Principles of Interferometry III

Jamie Stevens | ATCA Senior Systems Scientist

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Outline

- Closer to reality!
  - Actual bandwidth
  - Time delays and Earth rotation
  - Time averaging
  - Telescope hardware design
- Coordinate systems

In this talk, perfection will be assumed. You'll hear about where this assumption breaks down, and which later talks will deal with it.

The Visibility

Recall, the visibility is what an interferometer "sees":

$$V_\nu(\mathbf{b}) = \iint I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{b}.\mathbf{s}/c} d\Omega$$

This is the "ideal" equation, but in practice we can't assume:

- Monochromaticity: we can't just receive one specific wavelength
- Stationary sources: Earth keeps rotating
- Instantaneity: averaging over time will be required

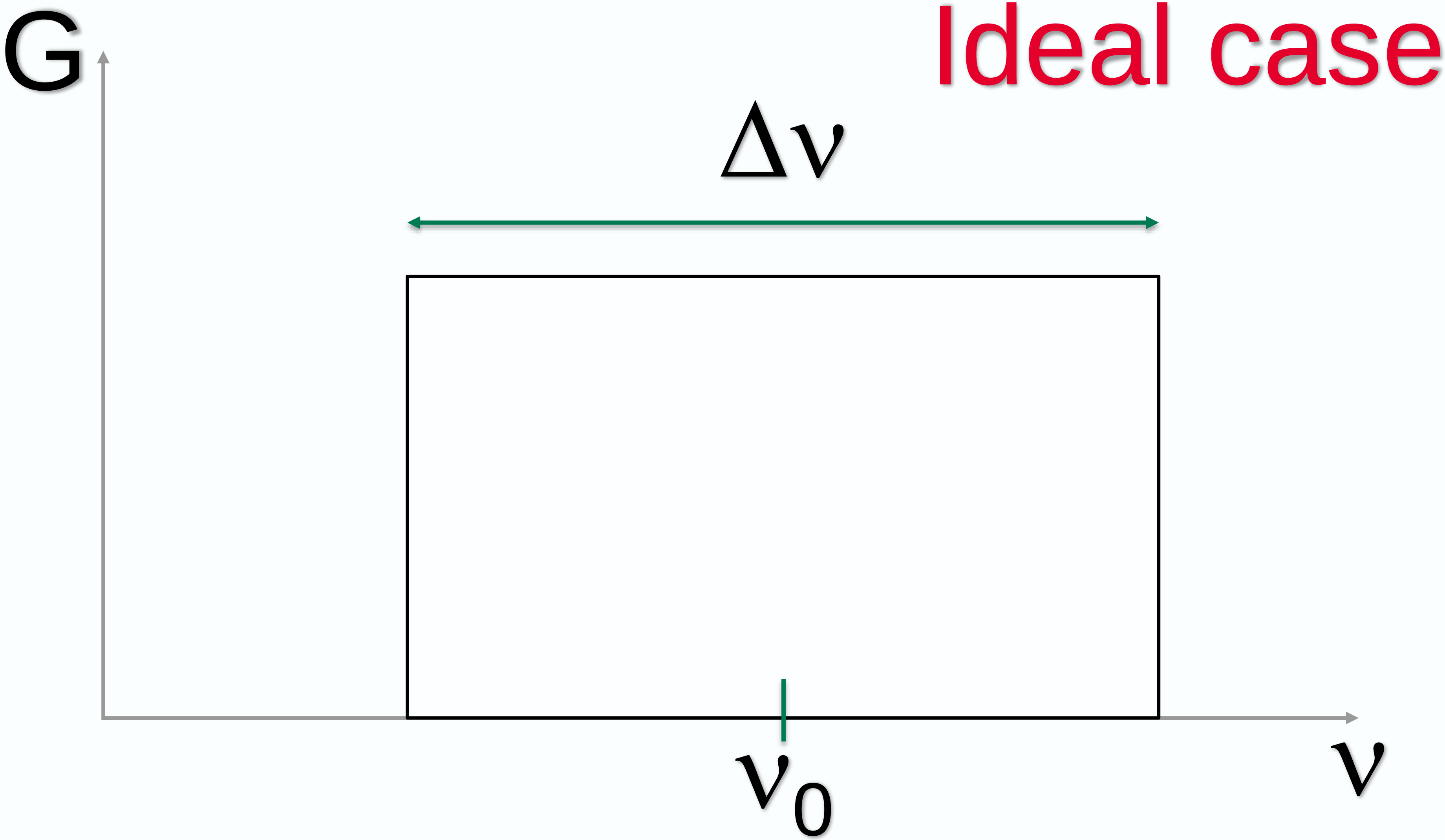
System Bandwidth

A receiving system will let in a range of frequencies. What does that mean for our interferometer?

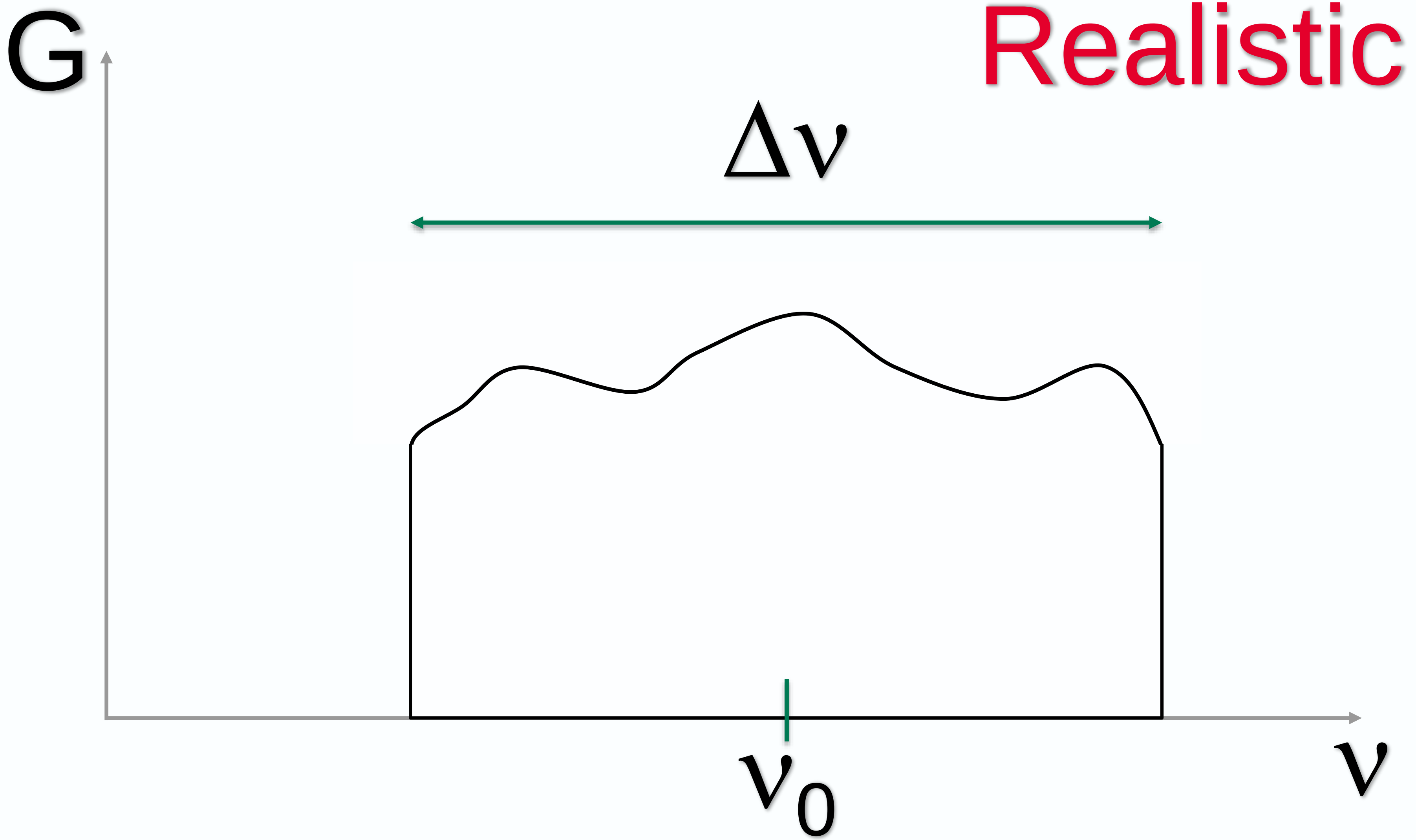
We define the frequency response as the amplitude and phase variation of the signal as a function of frequency.

The receiving system will not respond the same way to all frequencies, and neither will many parts of the Universe between the telescope and the source.

