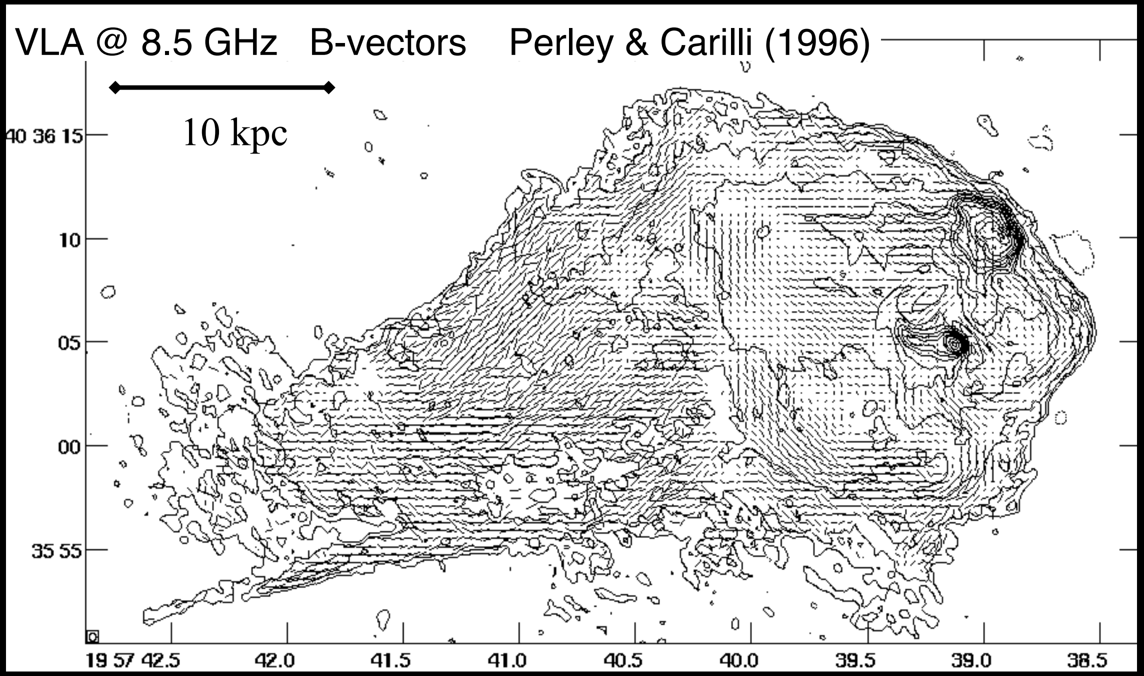
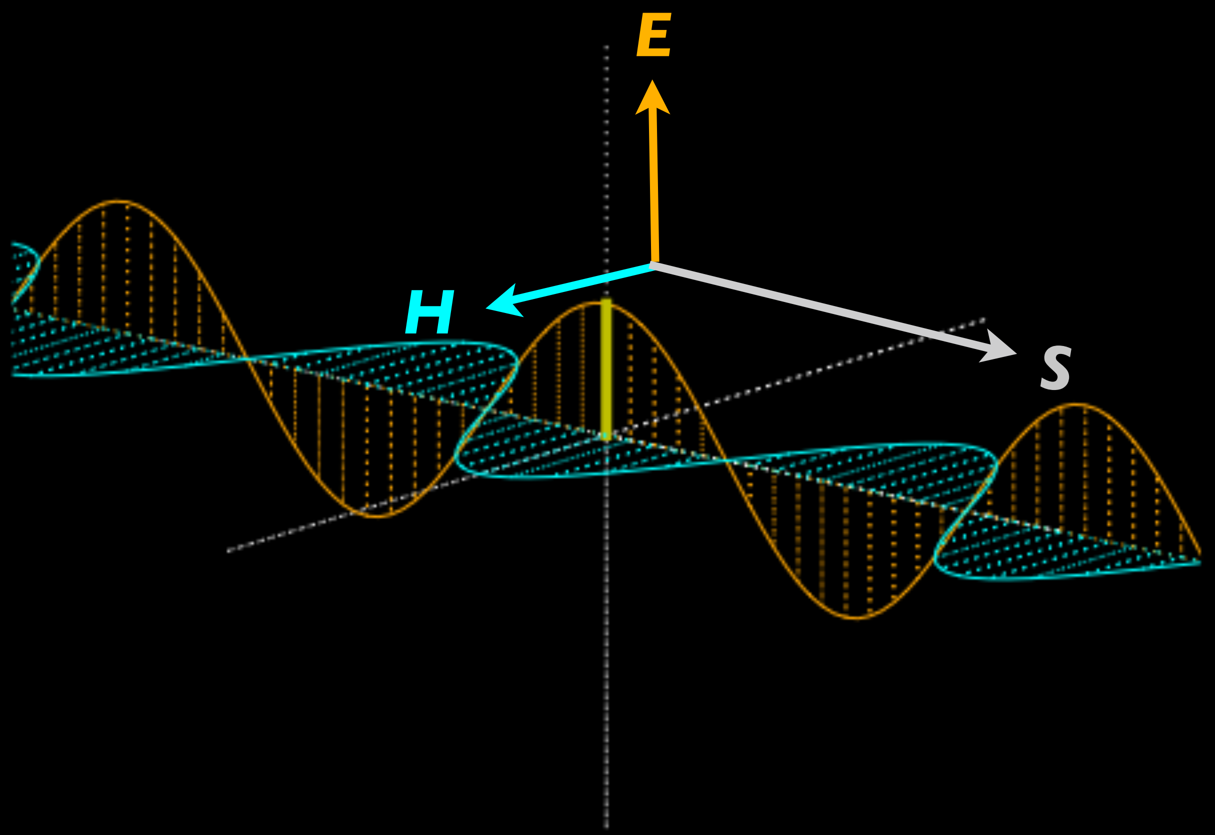


Polarimetry

Dave McConnell, CASS
Radio Astronomy School, Narrabri
30 September 2015

Electro-magnetic waves are polarized

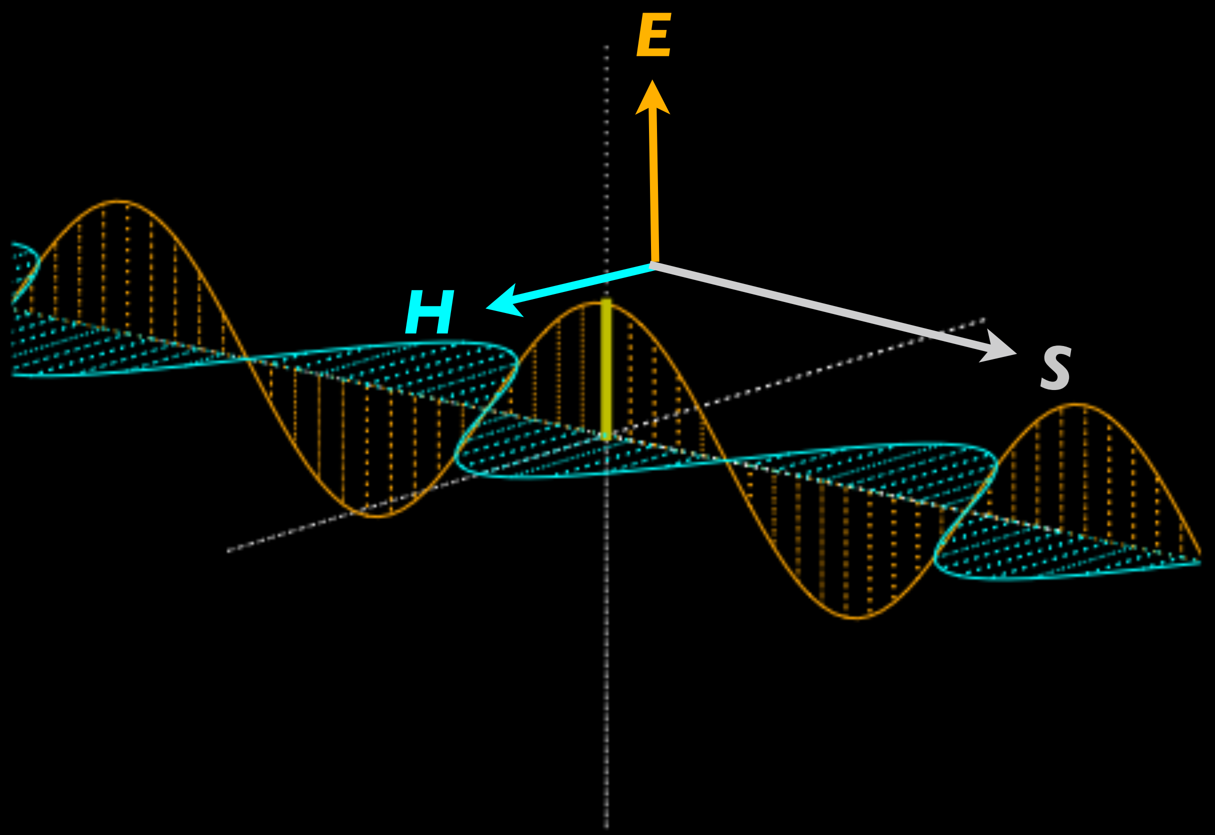


- E/M waves have direction, amplitude, frequency and polarization

Poynting vector

$$\boldsymbol{S} = c/4\pi\,(\boldsymbol{E} \times \boldsymbol{H})$$

Electro-magnetic waves are polarized



- E/M waves have direction, amplitude, frequency and polarization

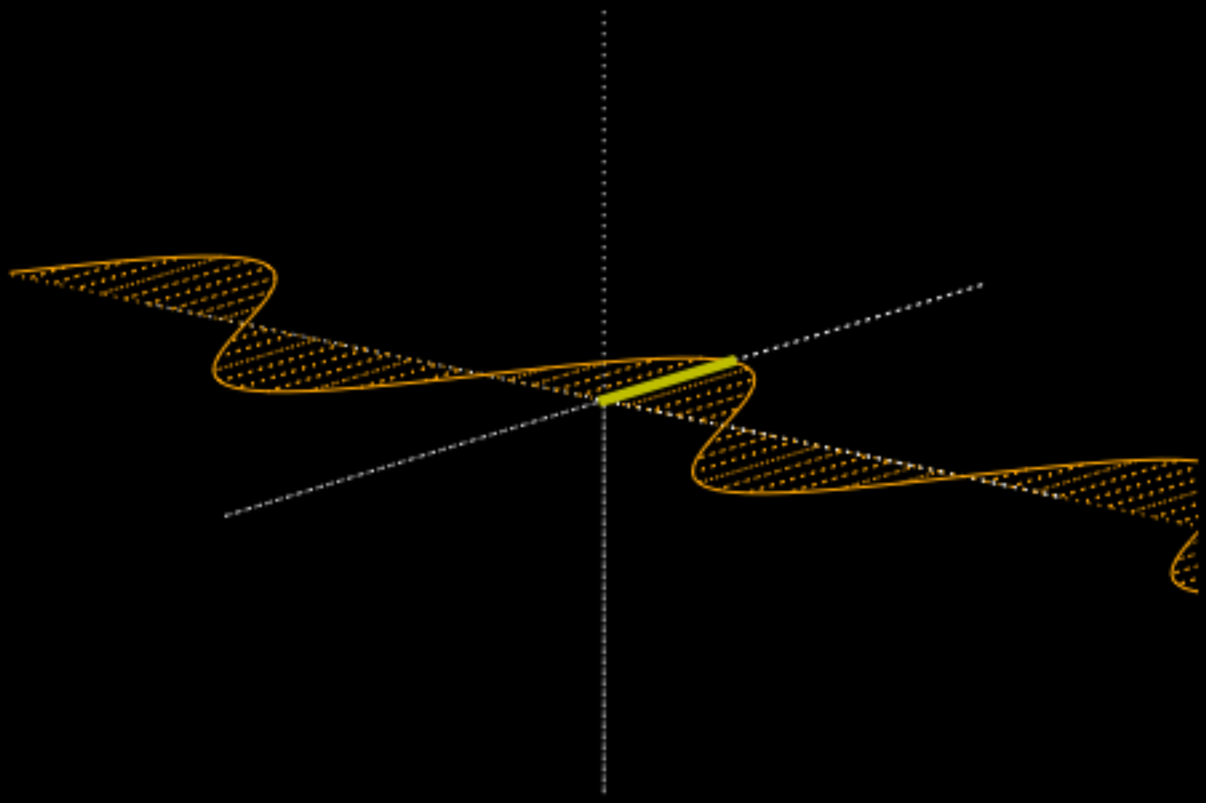
Poynting vector

$$\boldsymbol{S} = c/4\pi\,(\boldsymbol{E} \times \boldsymbol{H})$$

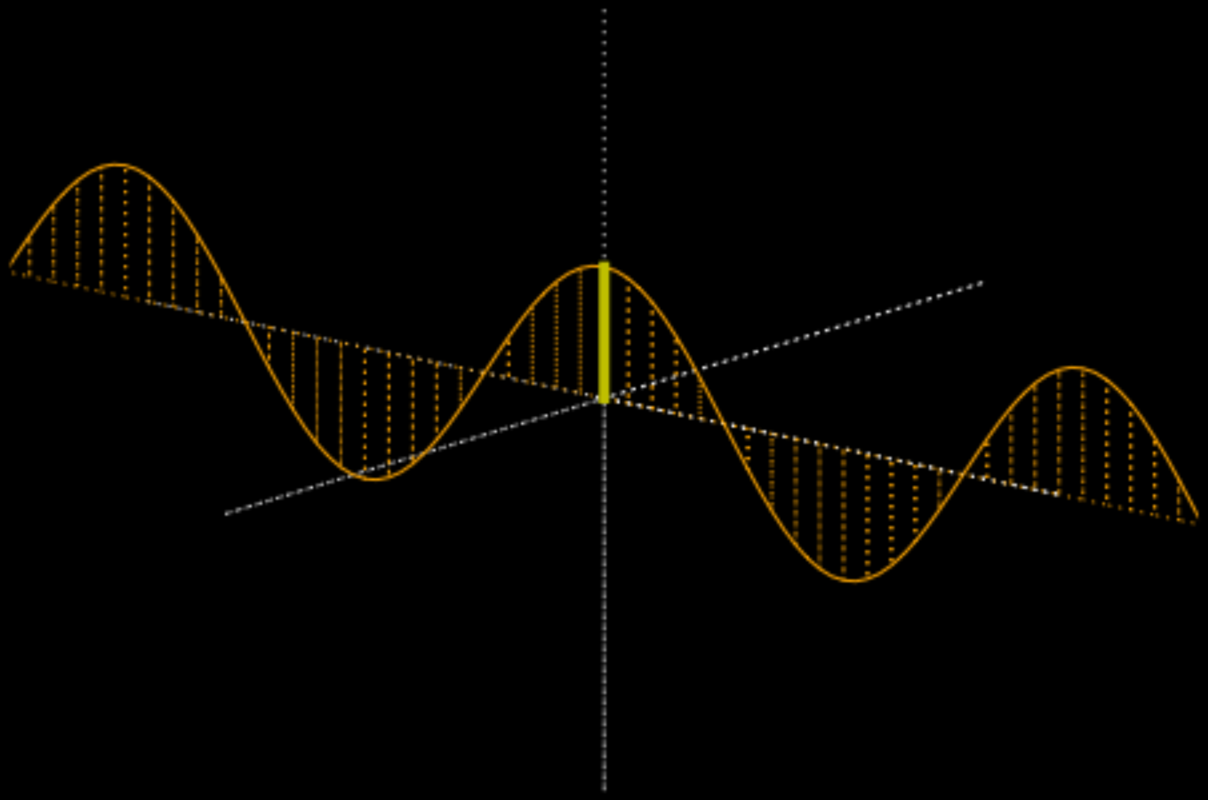
Outline of lecture

- Polarization - what is it?
- How is it described
- Origins of polarized light
- How is it important to astrophysics
- How is it measured

