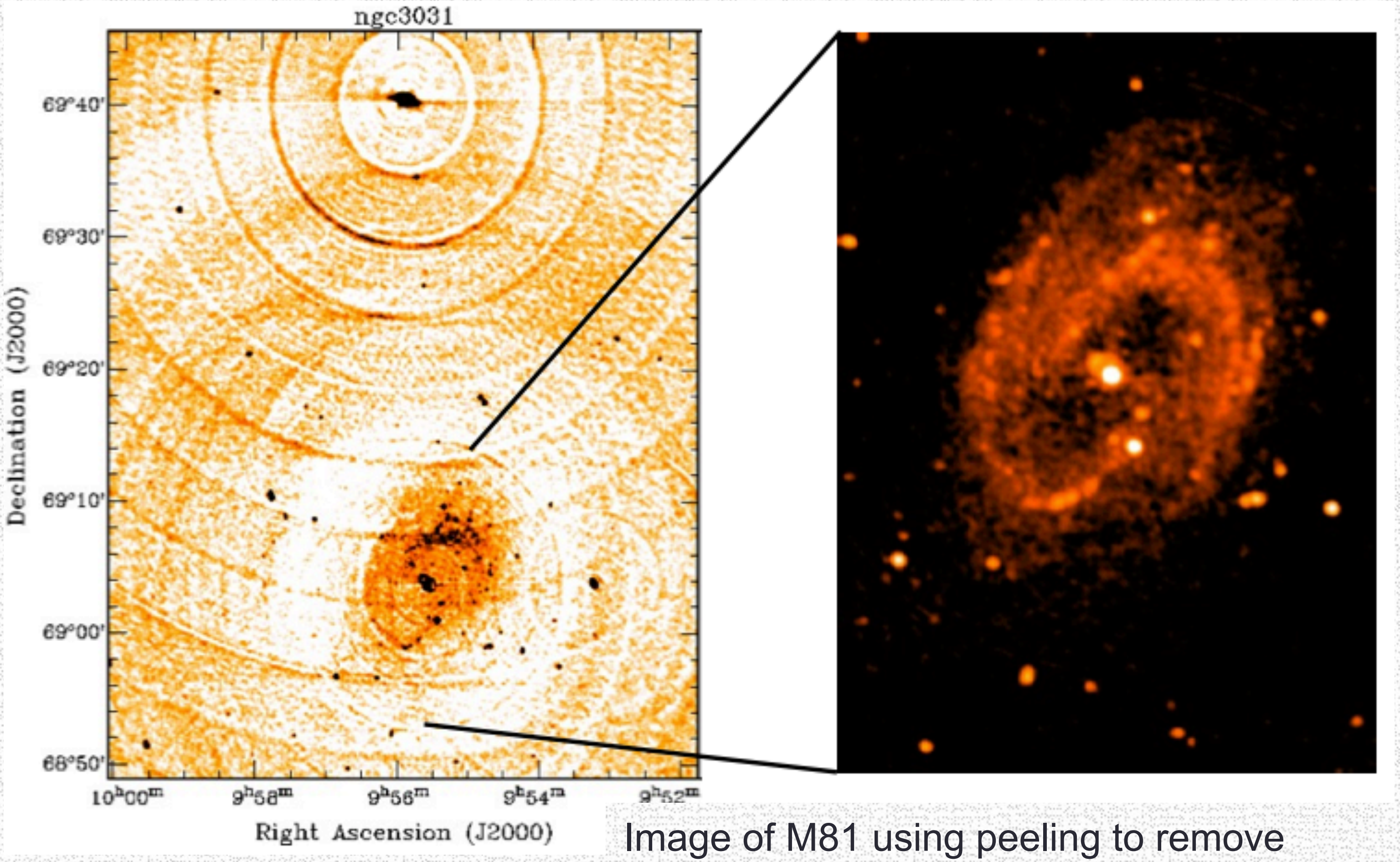
