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Netherlands Institute for Radio Astronomy

Calibration

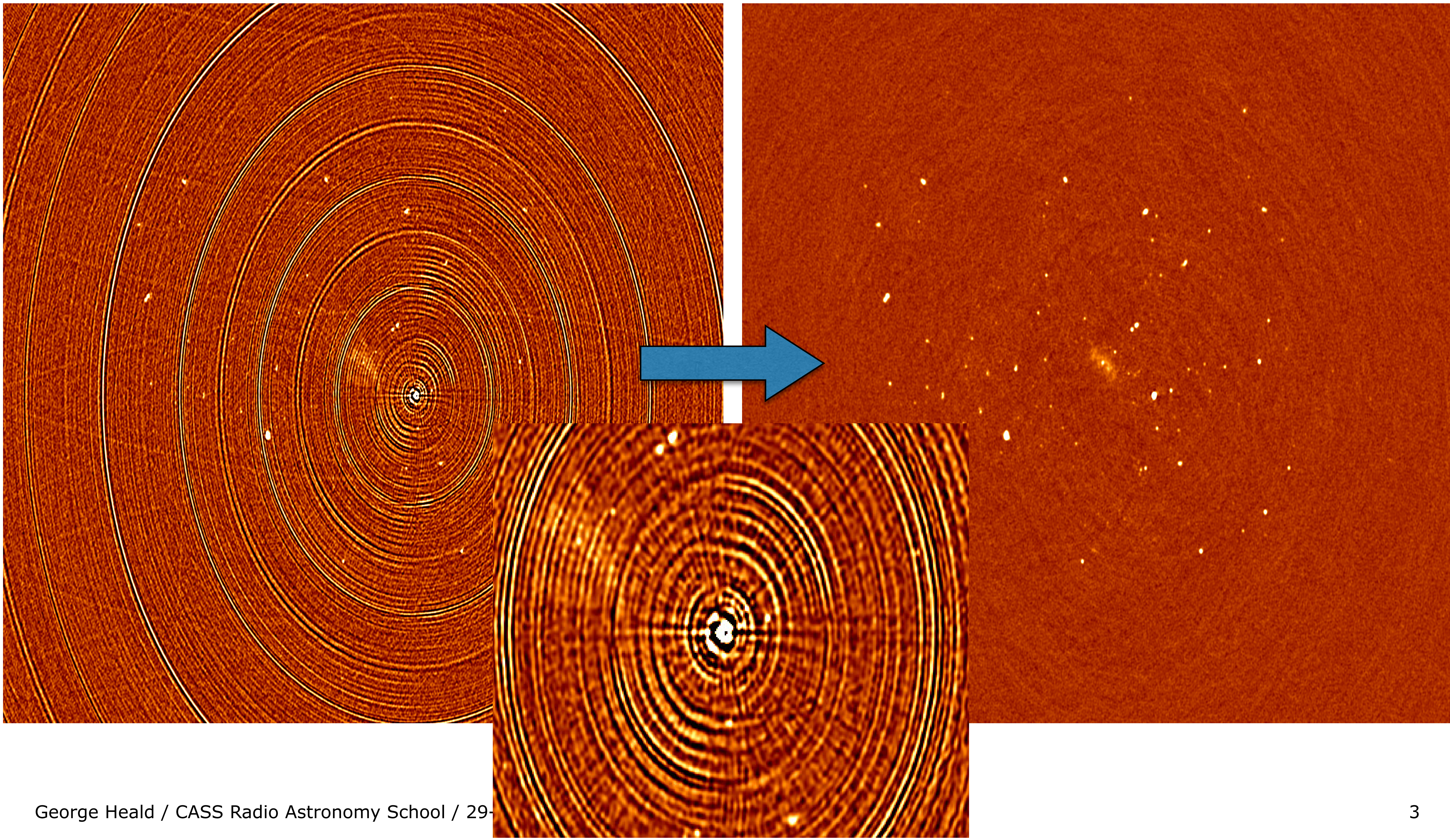
George Heald (ASTRON)
29 September 2015

Topics covered in this lecture

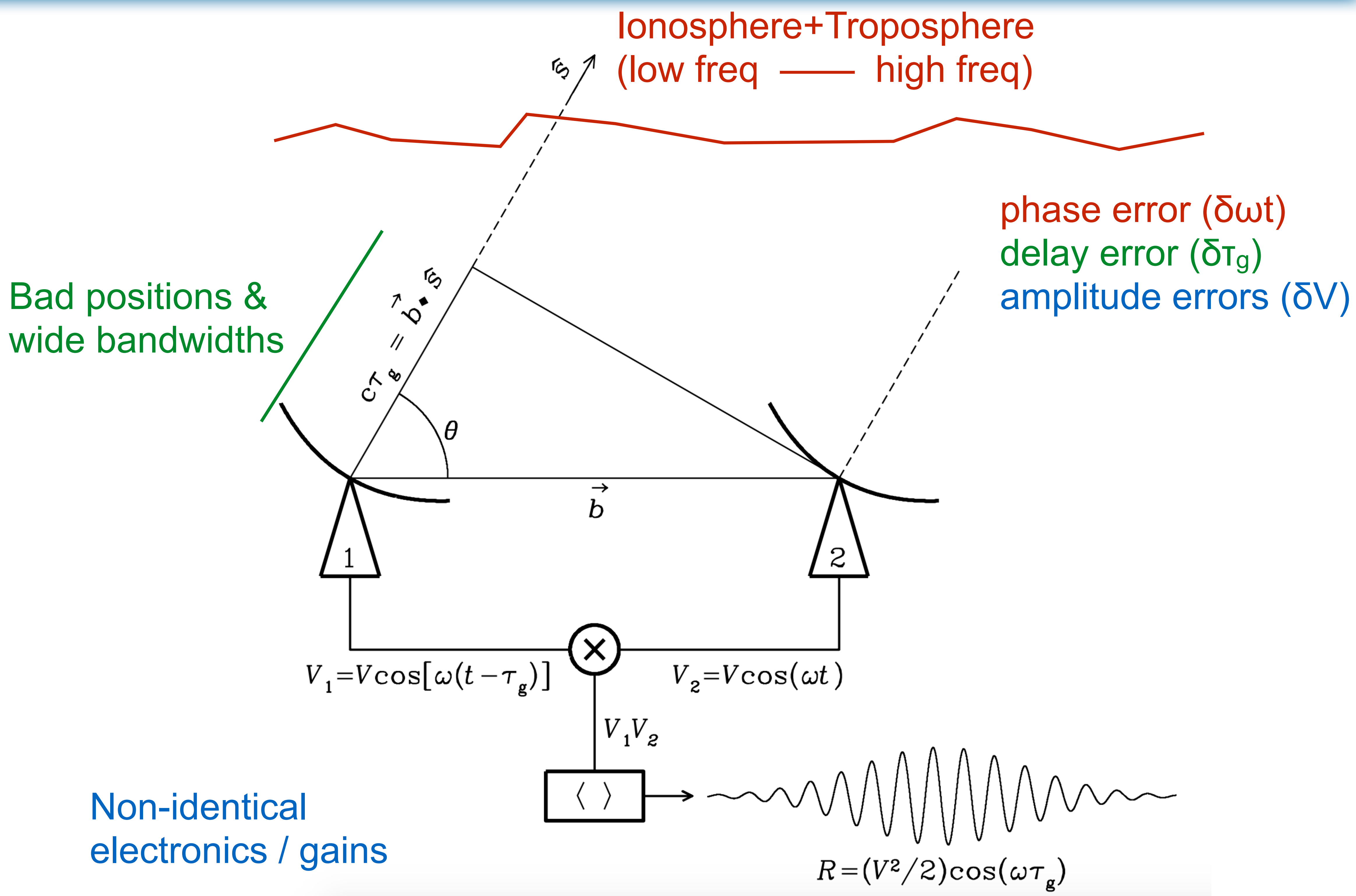
- Why calibration is needed
- The measurement equation
- Rules of thumb
- Basics:
  - Tsys, delays
  - primary and secondary calibration
  - gains, bandpass, leakage
  - closure relation
- Self-calibration

Why calibration is needed

- Not the case that the FT of observed visibilities will provide a nice image!
- Example image demonstrating effect of calibration:



Sketch of an ideal interferometer



Solve for these issues using calibration

Why calibration is needed

- Relevant physical effects:

Antenna/feed
- System temperature
- Primary beam
- Pointing
- Position (location)

Atmosphere
- Ionosphere
- Troposphere
- water vapor

Digitiser/Correlator
- Auto-leveling
- Baseline errors

LNA+conversion chain
- Clock
- Gain, phase, delay
- Frequency response

- Key factors: corresponding timescales, frequency dependence, polarimetric properties, **order in signal path**, …

The Measurement Equation

- The measurement equation (Hamaker, Bregman & Sault) is a matrix formalism for expressing the polarimetric response of an interferometer
- Introducing the coherency matrix **C**, which describes the intensity distribution:

$$\mathbf{C}_{pq} = \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} = \langle \vec{E}\vec{E}^{\dagger} \rangle$$

- and the "Jones matrix" J which contains all of the information about what happens to the signal, from the source through the correlator,

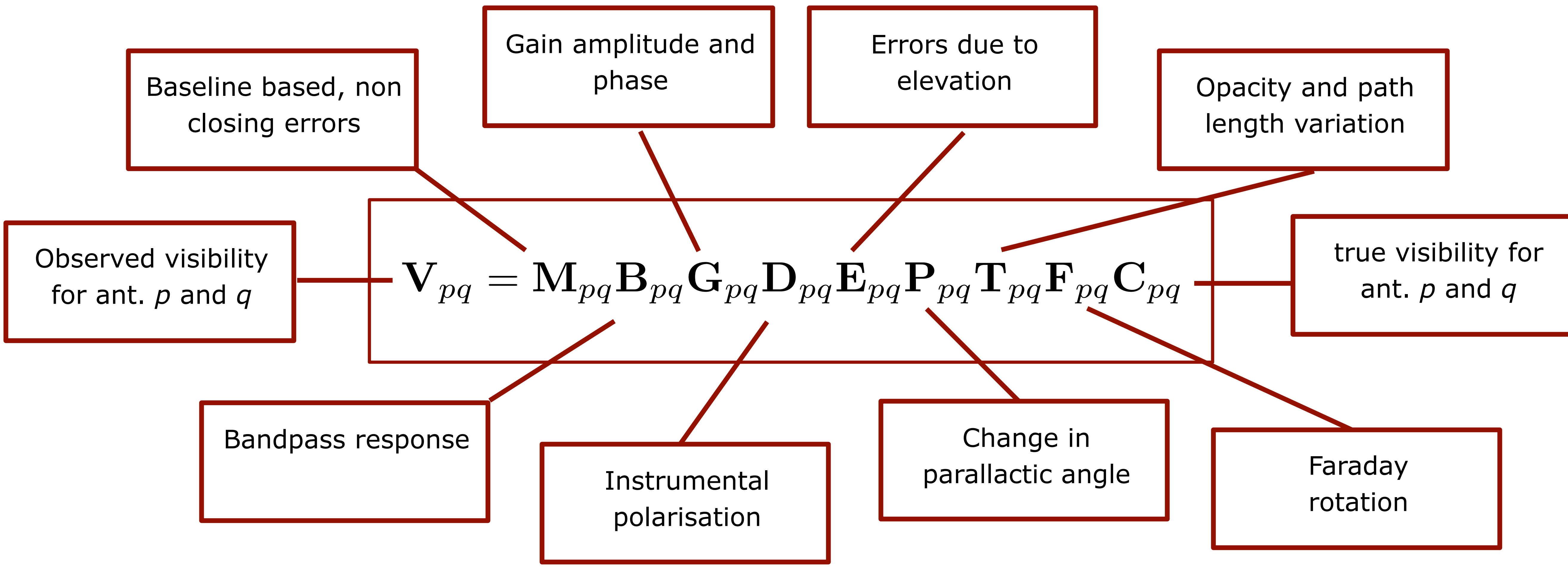
$$\mathcal{V}_p = \mathbf{J}_p \vec{E} \qquad\qquad \mathcal{V}_q = \mathbf{J}_q \vec{E}$$

- then with a bit of math we can write down the measurement equation:

$$\mathbf{V}_{pq} = \mathbf{J}_p \mathbf{C}_{pq} \mathbf{J}_q^{\dagger} = \mathbf{J}_{pq} \mathbf{C}_{pq}$$

Introduction to the RIME

- Within, for example CASA, the full radio interferometry measurement equation can be written as,



Calibration solves for each Jones matrix (when required) given a **model** for the sky.