Polarized waves: linear

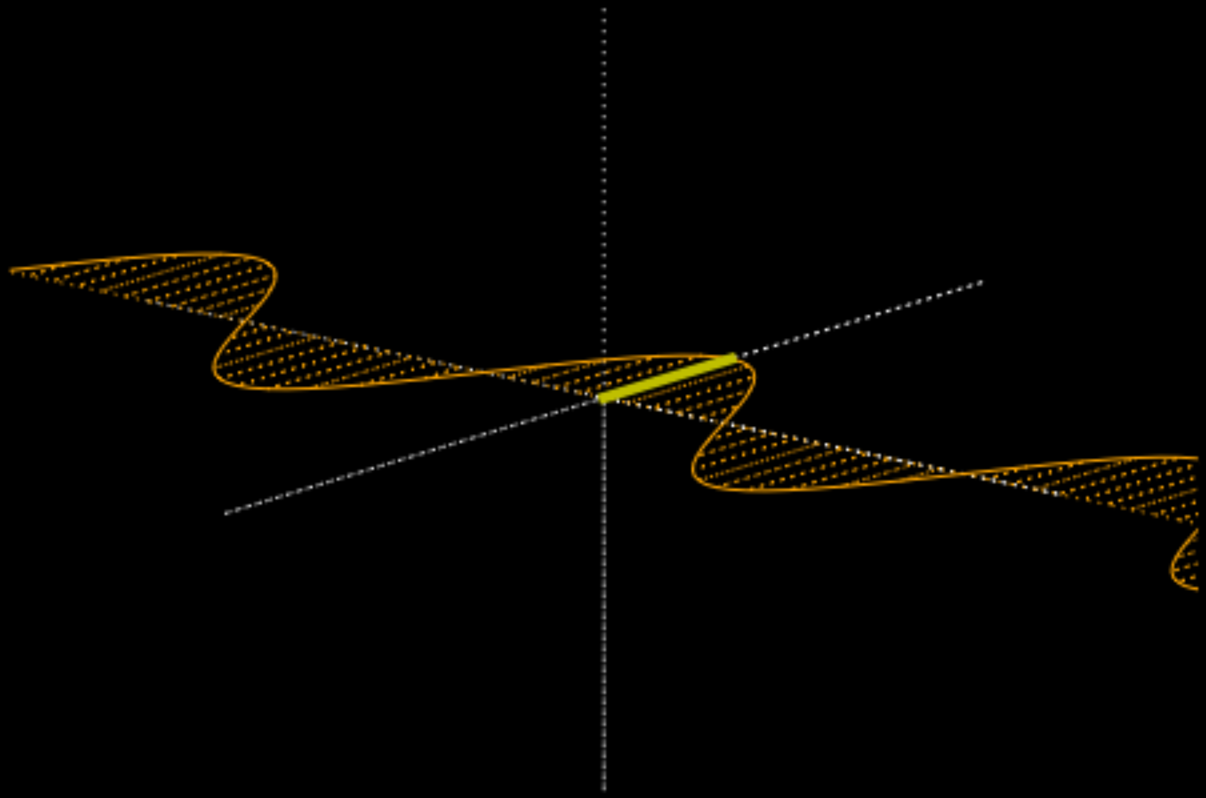


$$\mathbf{E} = E_x \cos(\omega t - kz)\underline{\hat{x}}$$

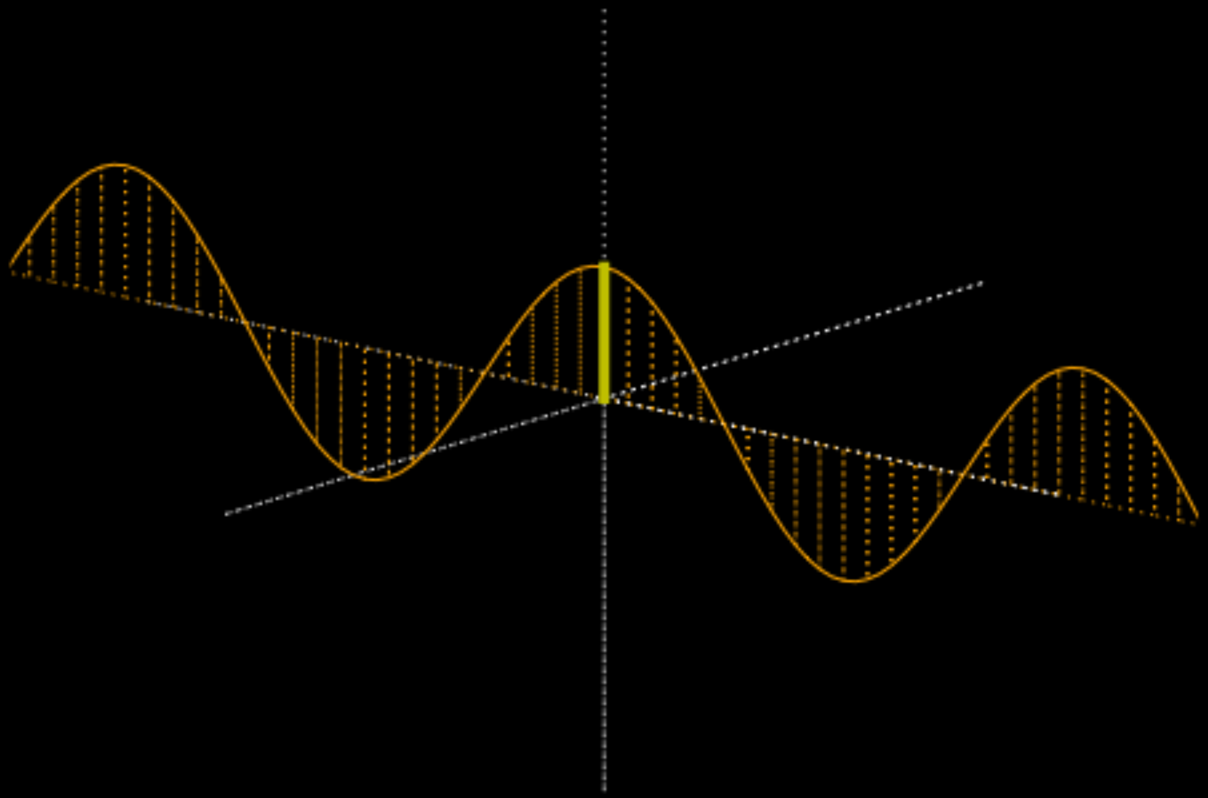


$$\mathbf{E} = E_y \cos(\omega t - kz)\underline{\hat{y}}$$

Polarized waves: linear

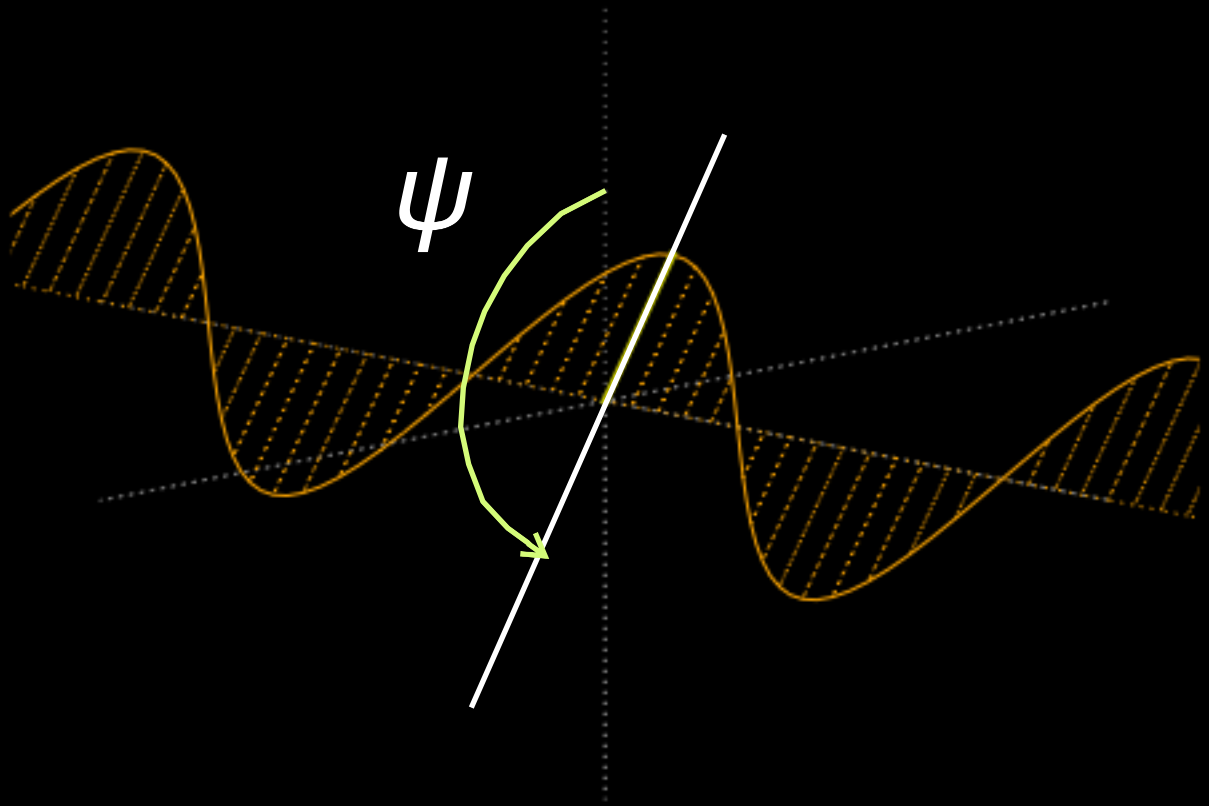


$$\mathbf{E} = E_x \cos(\omega t - kz)\underline{\hat{x}}$$



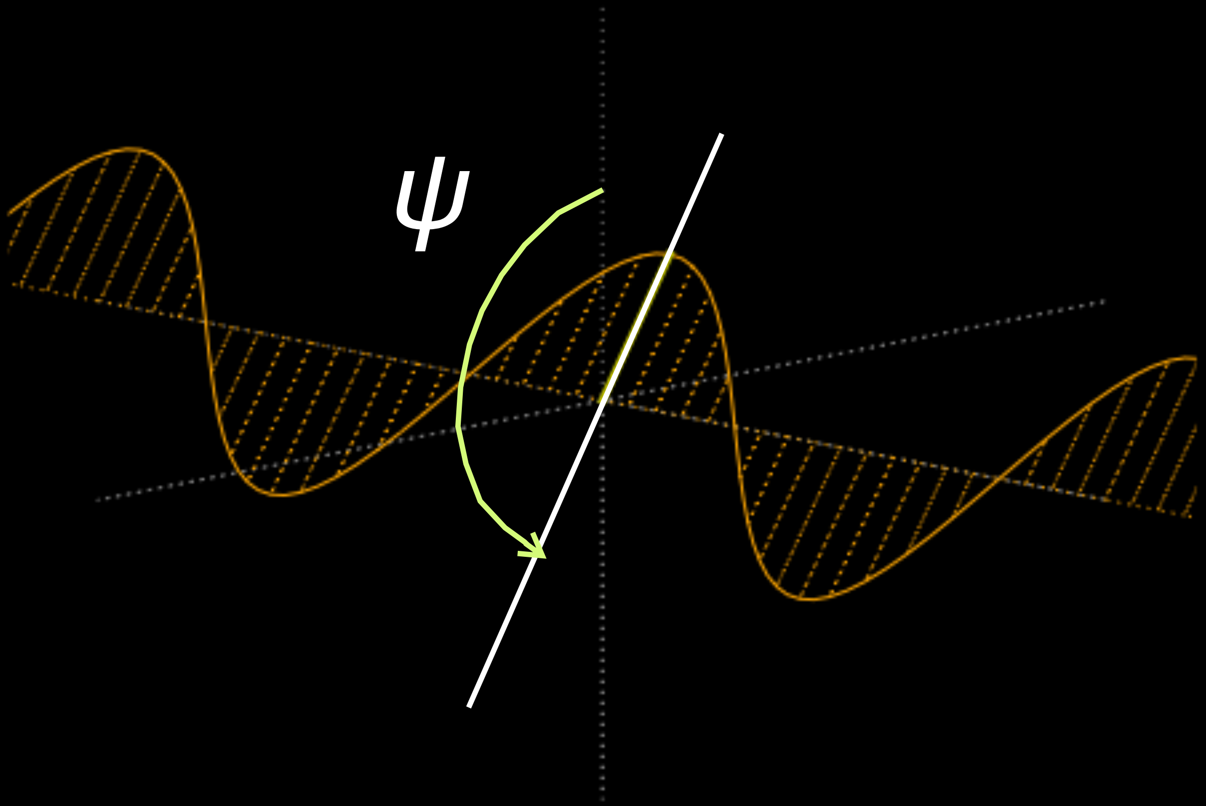
$$\mathbf{E} = E_y \cos(\omega t - kz)\underline{\hat{y}}$$

… at any angle ψ



$$\mathbf{E} = E_x \cos(\omega t - kz)\underline{\hat{x}} \\ + E_y \cos(\omega t - kz)\underline{\hat{y}}$$

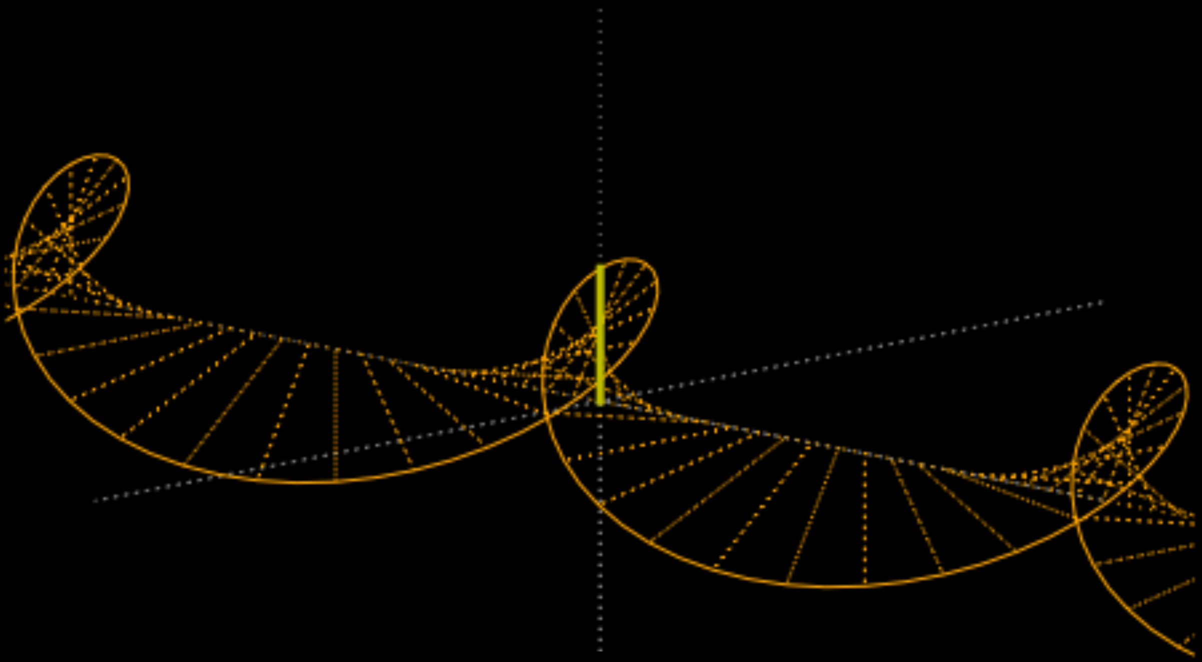
… at any angle ψ



$$\mathbf{E} = E_x \cos(\omega t - kz)\underline{\hat{x}} \\ + E_y \cos(\omega t - kz)\underline{\hat{y}}$$

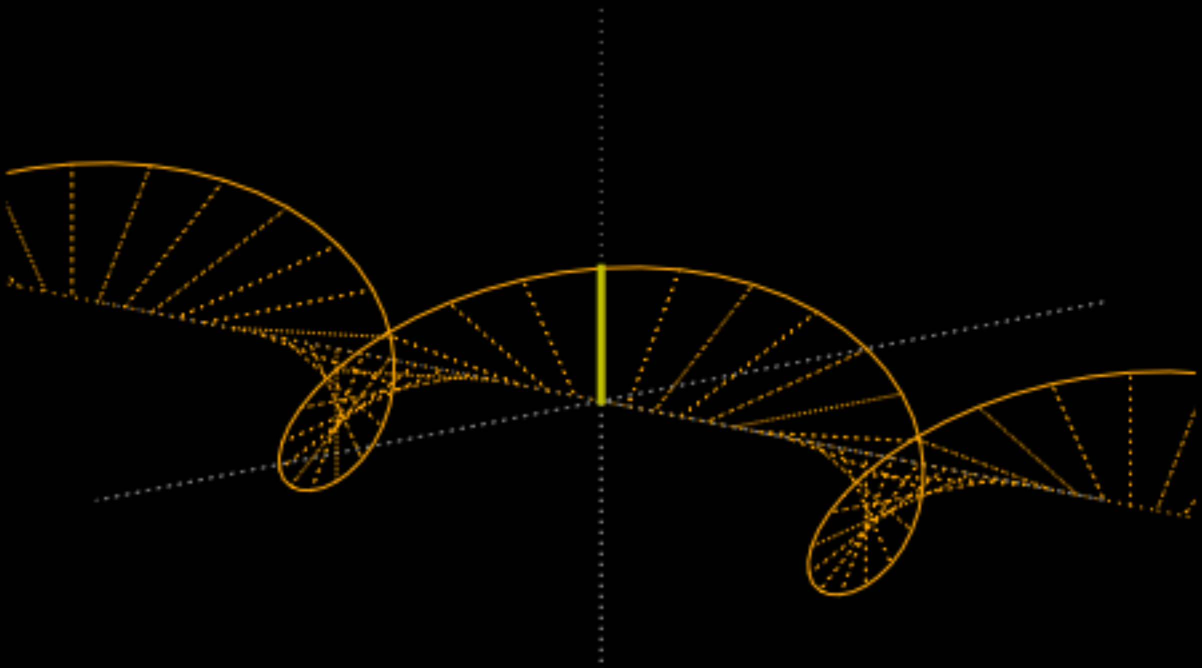
or Circular - LCP, RCP

Left



$$\mathbf{E} = E\cos(\omega t - kz)\underline{\hat{x}} \\ + E\cos(\omega t - kz + \frac{\pi}{2})\underline{\hat{y}}$$