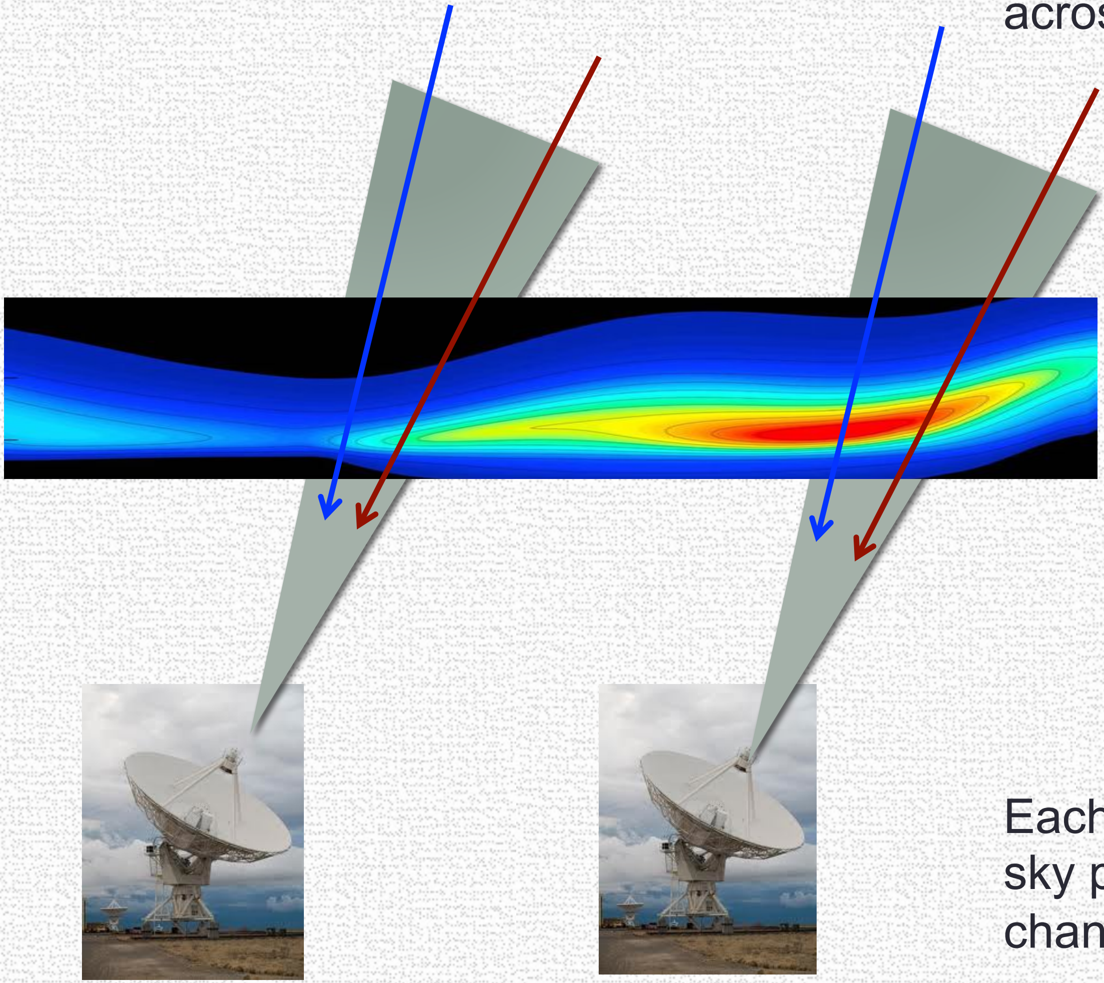


