CSIRO

Principles of Interferometry I

CASS Radio Astronomy School

R. D. Ekers

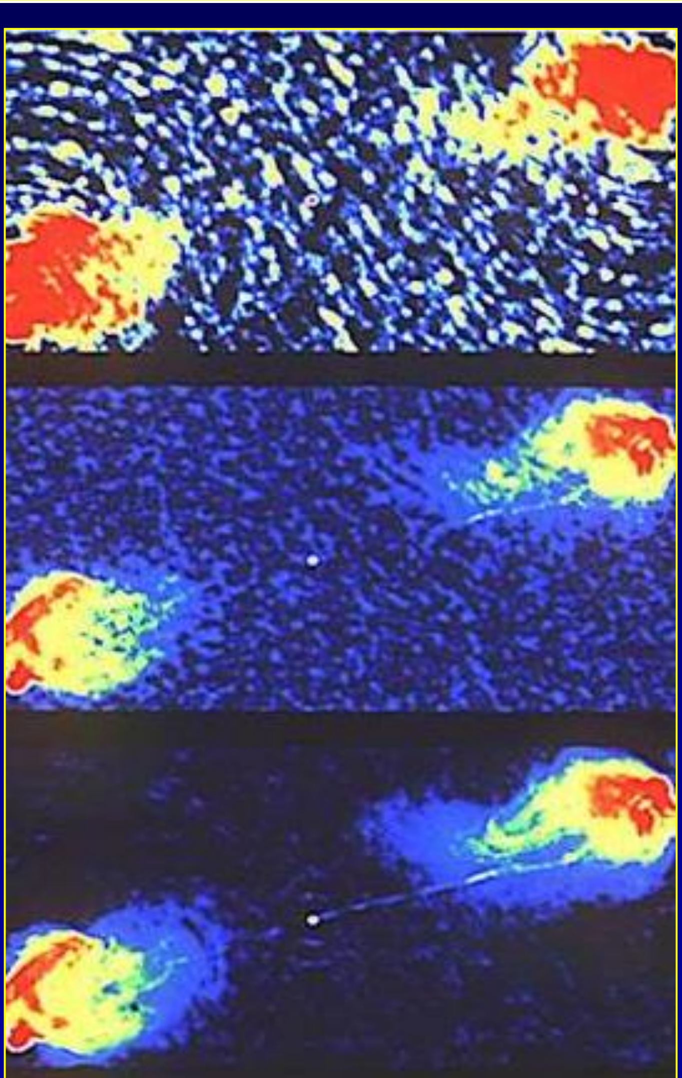
28 Sep 2015

WHY?

- Importance in radio astronomy
  - ATCA, VLA, WSRT, GMRT, MERLIN, IRAM…
  - VLBA, JIVE, VSOP, RADIOASTON
  - ALMA, LOFAR, MWA, ASKAP, MeerKat, SKA

Radio Image of
Ionised Hydrogen in Cyg X
CGPS (Penticton)

Cygnus A



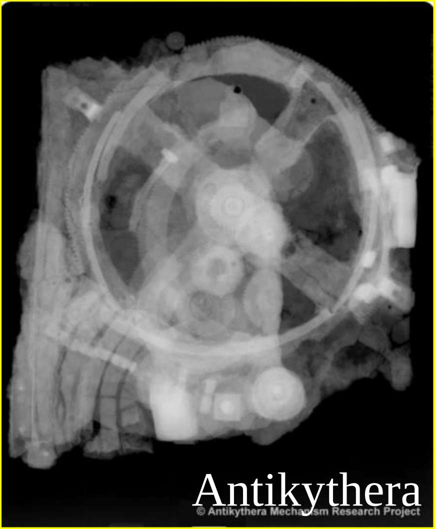
- ⇐ Raw data
  - VLA continuum
- ⇐ Deconvolution
  - correcting for gaps between telescopes
- ⇐ Self Calibration
  - adaptive optics

WHY?

- Importance in radio astronomy
  - ATCA, VLA, WSRT, GMRT, MERLIN, IRAM…
  - VLBA, JIVE, VSOP, RADIOASTRON
  - ALMA, LOFAR, ASKAP, SKA
- National Facilities
  - ✓ Easy for non-experts to use
  - ✘ don’t know what you are doing
- Cross fertilization
- Doing the best science

Indirect Imaging Applications

- Interferometry
  - radio, optical, IR, space…
- Fourier synthesis
  - Earth rotation, SAR, X-ray crystallography
- Axial tomography (CAT)
  - NMR, Ultrasound, PET, X-ray tomography
- Seismology
- Fourier filtering, pattern recognition
- Adaptive optics, speckle



Antikythera

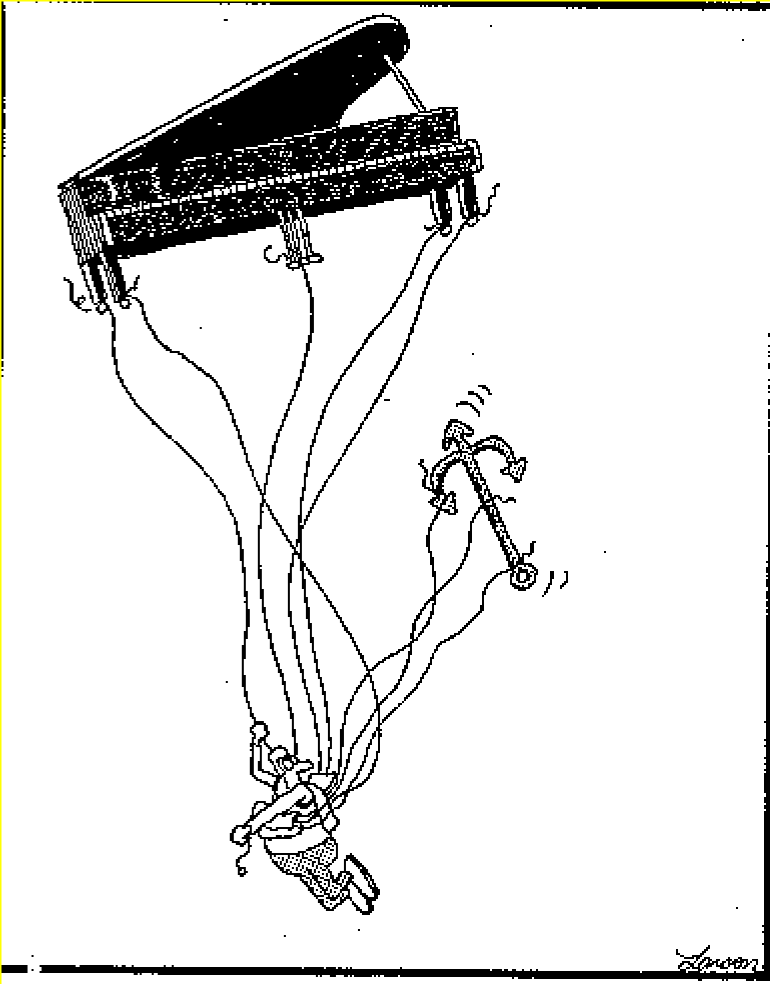
© Antikythera Mechanism Research Project

Doing the best science

- The telescope as an analytic tool
  - how to use it
  - integrity of results
- Making discoveries
  - Most discoveries are driven by instrumental developments
  - recognising the unexpected phenomenon
  - discriminate against errors
- Instrumental or Astronomical specialization?

HOW ?