Calibration

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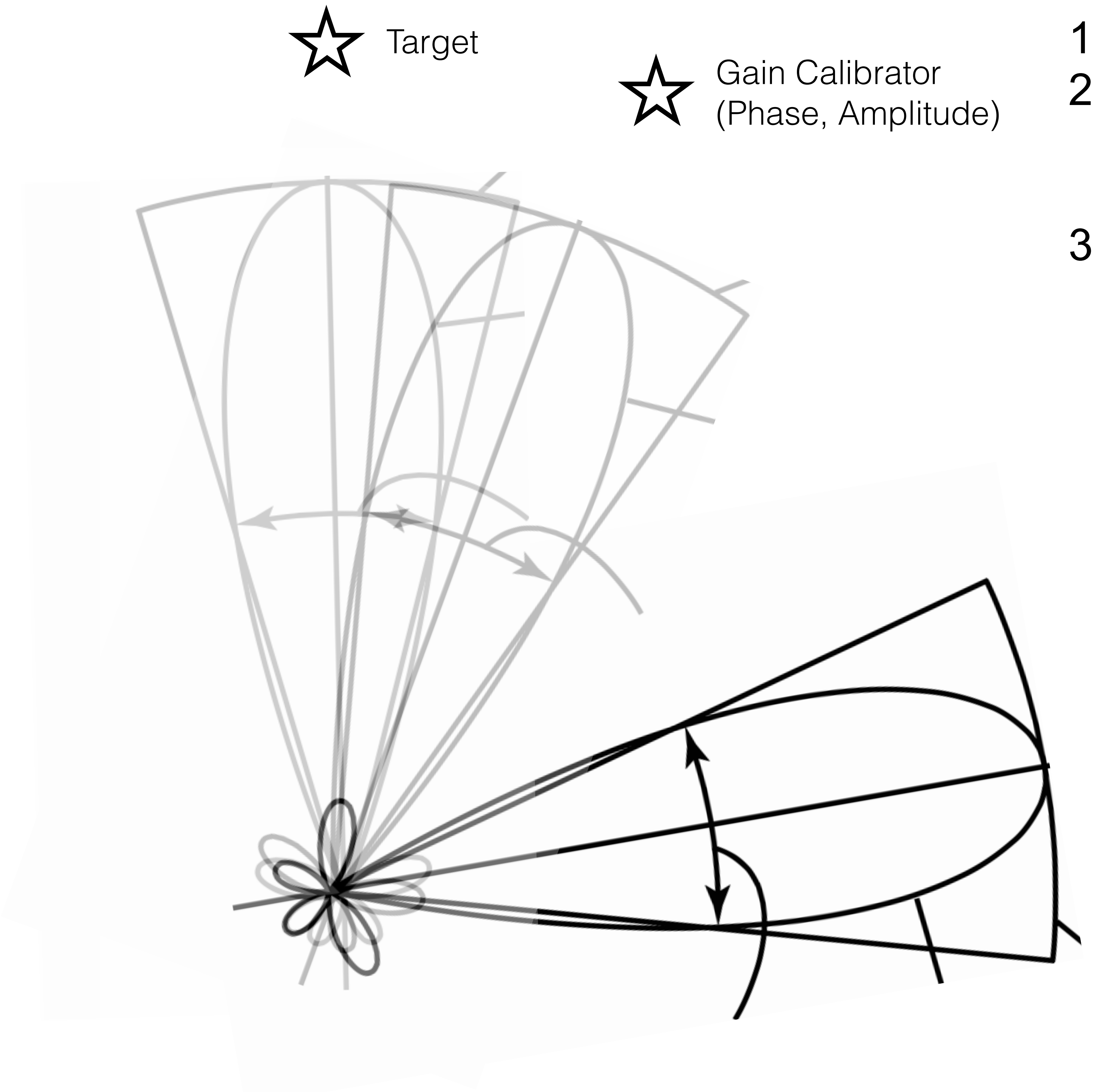
- Calibration is the process of perfecting the sky and instrument models

$$\mathbf{V}_{pq} = \underbrace{\mathbf{M}_{pq}\mathbf{B}_{pq}\mathbf{G}_{pq}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}}_{\text{instrument}}\underbrace{\mathbf{C}_{pq}}_{\text{sky}}$$

- Each of these terms is expressed mathematically in a parameterized fashion
- Parameters determined through fitting (typically least-squares) N(N-1)/2 equations; N unknowns … an overdetermined problem for N>3
- Note, fundamental assumption:
  Calibration parameters are antenna based

- Three levels:
  - Primary Calibration: use of a “known” standard source to determine time- and direction-independent quantities e.g. B_{pq}
  - Secondary Calibration: estimate local time-dependent conditions with nearby calibrator
  - Self-Calibration: use of the target field itself to determine highly time-dependent quantities, e.g. the phase of G_{pq}

Calibration strategy



1. Observe **source**
2. Observe **calibrator** to measure gains (amplitude and phase) as a function of time.
3. Observe **bright calibrator** of known flux-density and spectrum to measure absolute flux calibration, band-pass and residual delays

Calibrator models

$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\mathbf{B}_{pq}\mathbf{G}_{pq}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\boxed{\mathbf{C}_{pq}}$$

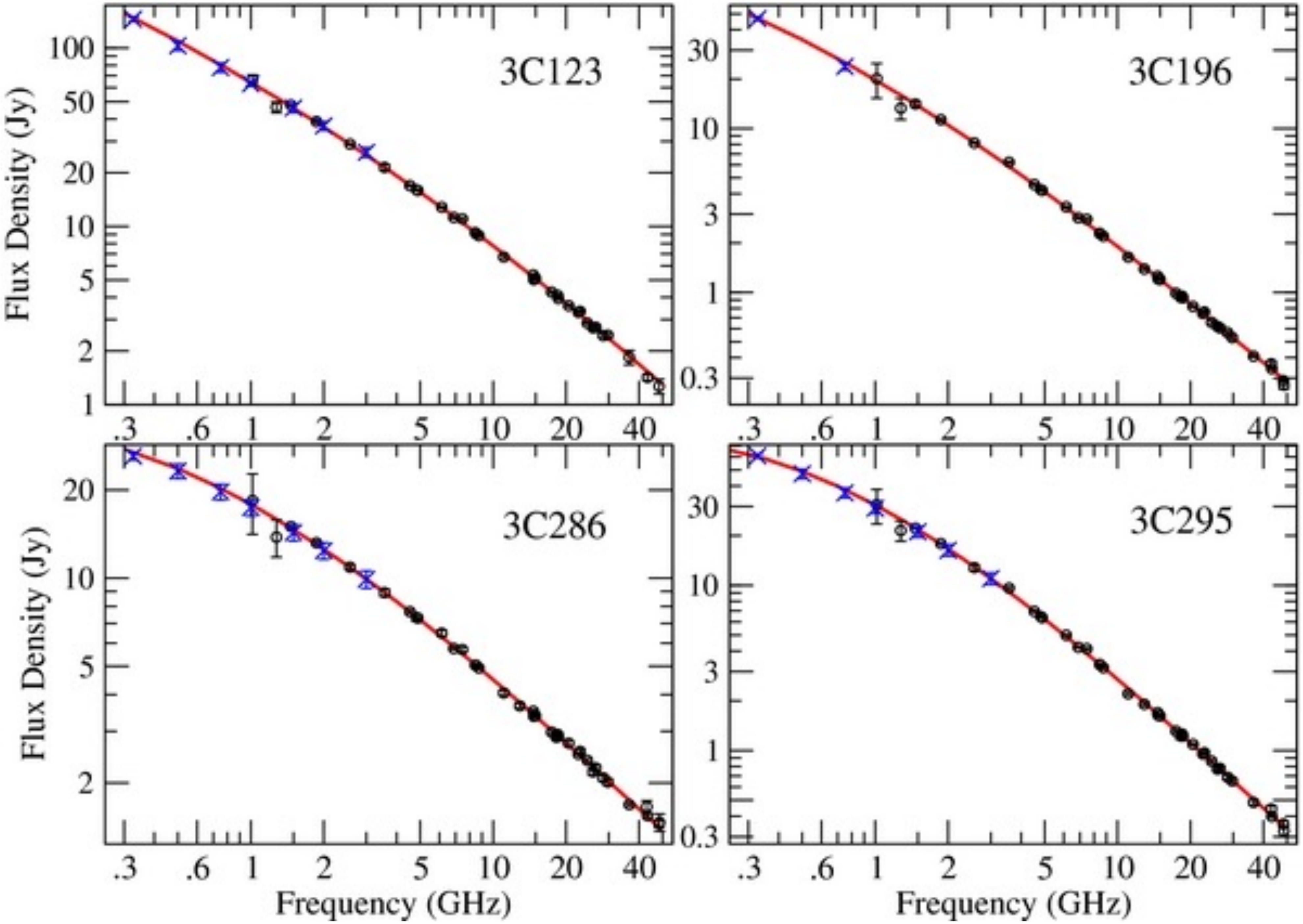
- Good calibration relies on good calibrator models
  - Incorrect or incomplete source models will manifest as calibration errors

$$\mathbf{C}_{pq} = \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix}$$

- Primary/secondary calibrators should have
  - Excellent positions (for astrometry)
  - Proper source size ("just compact enough") - standard calibrator lists
    - Compact enough to be unresolved on the longest baselines
    - Not so compact that the source is variable
  - Well-understood flux density (for flux scale) and spectral shape (for bandpass)
  - For polarization calibration: well understood polarimetric properties (including Faraday rotation measure, where appropriate)

Calibrator models

- Primary calibration derived from dedicated short observations of standard sources, e.g.:



Perley & Butler (2013)

Calibrator models

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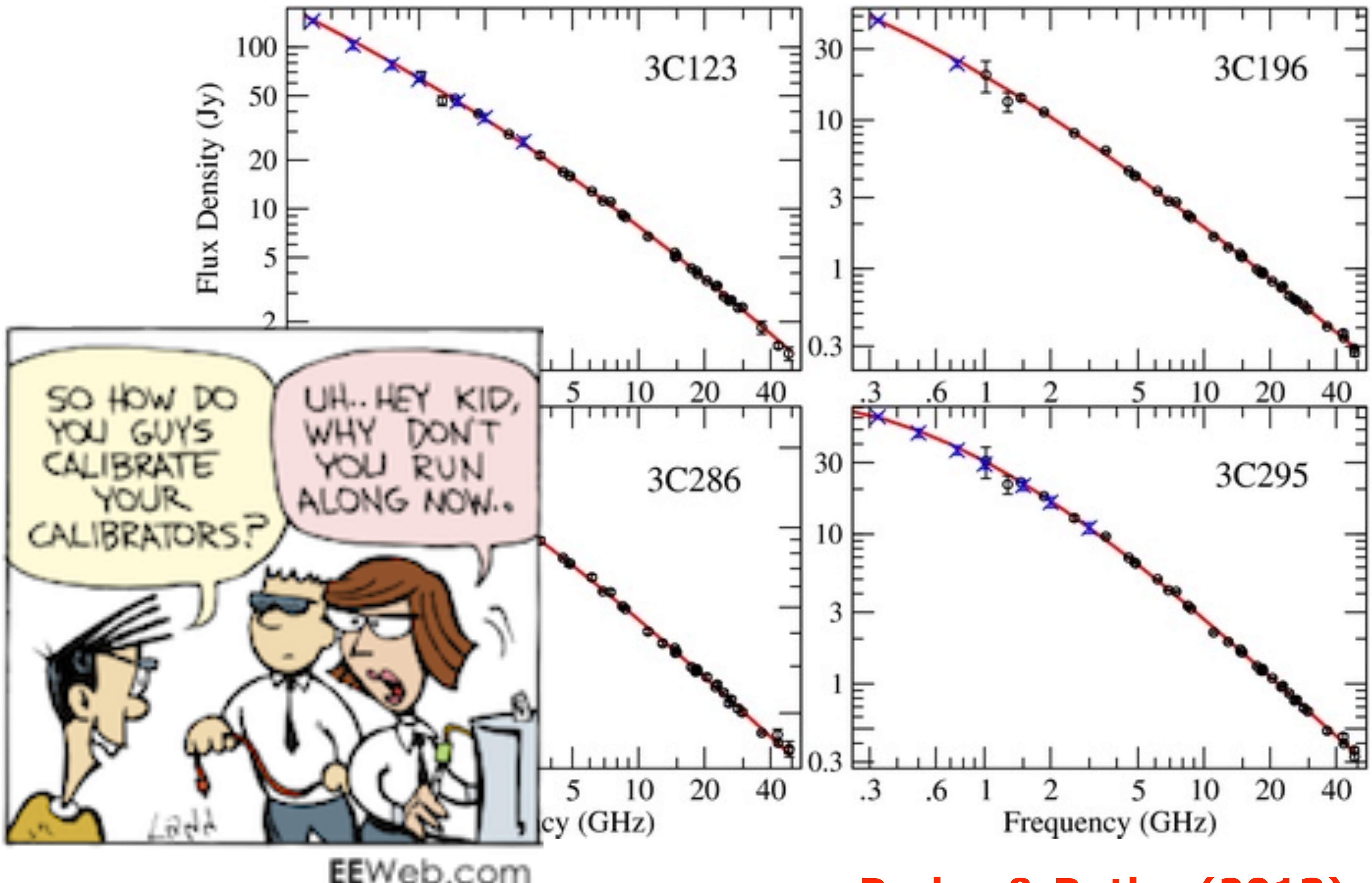

Perley & Butler (2013)