Image of M81 using peeling to remove sidelobes of the distant source. After peeling the DR is as expected from thermal noise.

[Tom Oosterloo, ASTRON]

Worst case atmosphere!

Antennas may view the sky through different parts of the ionosphere/ atmosphere, which may also vary across the field of view

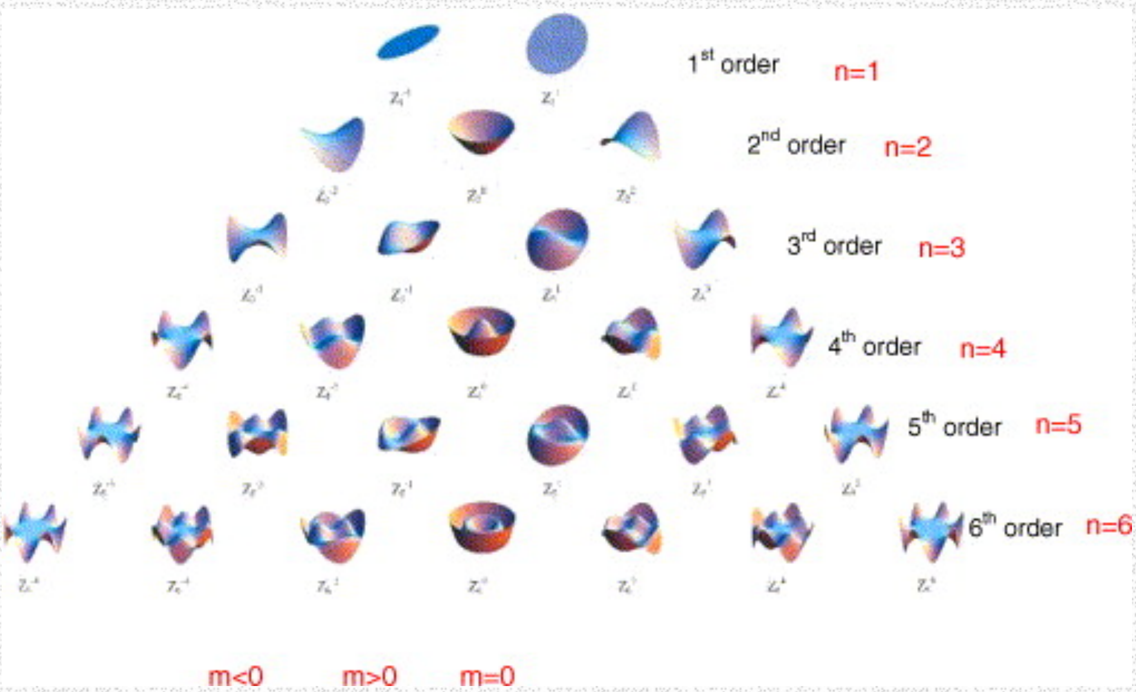


Different antennas see the field-of-view through different atmospheres

For each antenna, the atmosphere varies across the field-of-view.

Each antenna sees sources in offset sky positions! And these offsets change with time.

Measurement equations need to include direction dependent free parameters to model the atmosphere – phase screen