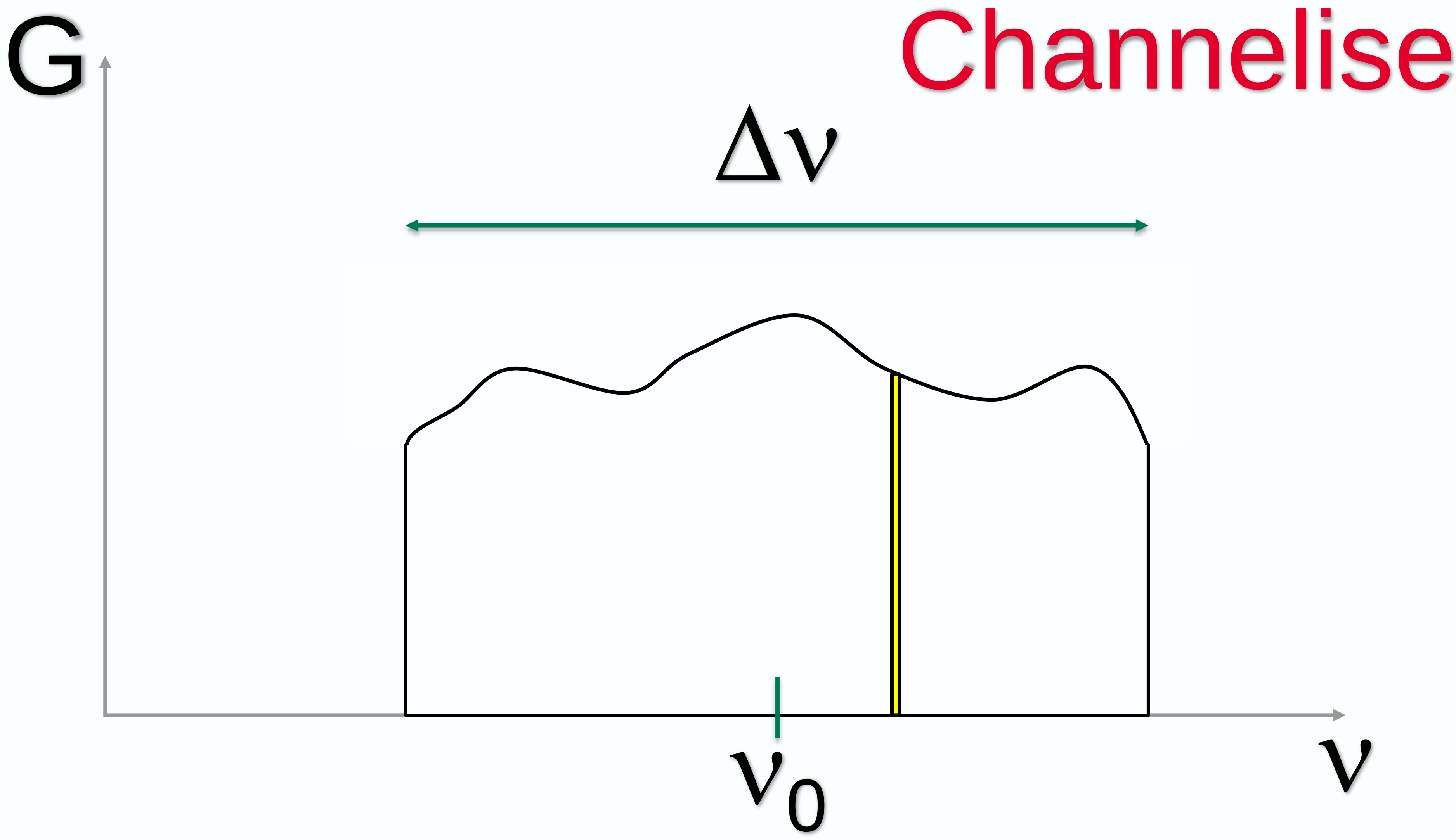
Frequency Response Function



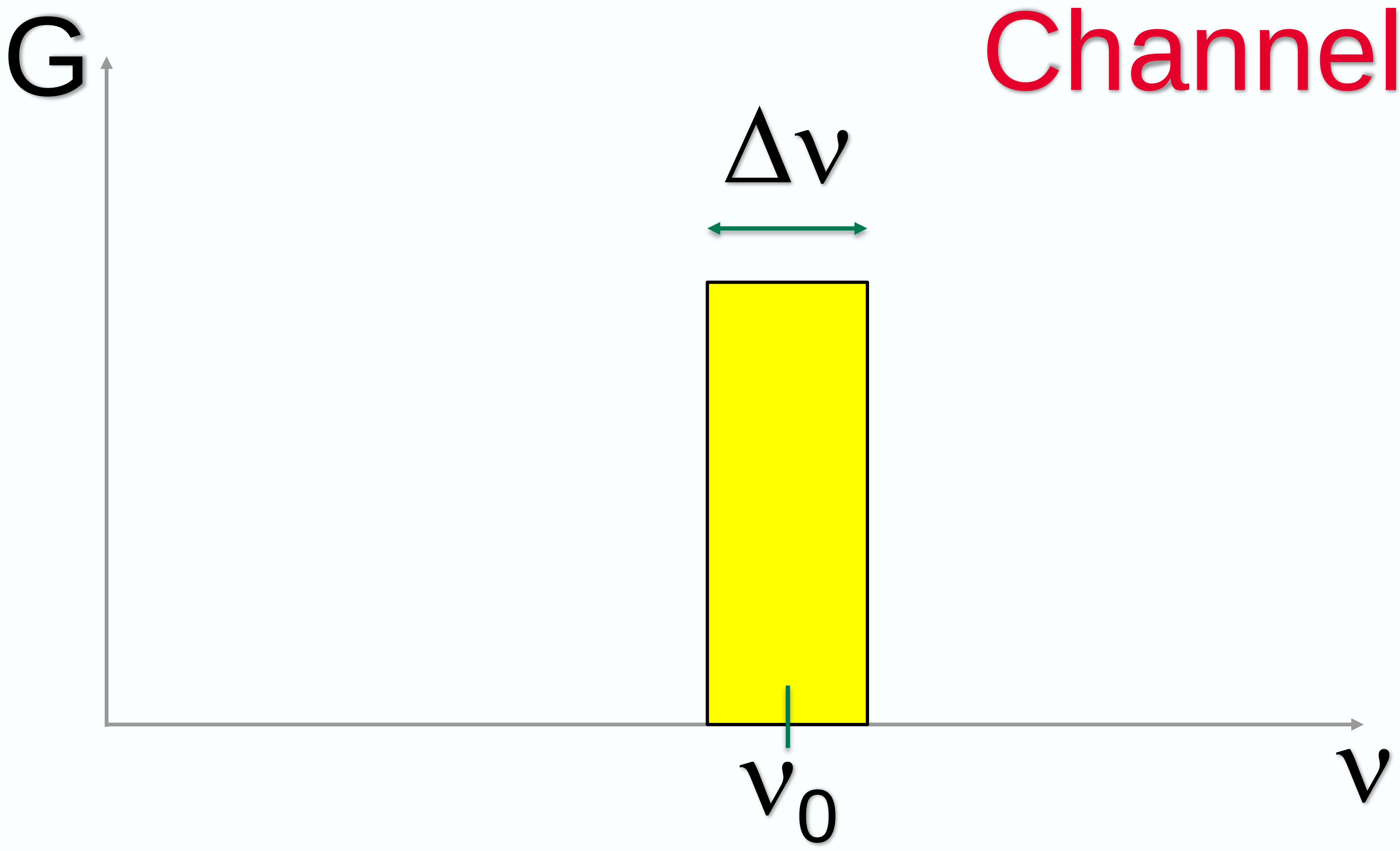
Frequency Response Function



Frequency Response Function



Frequency Response Function



Finite Bandwidth Response

Easiest to examine in 1D. Recall the fringe function:

$$F = \cos(2\pi\nu\tau_g)$$

You would have a different fringe function for every different frequency in your bandwidth. The effect is that each frequency's fringes interfere with each other.

$$\frac{1}{\Delta\nu}\int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2}\cos\left(2\pi\nu\tau_g\right)d\nu = \cos\left(2\pi\nu_0\tau_g\right)\frac{\sin\left(2\pi\Delta\nu\tau_g\right)}{\left(2\pi\Delta\nu\tau_g\right)}$$

We identify the $\mathrm{sinc}(x) = \sin(\pi x)\,/\pi x$ function, which acts as an envelope for the fringe function on the sky.