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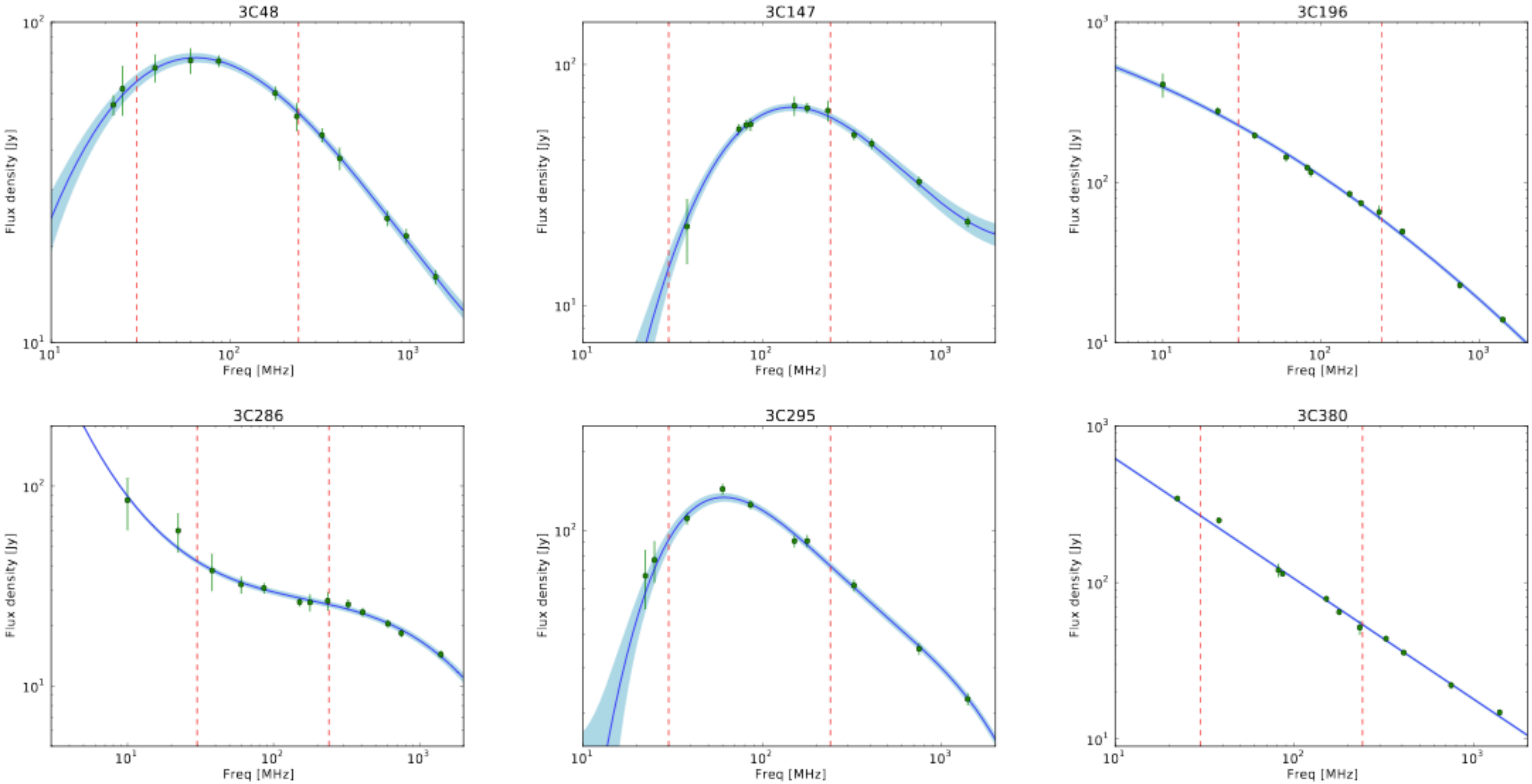


Scaife & Heald (2012)

Rules of thumb before calibrating

- Don’t be afraid to flag bad data!
  - Corrupted data can reduce the image quality significantly
  - Effect of missing data (even 25%) is often minor and easily corrected in deconvolution
  - Pay attention to the observing logs!

- Consider physical effects when choosing calibration parameters
  - What are the timescales and frequency dependence?

- Visualize your data!

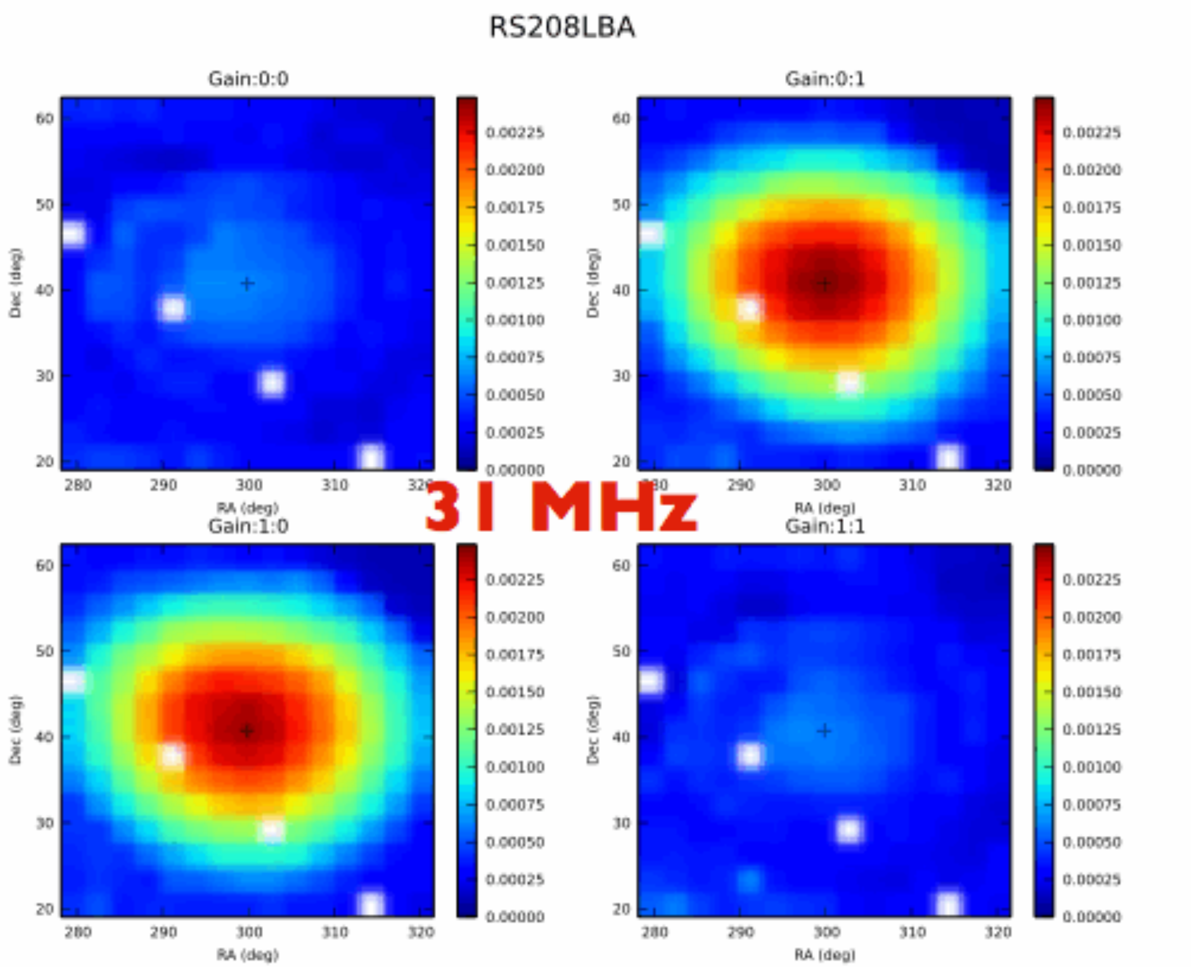
Faraday rotation

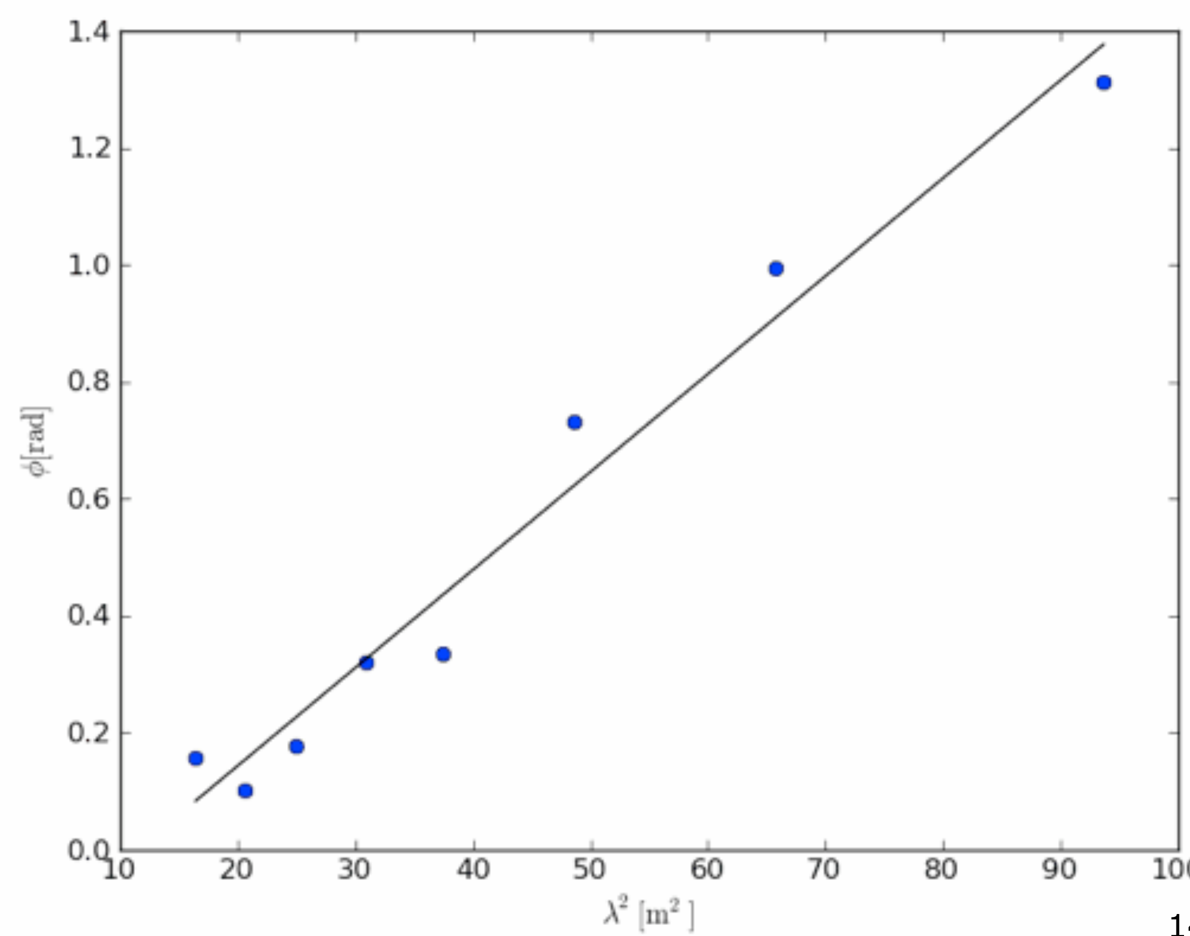
$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\mathbf{B}_{pq}\mathbf{G}_{pq}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

- Jones matrix representation of Faraday rotation:

$$F(\nu) = \begin{pmatrix} \cos(\phi\frac{c^2}{\nu^2}) & -\sin(\phi\frac{c^2}{\nu^2}) \\ \sin(\phi\frac{c^2}{\nu^2}) & \cos(\phi\frac{c^2}{\nu^2}) \end{pmatrix}$$

- Rotates the linear polarization coordinate frame as a function of frequency





Polarization leakage

$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\mathbf{B}_{pq}\mathbf{G}_{pq}\boxed{\mathbf{D}_{pq}}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

- "Leakage" of signal from X into Y (or R into L) and vice versa. Matrix form:

$$\mathbf{D} = \begin{pmatrix} 1 & D_y \\ D_x & 1 \end{pmatrix}$$

Recall that

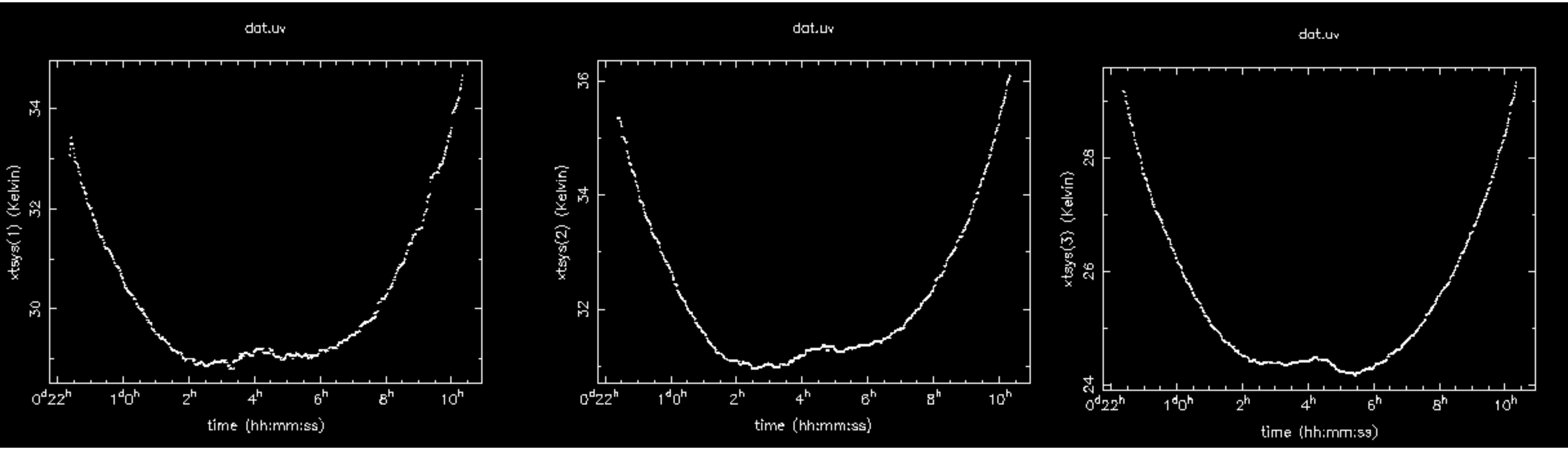
$$\mathcal{V}_p = \mathbf{J}_p\vec{E}$$

so this "D-term" encapsulates the fact that some electric field from one polarization state ended up in the measured voltage corresponding to the other polarization state. This can happen if linear dipoles are not perfectly orthogonal, or if there is crosstalk.