- Don’t Panic!
  - Many entrance levels

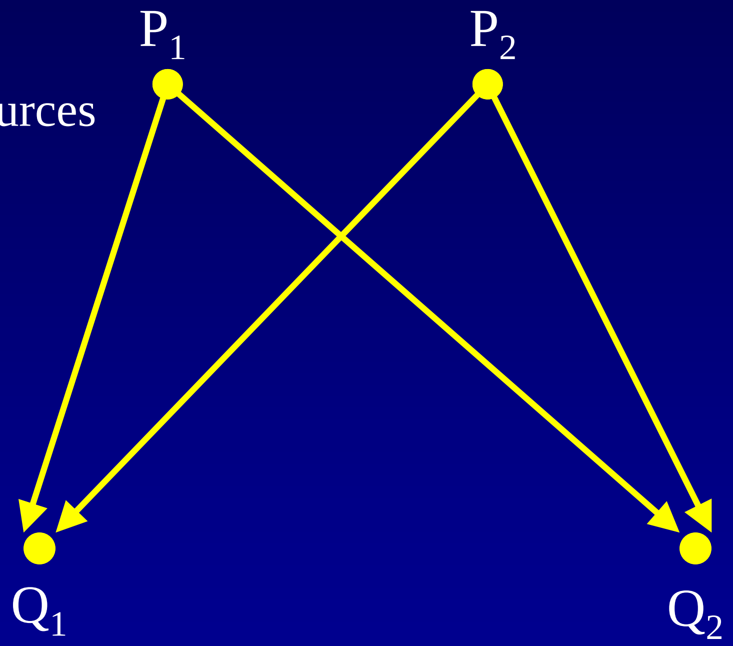


Murray didn’t feel the first pangs of real panic until he pulled the emergency cord.

Basic concepts

- Importance of analogies for physical insight
- Different ways to look at a synthesis telescope
  - Engineers model
    - Telescope beam patterns…
  - Physicist electromagnetic wave model
    - Sampling the spatial coherence function
    - Barry Clark *Synthesis Imaging chapter1*
    - Born & Wolf *Physical Optics*
  - Quantum model
    - Radhakrishnan *Synthesis Imaging last chapter*

Spatial Coherence



P_1 & P_2
spatially incoherent sources

At distant points Q_1 & Q_2
The field is partially coherent

van Cittert-Zernike theorem

The spatial coherence function is the Fourier Transform of the brightness distribution

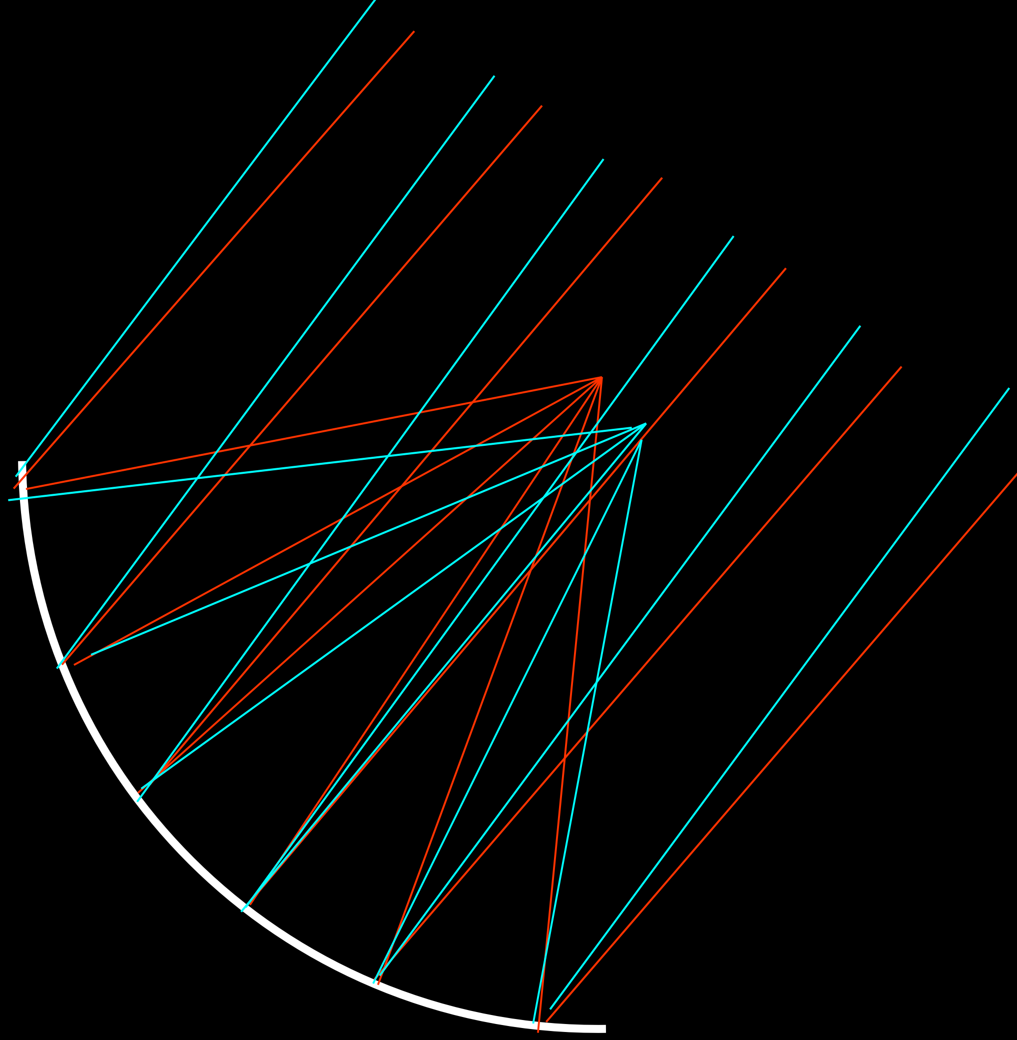
Physics: propagation of coherence

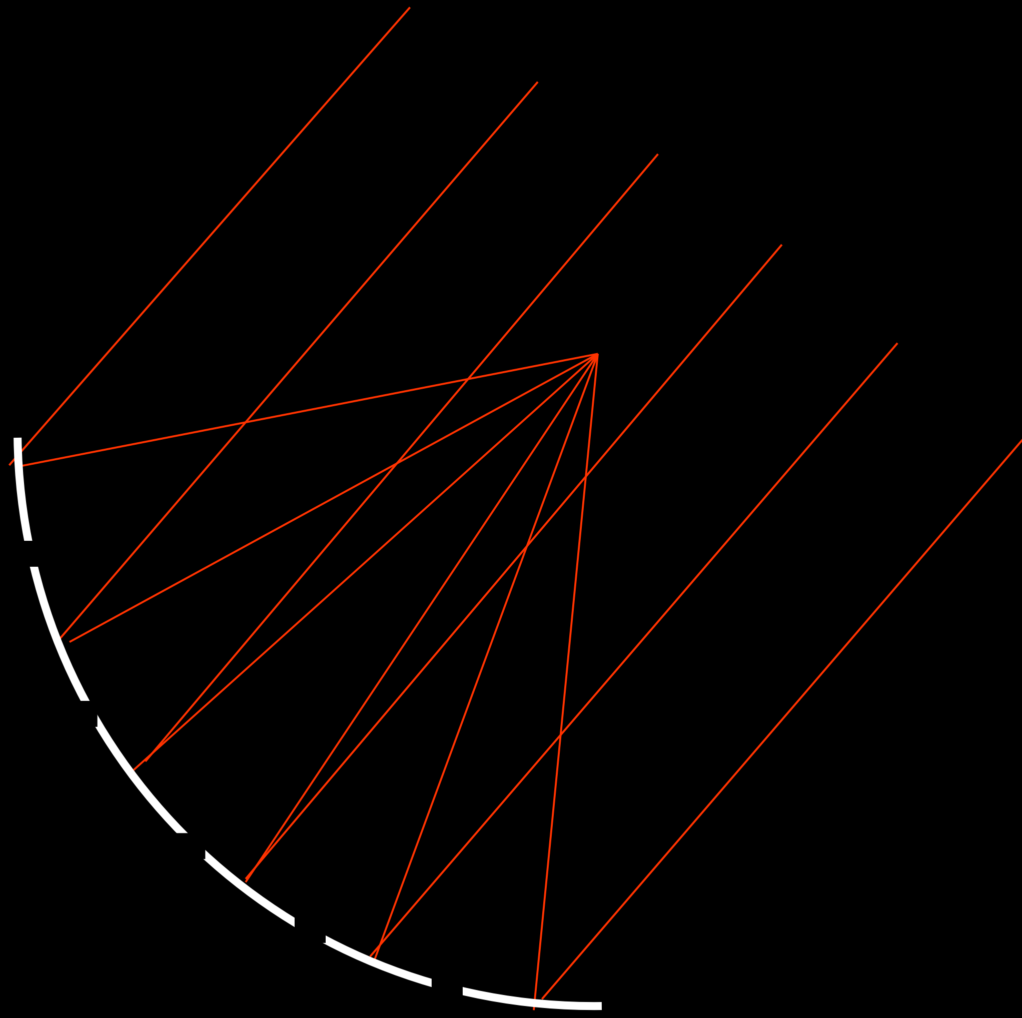
- Radio source emits independent noise from each element
  - Electrons spiraling around magnetic fields
  - Thermal emission from dust, *etc.*
- As electromagnetic radiation propagates away from source, it remains coherent
- By measuring the correlation in the EM radiation, we can work backwards to determine the properties of the source
- Van Cittert-Zernicke theorem states that the
  - *Sky brightness and Coherence function are a Fourier pair*
- Mathematically:

$$V(u,v) = \int I(l,m) \cdot e^{j.2\pi.(ul+vm)} dl.dm$$

Analogy with single dish

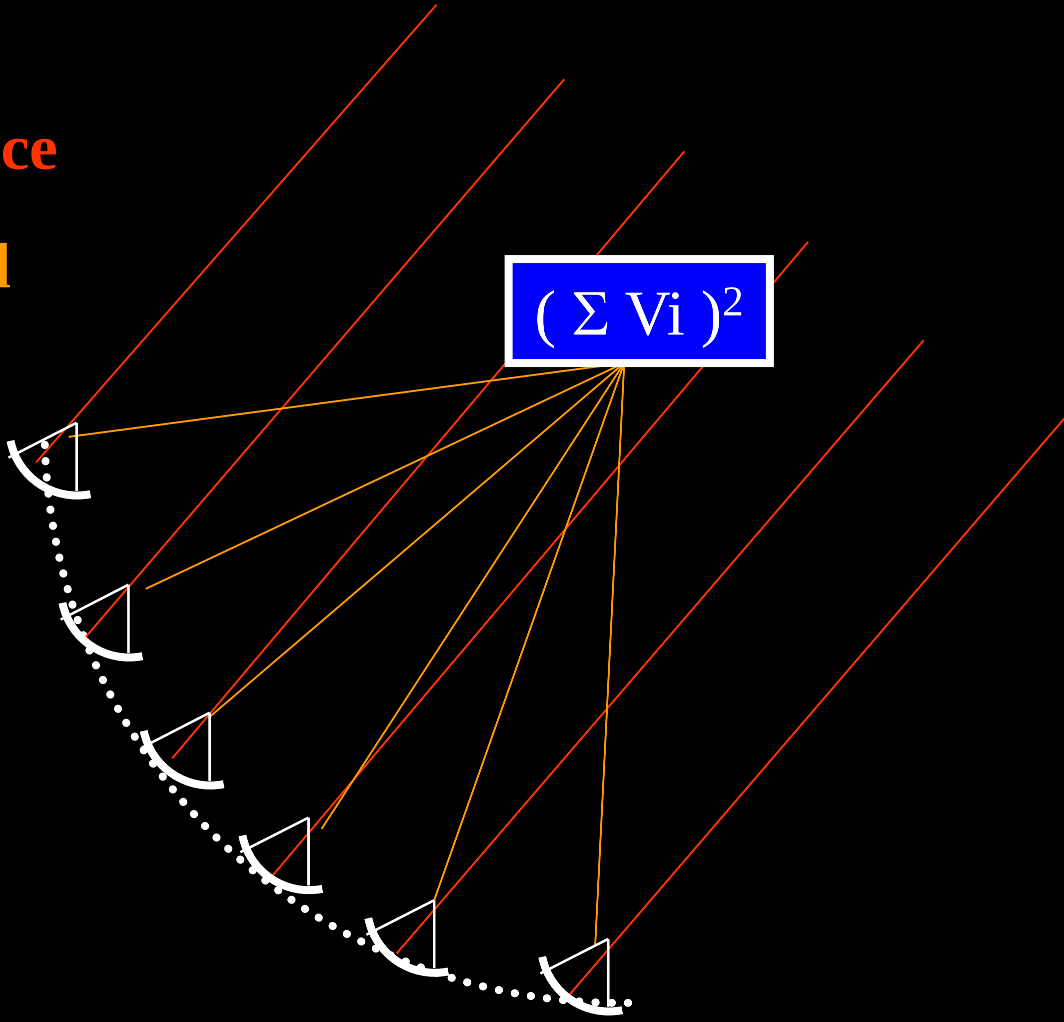
- Big mirror decomposition

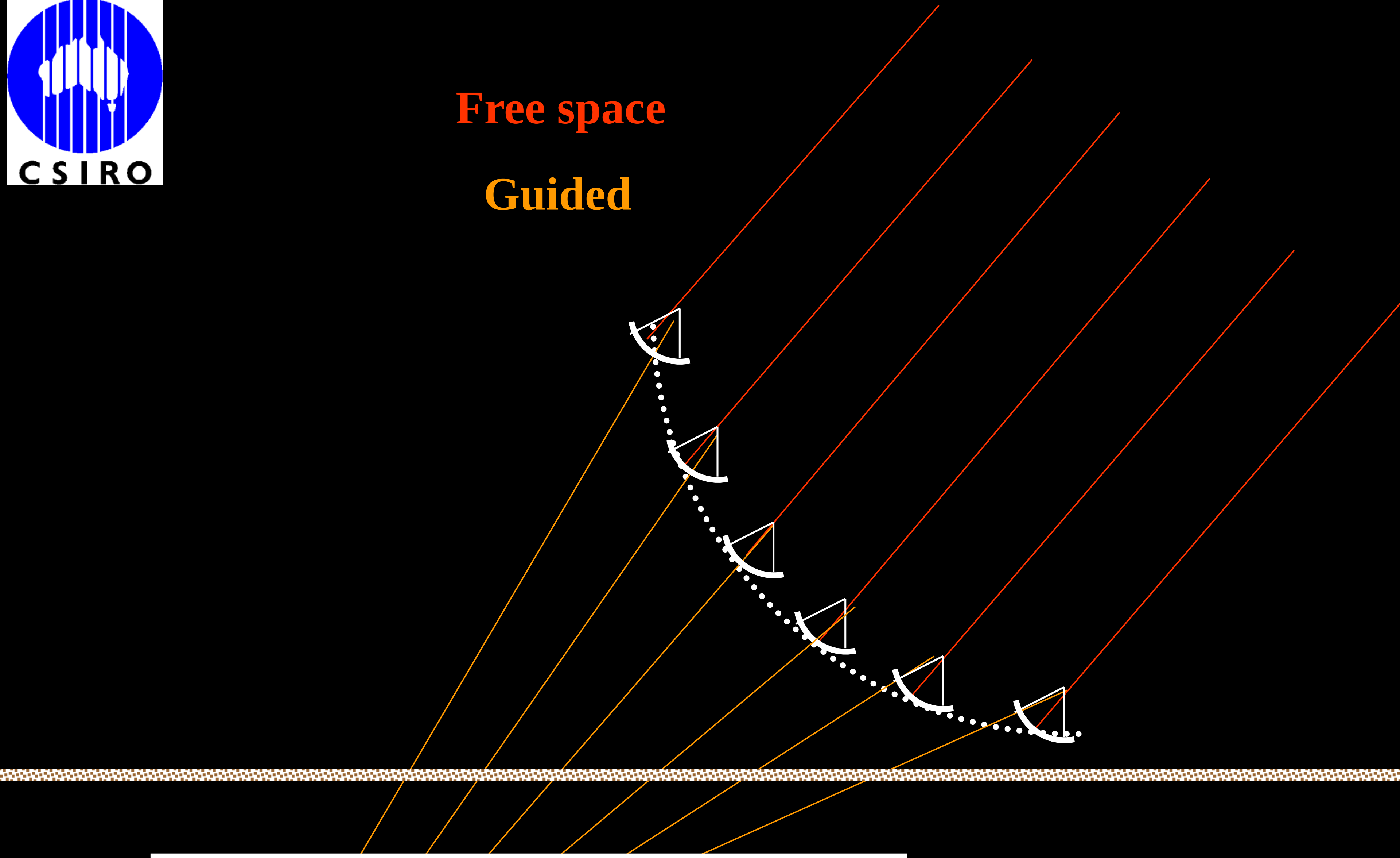




Free space

Guided





Free space

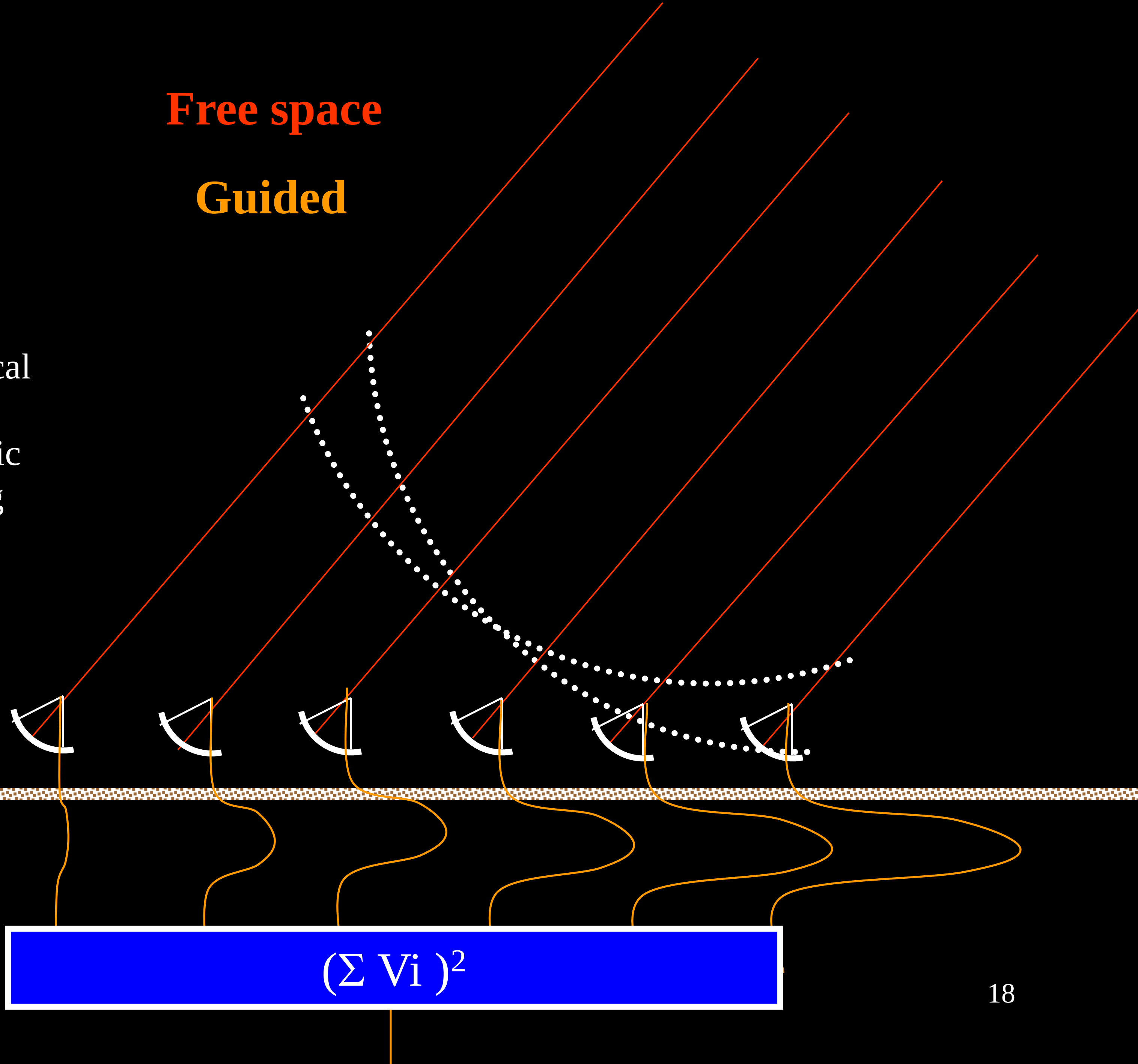
Guided

$(\Sigma V_i)^2$

Free space

Guided

Mechanical
v
Electronic