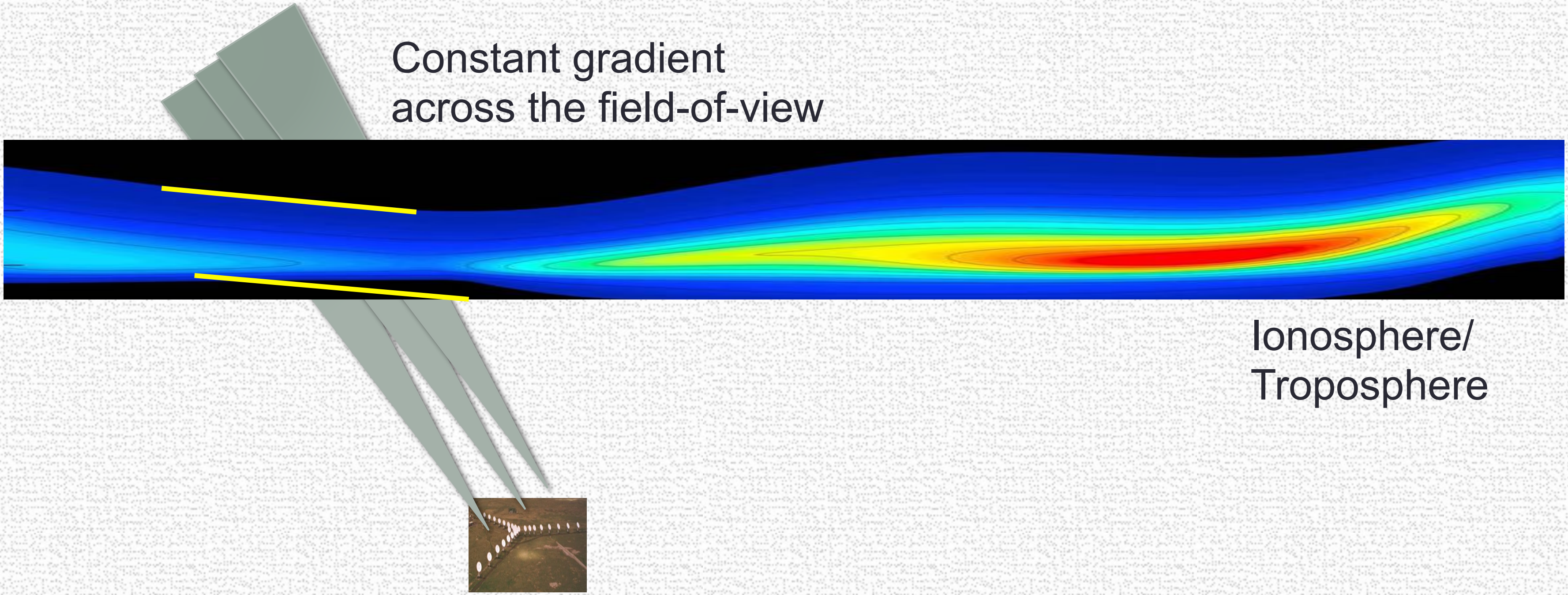
'Field-based calibration'

Low order Zernike polynomial functions are used to model the phase error across the field of view.

The polynomial coefficients functions of time.

The parameters are fitted using the position errors on strong sources

Complexity of the problem depends on height of atmosphere above array, extent of the array and field of view of antennas