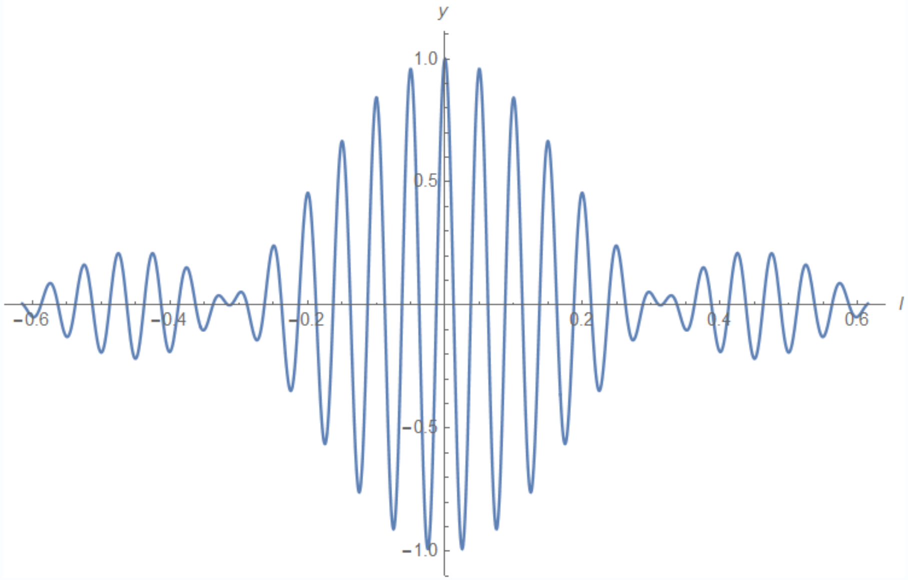
Example

For a square bandpass, the fringe pattern goes to 0 when $\Delta\nu\tau_g = 1$, which gives:

$$l = \sin\theta = \frac{c}{D\Delta\nu}$$

For ATCA's longest baseline, $D = 6000$m, and normal continuum resolution, $\Delta\nu = 1$ MHz, the first fringe null is 2.864 degrees away from the fringe maximum.

For our shortest baseline, $D = 31$m, and coarsest continuum resolution, $\Delta\nu = 64$ MHz, first fringe null is 8.691 degrees away.



For ATCA's lowest frequency, the primary beam FWHM is 0.71 degrees.

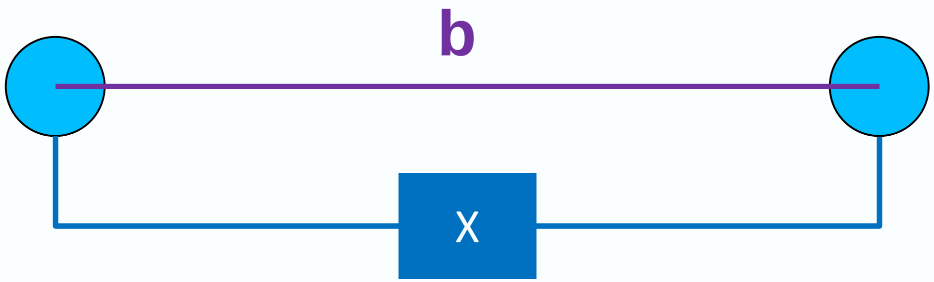
Moving the fringes

We have a fringe pattern, attenuated by bandwidth, for each interferometer baseline. Only at the very centre of these fringe patterns is the response uniform across all baselines.

We want to point these fringe pattern centres at the source we want to look at: how can we achieve this?

Answer: add some time delay.

Time Delay Illustrated



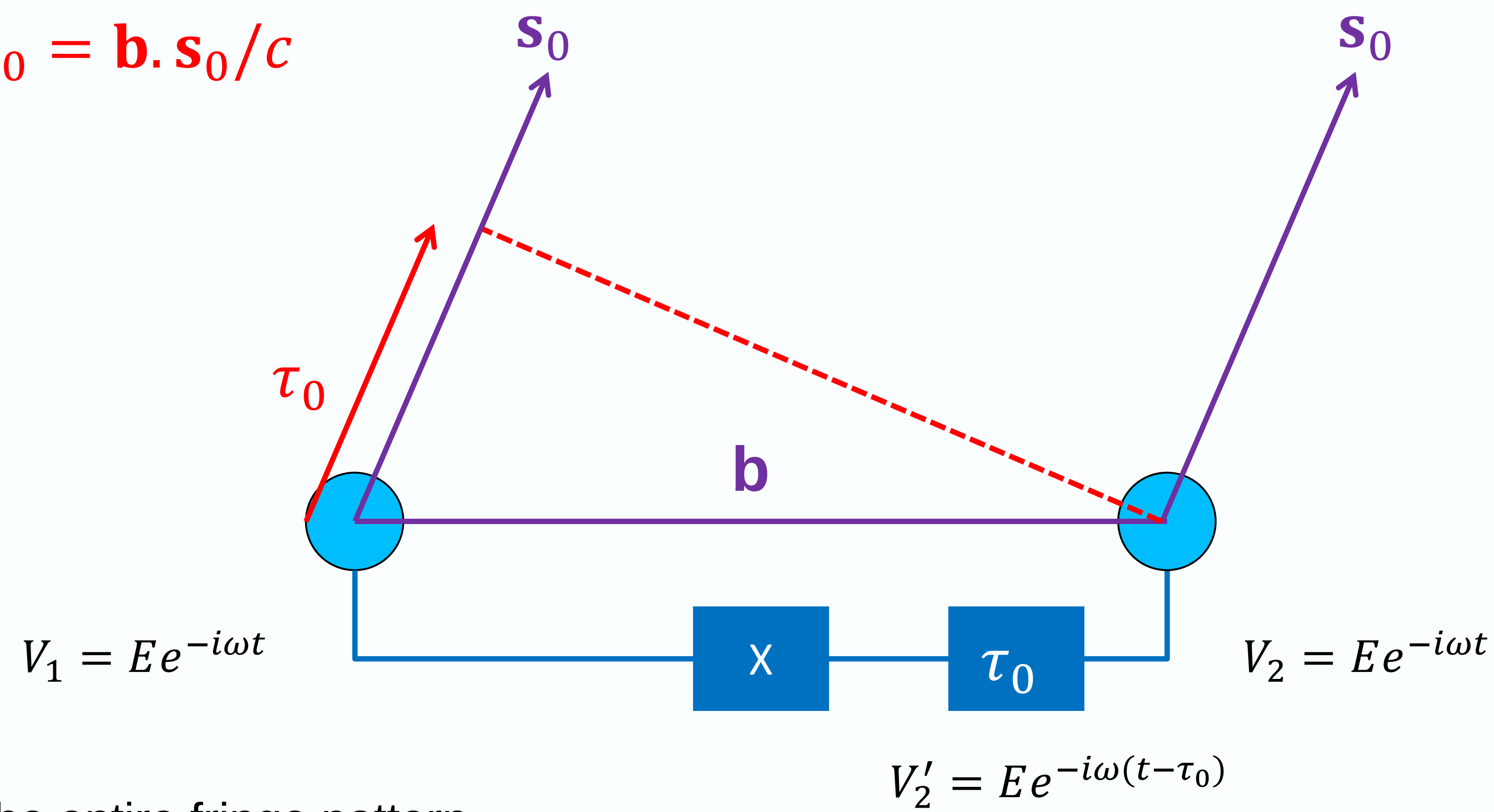
$V_1 = Ee^{-i\omega t}$

$V_2 = Ee^{-i\omega t}$

Fringe pattern maximum comes from waves that hit both receivers at the same time t **– at the meridian**