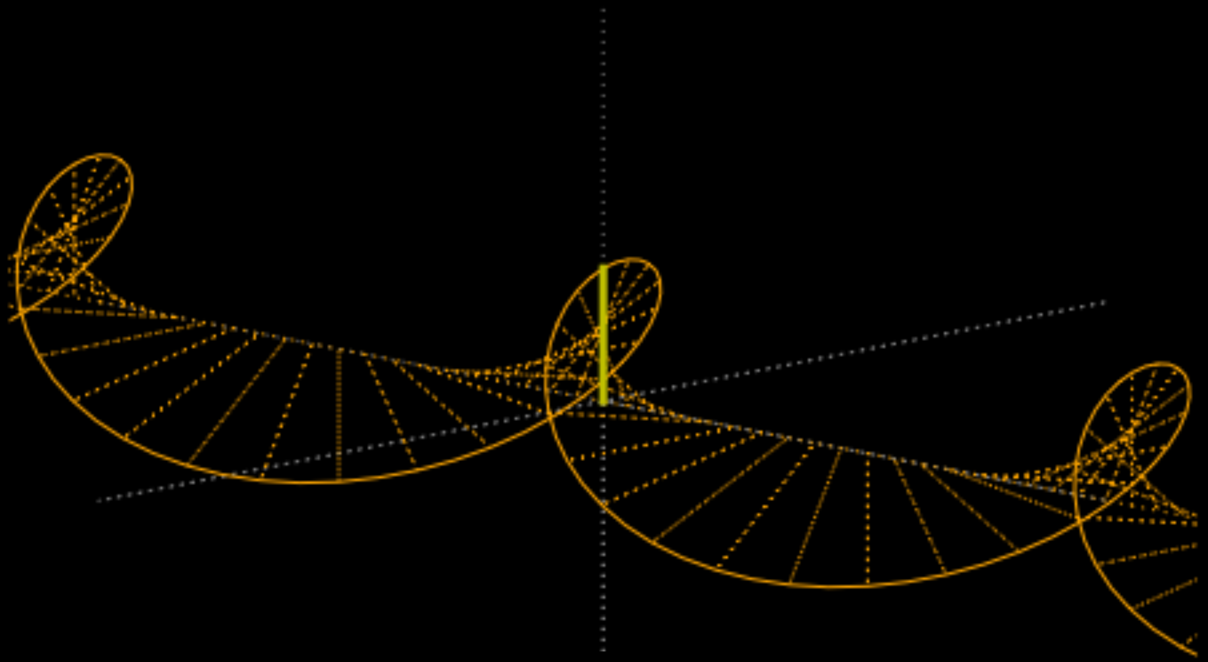
Right



$$\mathbf{E} = E\cos(\omega t - kz)\underline{\hat{x}} \\ + E\cos(\omega t - kz - \frac{\pi}{2})\underline{\hat{y}}$$

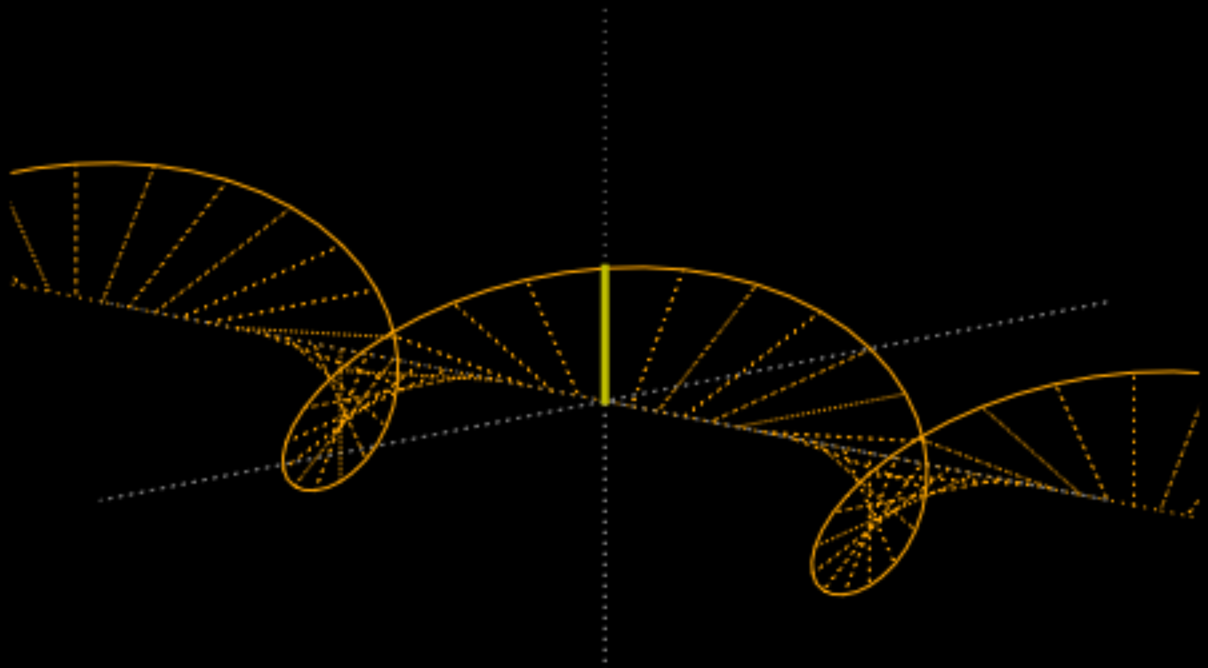
or Circular - LCP, RCP

Left



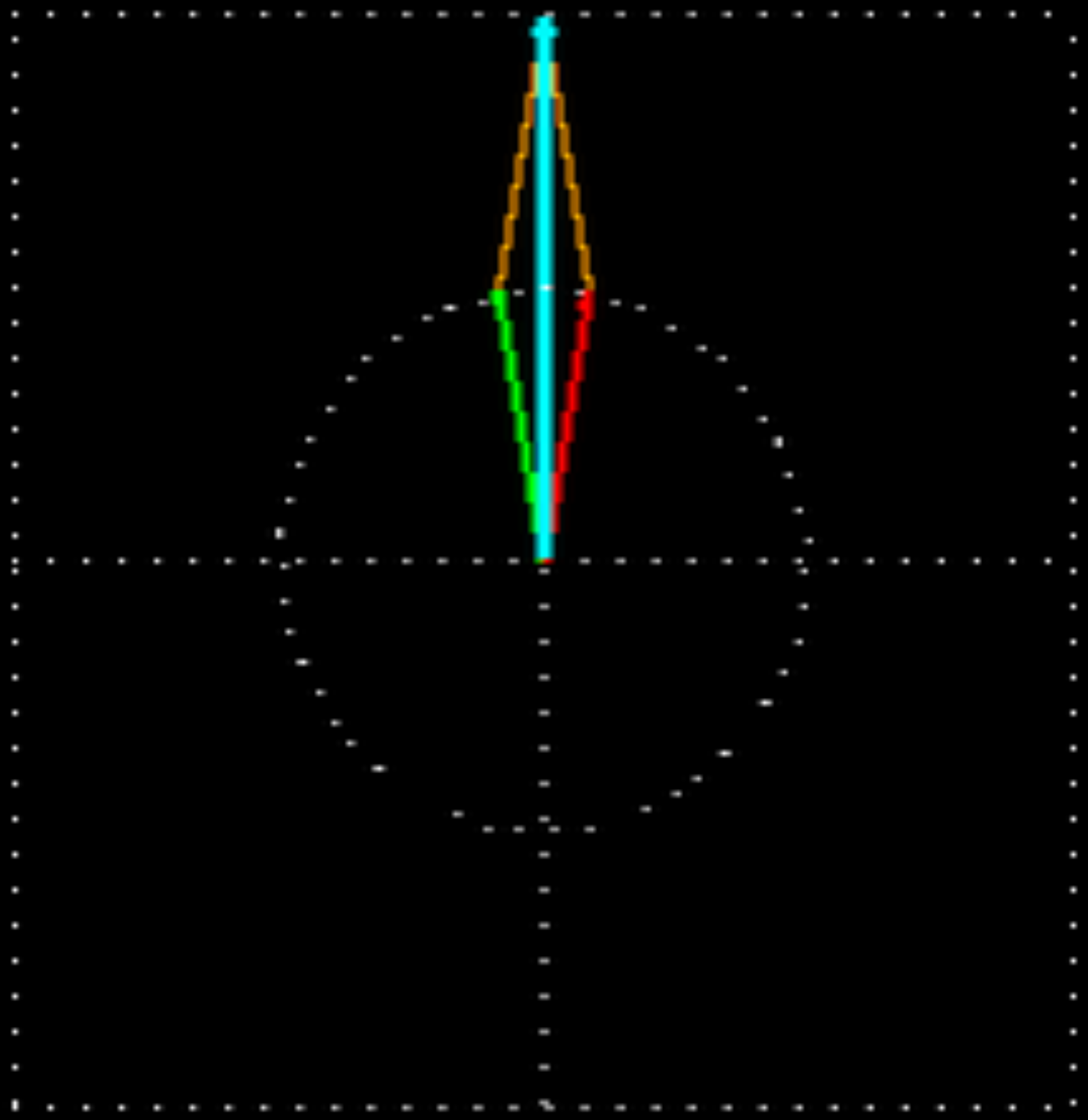
$$\mathbf{E} = E\cos(\omega t - kz)\underline{\hat{x}} + E\cos(\omega t - kz + \frac{\pi}{2})\underline{\hat{y}}$$

Right

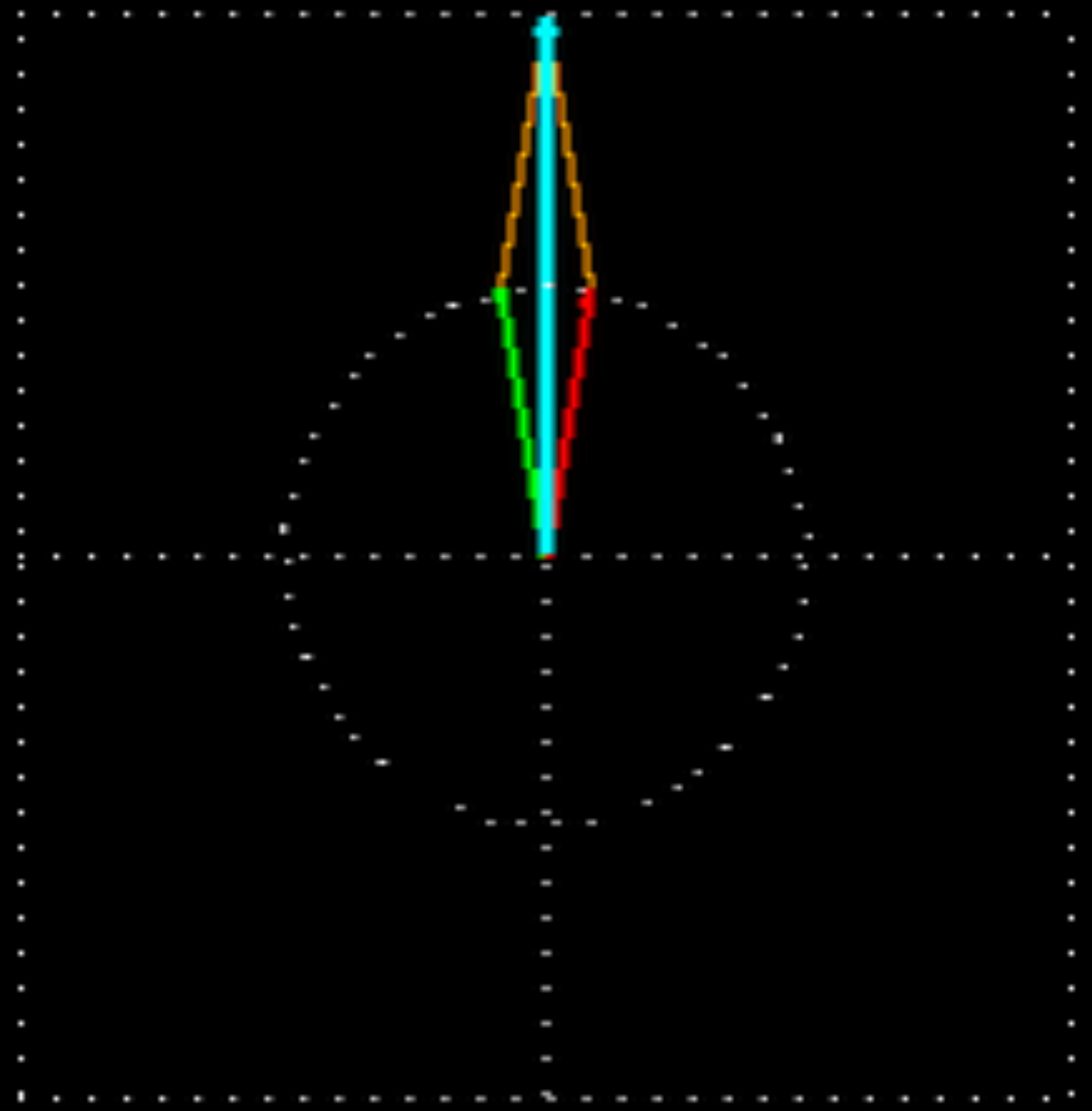


$$\mathbf{E} = E\cos(\omega t - kz)\underline{\hat{x}} + E\cos(\omega t - kz - \frac{\pi}{2})\underline{\hat{y}}$$

Linear as sum of circulars



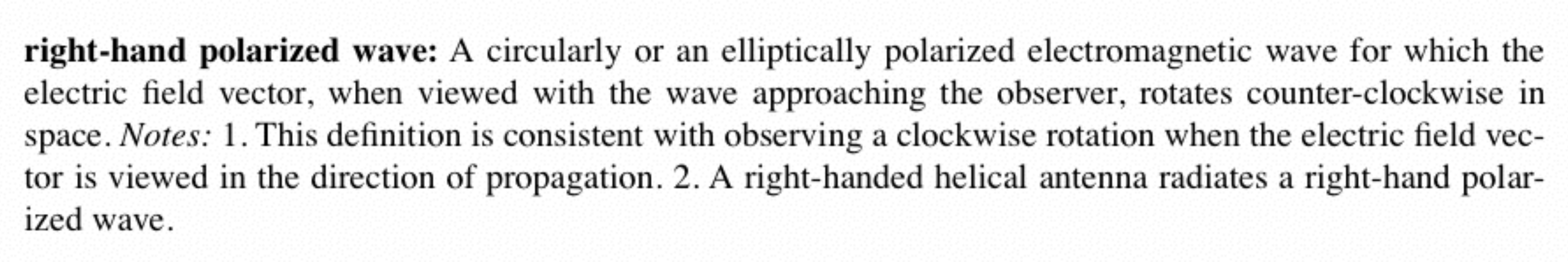
Linear as sum of circulars



Other combinations

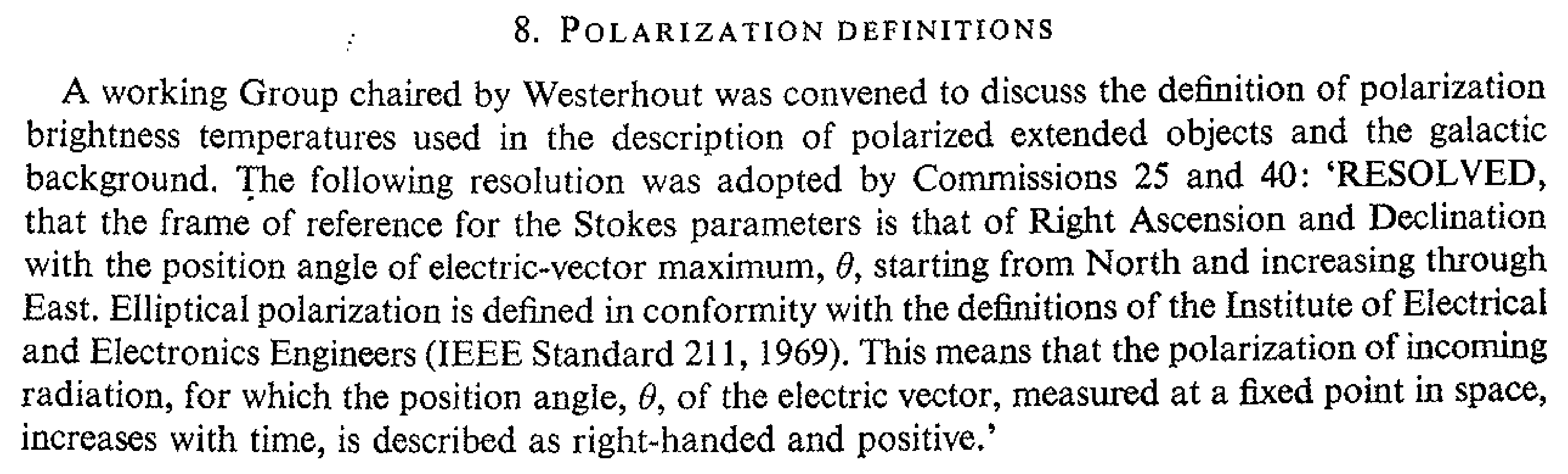
- The sum of two circular waves of unequal amplitude will have elliptical polarization.
- The sum of two orthogonal linears with phase difference $0 < \delta < \pi/2$ will also have elliptical polarization.

IEEE Standard 211, 1969



right-hand polarized wave: A circularly or an elliptically polarized electromagnetic wave for which the electric field vector, when viewed with the wave approaching the observer, rotates counter-clockwise in space. *Notes:* 1. This definition is consistent with observing a clockwise rotation when the electric field vector is viewed in the direction of propagation. 2. A right-handed helical antenna radiates a right-hand polarized wave.