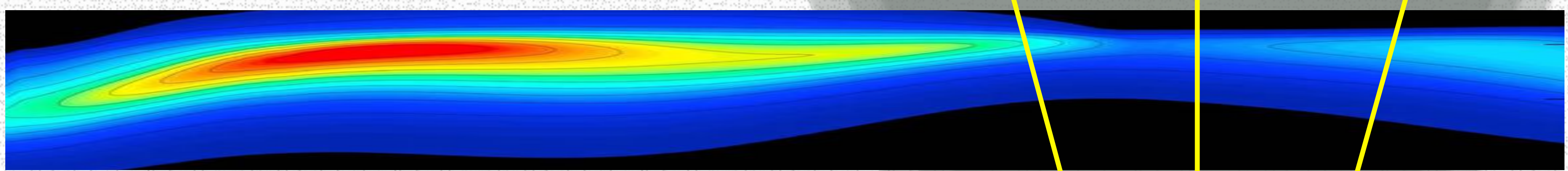


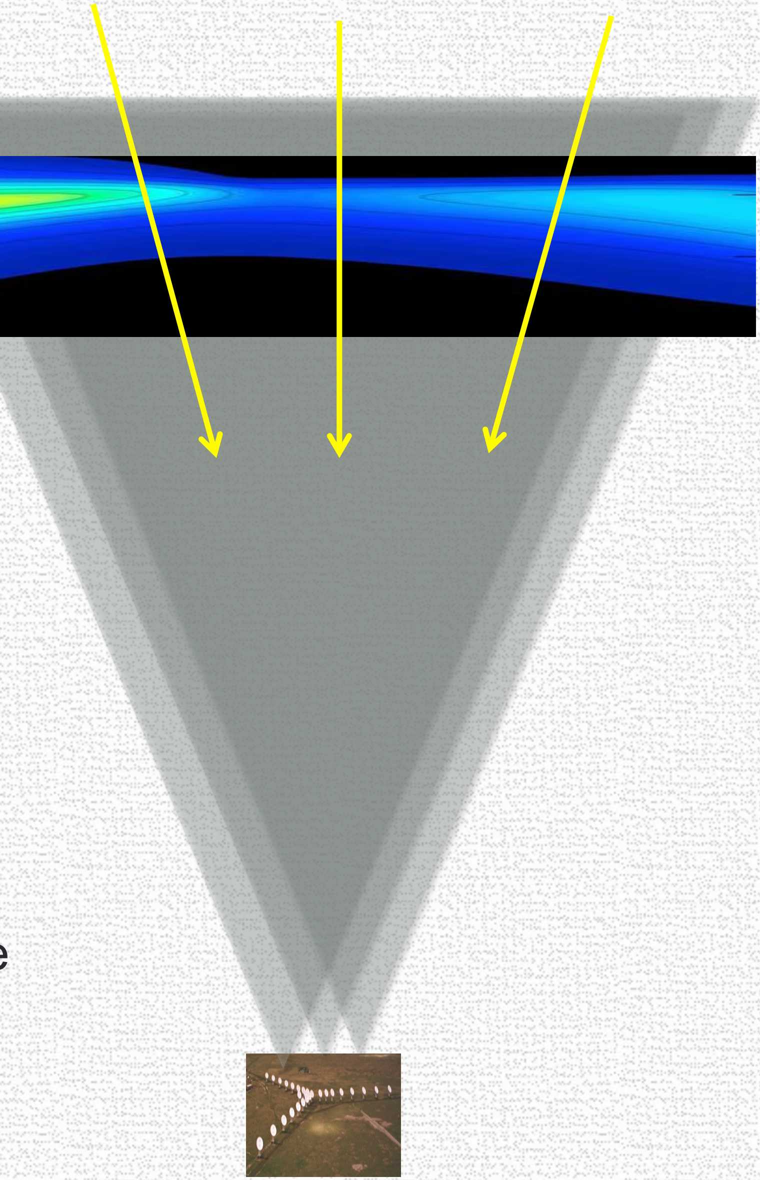
If all sources are seen by the array through the same linear atmosphere wedge – all sources in the field have the same position offset – which may differ from the offset for the calibrator.

Normal self-calibration with model at the right position can correct the error

Complexity of the problem depends on height of atmosphere above array, extent of the array and field of view of antennas