Time Delay Illustrated

$\tau_0 = \mathbf{b}.\mathbf{s}_0/c$

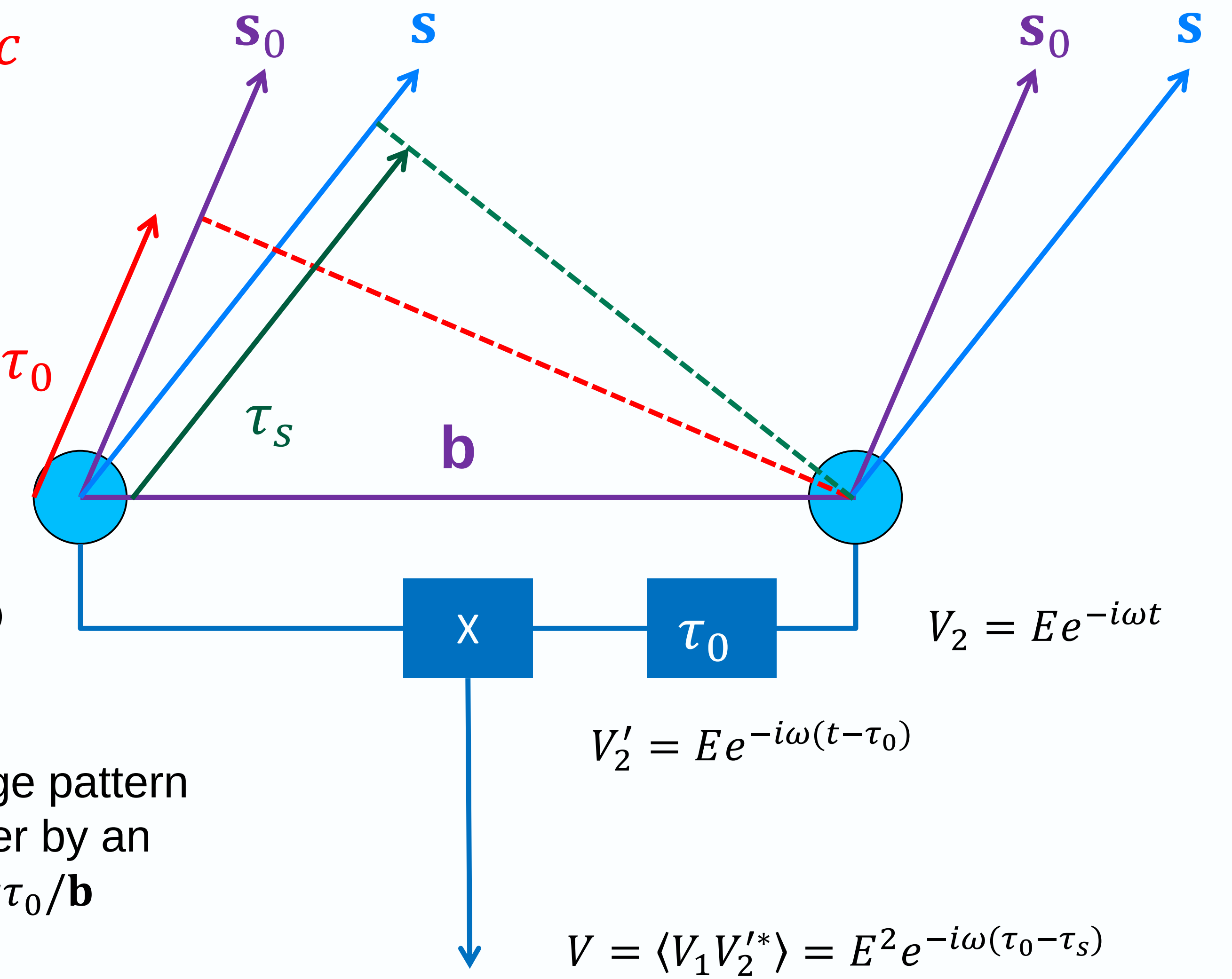


The entire fringe pattern has moved over by an angle $\sin\theta = c\tau_0/\mathbf{b}$

Time Delay Illustrated

$\tau_0 = \mathbf{b}.\mathbf{s}_0/c$

$\tau_s = \mathbf{b}.\mathbf{s}/c$



$V_1 = Ee^{-i\omega(t-\tau_s)}$

$V_2 = Ee^{-i\omega t}$

$V_2' = Ee^{-i\omega(t-\tau_0)}$

$V = \langle V_1 V_2'^* \rangle = E^2 e^{-i\omega(\tau_0-\tau_s)}$

The entire fringe pattern has moved over by an angle $\sin\theta = c\tau_0/\mathbf{b}$

Time Delay Accuracy

- Need to match τ_0 as closely as possible to τ_s to minimise coherence loss.
- To keep phase differences to much less than 1 radian, we need to have $(\tau_0 - \tau_s) \ll 1/\nu$, which for ATCA observations requires nanosecond accuracy.
- If the proper delay is inserted in one arm of the baseline, we can keep our fringe response function maximised toward our source. If the source is moving wrt our baseline, then we'll need to continually change our delay.

Earth Rotation Compensation

Usually, sources won't be moving with respect to our baseline – they are a long way away. But the Earth makes our baseline move with respect to the source.

The rate at which Earth's rotation moves a baseline's fringe pattern over a source at declination δ is

$$\nu_f = u\omega_E \cos\delta \,,$$

where $u = D/\lambda$ (the E-W baseline in wavelengths), and

$\omega_E = 7.3 \times 10^{-5}$rad/s (the angular rotation rate of the Earth).

For a frequency of 50 GHz, baseline length of 6km, $u \approx 10^6$, and for a source at the equator, $\nu_f \approx 73$ rad/s.

Time Averaging

Now we have the interferometer fringe pattern sitting stationary towards the source we're interested in.

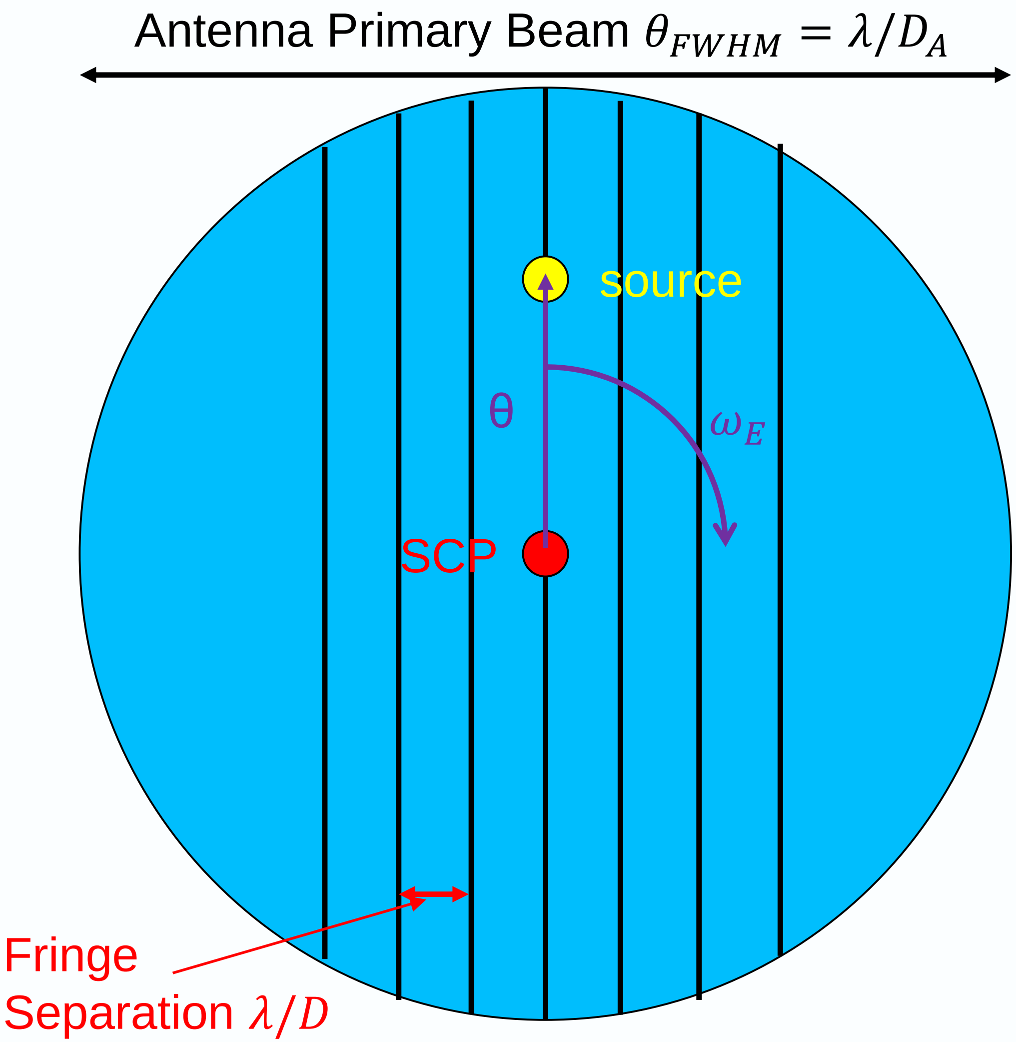
Can we now just start averaging the output of the interferometer over time to decrease the random noise?

Yes!

(*Provided that all you are interested in is the point source at the centre of your field*)

All the other stuff in your field will be moving with respect to your fringe pattern.

Time Smearing



Sources rotate about the pole at angular rate:

$$\omega_E = 7.3 \times 10^{-5} \text{ rad/s}$$

Source at angular distance θ will move by λ/D due to Earth rotation in time:

$$t = \frac{\lambda}{D} \frac{1}{\omega_E \sin\theta}$$

For $\theta = \theta_{FWHM}$,

$$t \approx \frac{D_A}{\omega_E D}$$

Time Smearing

How long can we integrate before a source at the edge of the primary beam goes through the coherence spacing?

For ATCA, $D_A = 22$m, and $D = 6000$m, so $t \approx \frac{D_A}{D\omega_E} = 50$ seconds.

If you needed to keep the whole hemisphere coherent at 2100 MHz:

$$t \approx \frac{\lambda}{D\omega_E} = 326 \text{ milliseconds}$$

Your visibility amplitudes will be severely attenuated if you average longer than these times.

ATCA sets its integration time to 10 seconds (maximum) to avoid these losses.

Intermediate Frequencies

We can't always work at the sky frequency.