IAU resolution, 1973

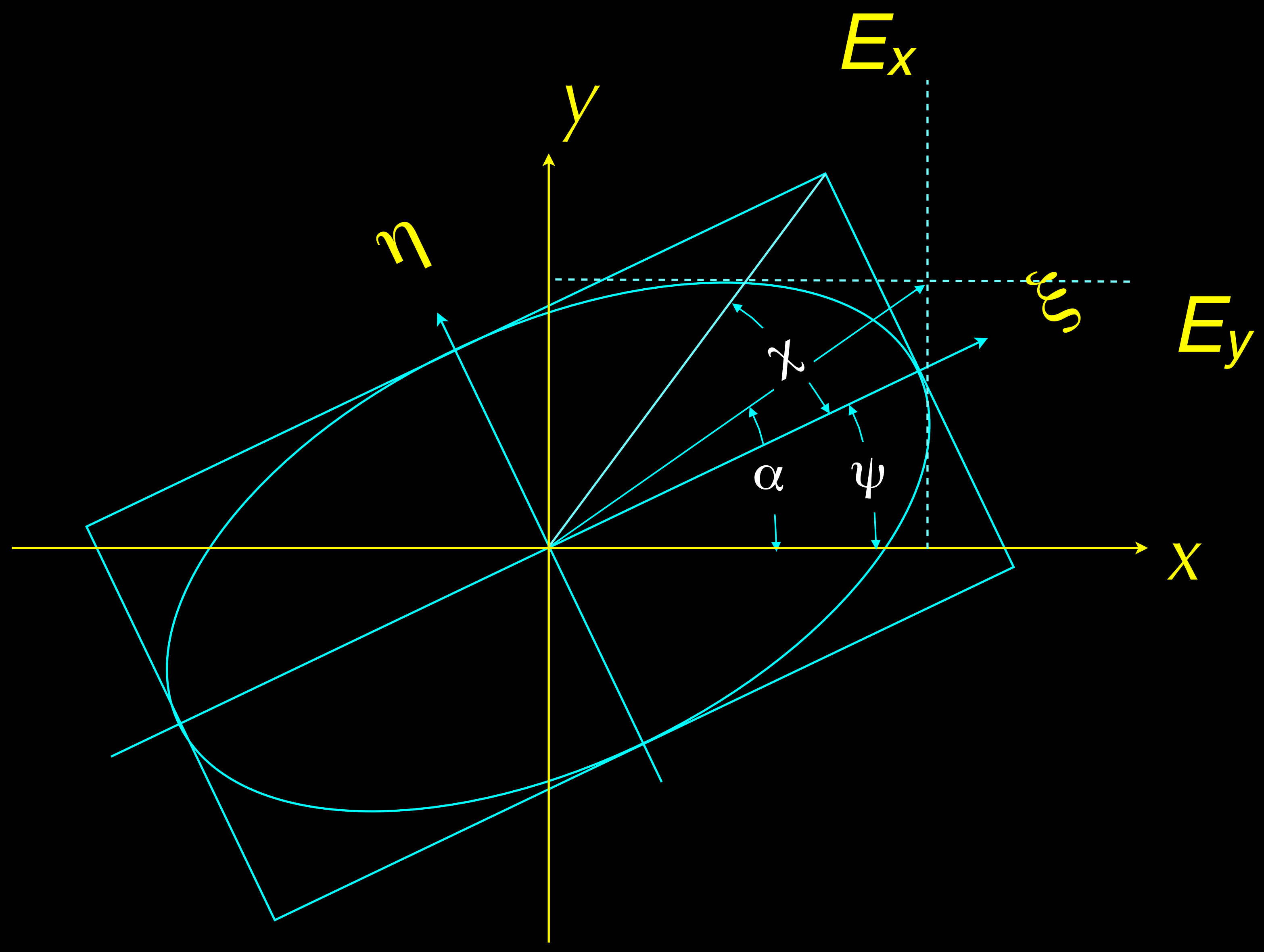


8. POLARIZATION DEFINITIONS

A working Group chaired by Westerhout was convened to discuss the definition of polarization brightness temperatures used in the description of polarized extended objects and the galactic background. The following resolution was adopted by Commissions 25 and 40: 'RESOLVED, that the frame of reference for the Stokes parameters is that of Right Ascension and Declination with the position angle of electric-vector maximum, θ, starting from North and increasing through East. Elliptical polarization is defined in conformity with the definitions of the Institute of Electrical and Electronics Engineers (IEEE Standard 211, 1969). This means that the polarization of incoming radiation, for which the position angle, θ, of the electric vector, measured at a fixed point in space, increases with time, is described as right-handed and positive.'


N
θ
E
SCP
W

Polarization ellipse



- $\tan \alpha = E_y / E_x$
- $\tan 2\psi = \tan 2\alpha \cos \delta$
- $\sin 2\chi = \sin 2\alpha \sin \delta$
- ψ is the position angle
- χ is the ellipticity

Stokes description

- Defined by George Stokes in 1852
- Adopted for astronomy by Chandrasehkar (1949) in the solution of radiative transfer problems.

Stokes parameters

$$
\begin{aligned}
I &= E_x^2 + E_y^2 \\
Q &= E_x^2 - E_y^2 \\
U &= 2E_xE_y\cos(\delta) \\
V &= 2E_xE_y\sin(\delta)
\end{aligned}
$$

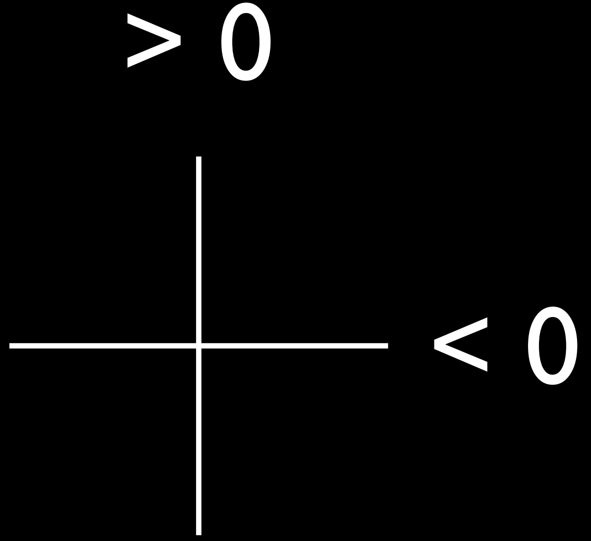
- For monochromatic waves
- I : total intensity
- Q : linear
- U : linear
- V : circular
- $I^2 = Q^2 + U^2 + V^2$

Stokes parameters

$$\begin{aligned} I &= E_R^2 + E_L^2 \\ V &= E_R^2 - E_L^2 \\ Q &= 2E_R E_L \cos(\delta) \\ U &= 2E_R E_L \sin(\delta) \end{aligned}$$

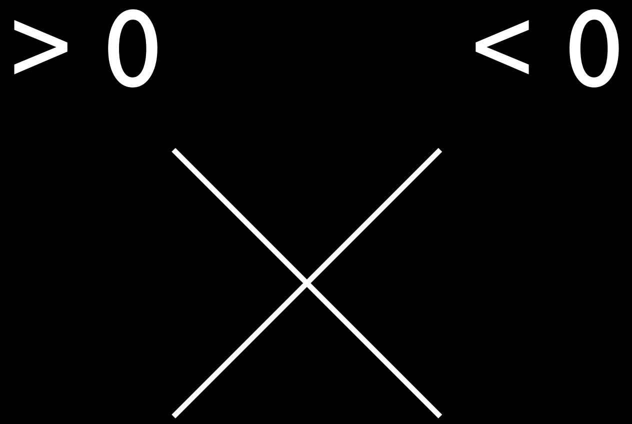
- For monochromatic waves
- I : total intensity
- Q : linear
- U : linear
- V : circular
- $I^2 = Q^2 + U^2 + V^2$

Linear: Q and U



Q

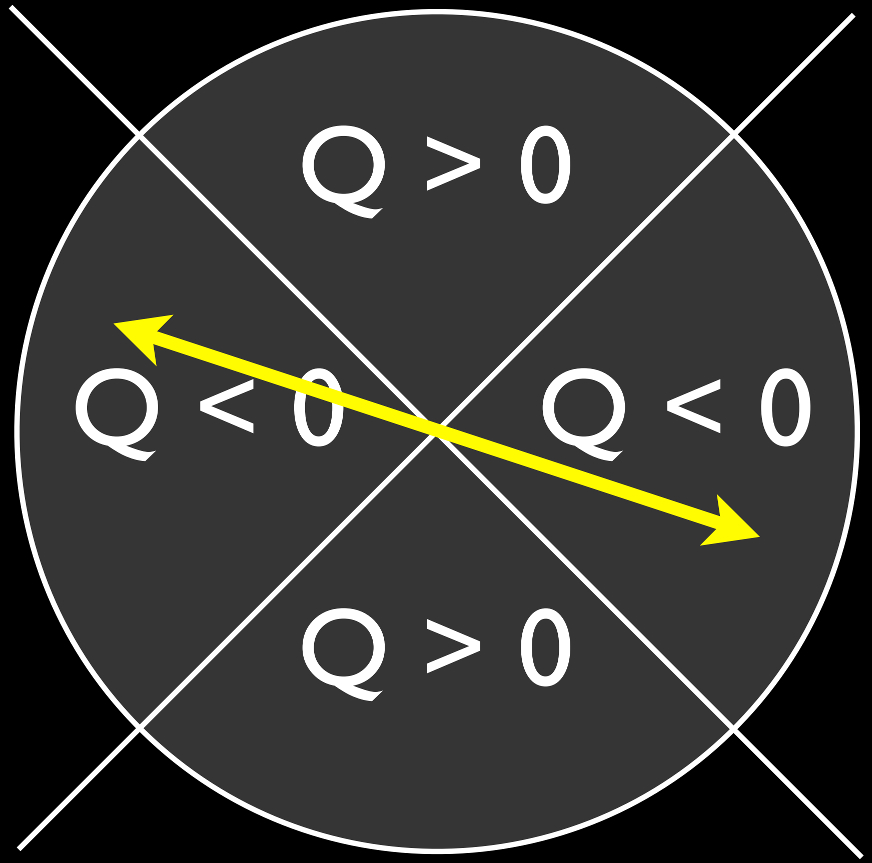
(U = 0)

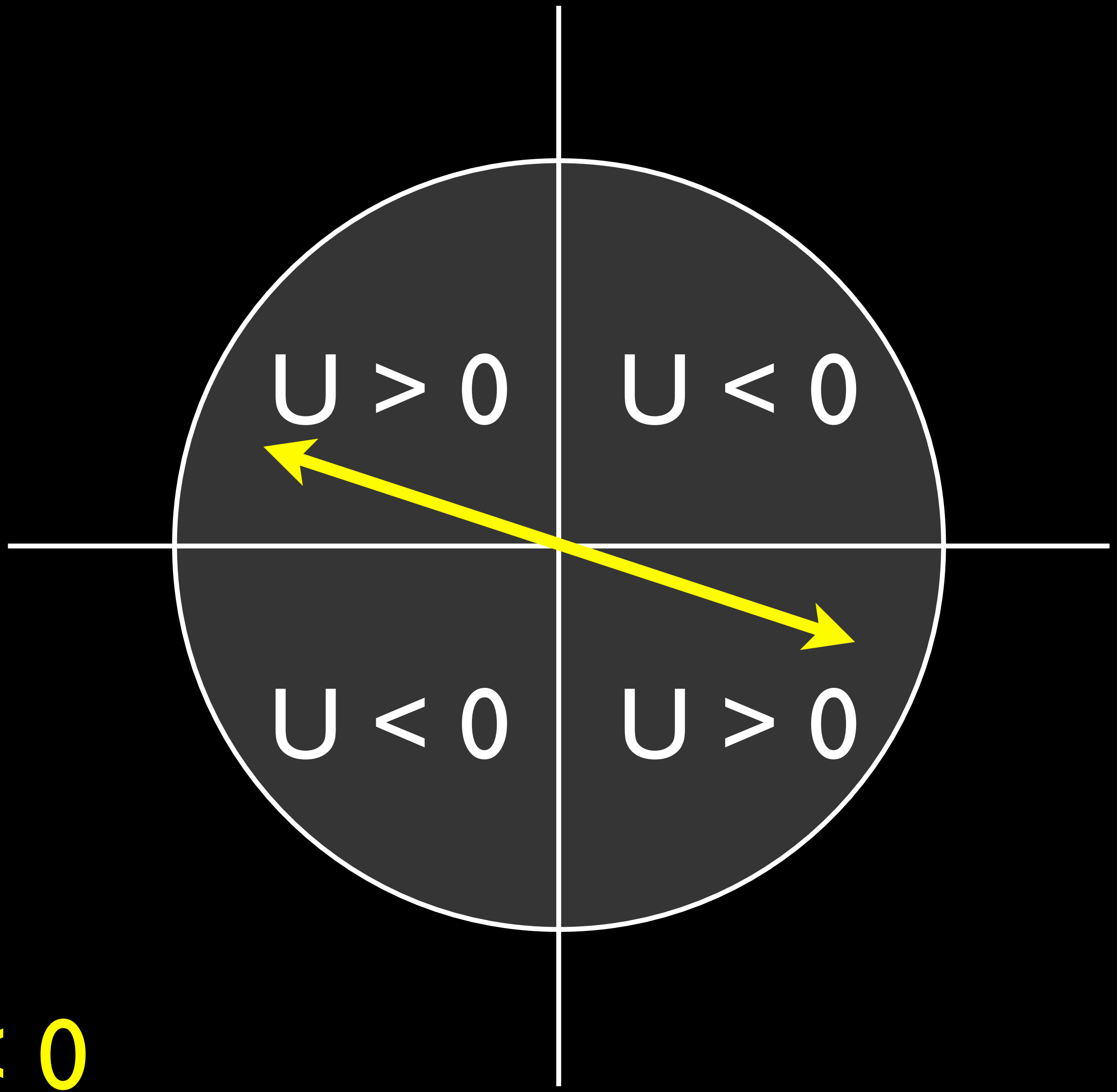


U

(Q = 0)