steering



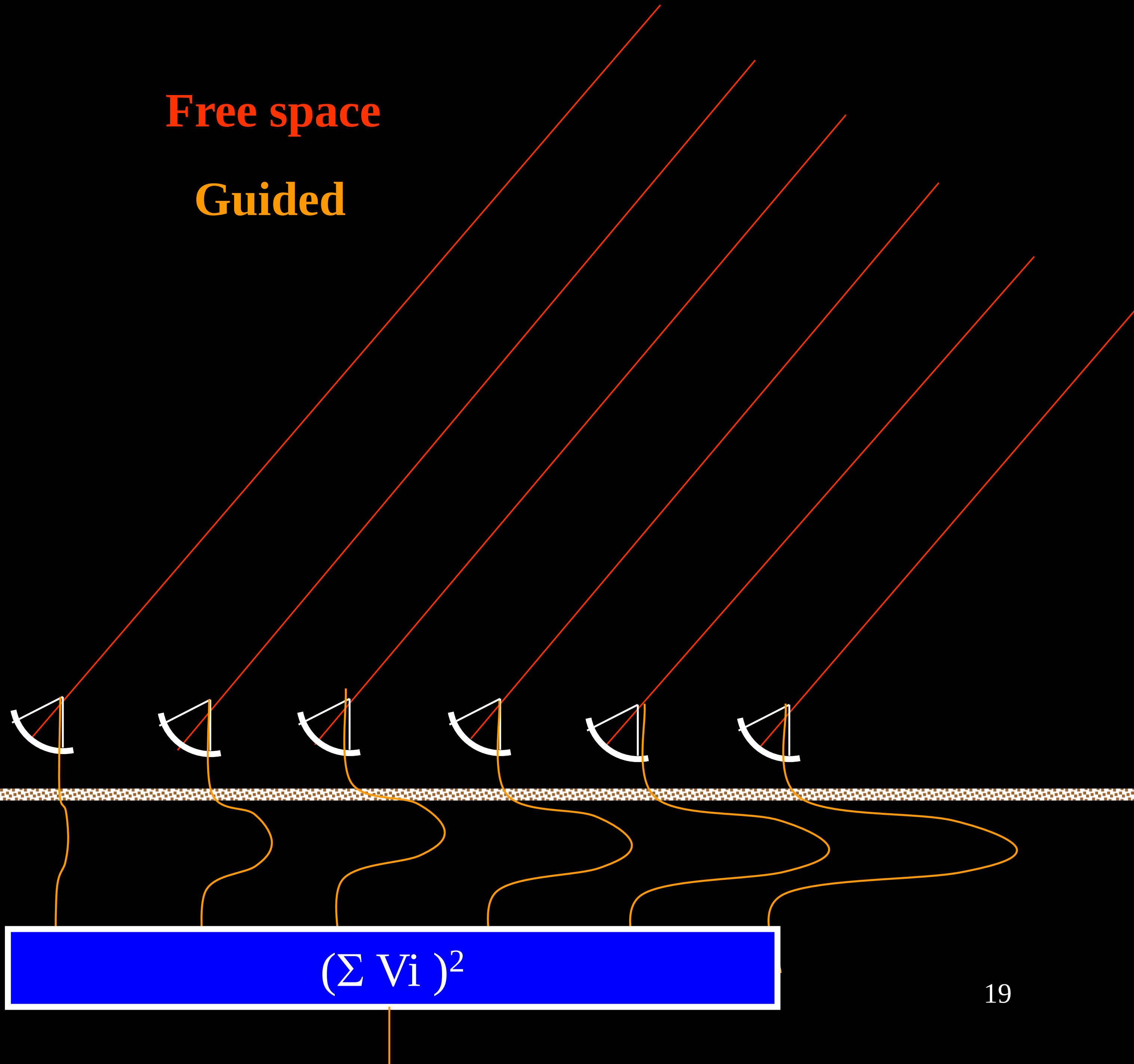
Delay

Phased array

$(\Sigma \mathrm{V}i)^2$

Free space

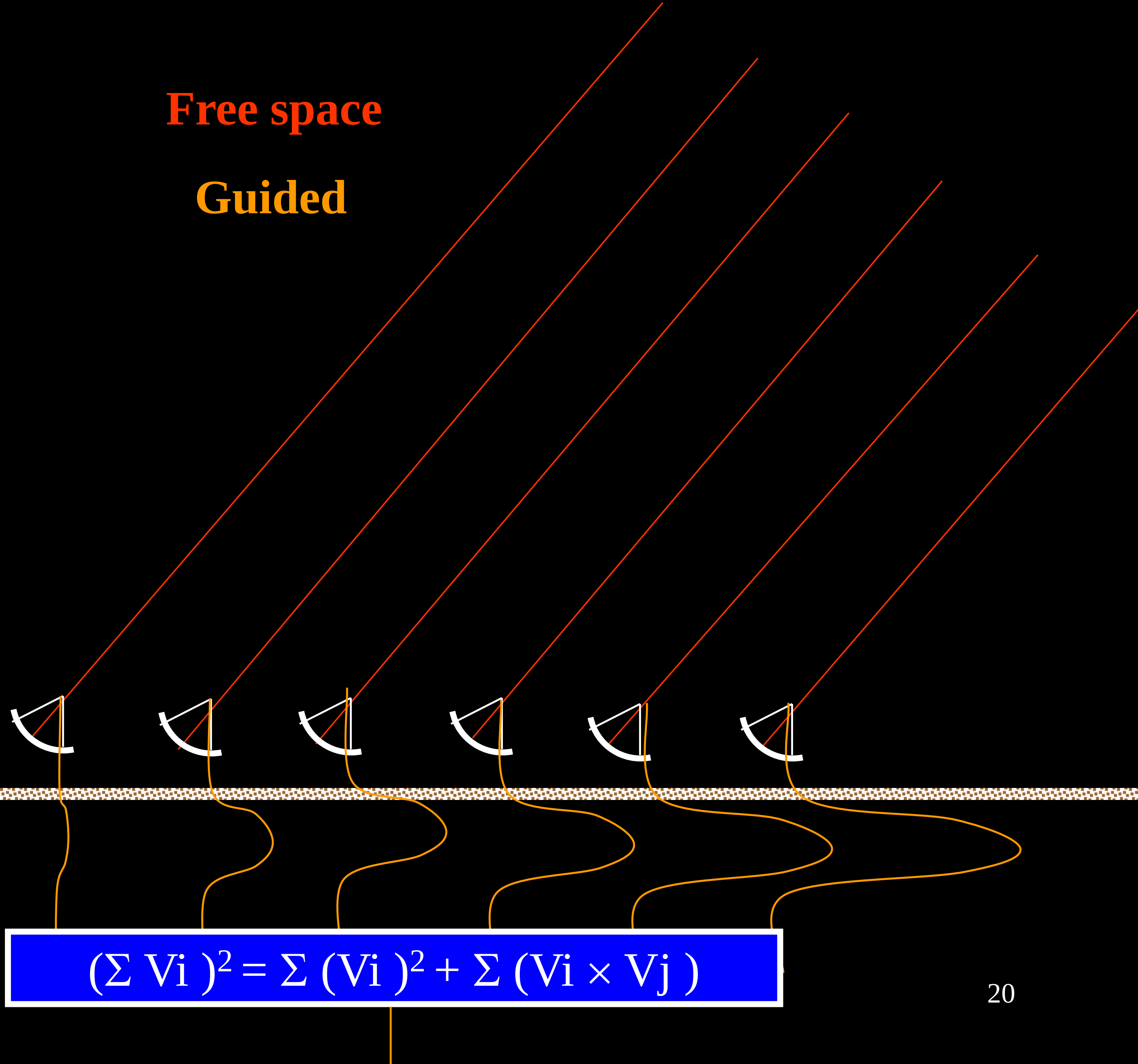
Guided



Delay

Phased array

$(\Sigma \; Vi)^2$



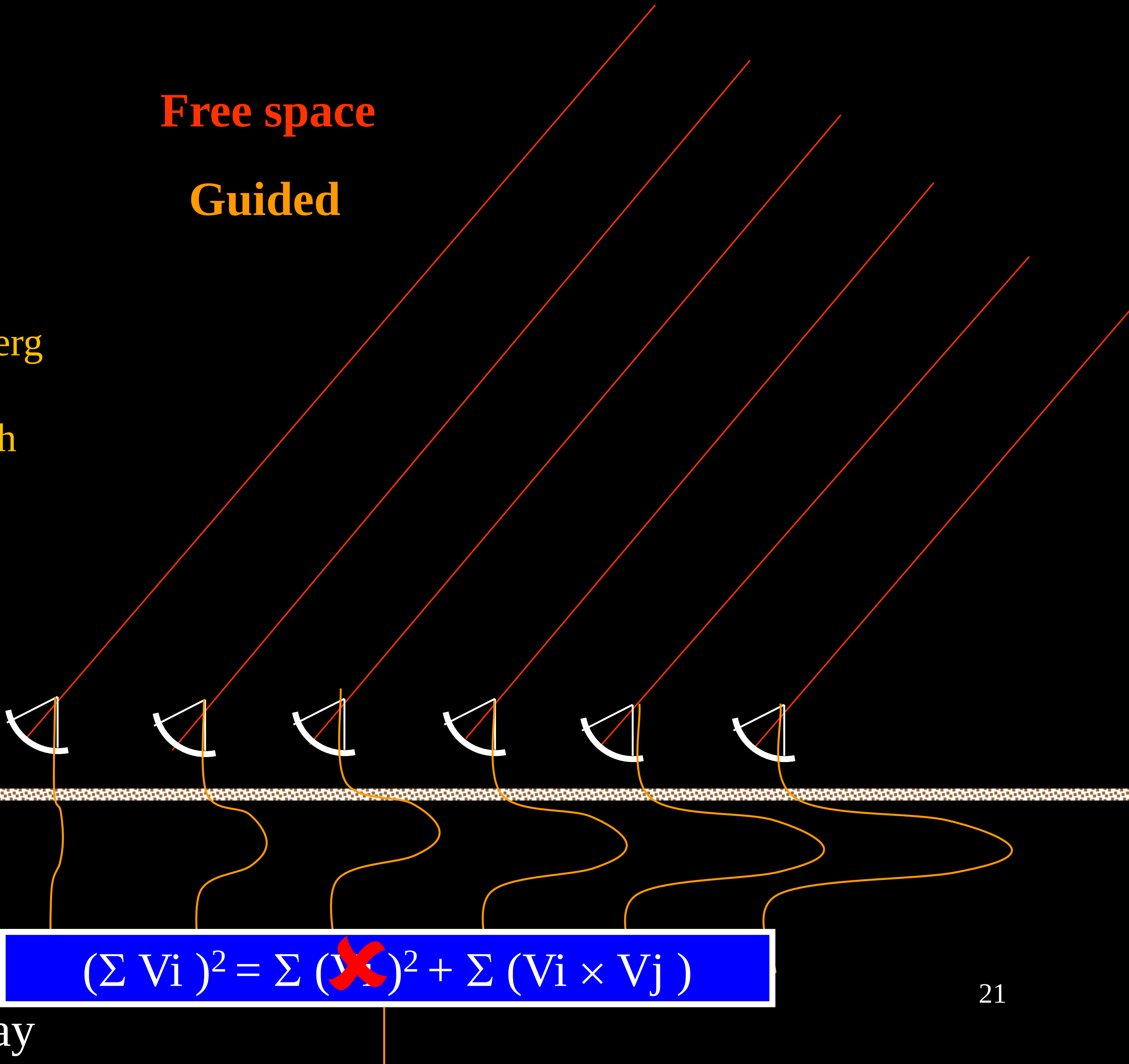
Free space

Guided

Delay

Phased array

$$(\Sigma\ Vi)^2 = \Sigma\ (Vi)^2 + \Sigma\ (Vi \times Vj)$$