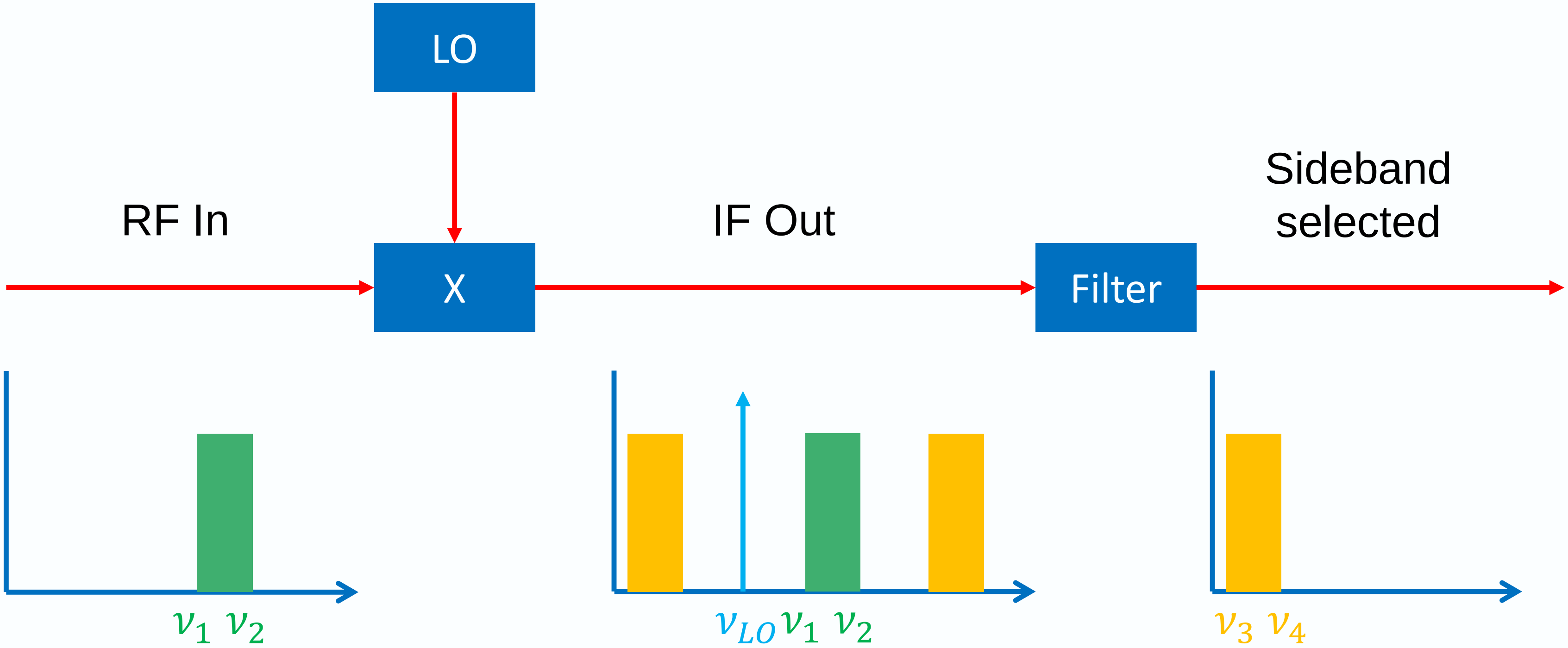
- Electronics work better at lower frequencies.
- Different frequencies would require different systems, making the telescope more complicated.

Radio frequency (RF) signals can be mixed with other radio frequency signals to make intermediate frequency (IF) signals, with almost no loss of information.

How does this frequency mixing affect our interferometer?

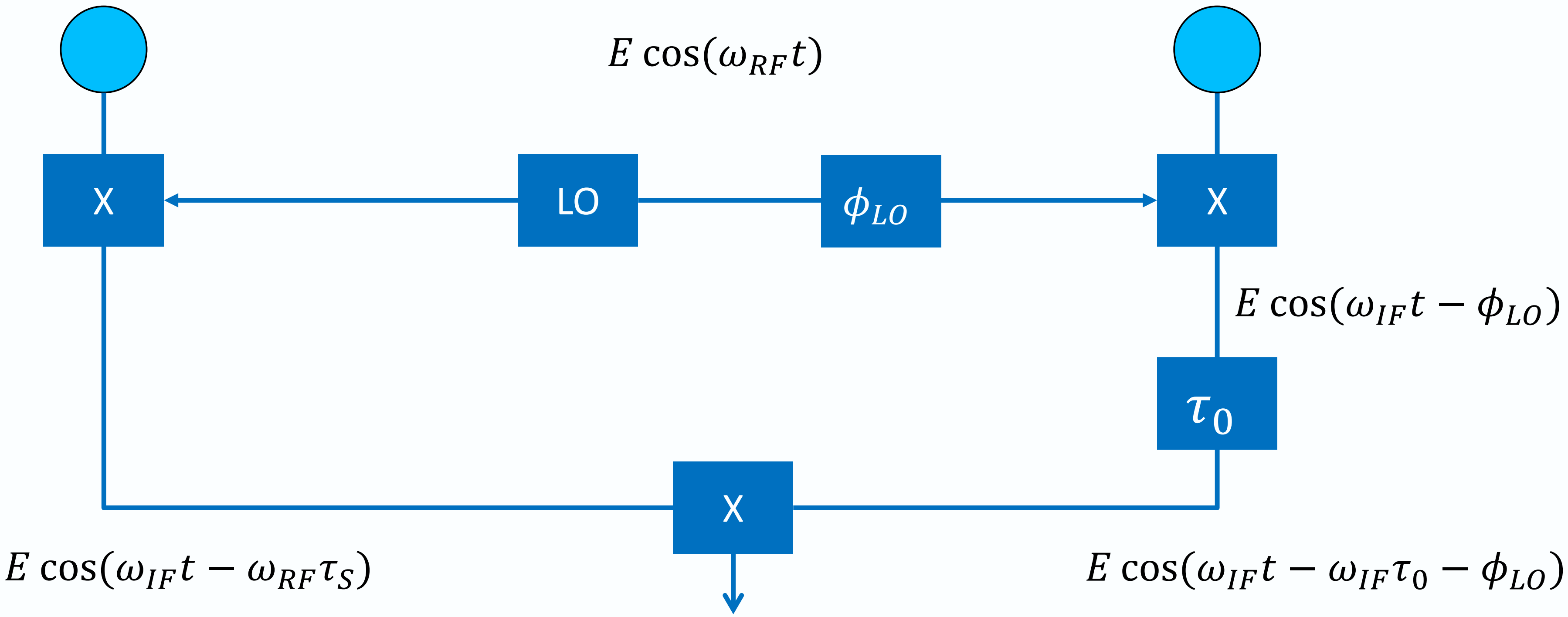
Frequency Conversion

In order to insert RF range ν_1 to ν_2 into the sampling system, which accepts the frequency range ν_3 to ν_4, we have to multiply the RF signal with a single frequency coming from a local oscillator (LO).



IF Delay System

Let’s look at our delay system again, but now with the LO included.



$$V = E^2 e^{-i(\omega_{RF}\tau_S - \omega_{IF}\tau_0 - \phi_{LO})}$$

Recovering the correct visibility phase

We've gone from having the RF interferometer phase:

$$\omega_{RF}(\tau_S - \tau_0)$$

To having the observed IF interferometer phase:

$$\omega_{RF}\tau_S - \omega_{IF}\tau_0 - \phi_{LO}$$

To make these the same, we need to set the LO phase to:

$$\phi_{LO} = \omega_{LO}\tau_0$$

(Recall, $\omega_{RF} = \omega_{IF} + \omega_{LO}$). This phase adjustment compensates for the delay being inserted at the IF, rather than at the RF.

Each telescope may do this differently. The old ATCA used to do this with the analogue LO, the ATCA with CABB does it digitally in the correlator.

Coordinate Systems & Geometry

You may have noticed that everything we have done here has been generalised in terms of two fundamental vectors.

$$\mathbf{b}$$

the baseline, which is the direction and separation of the interferometer elements

$$\mathbf{s}$$

a unit vector in the direction of the source being observed

We're now going to define the geometric coordinate frame for the interferometer.

2D Interferometer

A 2D interferometer is an easy start to the geometry, and is actually quite useful.

Our interferometer's coordinates will always be specified on the axes $(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})$, and the units of these axes will always be wavelengths.

A 2D interferometer is one in which all the elements lie in a single plane. In such a case, we can define all the baselines as:

$$\mathbf{b} = (\lambda u, \lambda v, \lambda w) = (\lambda u, \lambda v, 0)$$

Sky coordinates

The sky coordinate axes are labelled $(\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n})$.