Linear: Q and U

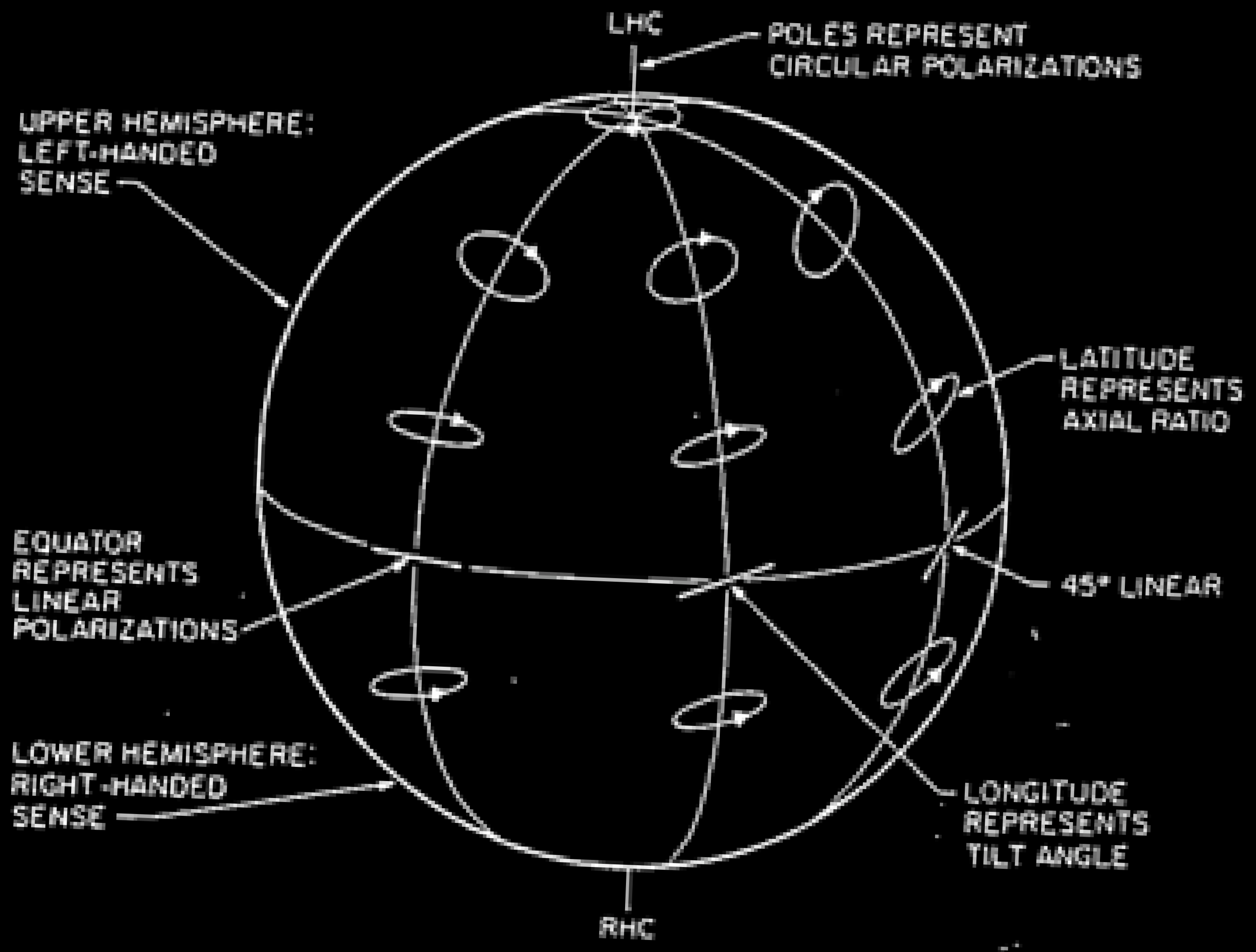




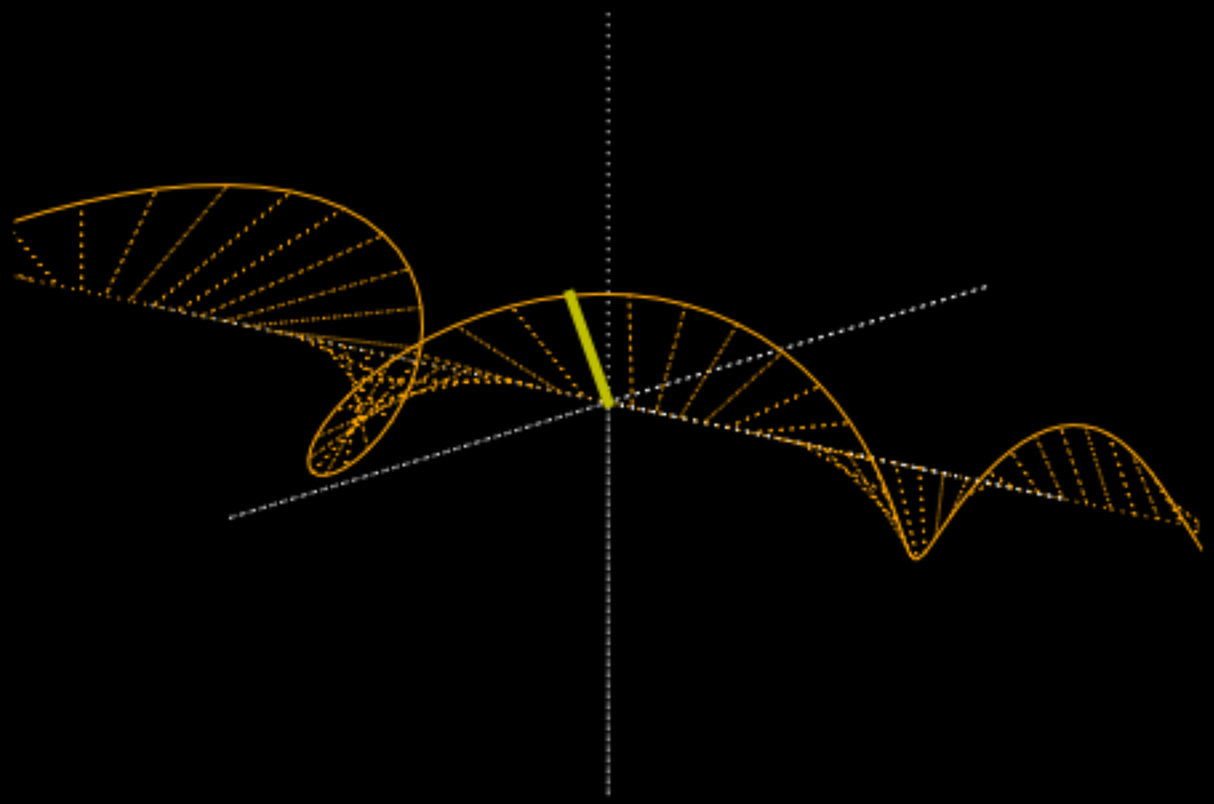
Q < 0

U > 0

The Poincaré sphere

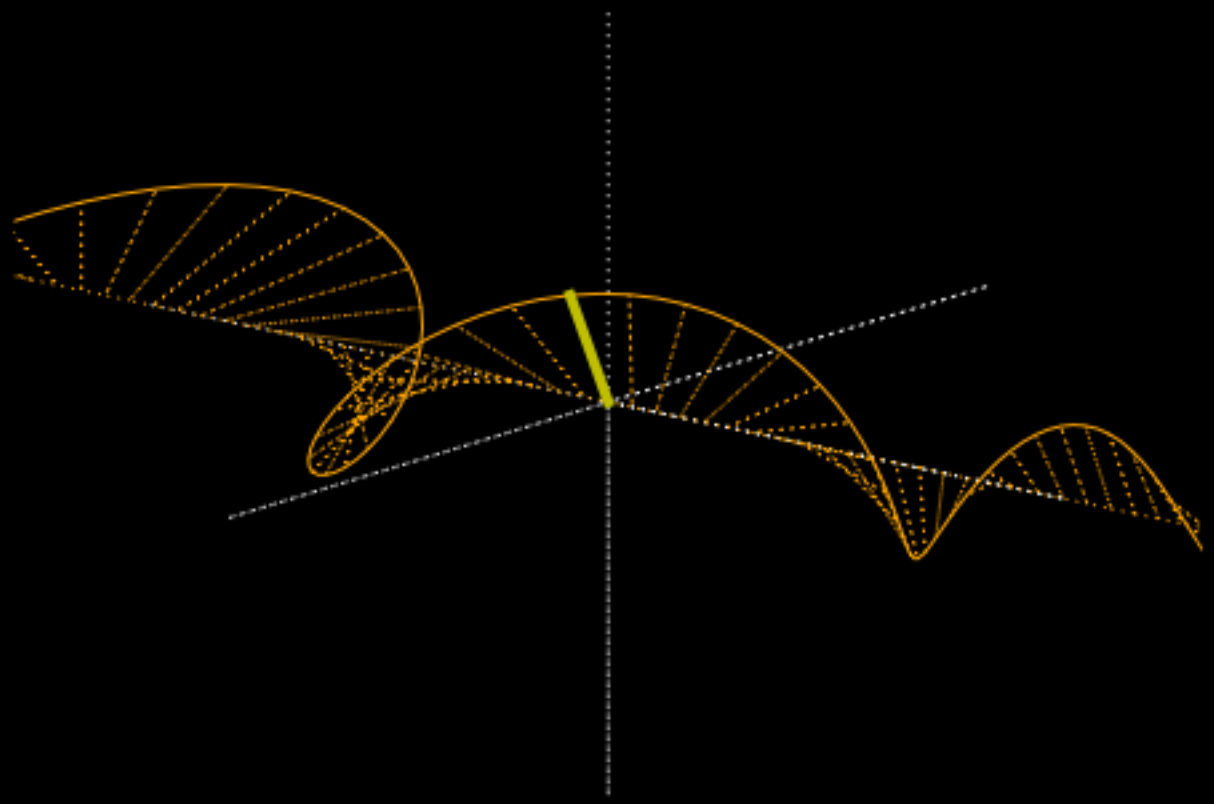


Partial polarization



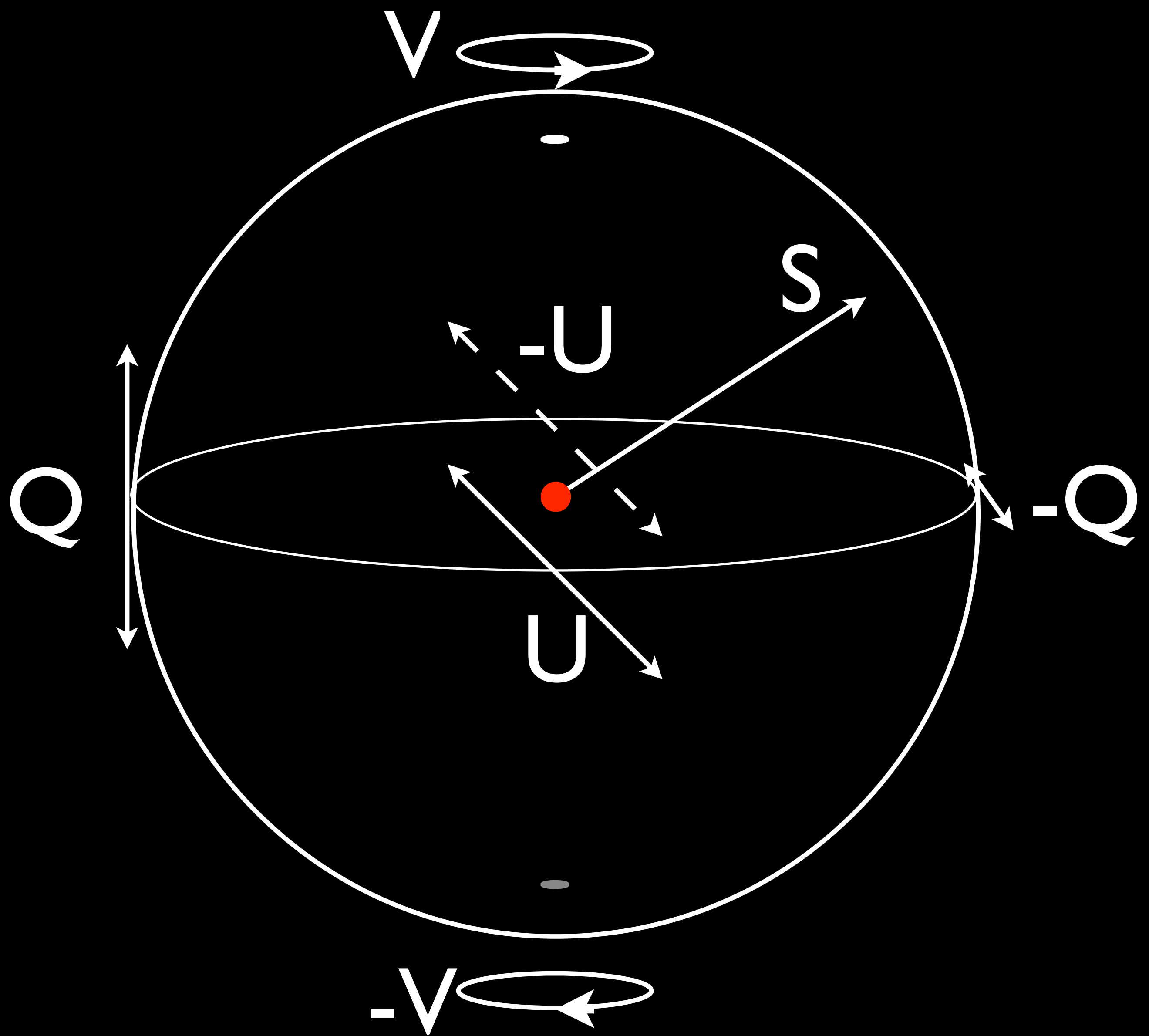
- Two monochromatic signals summed:
  - different frequencies
  - different polarization ellipses

Partial polarization



- Two monochromatic signals summed:
  - different frequencies
  - different polarization ellipses

Pancharatnam's extension



- Radius represents I
- S = (Q,U,V)
- $|S| \leq I$
- Unpolarized radiation (I - S) at centre .

Stokes parameters

$$
\begin{aligned}
I &= \langle E_x^2\rangle + \langle E_y^2\rangle \\
Q &= \langle E_x^2\rangle - \langle E_y^2\rangle \\
U &= 2\langle E_x E_y \cos(\delta)\rangle \\
V &= 2\langle E_x E_y \sin(\delta)\rangle
\end{aligned}
$$

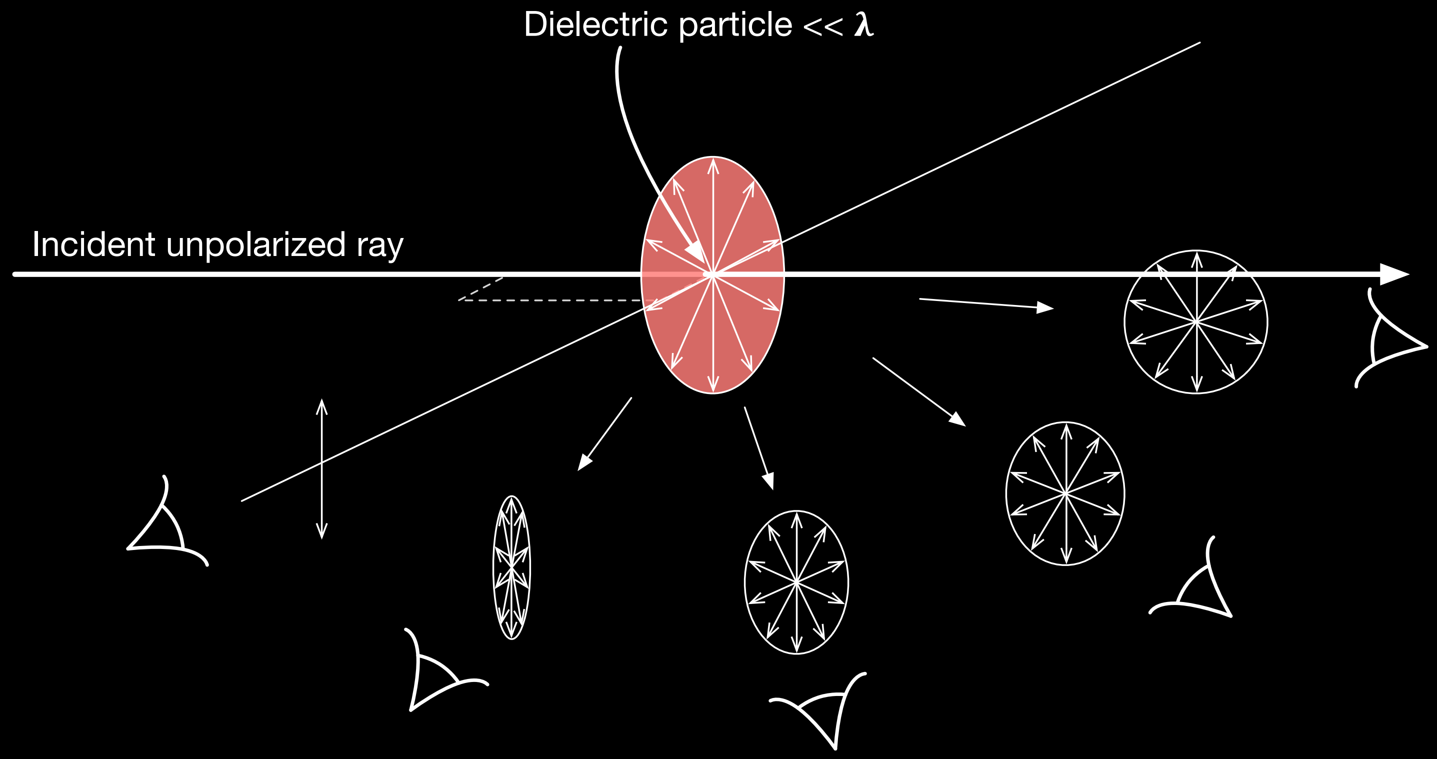
- For finite bandwidth radiation
- I : total intensity
- Q : linear
- U : linear
- V : circular
- $I^2 \geq Q^2 + U^2 + V^2$

Polarized light

- Loss of symmetry
- Polarized radiation
- Polarization-dependent propagation
  - reflection
  - scattering
  - birefringence

Scattering

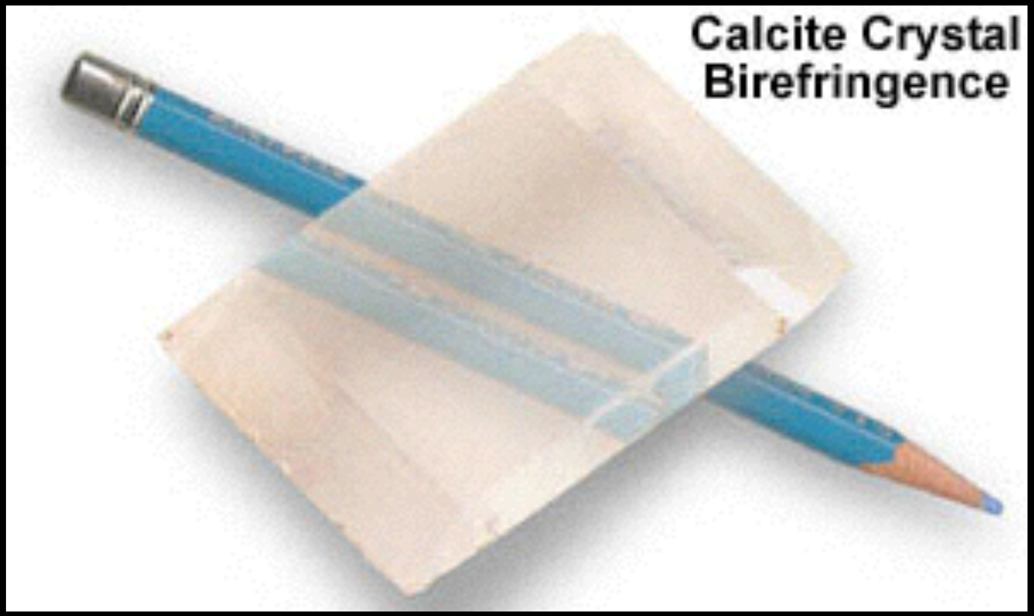
- Light from the day time sky is sun light "Rayleigh scattered" by molecules in the atmosphere.
- Sky light is polarized, maximally at 90 degrees to the sun.