Gains and bandpass

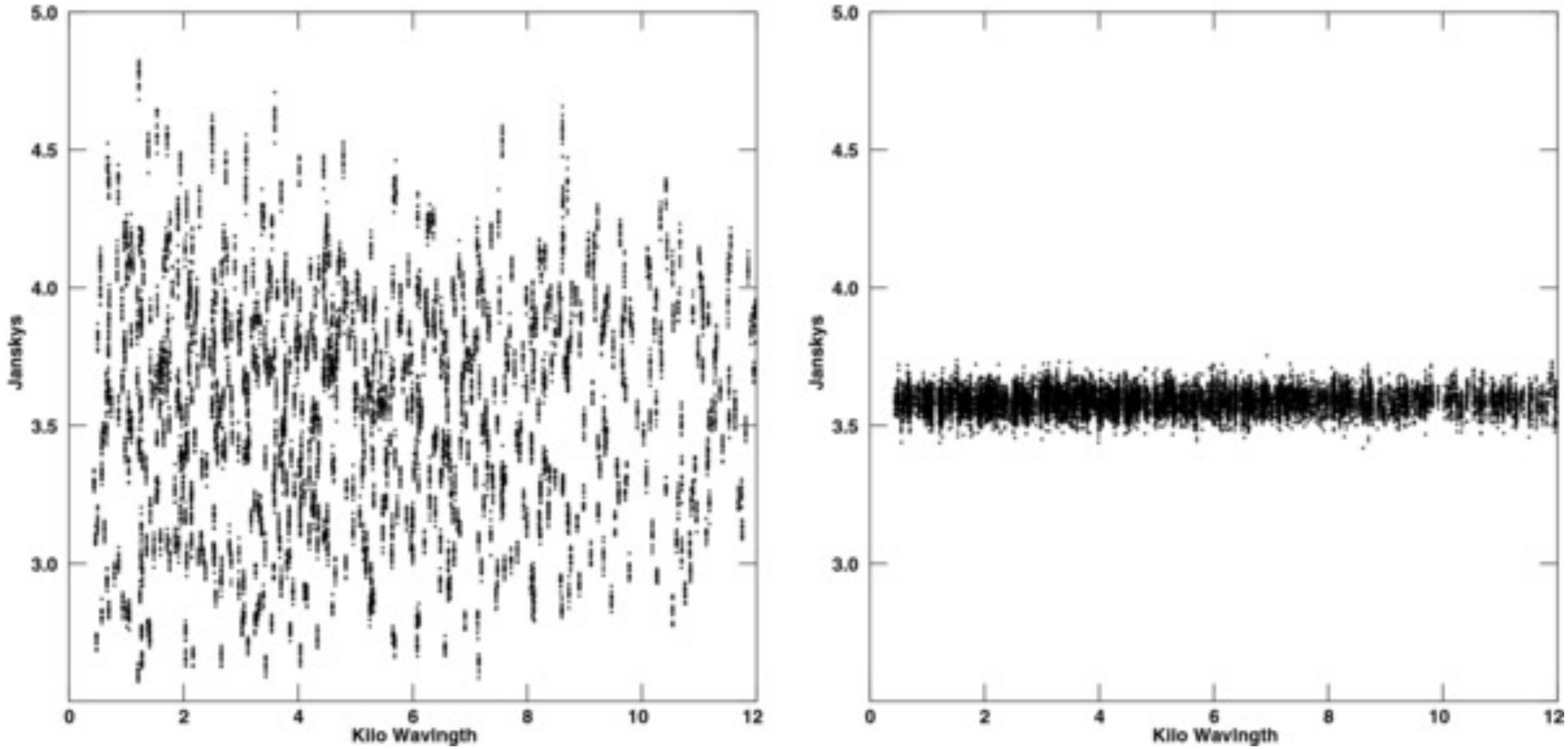
$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\mathbf{B}_{pq}\mathbf{G}_{pq}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

- Rule of thumb: don't solve for things that can be determined in other ways
  - Example: system temperature (Tsys), a time-dependent measure of the sensitivity of each antenna
    - Often measured directly (using on-dish calibrator source) and included with visibility data
    - When available, apply it first!
    - Example: 3 WSRT antennas (measured T_{sys} values for the X feed):



Online gain correction (JVLA)

- Variable system gain corrected online (via a calibrated signal injected in the receivers and detected in the correlator)



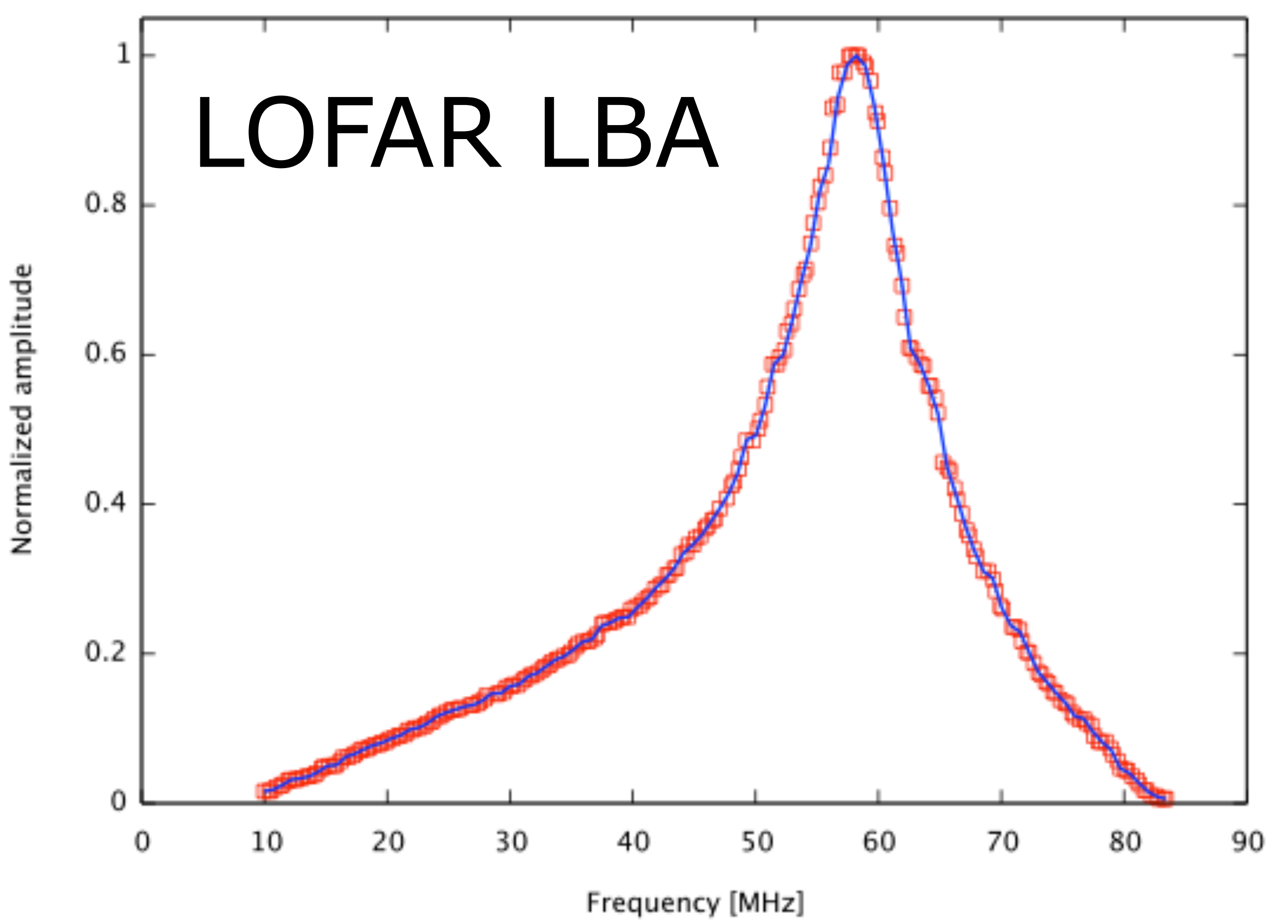
Perley & Butler (2013)

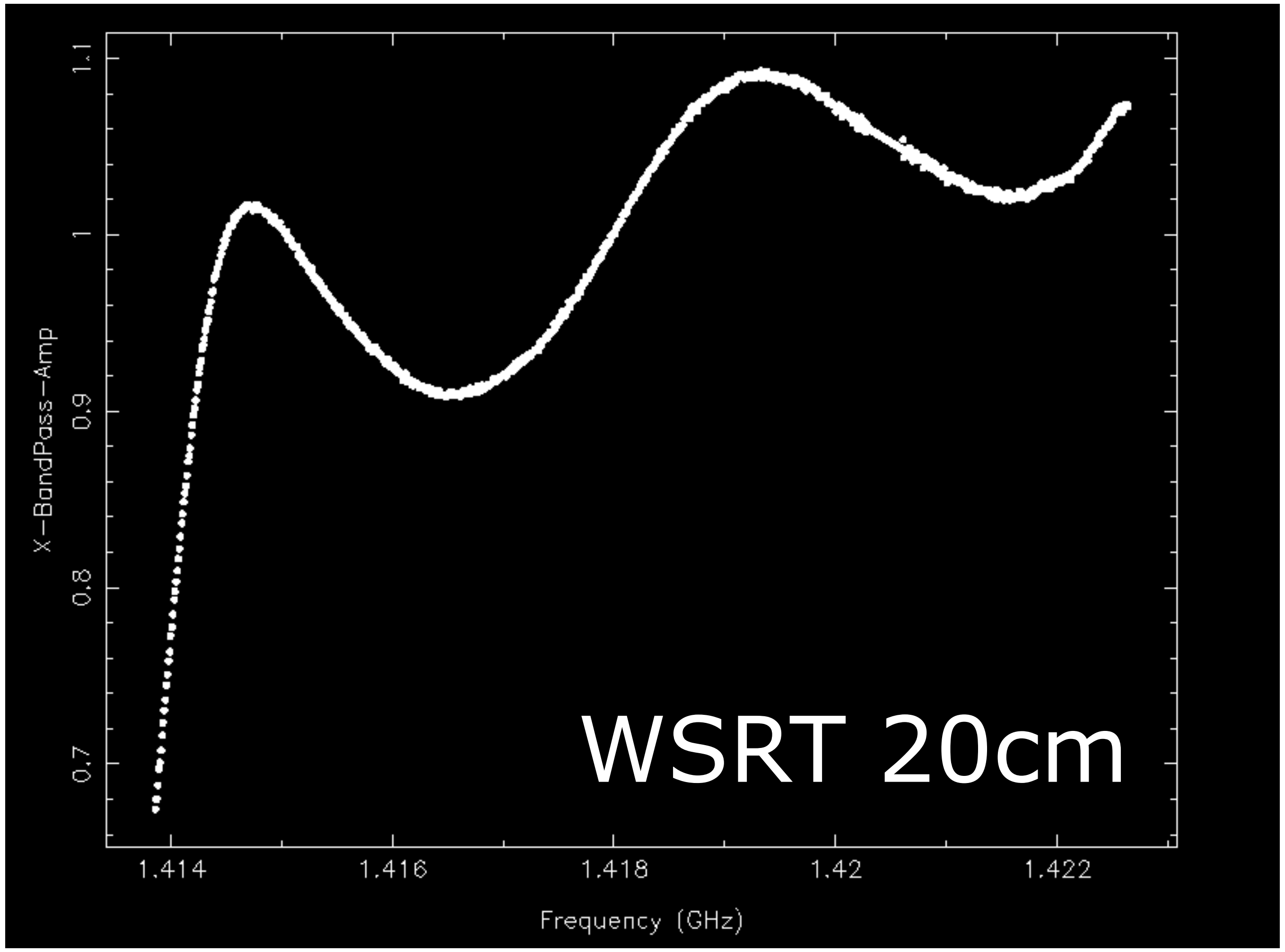
Bandpass calibration

$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\mathbf{B}_{pq}\mathbf{G}_{pq}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

- The bandpass captures the frequency dependent sensitivity across the observed frequency range (which has features due to filters, inherent sensitivity variations, and possibly signal processing artifacts)

$$\mathbf{B} = \begin{pmatrix} B_x(\nu) & 0 \\ 0 & B_y(\nu) \end{pmatrix}$$





$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\boxed{\mathbf{B}_{pq}\mathbf{G}_{pq}}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

- Everything else antenna-based that lead to amplitude and phase changes (often time-dependent)

$$\mathbf{G} = \begin{pmatrix} G_x(t) & 0 \\ 0 & G_y(t) \end{pmatrix}$$

where

$$G_x(t) = \underbrace{\left|G_x(t)\right|}_{\text{Amplitude}}\ \overbrace{e^{i\theta_x}}^{\text{Phase}}$$