The unit vector $\mathbf{s}$ has components:

$$\mathbf{s} = (l, m, n) = (l, m, \sqrt{1 - l^2 - m^2})$$

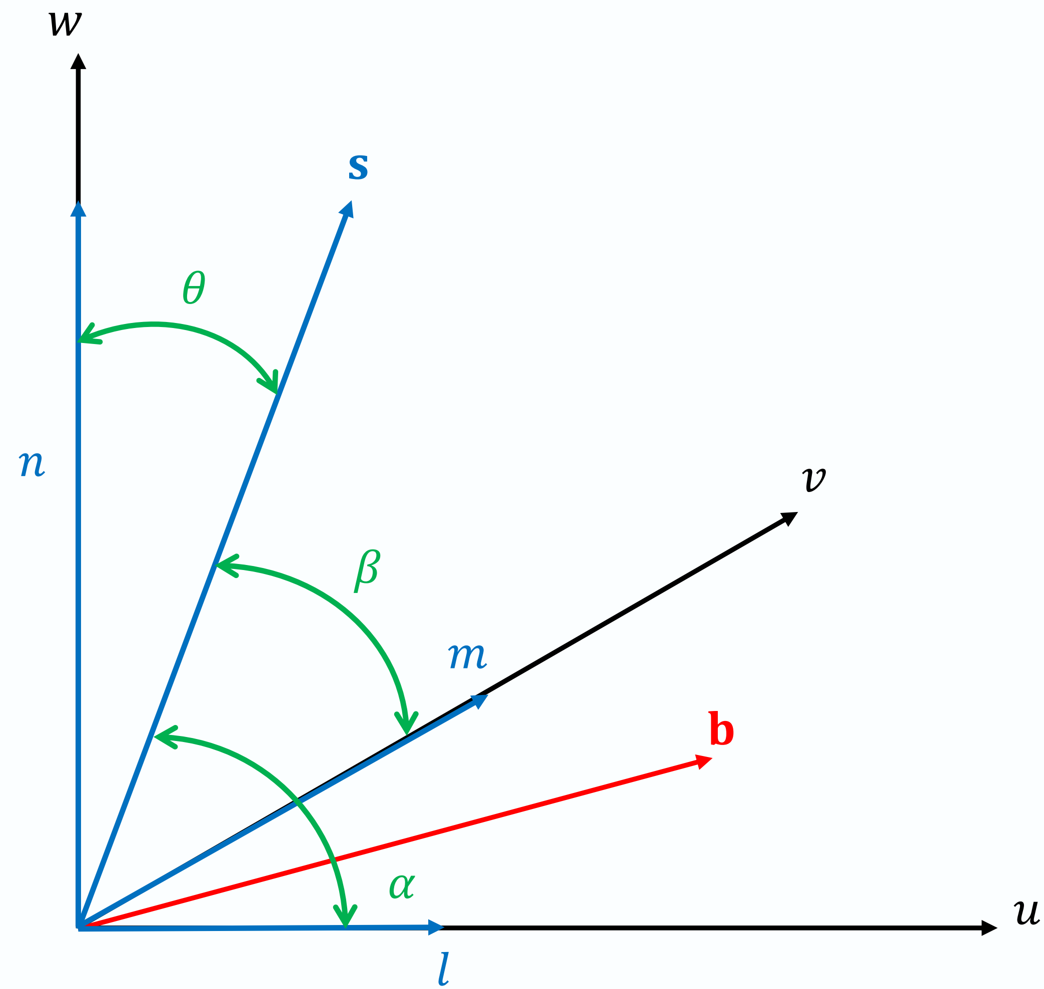
Direction Cosines – Source coordinates

The unit vector **s** is defined by its projections (l, m, n) on the (u, v, w) axes. The components are called the **Direction Cosines:**

$$l = \cos\alpha$$

$$m = \cos\beta$$

$$n = \cos\theta = \sqrt{1 - l^2 - m^2}$$



The Visibility - in real coordinates

Recall, the visibility:

$$V_\nu(\mathbf{b}) = \iint I_\nu(\mathbf{s}) e^{-2\pi i\nu \mathbf{b}.\mathbf{s}/c} d\Omega$$

In our coordinate system, we get $\nu\mathbf{b}.\mathbf{s}/c = ul + vm$. Thus,

$$V_\nu(u,v) = \iint I_\nu(l,m) e^{-i2\pi(ul+vm)} dldm$$

This is a Fourier transform between the projected brightness and the spatial coherence function, so we can state:

$$I_\nu(l,m) \Leftrightarrow V_\nu(u,v)$$

Josh will tell you how to do this!

Van Cittert-Zernicke Theorem

We measure the visibilities, and we want to recover the sky intensity that could cause those visibilities. So:

$$I_\nu(l,m) = \iint V_\nu(u,v)e^{i2\pi(ul+vm)}dudv$$

All we need to do is measure the visibilities over the entire continuum of (u,v) points, and we can recover what the sky must be… but of course that is impossible.

Later on (another talk), we'll see what has to be done to take real measurements and make an image of the sky.

2D Interferometers

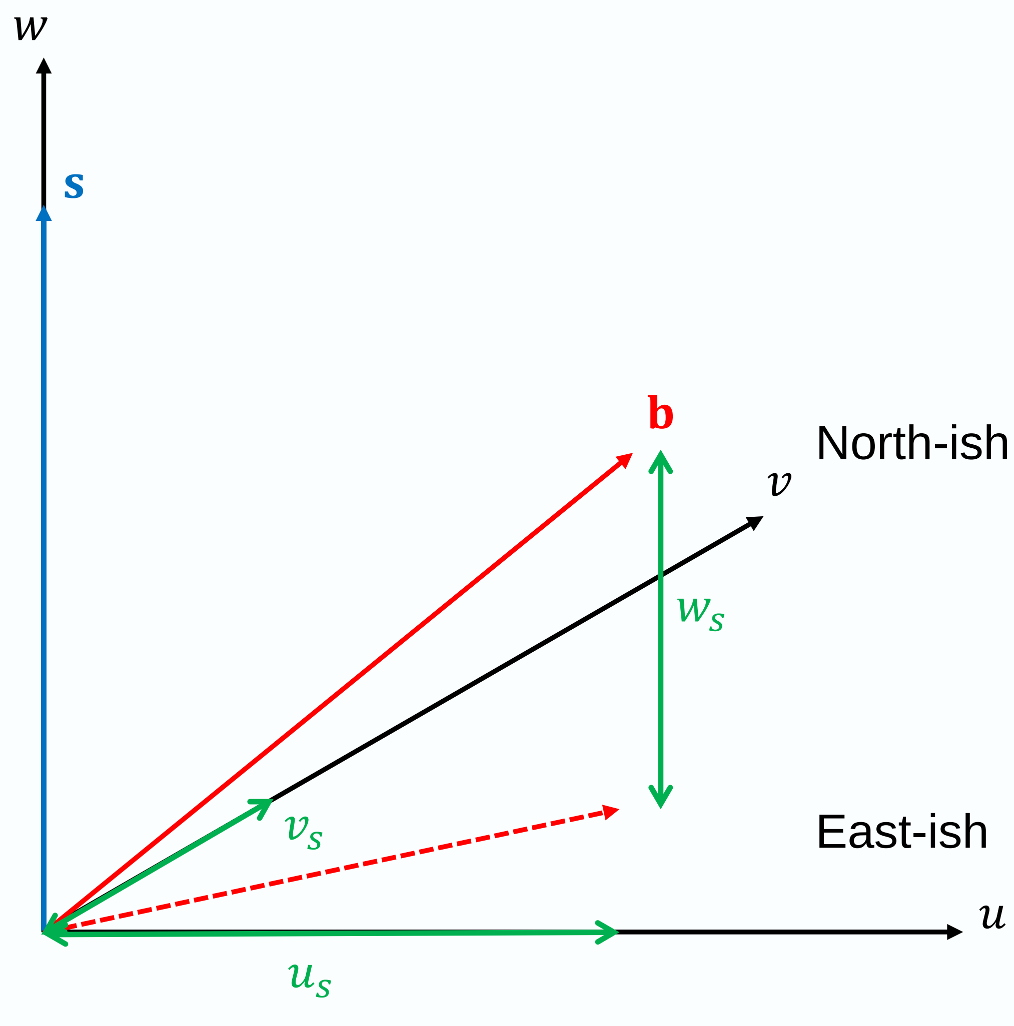
Any interferometer for which all baselines lie in a single plane can use the principles of 2D interferometry.

- ATCA and WSRT (Westerbork Synthesis Radio Telescope) are both East-West arrays, so they remain planar over time, with respect to the source.
  - For this plane, the w axis points toward the NCP. So anything close to 90 degrees away has poor resolution (on the celestial equator).
- VLA (Very Large Array) and GMRT (Giant Metrewave Radio Telescope) are both “instantaneous” 2D planar arrays; snapshot observations can be considered planar.
  - In a snapshot, the w axis points toward the zenith. So anything close to 90 degrees away has poor resolution (on the horizon).

General Coordinate System

Because the plane of a coplanar array rotates over time with respect to the source, we need to consider all three baseline components.

l and m now increase to the East and North respectively.



3D Interferometers

The interferometer is now measuring the coherence function through a volume. The visibility measured is thus:

$$V_\nu(u,v,w) = \iint I_\nu(l,m) e^{-i2\pi(ul+vm+wn)} dl dm$$

Notice that we are still integrating only over l and m, so this is not a Fourier transform.

We can use the LO again to do phase tracking, and adjust the phases by $e^{i2\pi w}$; this “stops” the fringes for the direction $l = m = 0$ (the phase centre). Thus:

$$V_\nu(u,v,w) = \iint I_\nu(l,m) e^{-i2\pi\left(ul+vm+w(\sqrt{1-l^2-m^2}-1)\right)} dl dm$$

3D to 2D

$$V_\nu(u,v,w) = \iint I_\nu(l,m) e^{-i2\pi\left(ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right)} dl dm$$

We can remove the w term if we can satisfy the condition:

$$w\left(1-\sqrt{1-l^2-m^2}\right) = w(1-\cos\theta) \sim w\theta^2/2 \ll 1$$

Which gives the further constraint:

$$\theta_{max} < \sqrt{1/2w} \sim \sqrt{\lambda/D} \sim \sqrt{\theta_{syn}}$$

Using this transform thus always results in errors in the images. Full 3D imaging works, but is expensive. Josh will say more about your options in his wide-field imaging talk.