Density gradient varies across the field-of-view



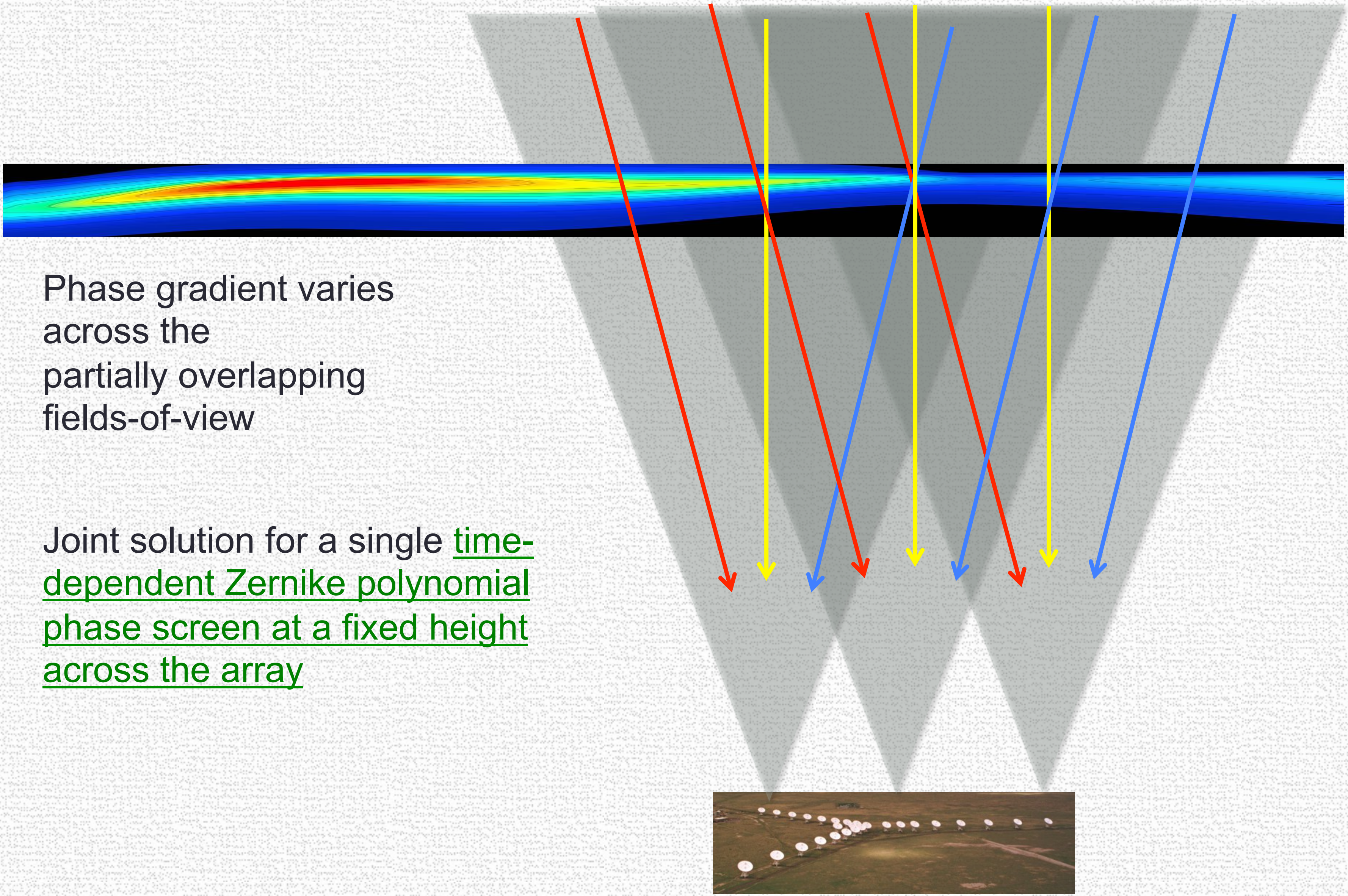


If different sources are seen by the array through different wedges of the atmosphere – the position offset varies across the field.

Field based calibration that solves for a single time-dependent Zernike polynomial phase screen across the field of view.

Corrections derived using strong sources in the field whose positions are known from high frequency surveys.

Complexity of the problem depends on height of atmosphere above array, extent of the array and field of view of antennas