Amplitude **Phase**

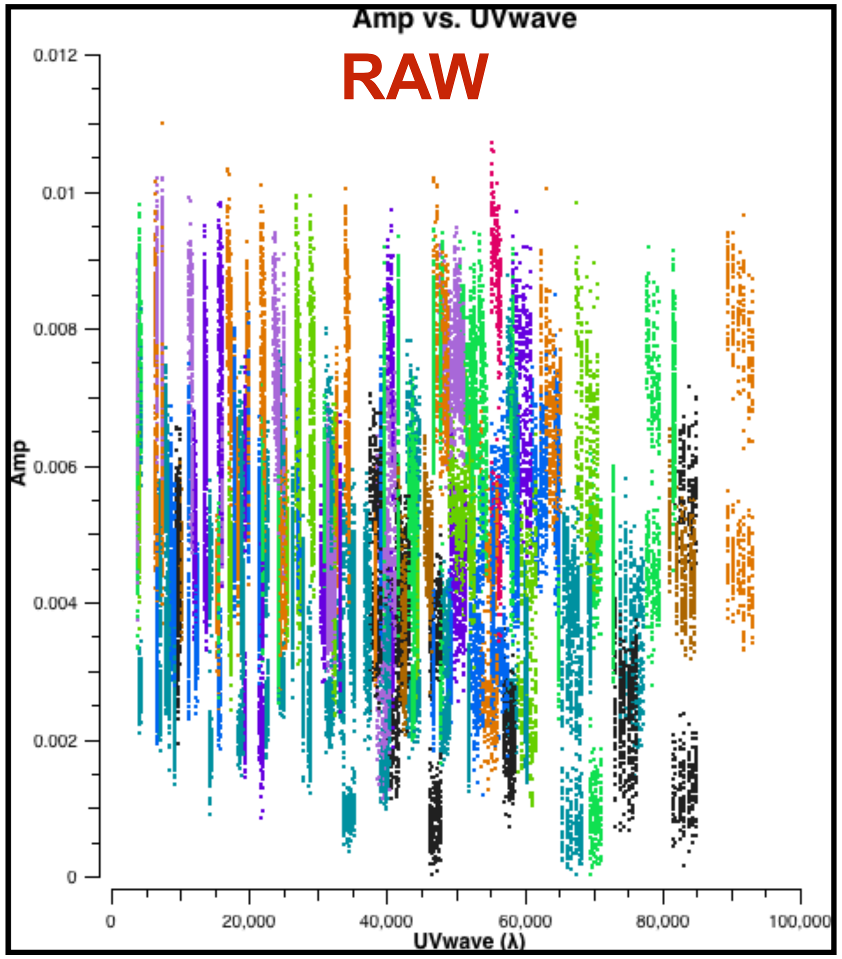
$$\mathbf{G}_{pq}\mathbf{C}_{pq} = \mathbf{G}_p\mathbf{C}_{pq}\mathbf{G}_q^\dagger$$

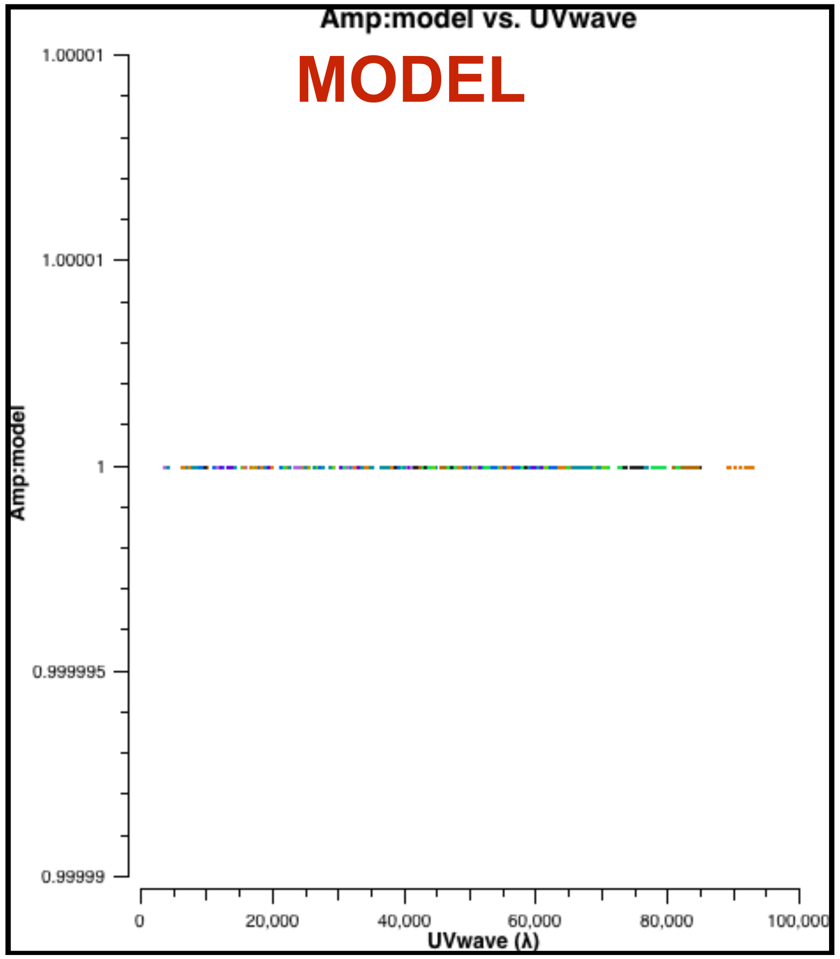
Example of gain amplitude calibration

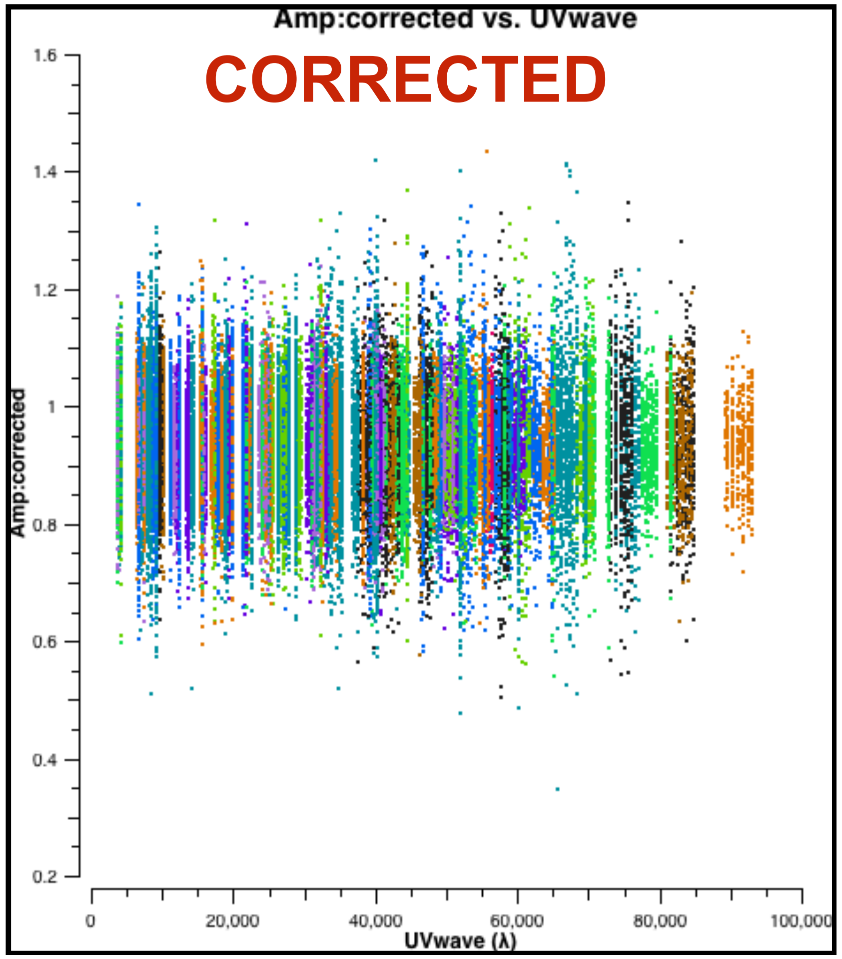
$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\boxed{\mathbf{B}_{pq}\mathbf{G}_{pq}}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

Here is an observed visibility function (amplitude), the ideal visibility function and the calibrated data (after solving the G_{pq} in the the measurement equation).

Main source of amplitude error: Variable gain in the amplifiers of the system.







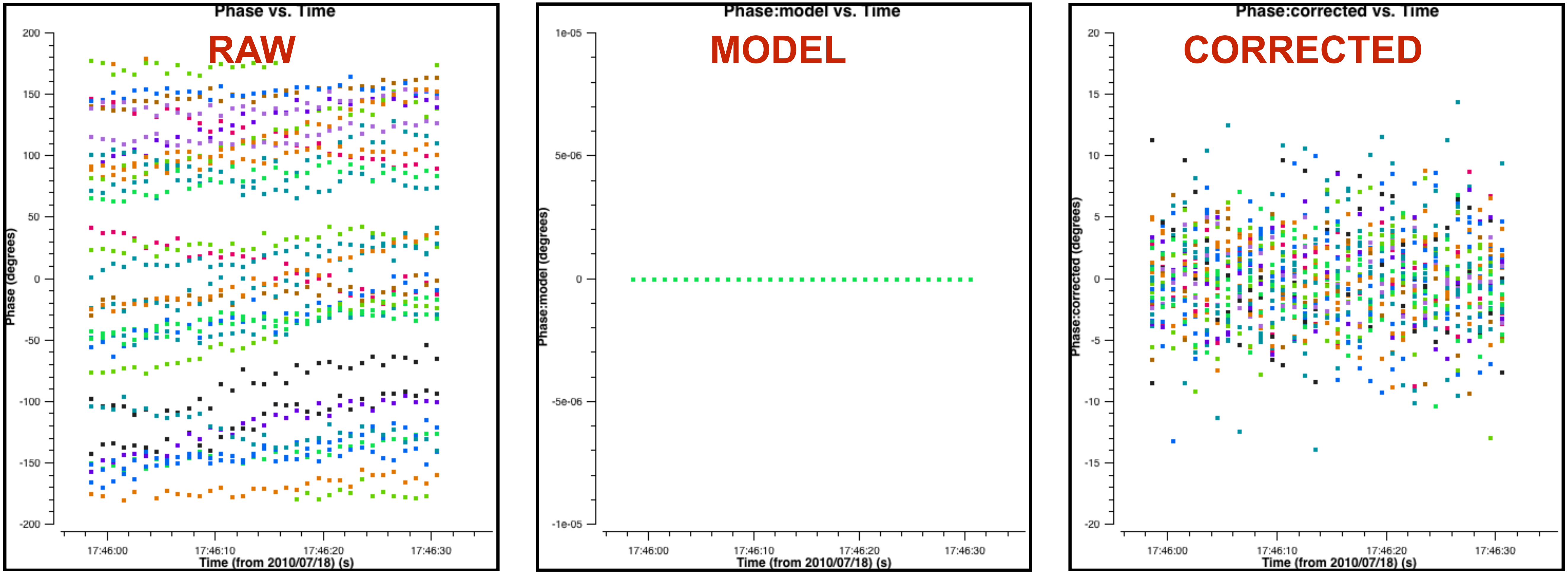
Each colour represents visibilities with a common antenna.

Example of gain phase calibration

$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\mathbf{B}_{pq}\mathbf{G}_{pq}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

Here is an observed visibility function (phase), the ideal visibility function and the calibrated data (after solving the G_{pq} in the the measurement equation).

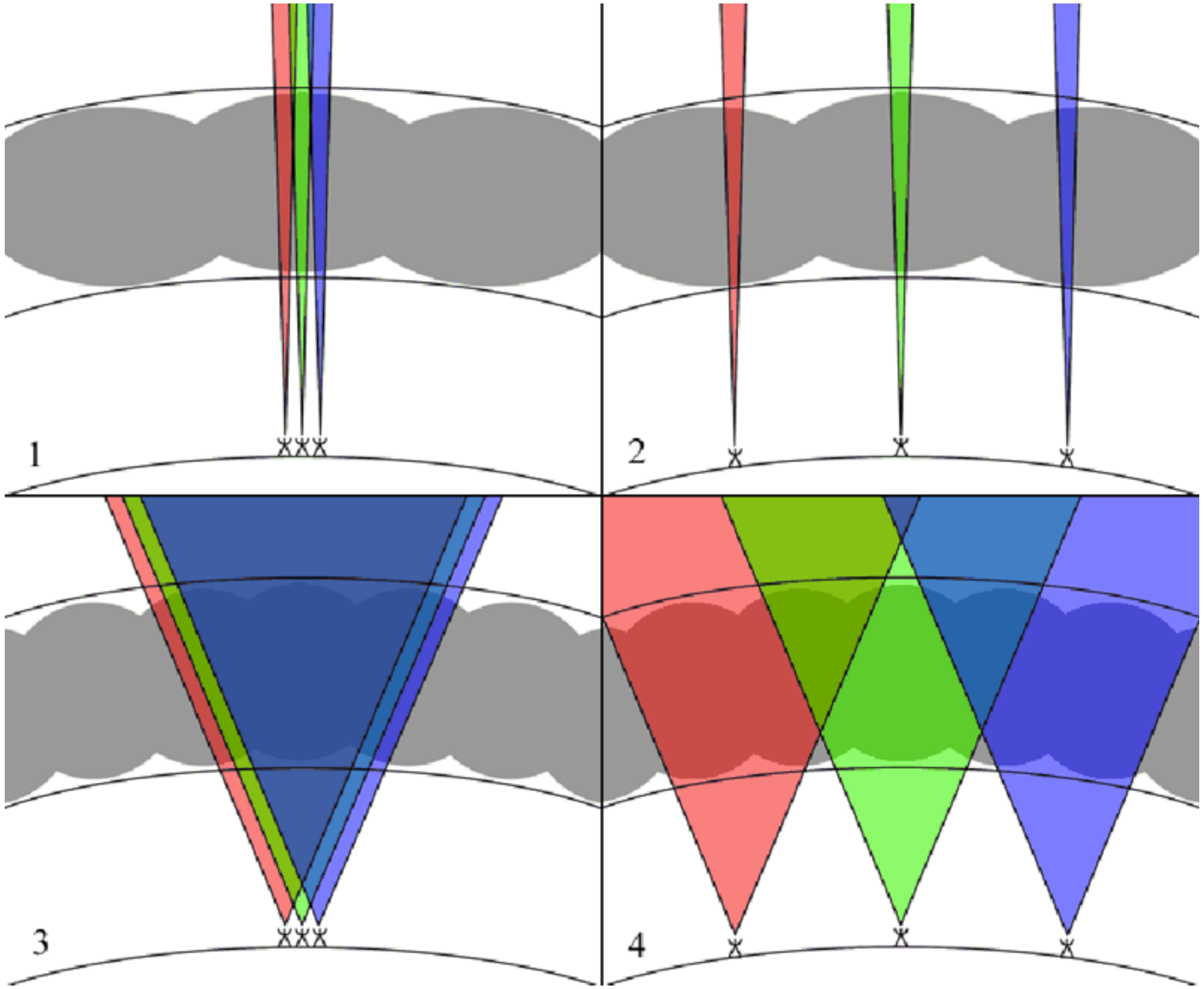
Main source of phase error: Variable ionosphere or troposphere + electronics.