Calculating (u, v)

How do we go from elements sitting on Earth to the appropriate (u, v) coordinates for the baseline?

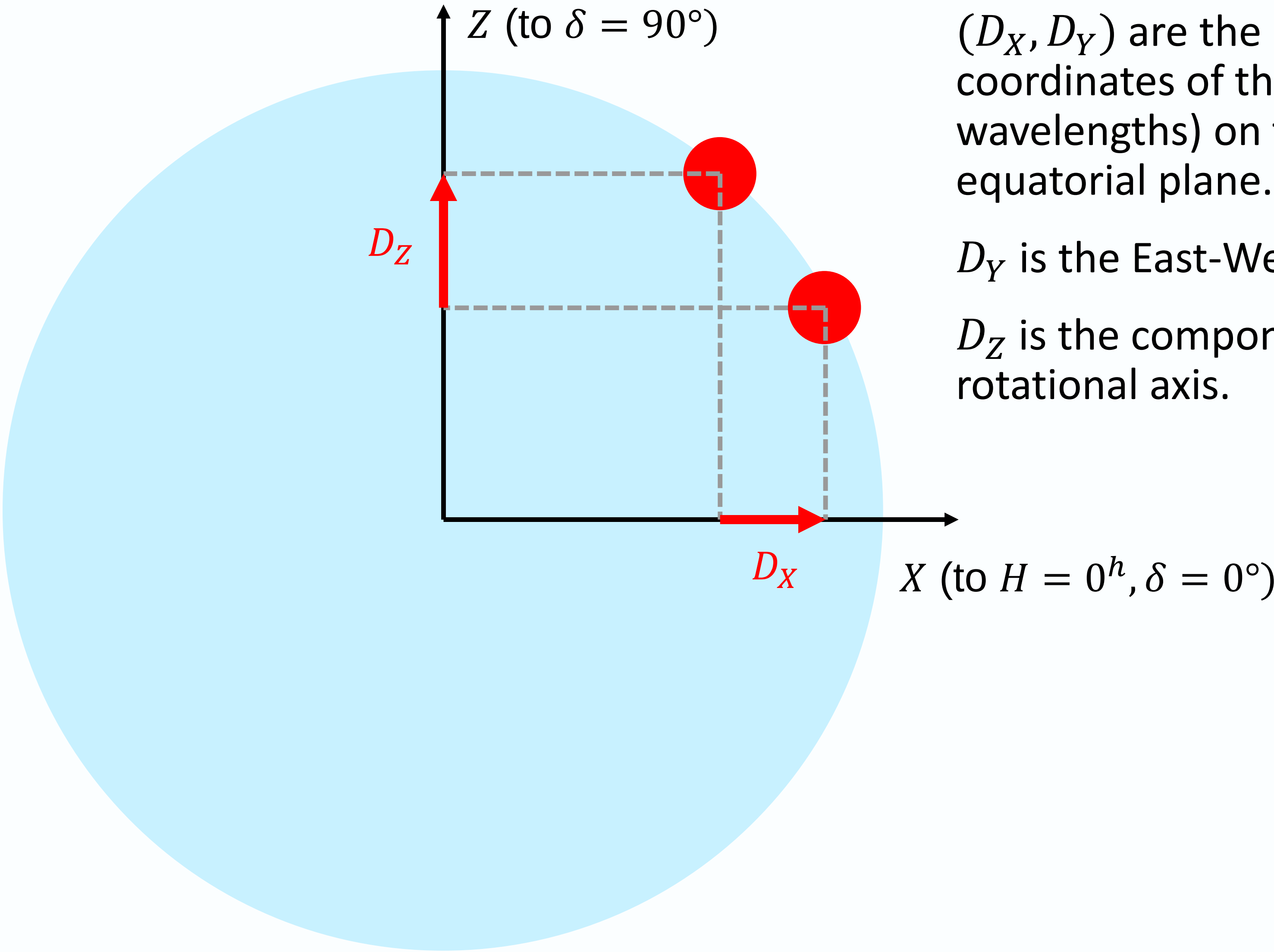
Recall, the standard geometry, applicable in all cases.

- w always points to the phase centre as the Earth rotates.
- u is orthogonal, and points East; v is orthogonal, and points North

The Earth-based coordinate system for each element has the axes (X, Y, Z).

- X points to $H = 0^h, \delta = 0°$; the intersection of the meridian and the celestial equator
- Y points to $H = -6^h, \delta = 0°$; 6 hours East, on the celestial equator
- Z points to $\delta = 90°$; the North celestial pole.

Earth Coordinates Illustrated



(D_X, D_Y) are the projected coordinates of the baseline (in wavelengths) on the Earth's equatorial plane.

D_Y is the East-West component

D_Z is the component along the rotational axis.

Earth Coordinates Simplified

We can express the transformation:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\sin H & \cos\delta \\ \cos\delta\cos H & -\cos\delta\sin H & \sin\delta \end{pmatrix} \begin{pmatrix} D_X \\ D_Y \\ D_Z \end{pmatrix}$$

The w coordinate is the delay distance (in wavelengths) between the two antennas. The geometric delay is:

$$\tau_g = \frac{w\lambda}{c} = w/\nu$$

The rate of change is called the fringe frequency:

$$\nu_F = \frac{dw}{dt} = -\omega_E u \cos\delta$$

Earth Rotation Revisited

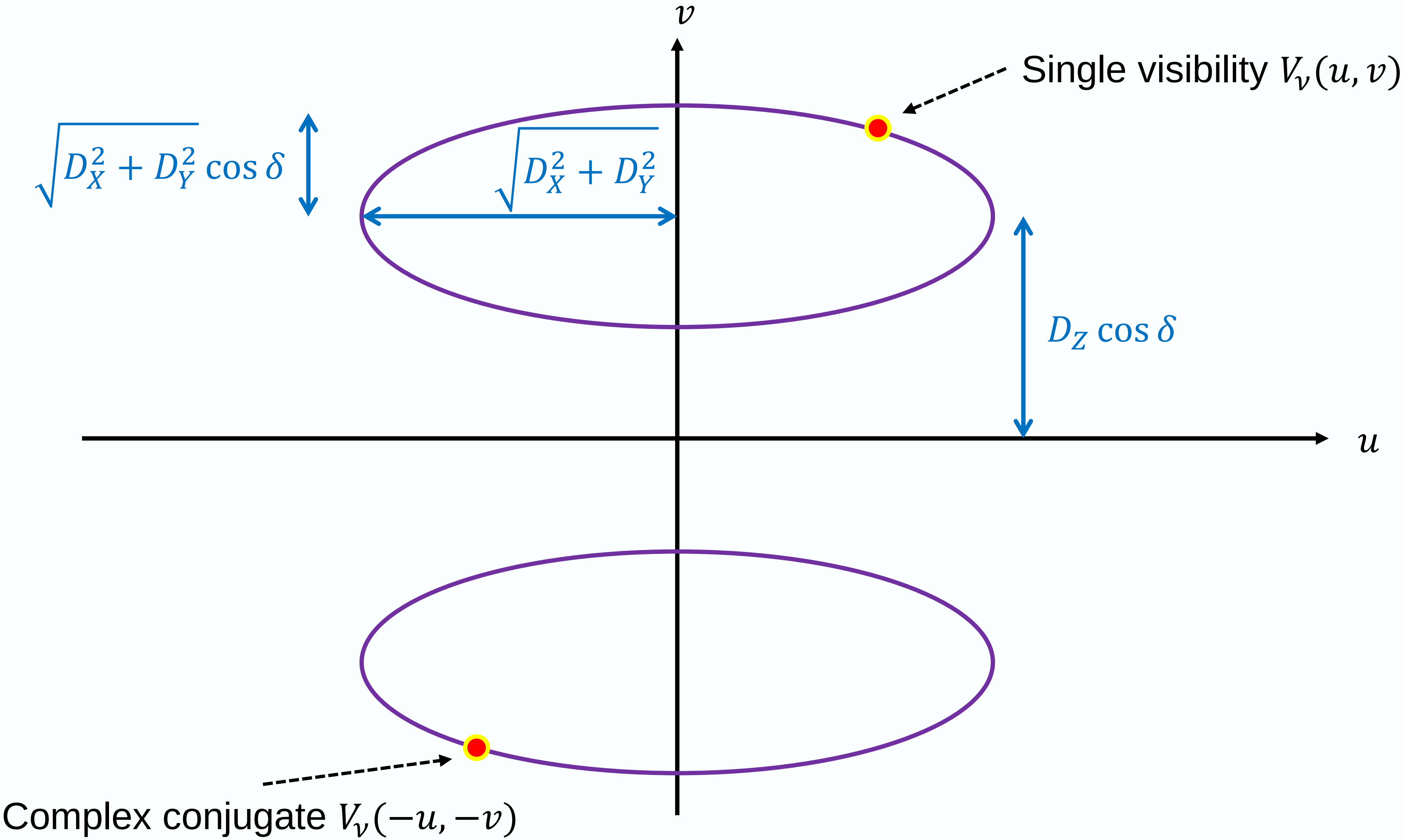
Our baseline will rotate with respect to the source as the Earth rotates. Using the extra information this provides is called “Earth Rotation Synthesis”.