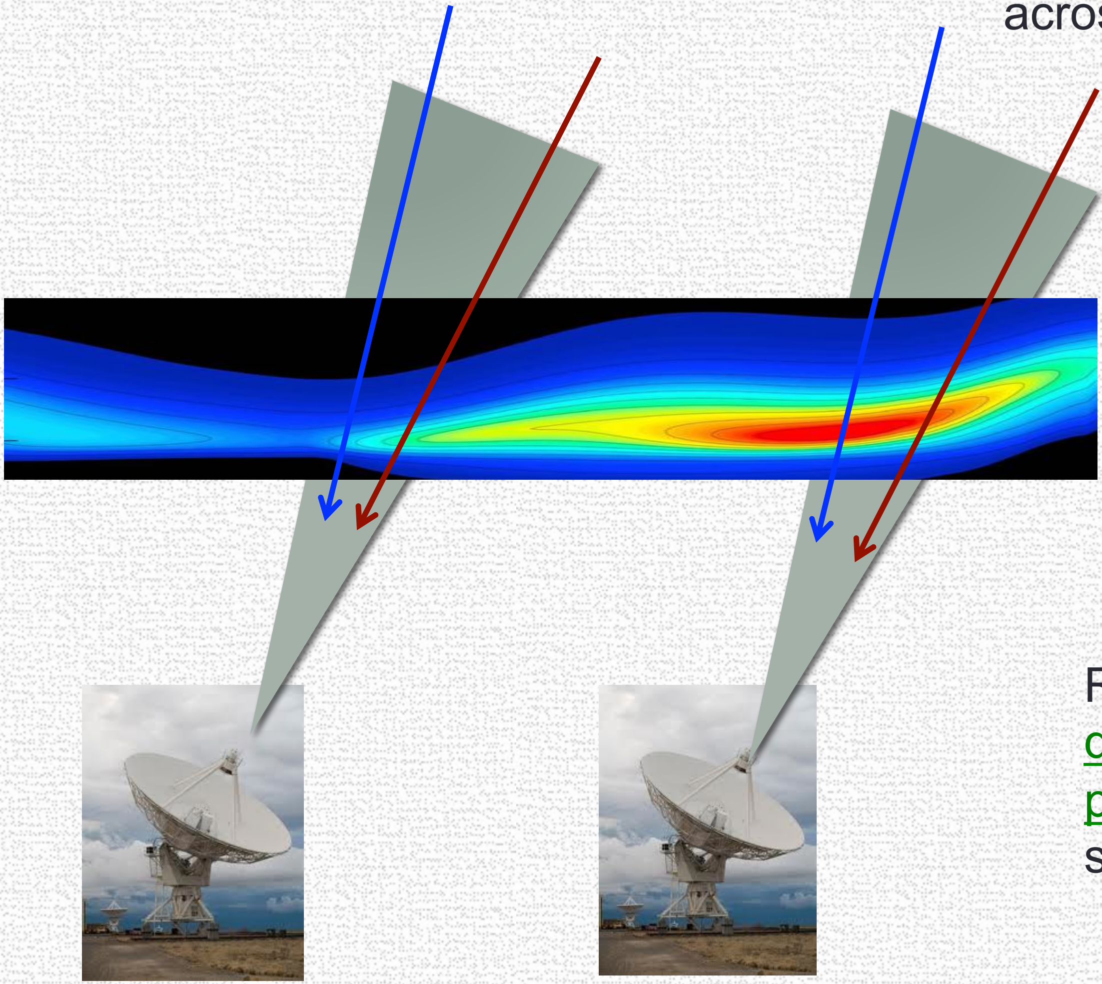


Phase gradient varies across the partially overlapping fields-of-view

Joint solution for a single time-dependent Zernike polynomial phase screen at a fixed height across the array

Worst case atmosphere!

Antennas may view the sky through different parts of the ionosphere/ atmosphere, which may also vary across the field of view



Requires solution for time-dependent Zernike polynomial phase screens : separate screen for each antenna.

Summary of issues that limit dynamic range

First: good dynamic range requires excellence in system design so that telescope related errors are minimum and internally calibrated

Second: good dynamic range requires a good site free of RFI and where atmosphere/ionosphere effects are minimum

- Amplitude/Phase calibration errors – antenna based errors – errors during the process of amplitude/phase referencing:
  Self-calibration
- Closure errors - baseline based multiplicative errors:
  Baseline-based corrections derived from calibrator field observations
- Sky-position dependent errors, which may also be time dependent – telescope beam shape and pointing errors & non-isoplanatic atmosphere:
  Peeling;
  Phase screen model as free parameters to be solved for.