More complex delay corrections require ‘fringe fitting’ : see VLBI lecture.

Gain phases

- Phase fluctuations caused by structure in atmosphere, troposphere, ionosphere

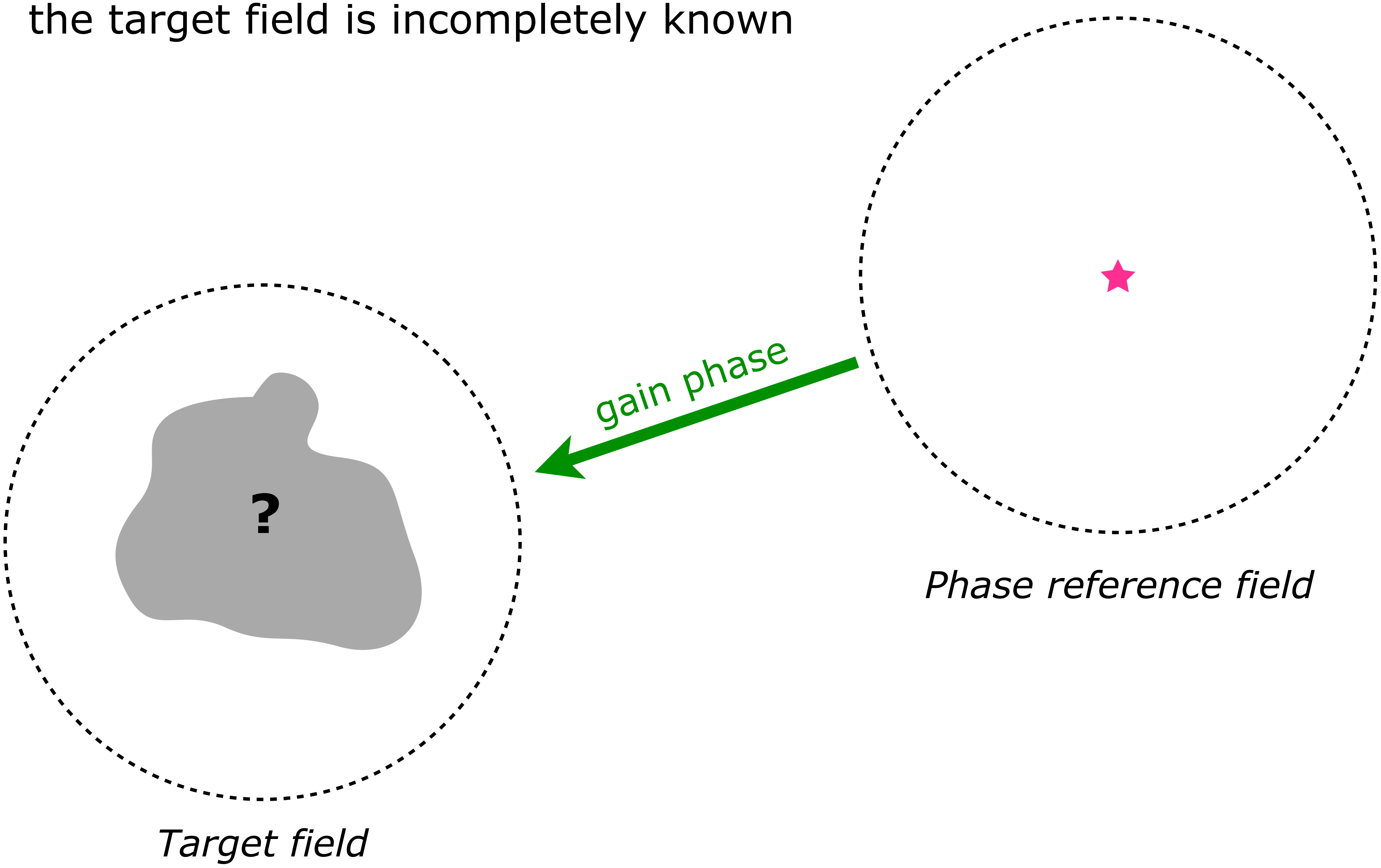


Intema et al. (2009)

- Local gain solutions can be determined from a nearby phase calibrator, or the target field itself if good model available (or can be constructed)

Secondary calibration

- Phase calibrator used in cases where either
  - number of antennas is small (so that the number of constraints is insufficient to give a robust solution), or
  - the target field is incompletely known



Self-calibration philosophy

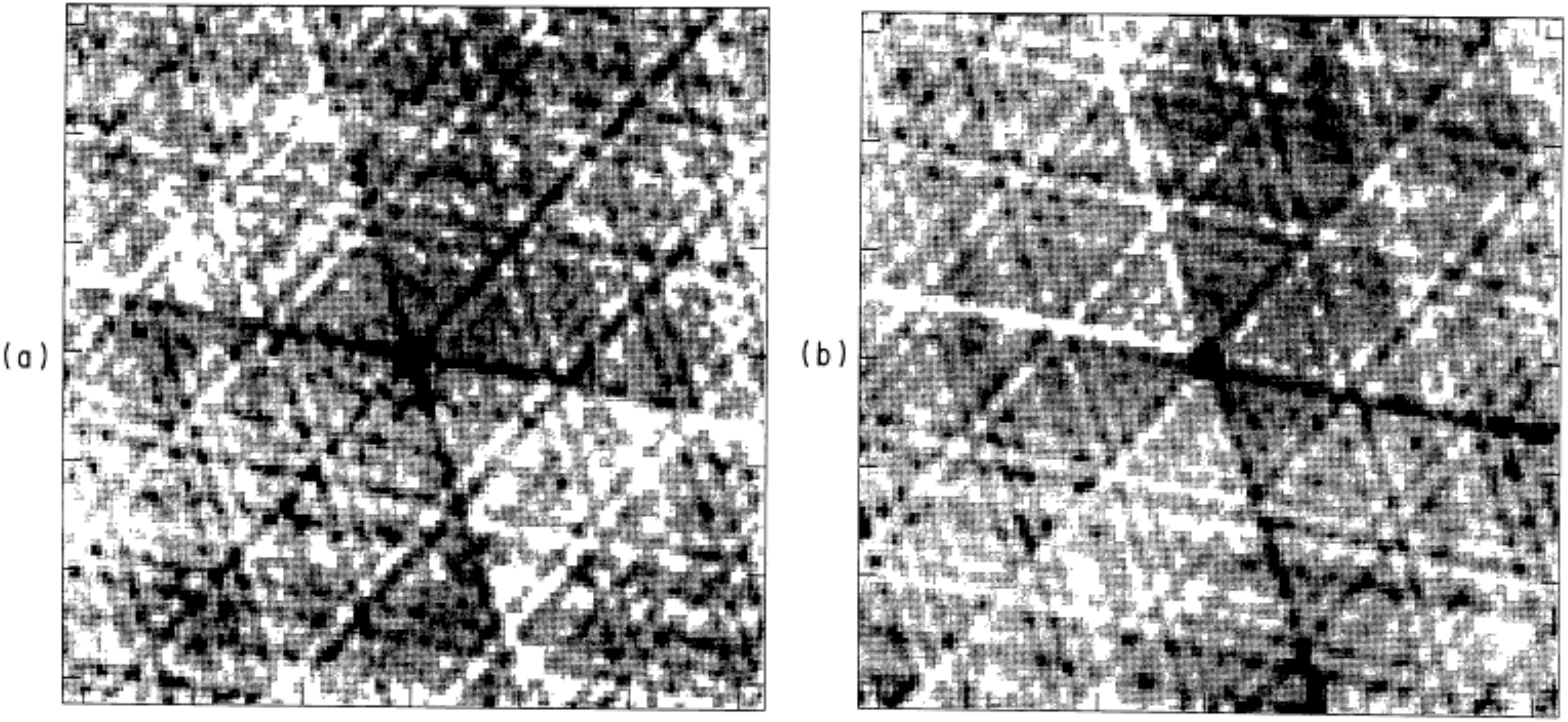
After **transferring the solutions** from a calibrator we may find that there are **residual errors** in our data.

Why?

Our calibrators are observed at a **different time** (except for simultaneous observations; in beam-calibration) and **position** on the sky than our target.

Common error patterns

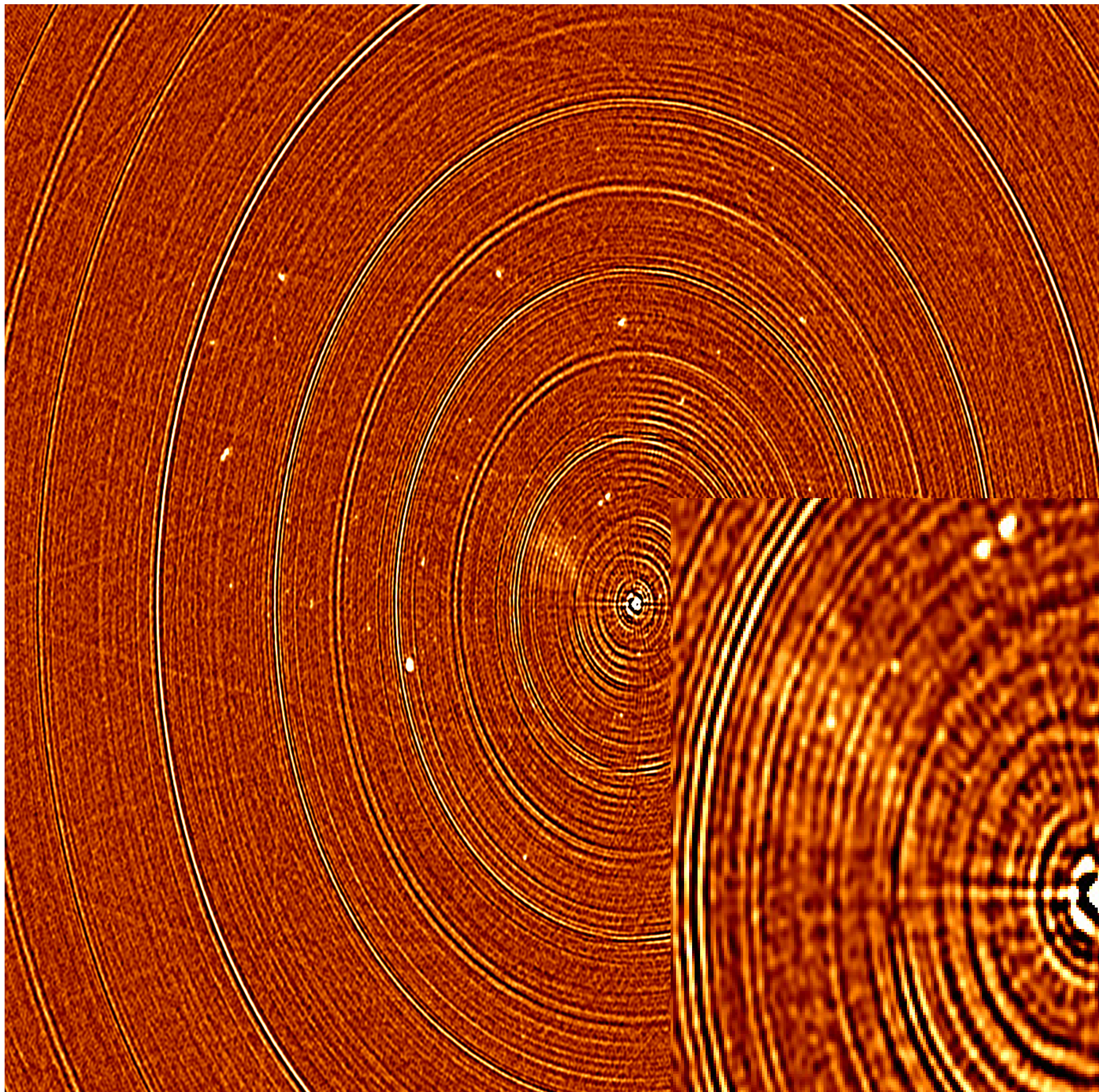
- Amplitude error gives symmetric pattern; phase error gives asymmetric pattern