As the baseline rotates, it traces out an ellipse in the (u, v) plane:

$$u^2 + \left(\frac{v - D_Z \cos\delta}{\sin\delta}\right)^2$$

We also benefit because once you have $V_\nu(u, v)$, you get $V_\nu(-u, -v)$ for free, through $V_\nu(-u, -v) = V_\nu^*(u, v)$.

(u, v) coverage illustrated



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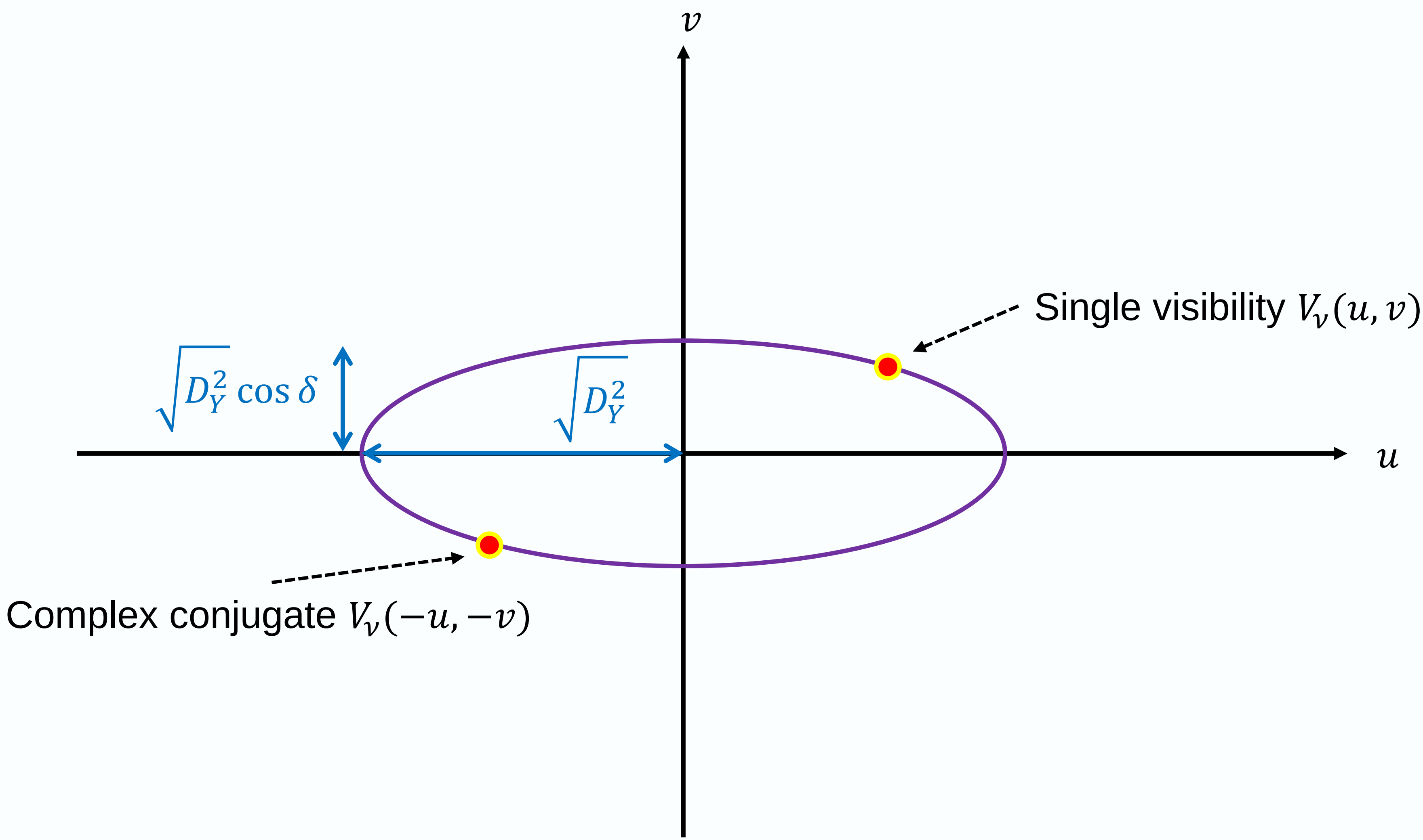
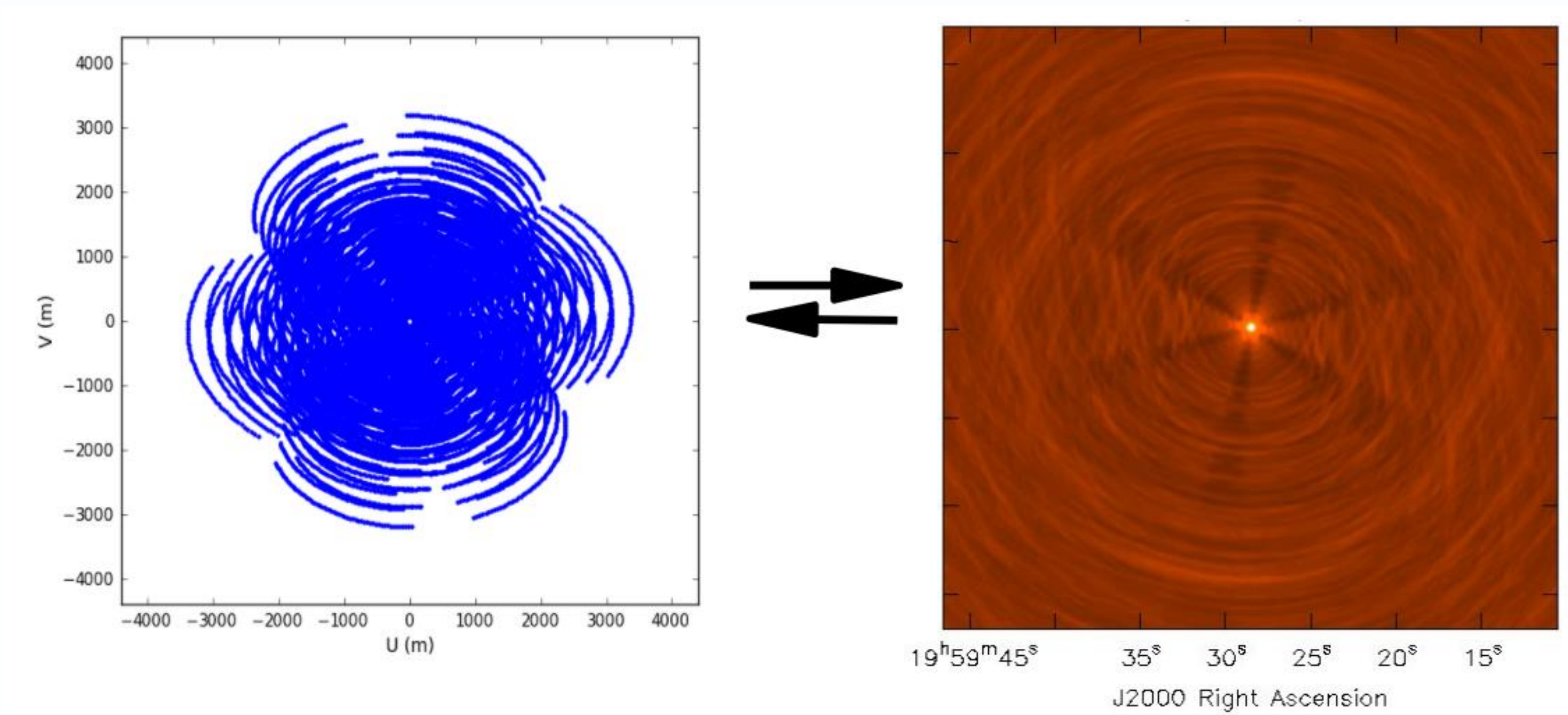
Because E-W baselines have $D_X = D_Z = 0$, their (u, v) ellipse will always be centered at $(u = 0, v = 0)$.

(u, v) coverage illustrated, an E-W array



The importance of (u, v) coverage

The (u, v) coverage of your observation is the Fourier transform of your observation's synthesised beam.



And that brings us to…

The rest of the school is about coping with real interferometers.

Building an interferometer: Christoph's receivers talk.

Recording the visibilities: Max's correlator talk.

Getting enough visibilities, in the right spots: Shari's strategies talk.

Distinguishing sky from instrument: George's calibration talk.

Calculating $I(l, m)$ without all the (u, v): Anna's imaging talk.

$I(l, m)$ is much more complicated than you think: Dave's and Anna's polarimetry talks.

Detecting problems in $V(u, v)$ using $I(l, m)$: Ron's error detection talk.

Thank you

Astronomy and Space Science
Jamie Stevens
ATCA Senior Systems Scientist

t +61 2 6790 4064
e jamie.stevens@csiro.au
w www.narrabri.atnf.csiro.au