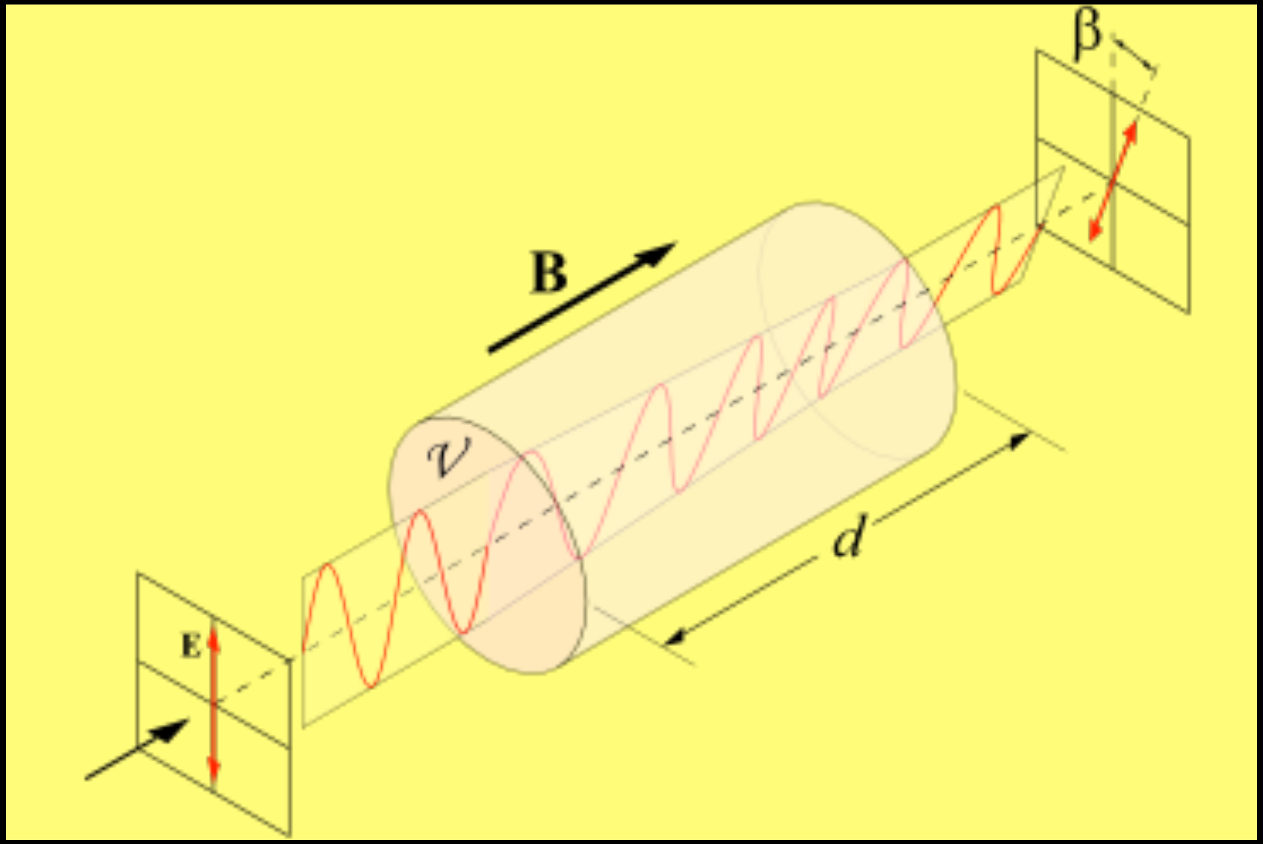


- The CMB is believed to be partially polarized because of Thomson scattering

Birefringence

Birefringence occurs when light passes through anisotropic material whose refractive index differs for the two polarization modes.



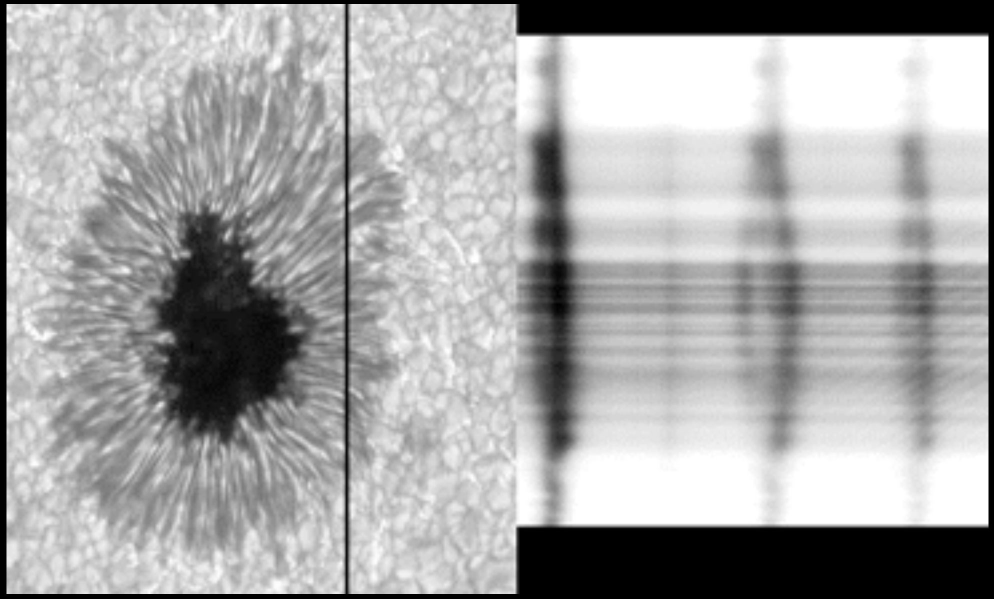


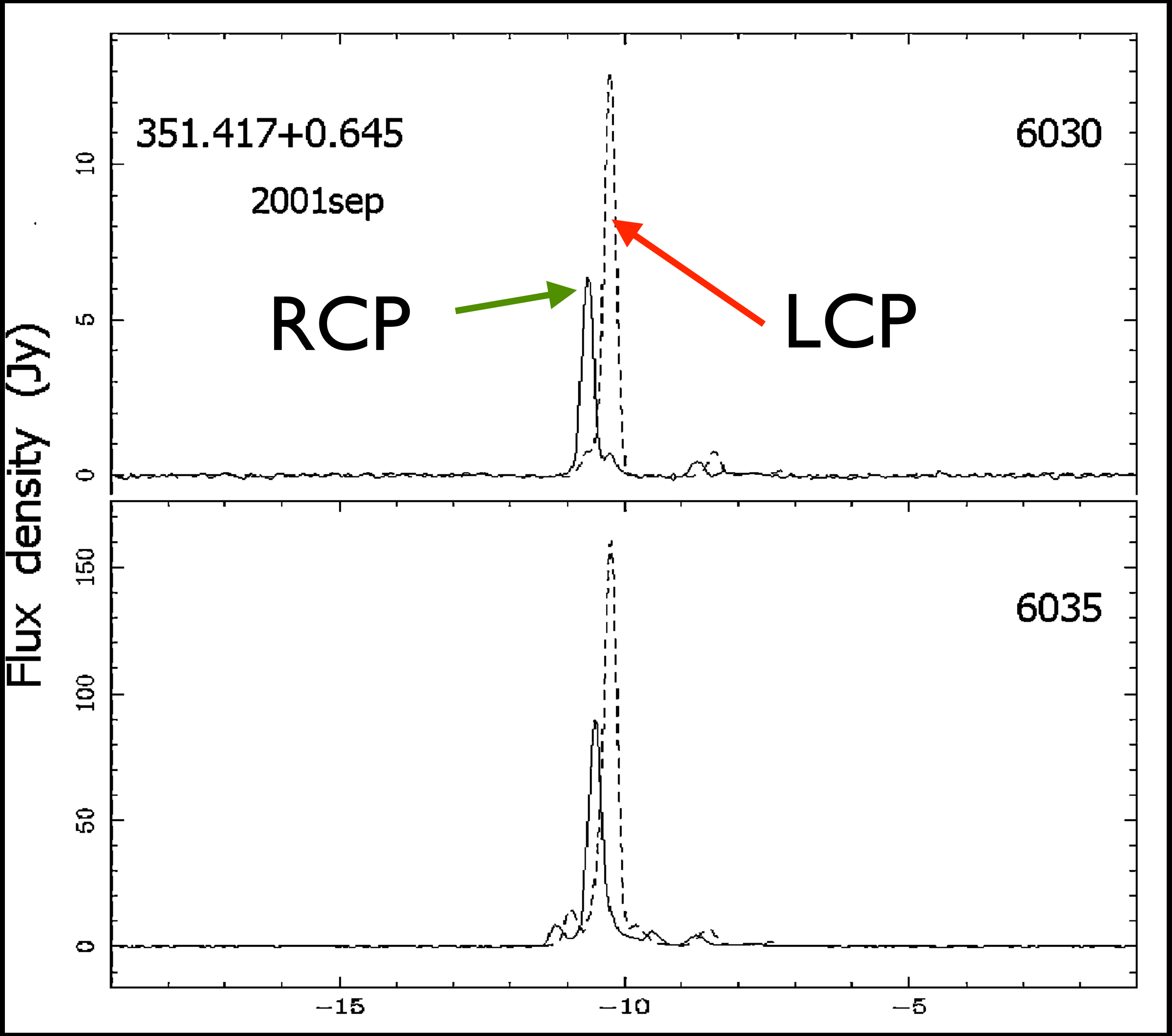
Linear modes birefringent

Circular modes birefringent

Zeeman Effect

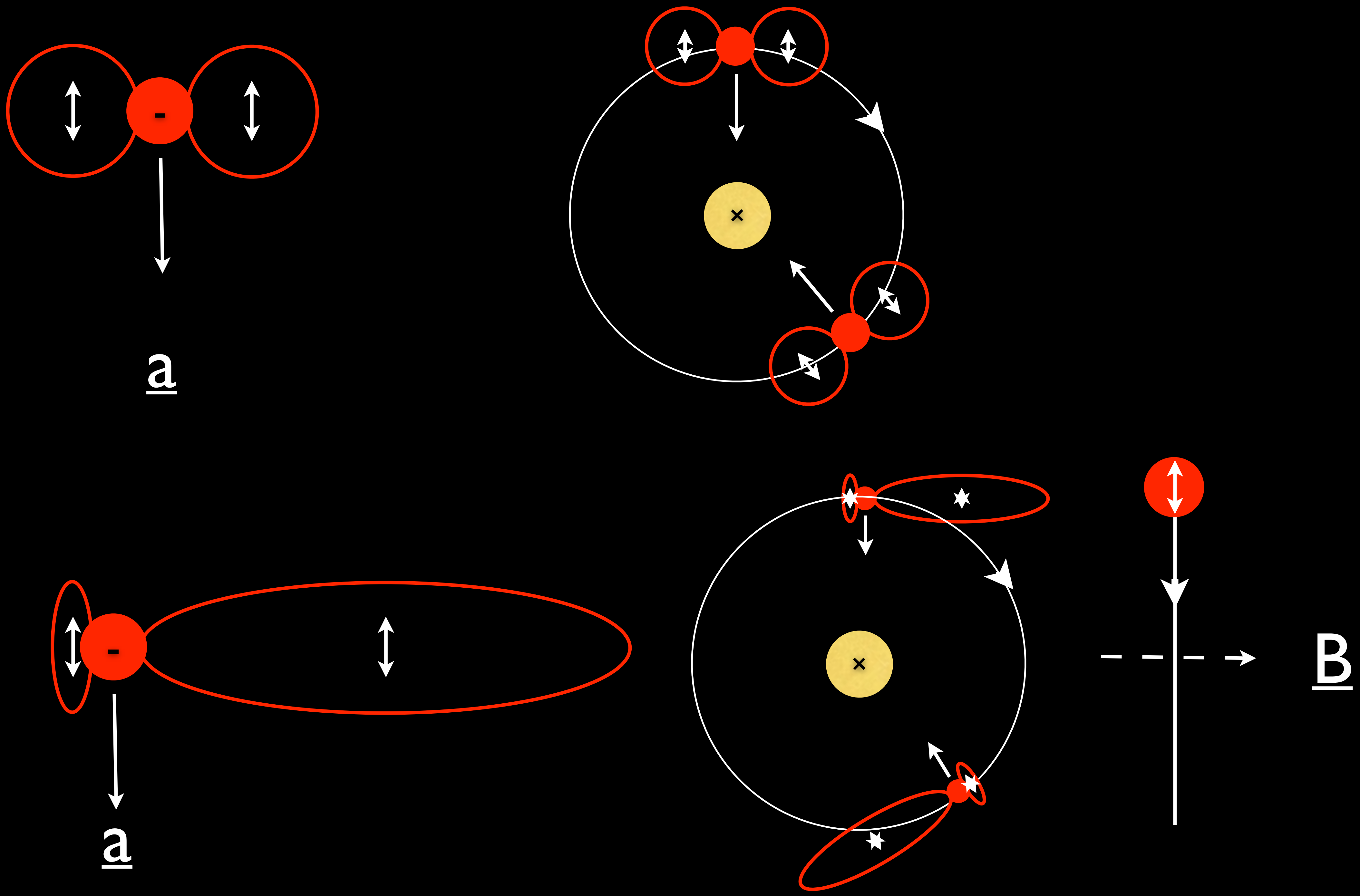
Atoms and molecules with a net magnetic moment will have their levels split in the presence of a magnetic field.





Caswell, 2003
6 GHz methanol maser

Cylotron & Synchrotron



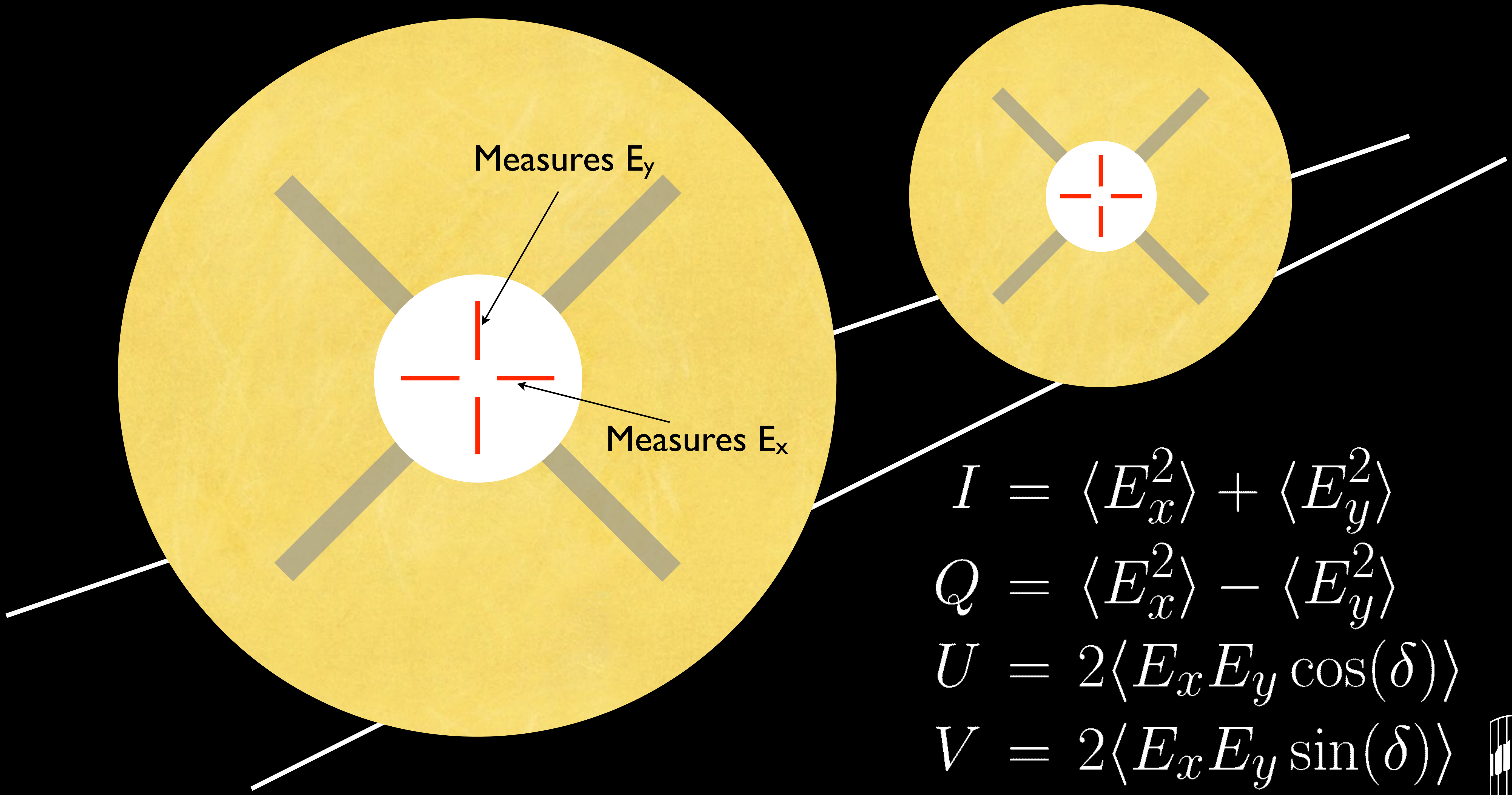
Faraday rotation

Magnetised plasmas are birefringent: the two circular modes have refractive index dependent on the parallel component of the magnetic field, the electron density, and the wave frequency.
The relative phase of the two modes changes along the propagation path, and so does the position angle ψ of the resultant linearly polarized radiation.

$$\psi = \mathrm{RM}\,\lambda^2$$

$$\mathrm{RM}^{(SI)} = \frac{e^3}{8\pi^2\varepsilon_0 m^2 c^3}\int_0^d n_e B\ \mathrm{d}s = 2.62\times 10^{-13}\int_0^d n_e B\ \mathrm{d}s,$$

How is it measured?