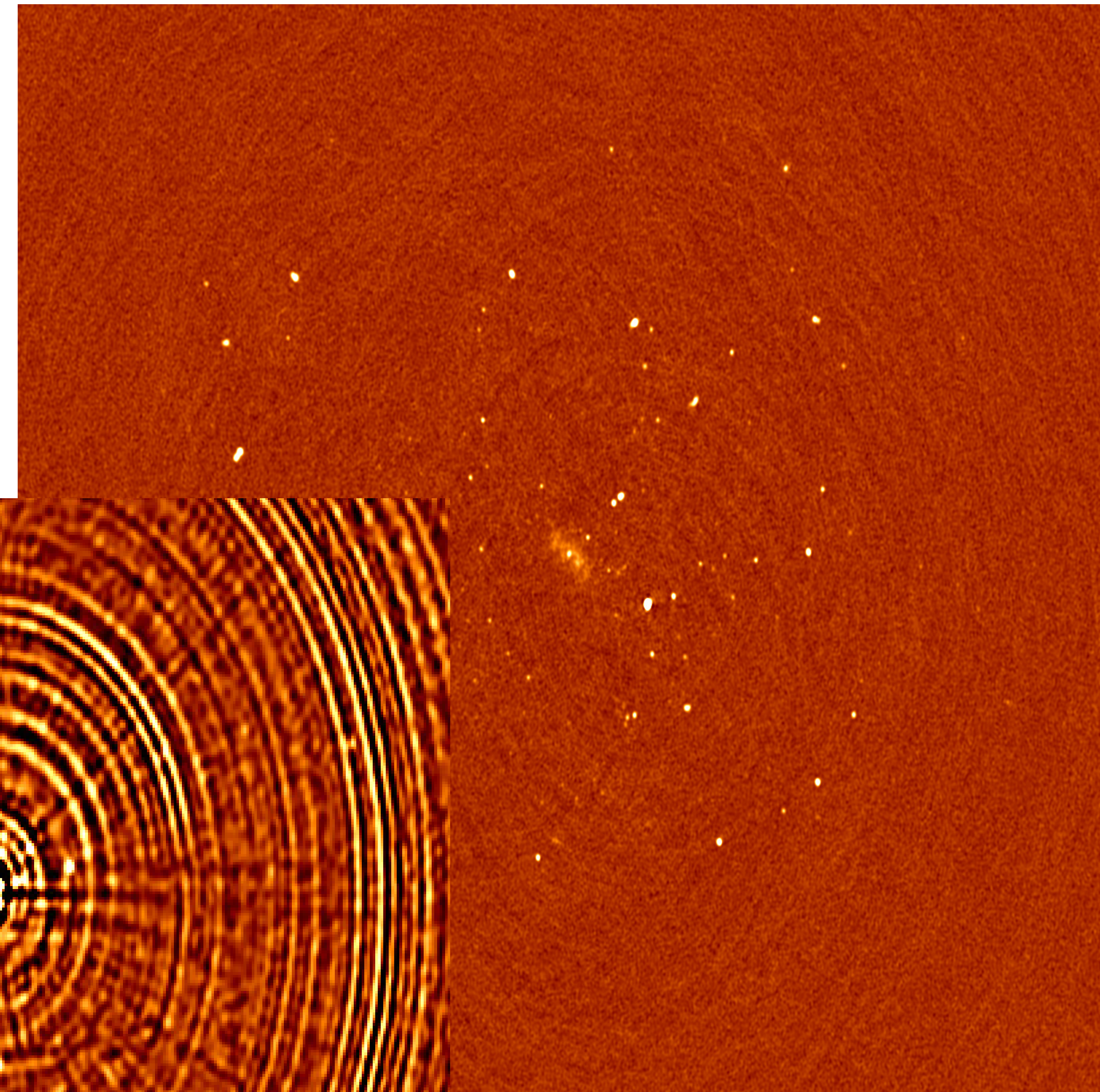


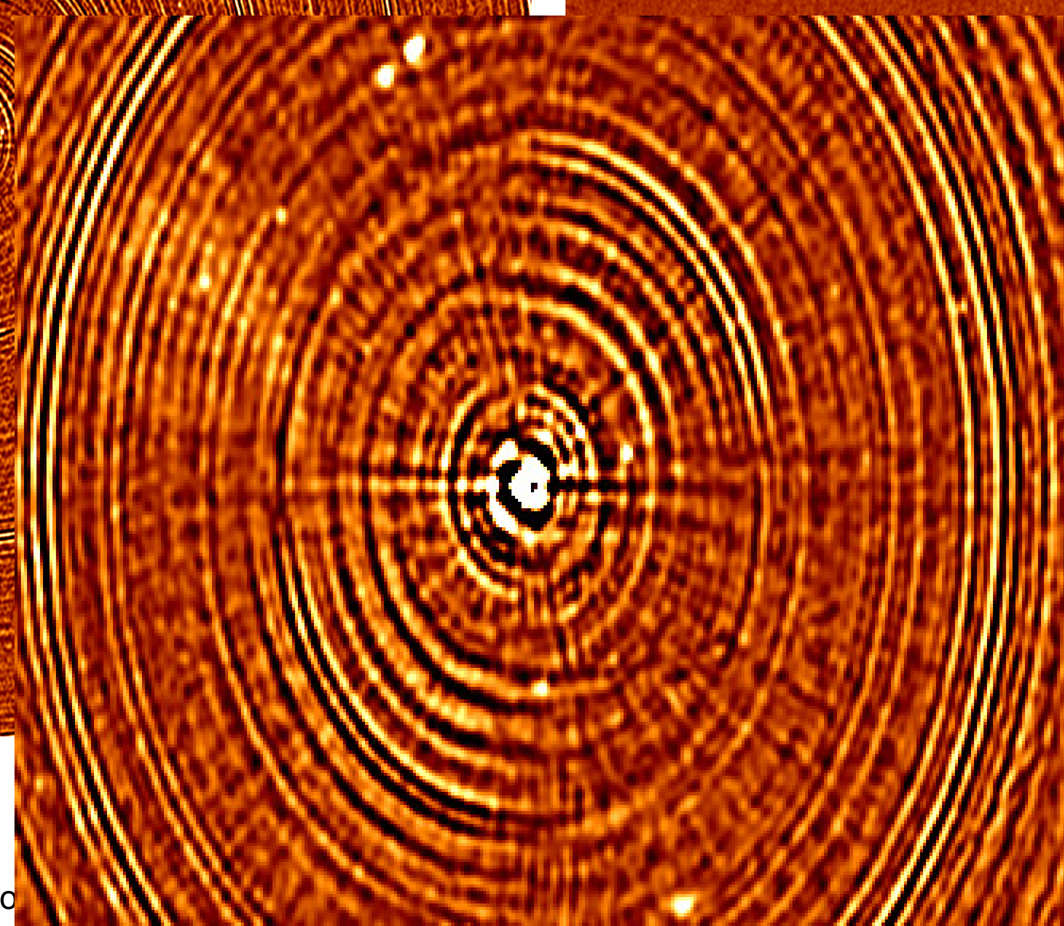
A 10° phase error is as bad as a 20% amplitude error.

Why calibration is needed

- Not the case that the FT of observed visibilities will provide a nice image!
- Example image demonstrating effect of calibration:







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Why?

Our calibrators are observed at a **different time** (except for simultaneous observations; in beam-calibration) and **position** on the sky than our target.

Use the process of self-calibration:

1) Make an image of your target (after applying calibrator solutions).
2) Use this model to calibrate the data over some solution interval.
3) Make an image of your target (after applying self-calibration solutions).
4) Use this model to calibrate the data over some solution interval.
5) Iterate this process until no major improvement on image quality.

Advantages:

1) Can correct for residual amplitude and phase errors.
2) Can correct for direction dependent effects (see later).

Disadvantages:

1) Errors in the model or low SNR can propagate into your self-calibration solutions, and you can diverge from the correct model.

Self calibration

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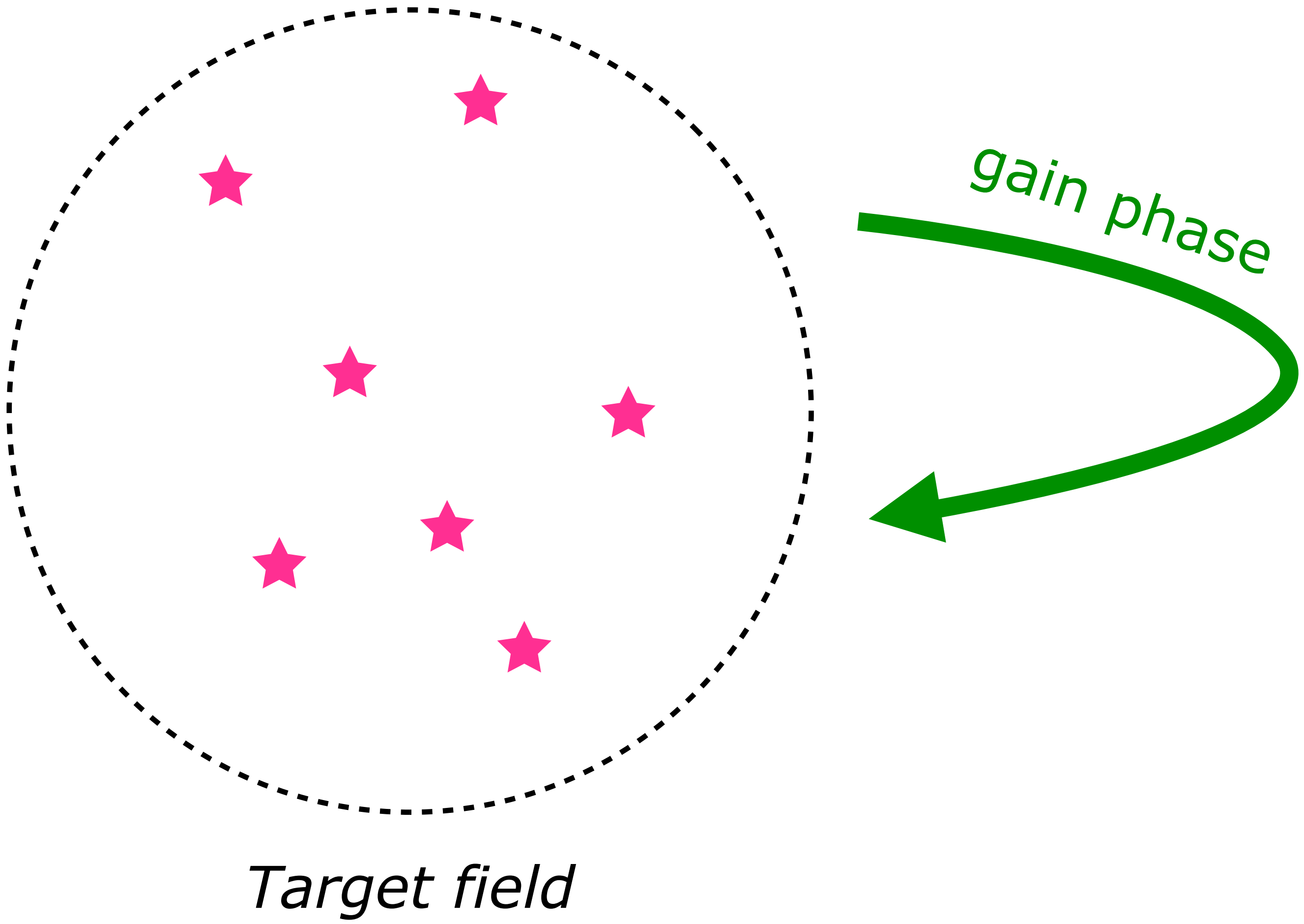
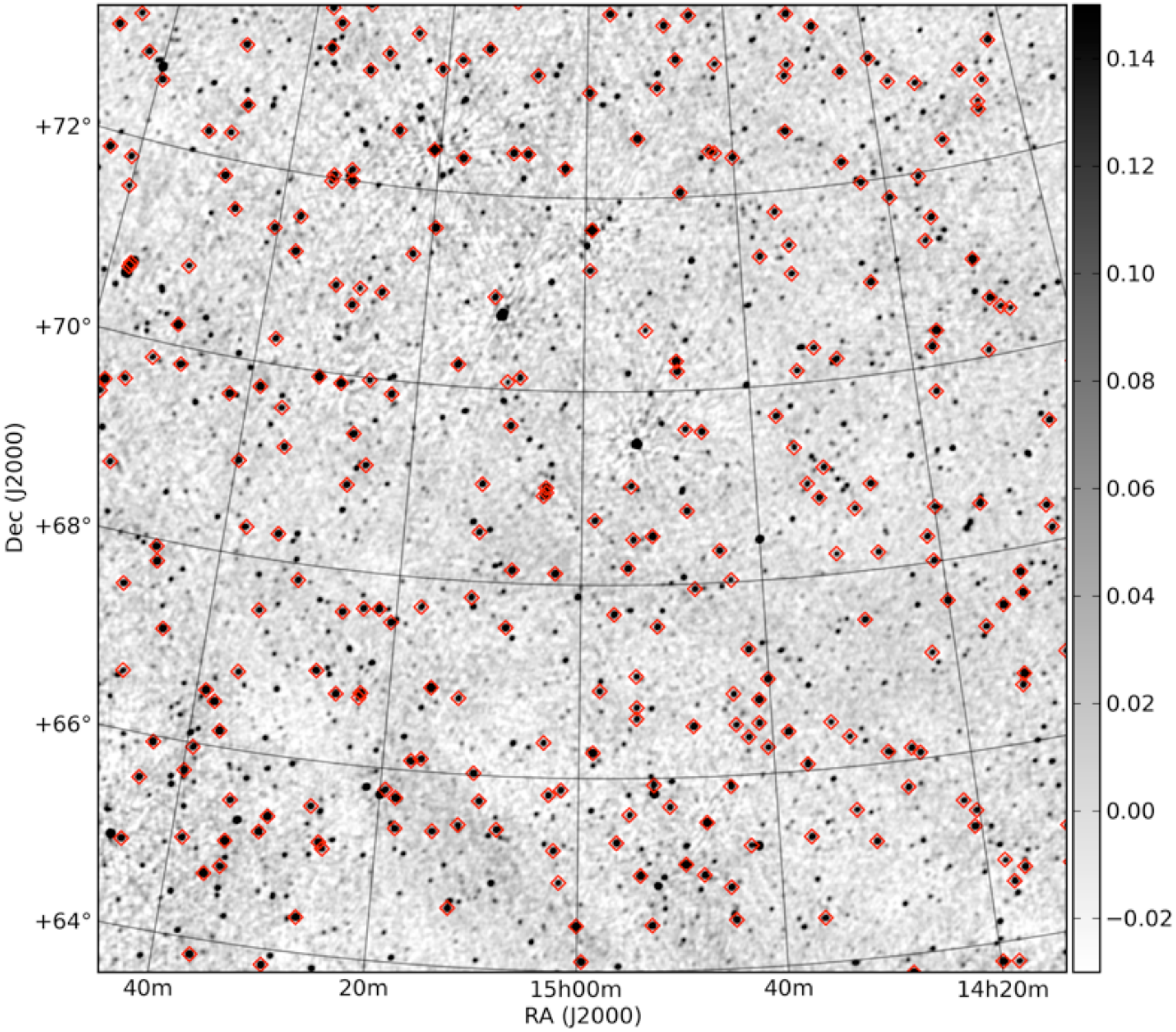
$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\mathbf{B}_{pq}\boxed{\mathbf{G}_{pq}}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

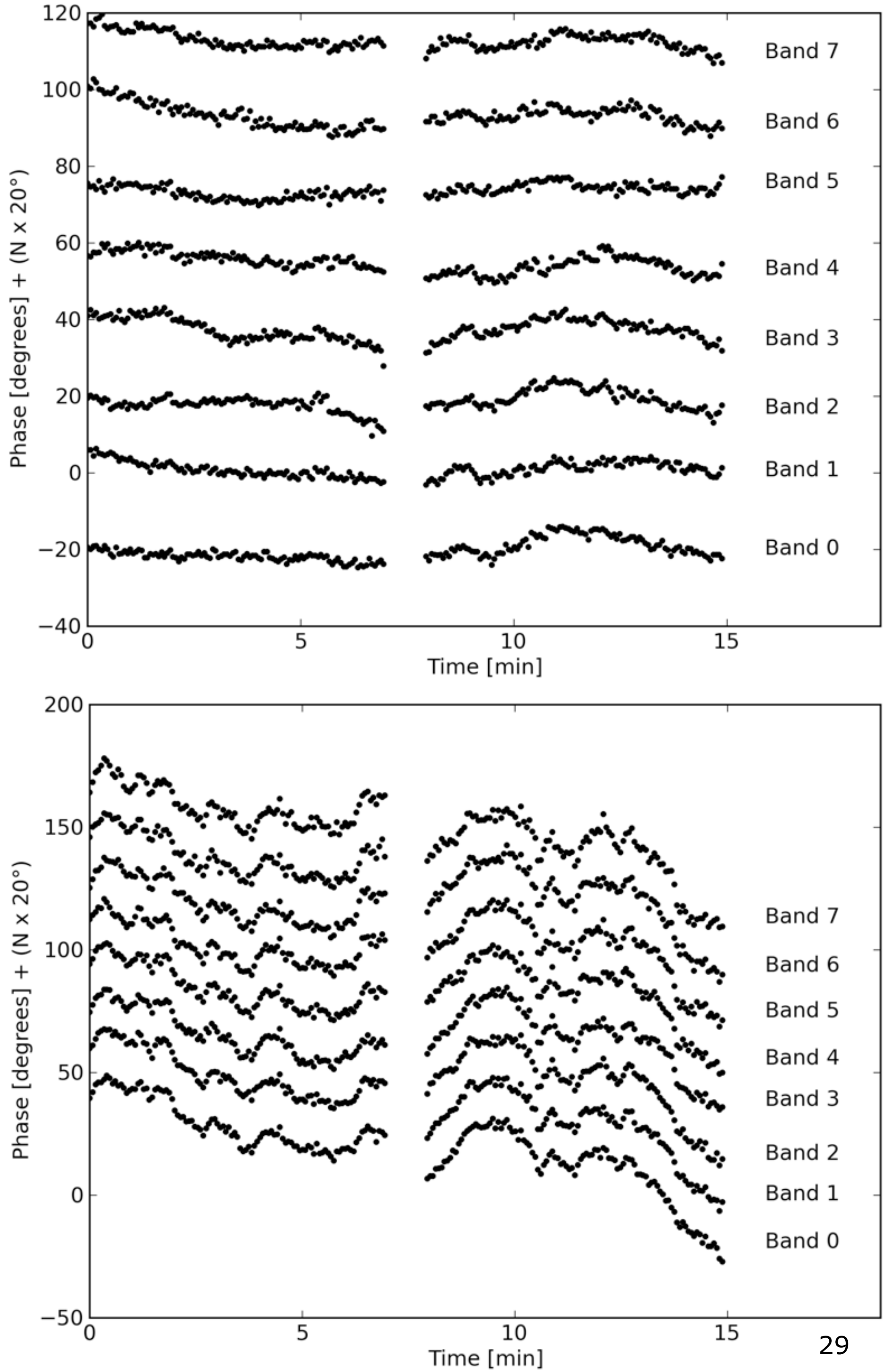
- Develop model of target field, use to estimate updated gain term



Example: LOFAR phase solutions

- From MSSS, at HBA frequencies





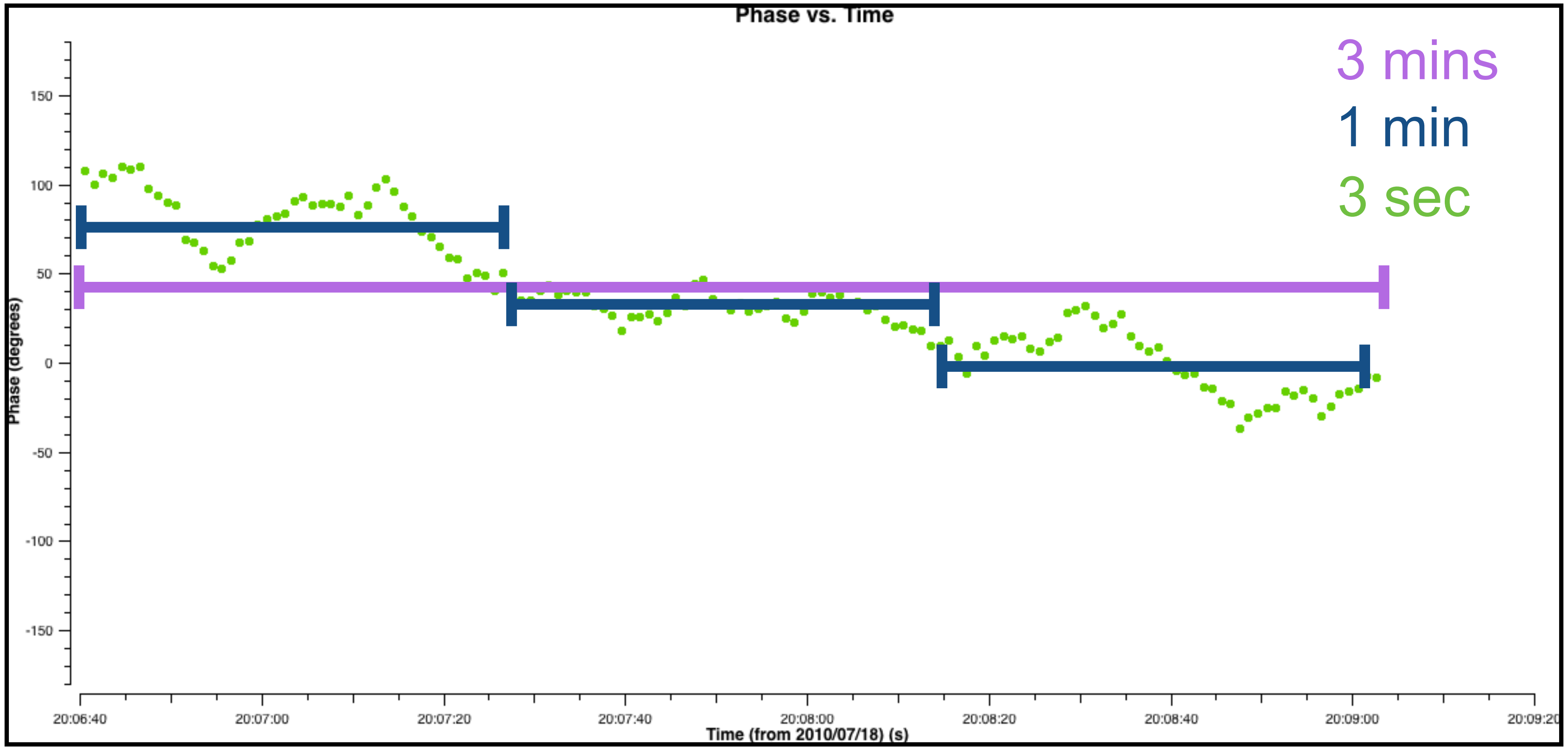
Heald et al (2015)

Selfcal rules of thumb

- Sky model used in each stage is fundamentally important!

$$V_{ij}^{\mathrm{obs}} = G_{ij} V_{ij}^{\mathrm{true}}$$

  - Balancing act: include as many real sources as possible, but do not include any fake sources!
- Choose solution intervals well (smallest possible timescale given S/N)

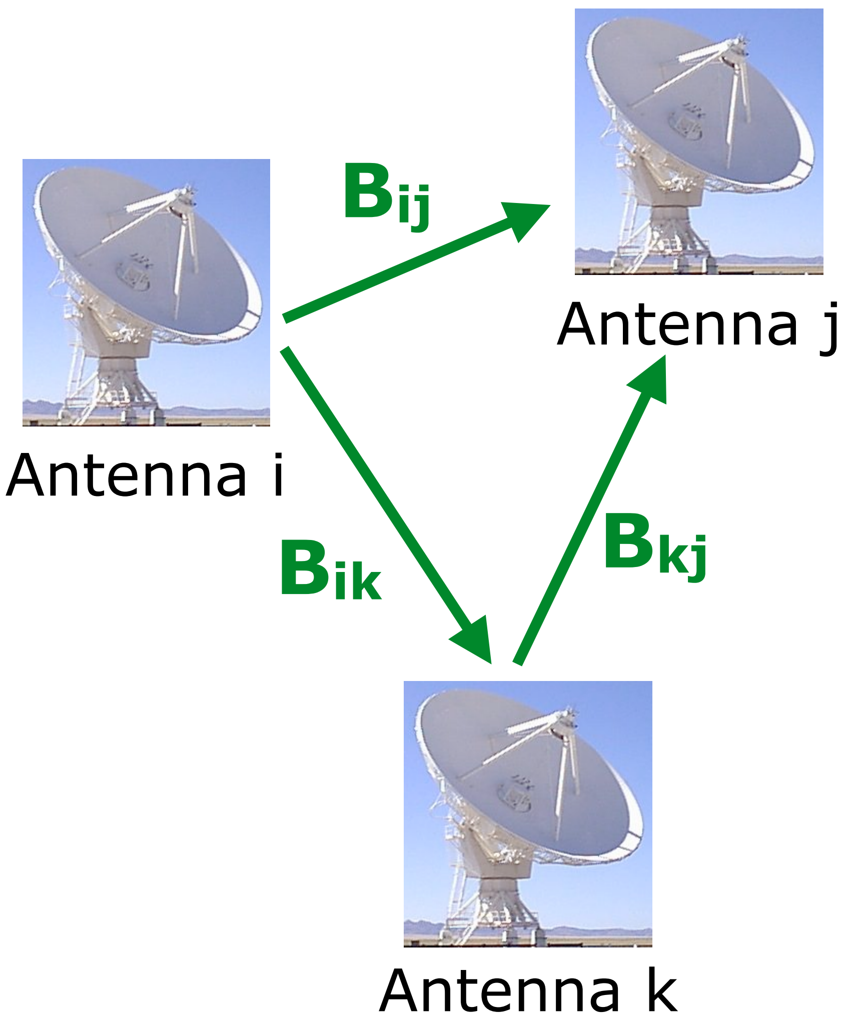


- As always: visualize your data (at every step!)

Closure

$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\mathbf{B}_{pq}\mathbf{G}_{pq}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

- The most problematic errors do not obey the general rule that we have assumed so far: calibration parameters should be antenna-based.
- However, baseline-dependent errors can occur. How to recognize these?



$$\mathbf{V}_{pq} = \mathbf{G}_{pq}\mathbf{C}_{pq} = \mathbf{G}_{p}\mathbf{C}_{pq}\mathbf{G}_{q}^{\dagger}$$

For one polarization:

$$\tilde{V}_{pq} = |G_p|\, e^{i\theta_p}\, V_{pq}\, |G_q|\, e^{-i\theta_q}$$

$$V_{pq} = |V_{pq}|\, e^{i\phi_{pq}}$$

$$\tilde{\phi}_{pq} = \phi_{pq} + \theta_p - \theta_q$$

$$\tilde{\phi}_{ij} + \tilde{\phi}_{ik} + \tilde{\phi}_{kj} = \phi_{ij} + \phi_{ik} + \phi_{kj}$$

This quantity, the "closure phase," is independent of antenna-based gain phases

A possible calibration strategy (*)

- Load data
- Inspect logs, flag bad data, flag for shadowing (look outside!)
- Calibrate primary calibrator (bandpass, gain, leakage)
  - inspect solutions
  - transfer bandpass, gain amplitude (flux scale) and leakage to secondary
- Calibrate secondary calibrator (gain)
  - inspect solutions
  - transfer bandpass, gains, and leakage to target
- Inspect target for bad data, flag if necessary
- Image, deconvolve, and selfcal if necessary

Summary

- Many effects including the atmosphere, delay errors and the electronics of the receiver systems will corrupt the signal from your target of interest.
- Standard calibration transfer techniques, using bright and simple sources can eliminate most of these effects.
- The Measurement Equation is a useful framework for understanding errors, and for determining calibration parameters.
- Residual errors can be removed using self-calibration providing you have sufficient signal-to-noise ratio, enough baselines, and an accurate model for your source.

Your calibration is only as good as your model since model errors will be absorbed into your calibration solutions.

- Direction dependent effects limit the quality of wide-field imaging due to time variable beam patterns, time- and direction-dependent ionosphere and our limited knowledge of the sky model (lecture on high dynamic range imaging).

Don't underestimate the value of continually viewing your data and calibration solutions!