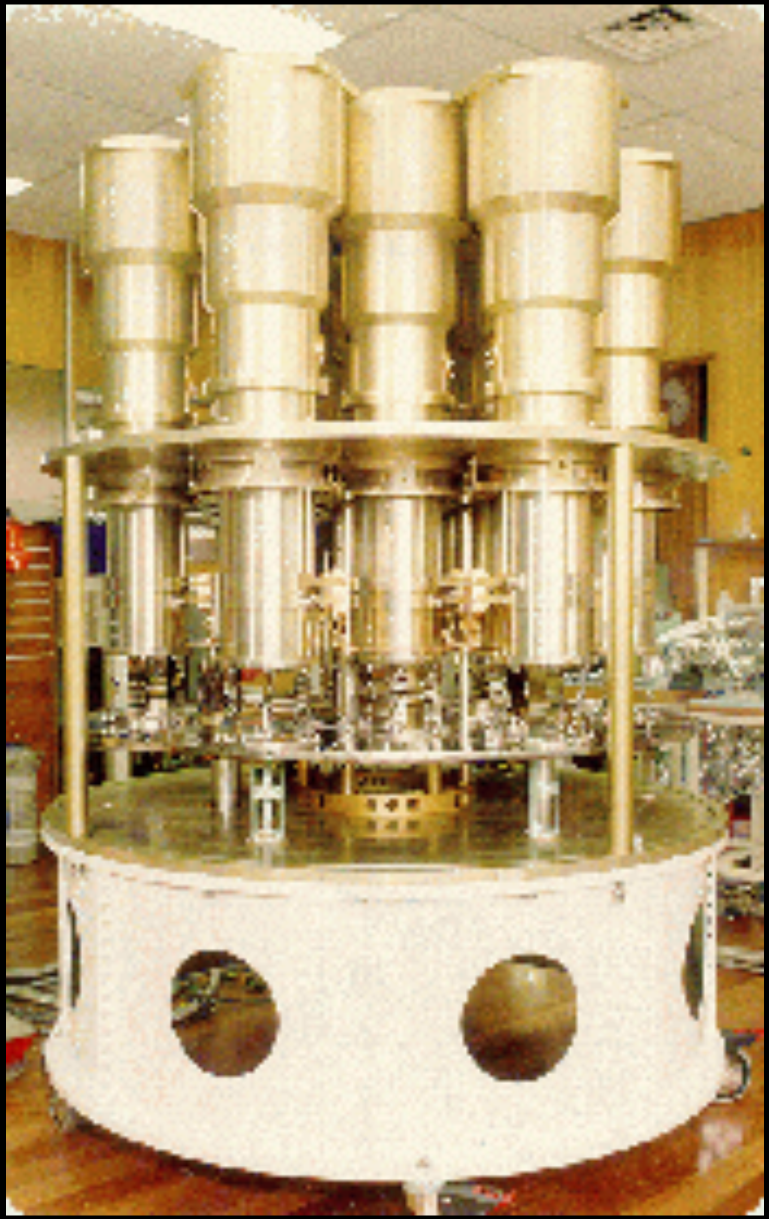


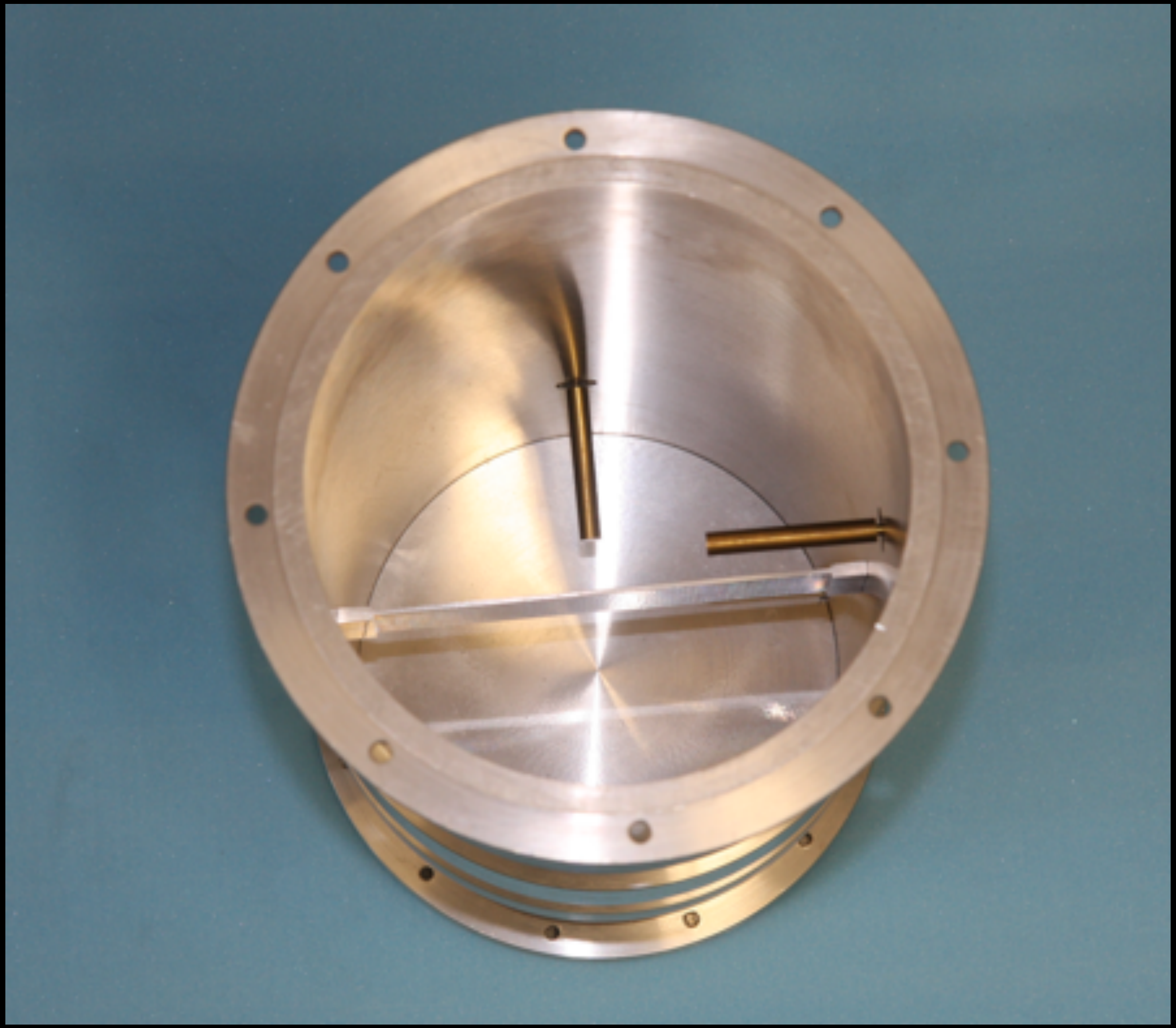
Measures E_y

Measures E_x

$$\begin{aligned} I &= \langle E_x^2 \rangle + \langle E_y^2 \rangle \\ Q &= \langle E_x^2 \rangle - \langle E_y^2 \rangle \\ U &= 2\langle E_x E_y \cos(\delta) \rangle \\ V &= 2\langle E_x E_y \sin(\delta) \rangle \end{aligned}$$

How is it measured?



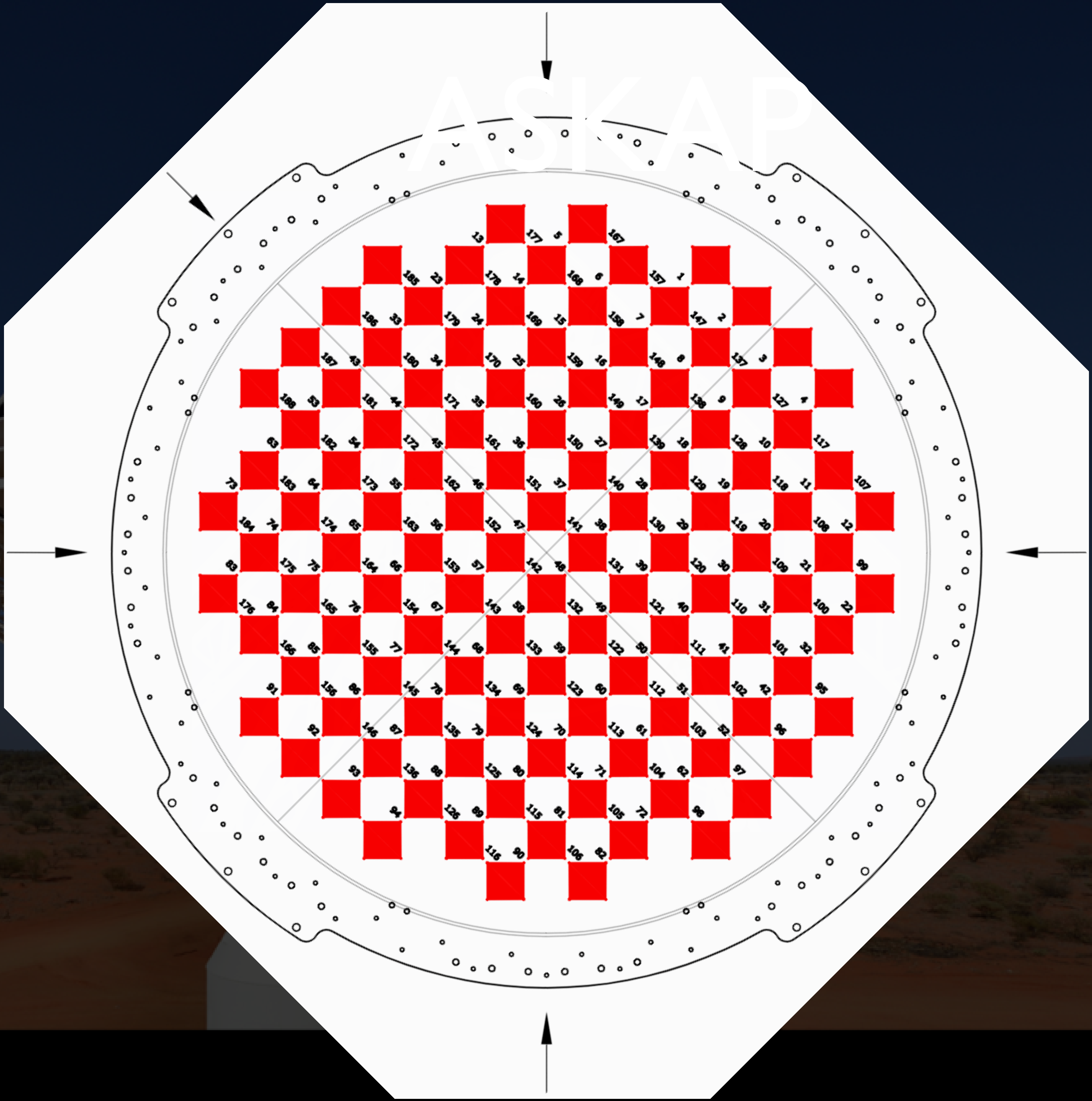


ASKAP

ASKAP

Phase Array Feed

ASKAP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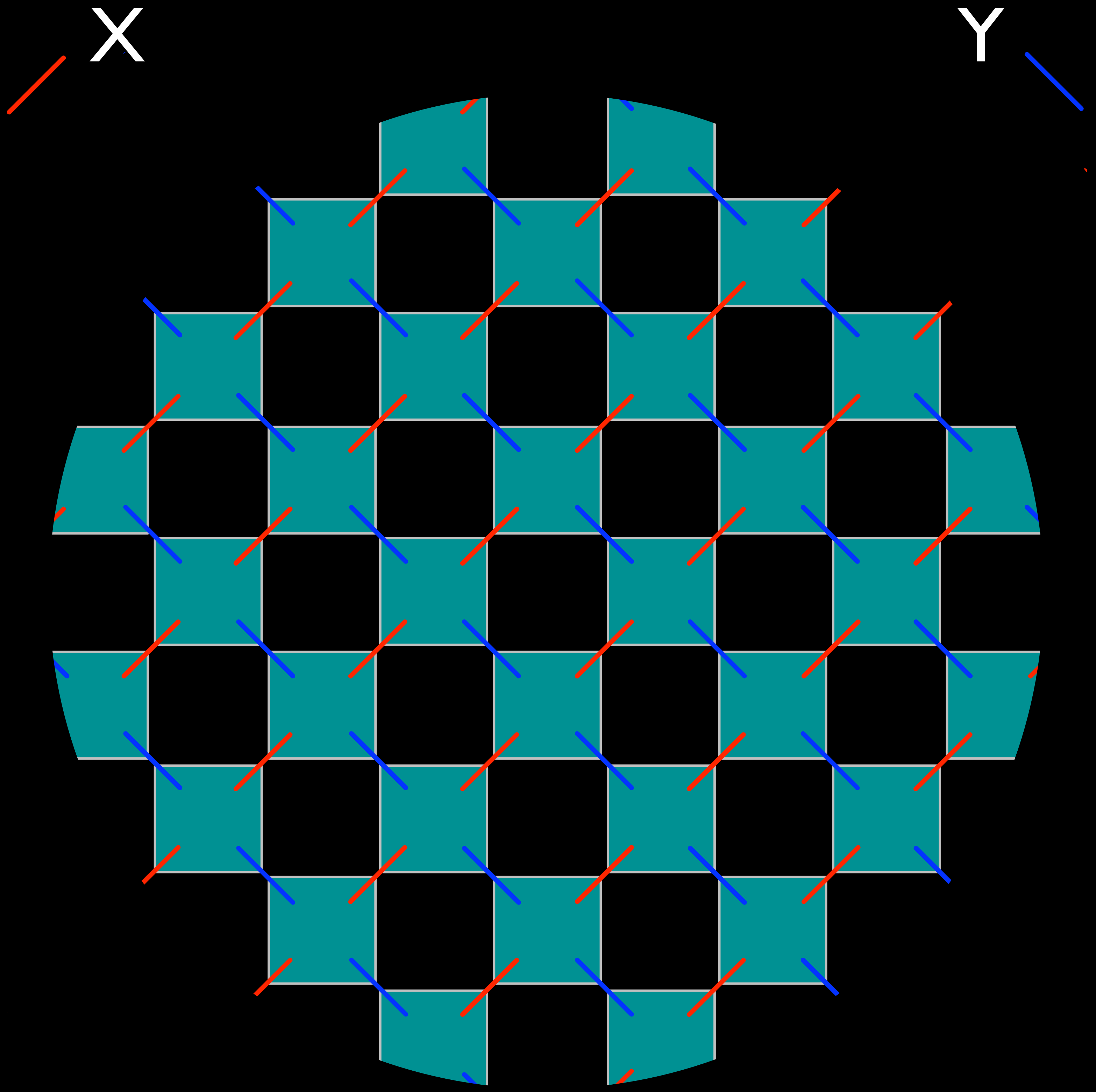

X
Y

Synthesis Imaging

- I, Q, U and V are equivalent for aperture synthesis imaging, but for the possibility (certainty) of negative Q, U, V.
- We can define and measure visibilities for each of the Stokes quantities: for antennas p and q : $\langle {E_x}^2 \rangle \Rightarrow \langle E_{xp} E_{xq} \rangle$,

$$\langle E_x\, E_y \rangle \Rightarrow \langle E_{xp} E_{yq} \rangle \text{ , etc}$$

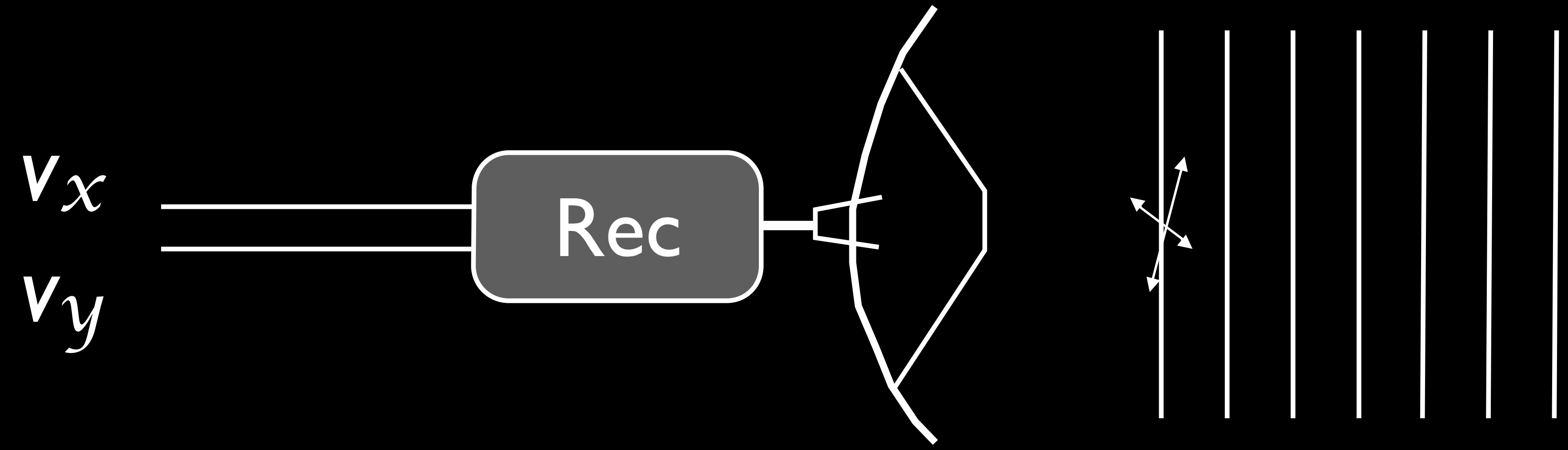
Why is it so difficult?

- Depolarization leads to weak signals
- Conceptual difficulties - it is complicated
- Instrumental effects can be significant and difficult to separate from the signal

Instrumental imperfections

- Leakage - a little E_x detected in y-feed, ...
- phase errors (in general, complex gain variations)
- polarization response varies within the beam

Jones calculus

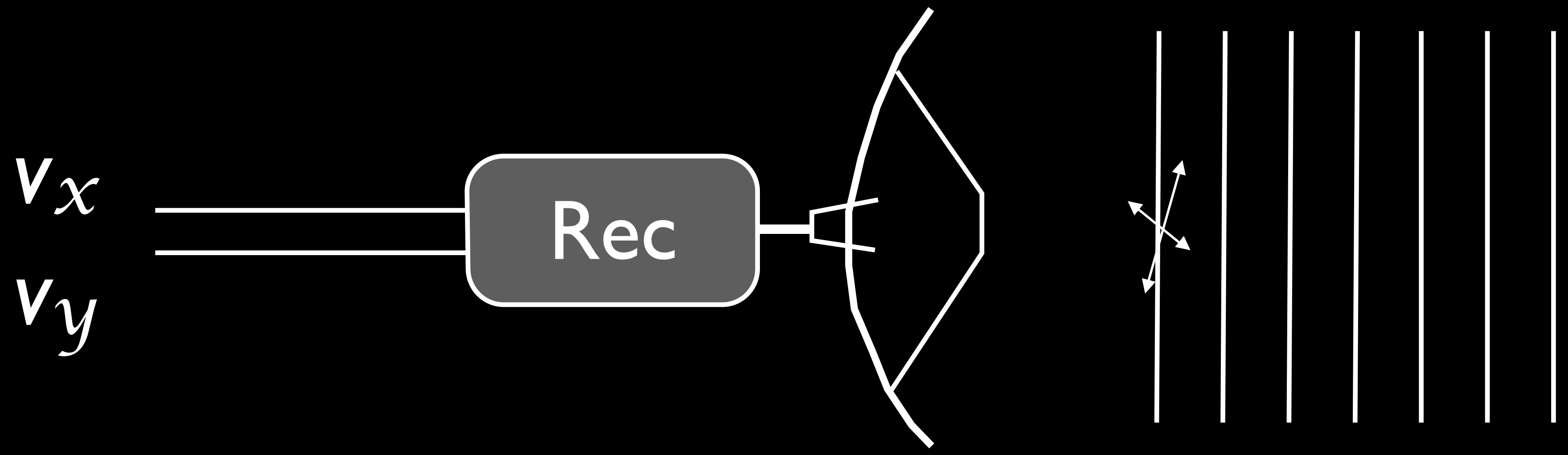


$$\begin{pmatrix} v_x \\ v_y \end{pmatrix}_o = \begin{pmatrix} G_{xx} & G_{xy} \\ G_{yx} & G_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix}_o = \begin{pmatrix} 1 & D_{i,x} \\ -D_{i,y} & 1 \end{pmatrix} \begin{pmatrix} G_{i,x} & 0 \\ 0 & G_{i,y} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$$

Jones calculus

$$v = Je$$



$$J = GDCPF$$

Ionospheric Faraday

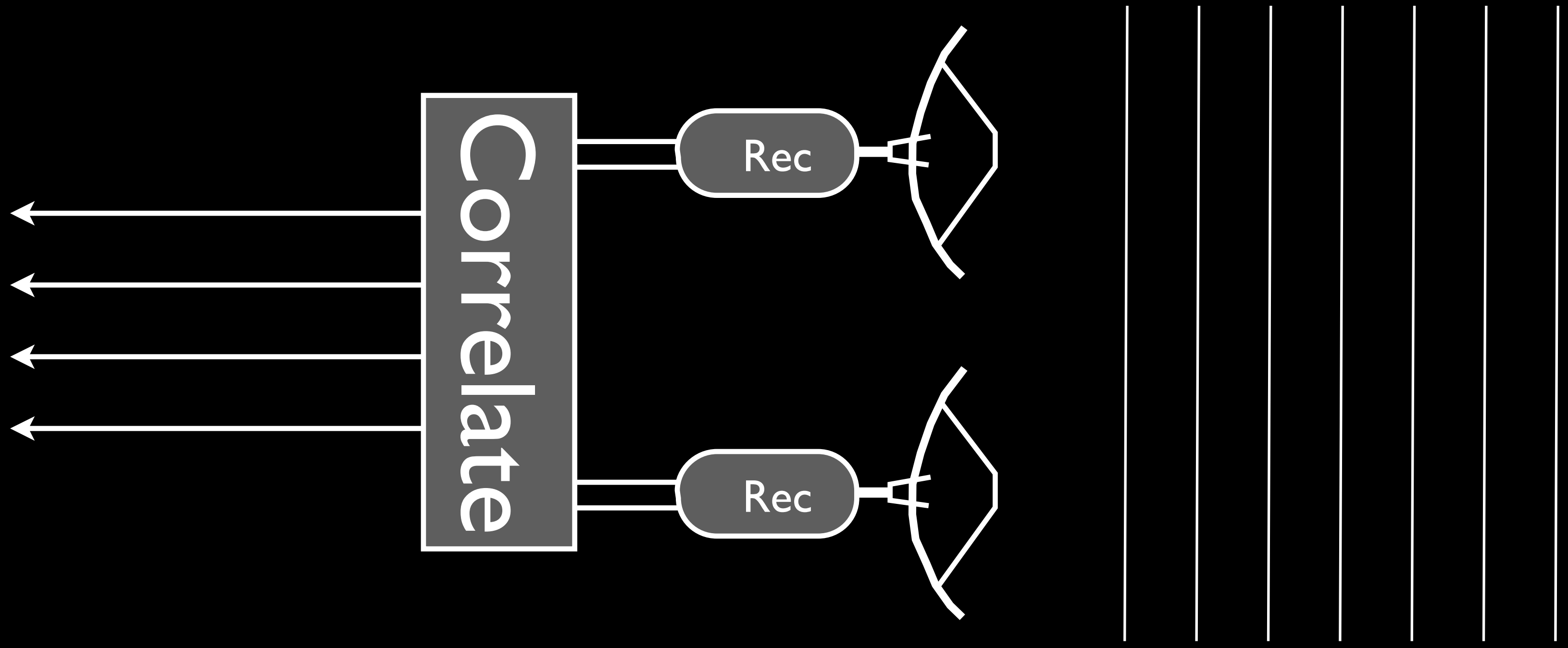
Parallactic angle

Properties of Feed

Feed errors

Complex receiver gains

Mueller calculus

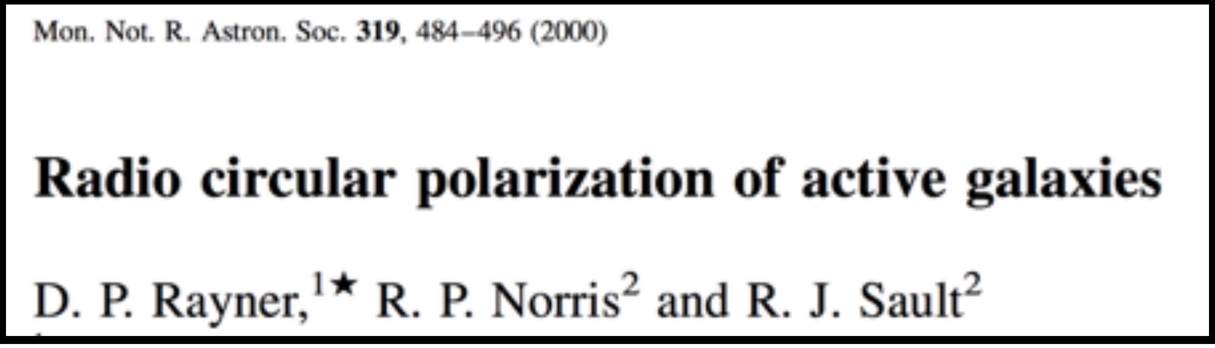


Measured visibilities

$$\begin{pmatrix} v_{xx} \\ v_{xy} \\ v_{yx} \\ v_{yy} \end{pmatrix} = \begin{bmatrix} \text{4 x 4 matrix} \end{bmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

ATCA excels

The ATCA is comparatively easy to use for polarimetry, thanks to its excellent properties (linear feeds, stable receivers) and to the miriad software package that uses the Jones and Mueller calculi to calibrate data with very little effort from the user.



Mon. Not. R. Astron. Soc. **319**, 484–496 (2000)

Radio circular polarization of active galaxies

D. P. Rayner,[1★] R. P. Norris[2] and R. J. Sault[2]

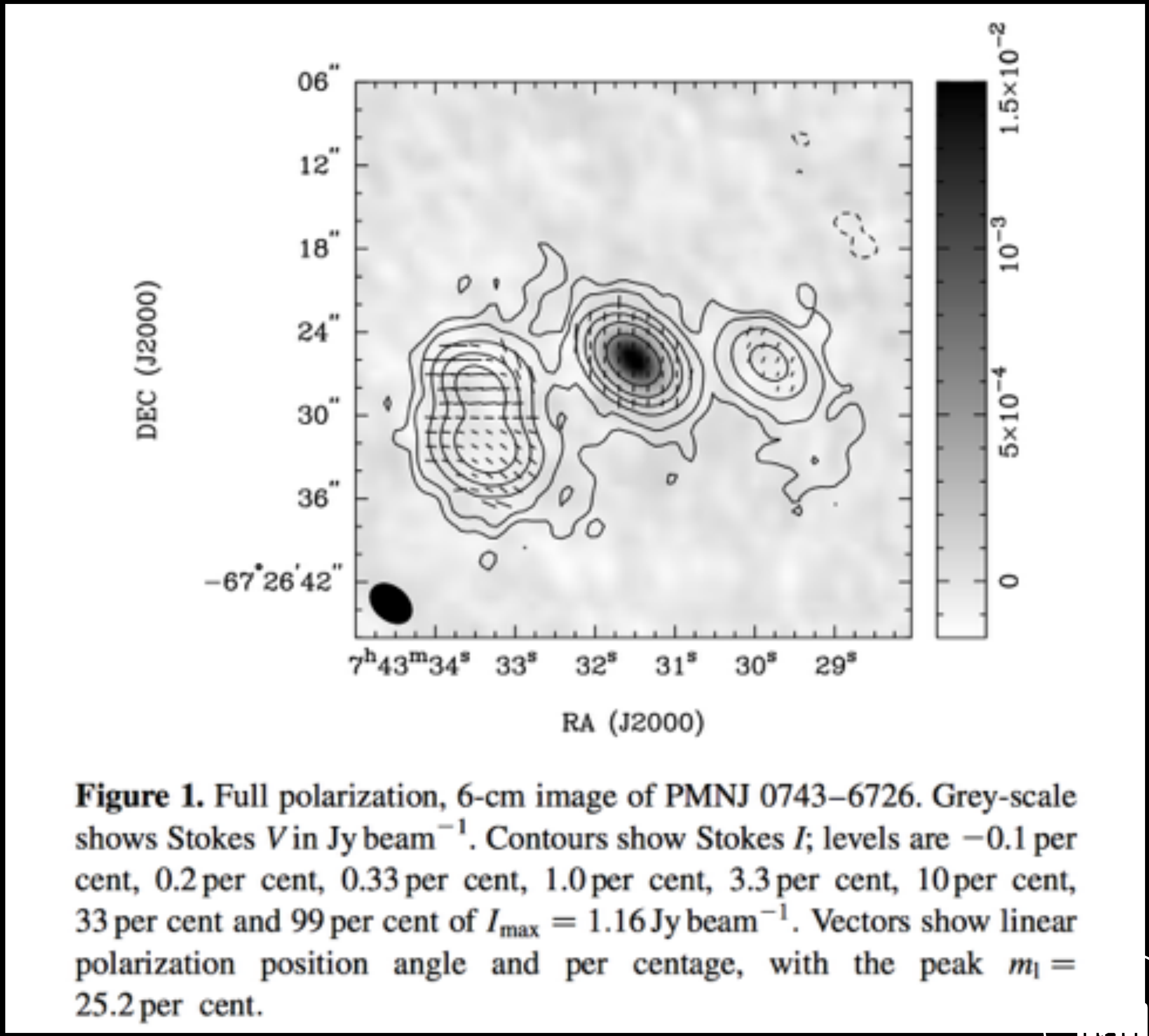


Figure 1. Full polarization, 6-cm image of PMNJ 0743−6726. Grey-scale shows Stokes *V* in Jy beam^{-1}. Contours show Stokes *I*; levels are −0.1 per cent, 0.2 per cent, 0.33 per cent, 1.0 per cent, 3.3 per cent, 10 per cent, 33 per cent and 99 per cent of $I_{\max} = 1.16$ Jy beam^{-1}. Vectors show linear polarization position angle and per centage, with the peak $m_l =$ 25.2 per cent.

References

- Radhakrishnan. Polarisation. URSI proceedings (1990) pp. 34
- Hamaker et al. Understanding radio polarimetry. I. Mathematical foundations. Astronomy and Astrophysics Supplement (1996) vol. 117 pp. 137
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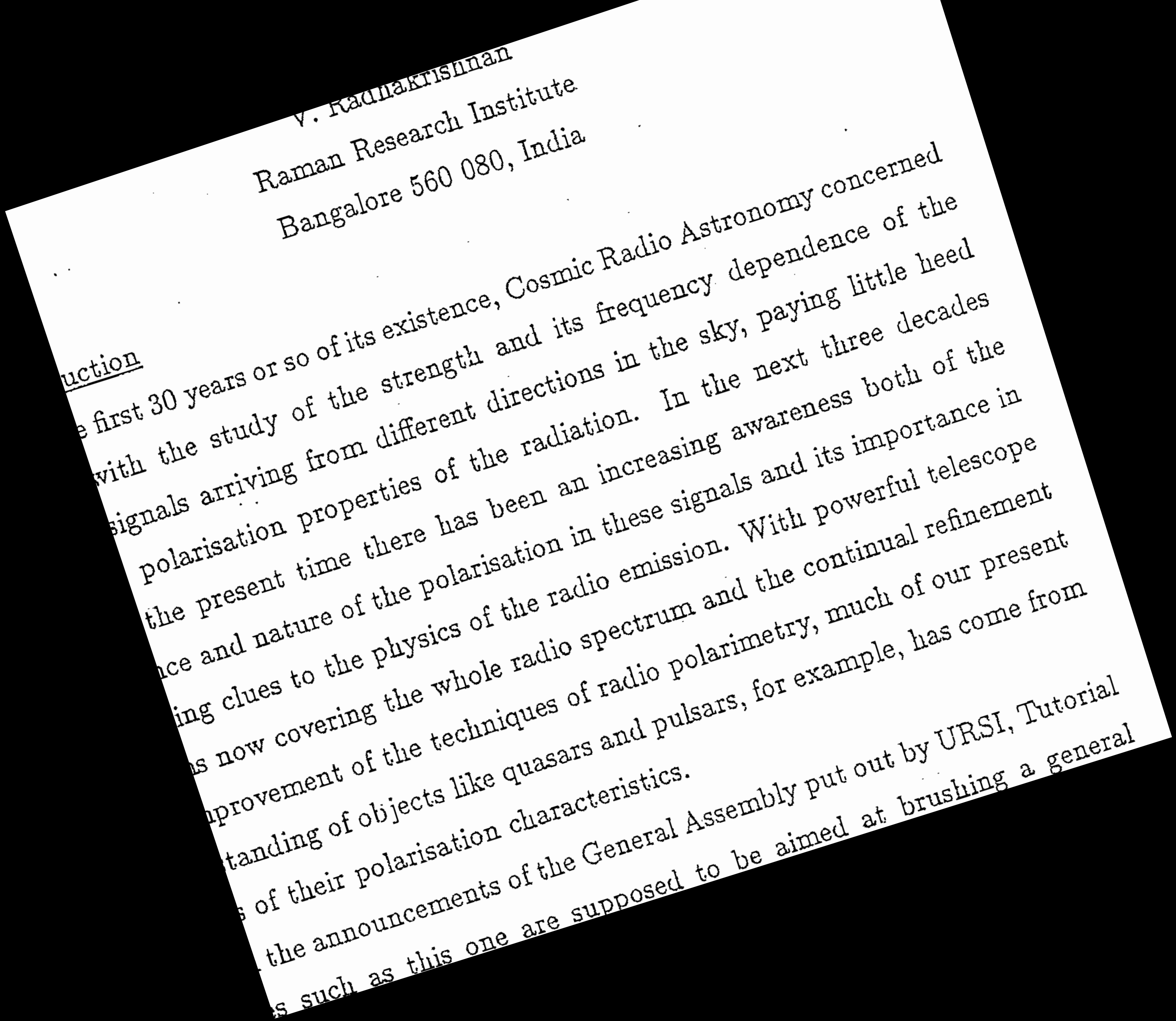

V. Radhakrishnan
Raman Research Institute
Bangalore 560 080, India

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e first 30 years or so of its existence, Cosmic Radio Astronomy concerned with the study of the strength and its frequency dependence of the signals arriving from different directions in the sky, paying little heed polarisation properties of the radiation. In the next three decades the present time there has been an increasing awareness both of the ce and nature of the polarisation in these signals and its importance in ing clues to the physics of the radio emission. With powerful telescope s now covering the whole radio spectrum and the continual refinement nprovement of the techniques of radio polarimetry, much of our present tanding of objects like quasars and pulsars, for example, has come from s of their polarisation characteristics.

the announcements of the General Assembly put out by URSI, Tutorial s such as this one are supposed to be aimed at brushing a general

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