Free space

Guided

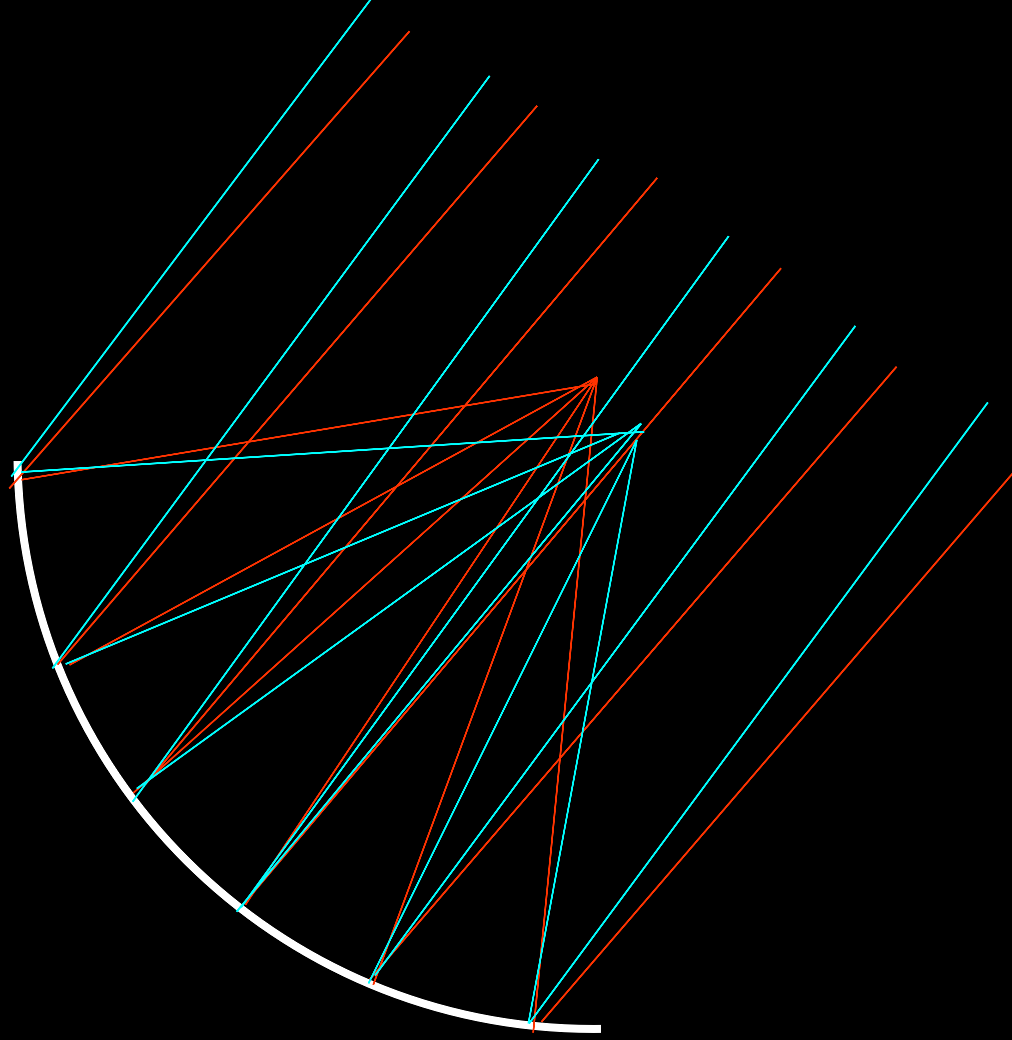
Ryle & Vonberg
(1946)
phase switch

Delay

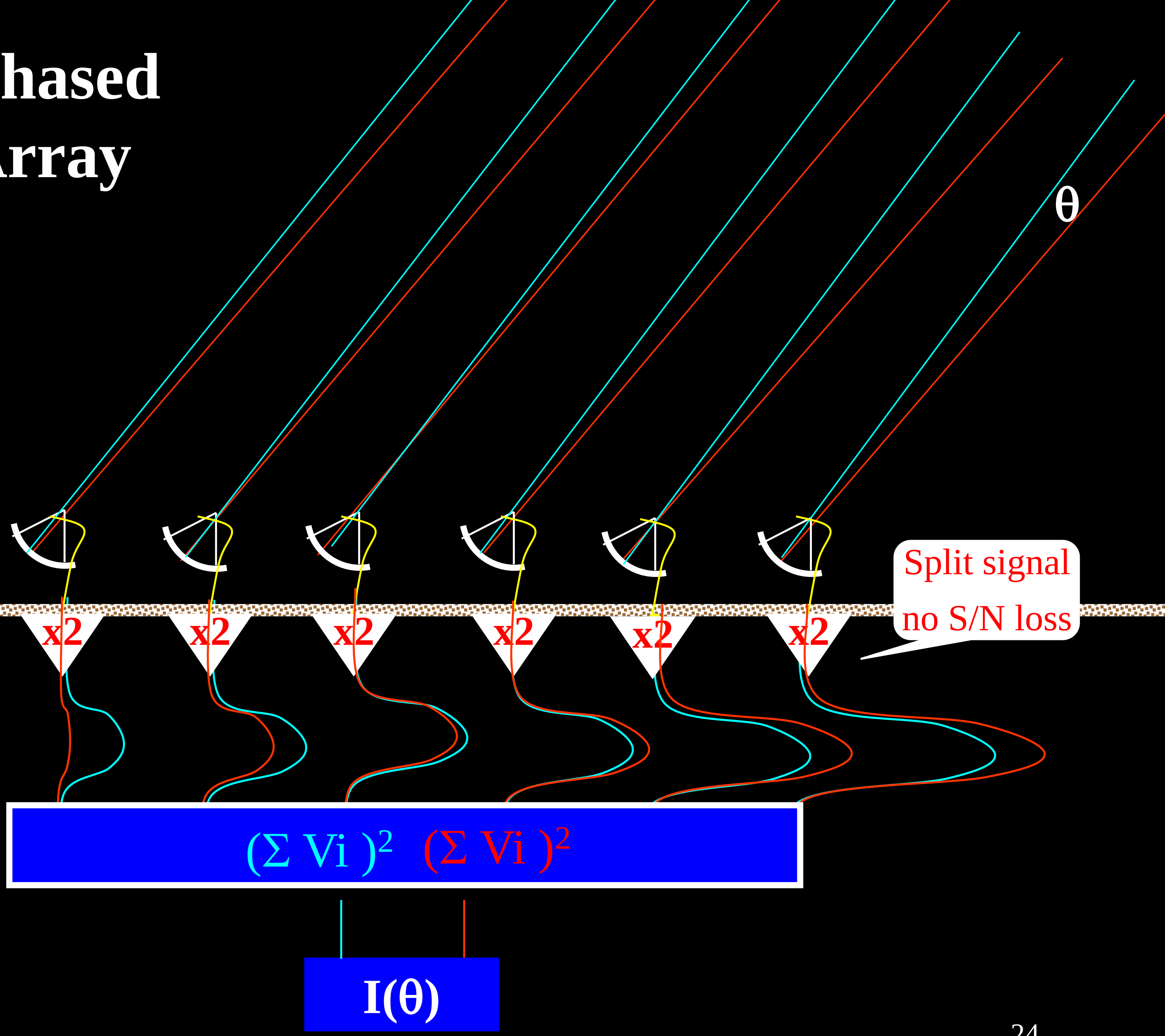
~~Phased array~~

Correlation array

$$(\Sigma\ Vi)^2 = \Sigma\ (\cancel{Vi})^2 + \Sigma\ (Vi \times Vj)$$



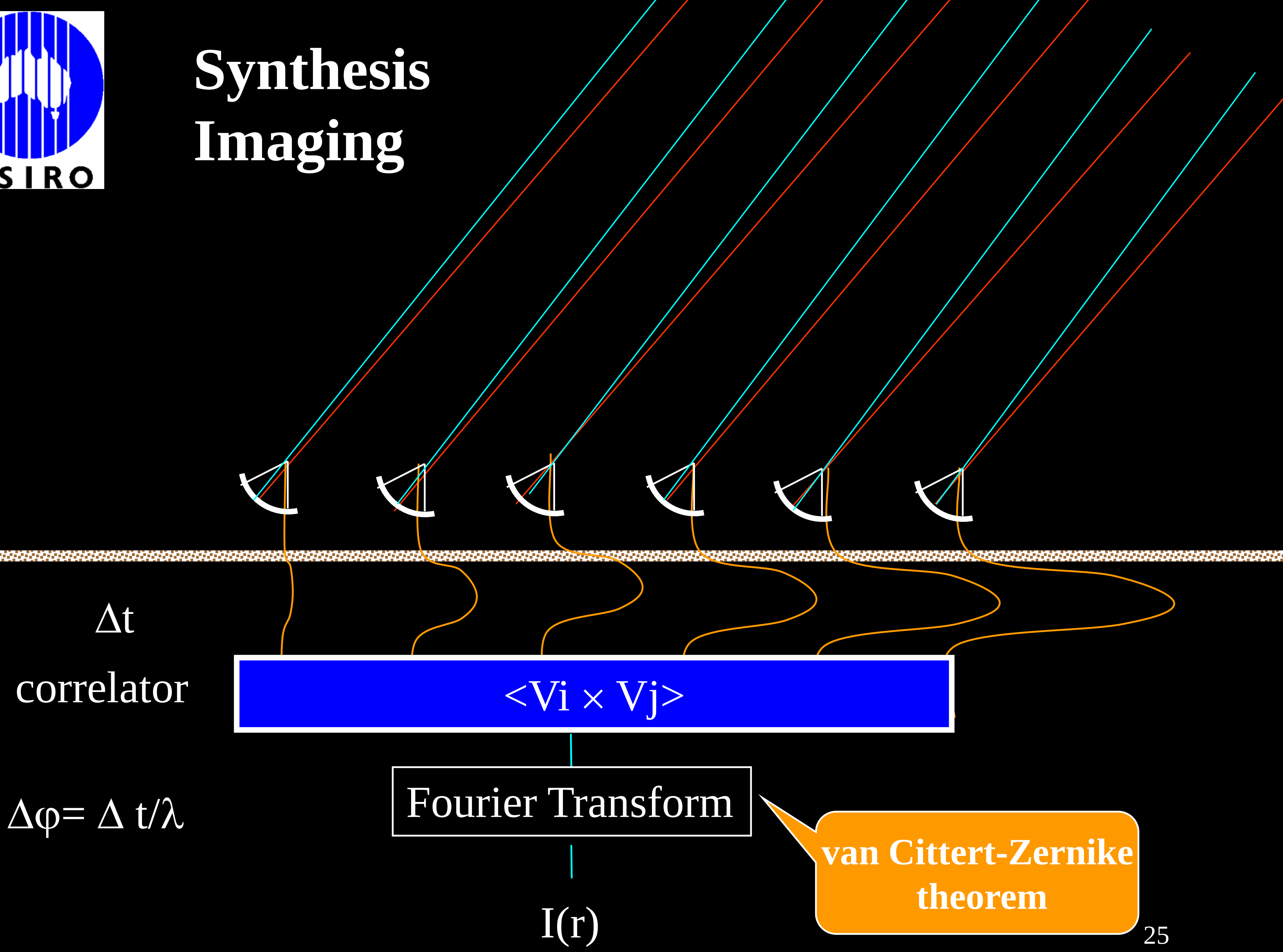
Phased Array



Split signal
no S/N loss

Phased array
Tied array
Beam former

Synthesis Imaging



Δt

correlator

$\langle V_i \times V_j \rangle$

Fourier Transform

$\Delta\varphi = \Delta t/\lambda$

I(r)

van Cittert-Zernike theorem

Physics: propagation of coherence

- Radio source emits independent noise from each element
  - Electrons spiraling around magnetic fields
  - Thermal emission from dust, *etc.*
- As electromagnetic radiation propagates away from source, it remains coherent
- By measuring the correlation in the EM radiation, we can work backwards to determine the properties of the source
- Van Cittert-Zernicke theorem states that the
  - *Sky brightness and Coherence function are a Fourier pair*
- Mathematically:

$$V(u,v) = \int I(l,m) \cdot e^{j.2\pi.(ul+vm)} dl.dm$$

Analogy with single dish

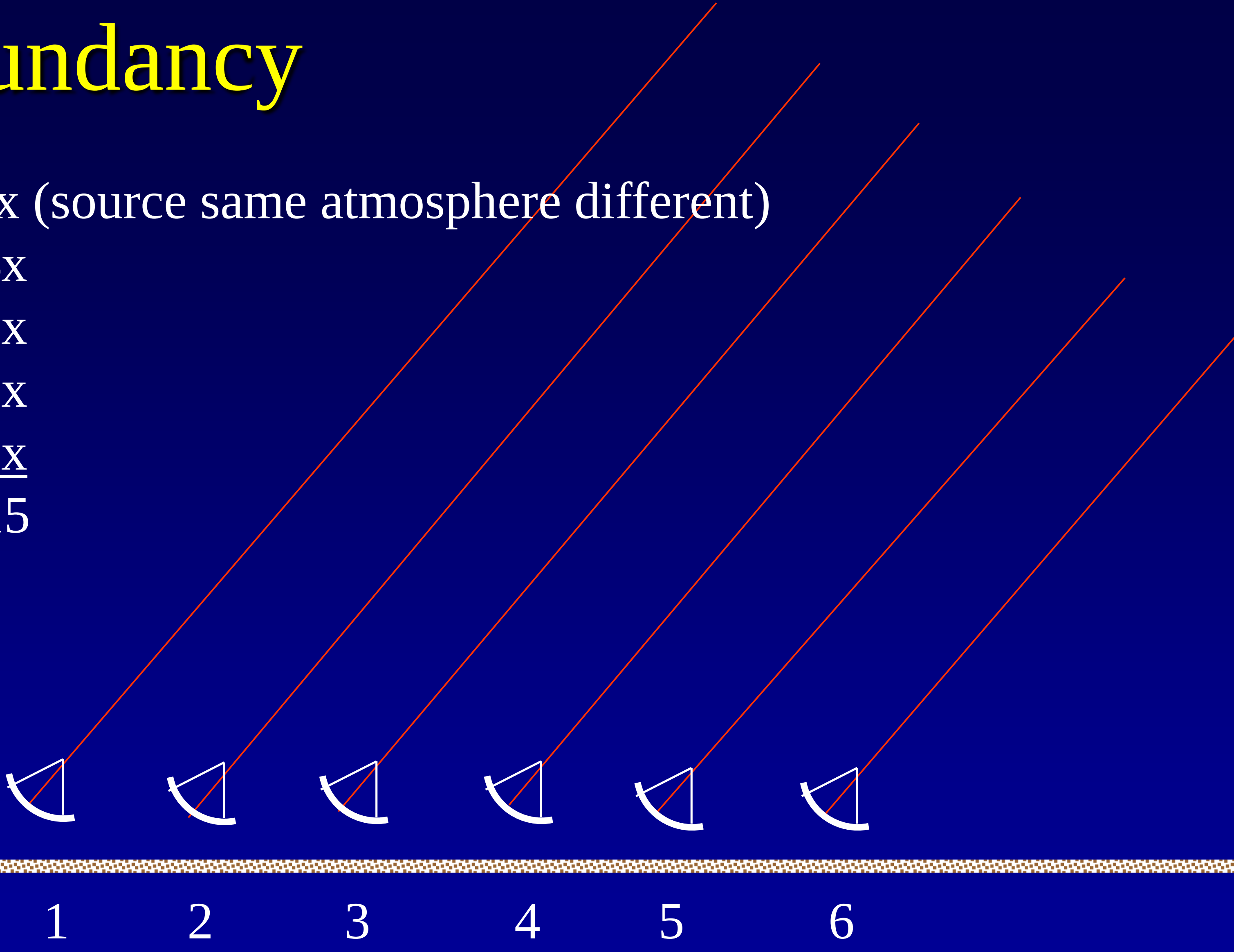
- Big mirror decomposition
- Reverse the process to understand imaging with a mirror
  - Eg understanding non-redundant masks
  - Adaptive optics
- Single dishes and correlation interferometers
  - Darrel Emerson, NRAO
  - http://www.gb.nrao.edu/sd03/talks/whysd_r1.pdf

Filling the aperture

- Aperture synthesis
  - measure correlations with multiple dishes
  - moving dishes sequentially
  - earth rotation synthesis
  - store all correlations for later use
- Partially unfilled aperture
  - some spacings missing
- Redundant spacings
  - some interferometer spacings occur twice
- Non-redundant aperture

Redundancy

1unit 5x (source same atmosphere different)
2units 4x
3units 3x
4units 2x
5units 1x
n(n-1)/2 = 15



Westerbork: 1970

- Grating array
- 14 x 25m dishes 3km
  - Four moveable
  - Ten fixed
  - 10 redundant spacings
  - Self calibration

WiFi
IEEE 802.11

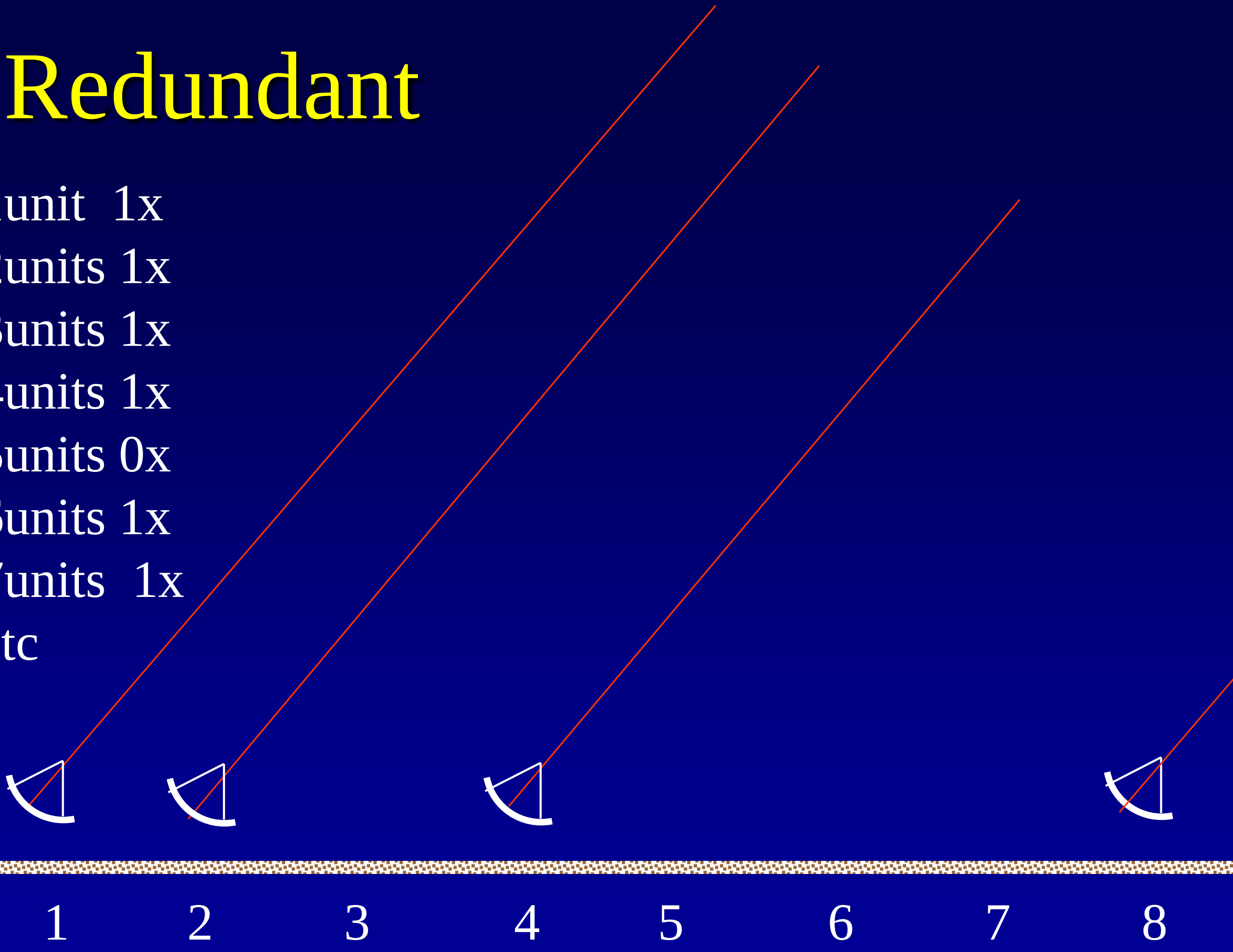
- **Inventors**: Dr John O'Sullivan et al
- **Country**: Australia
- **Invention**: Wireless LAN for high speed data transfer

The IEE 802.11 and Redundant Spacing analogy

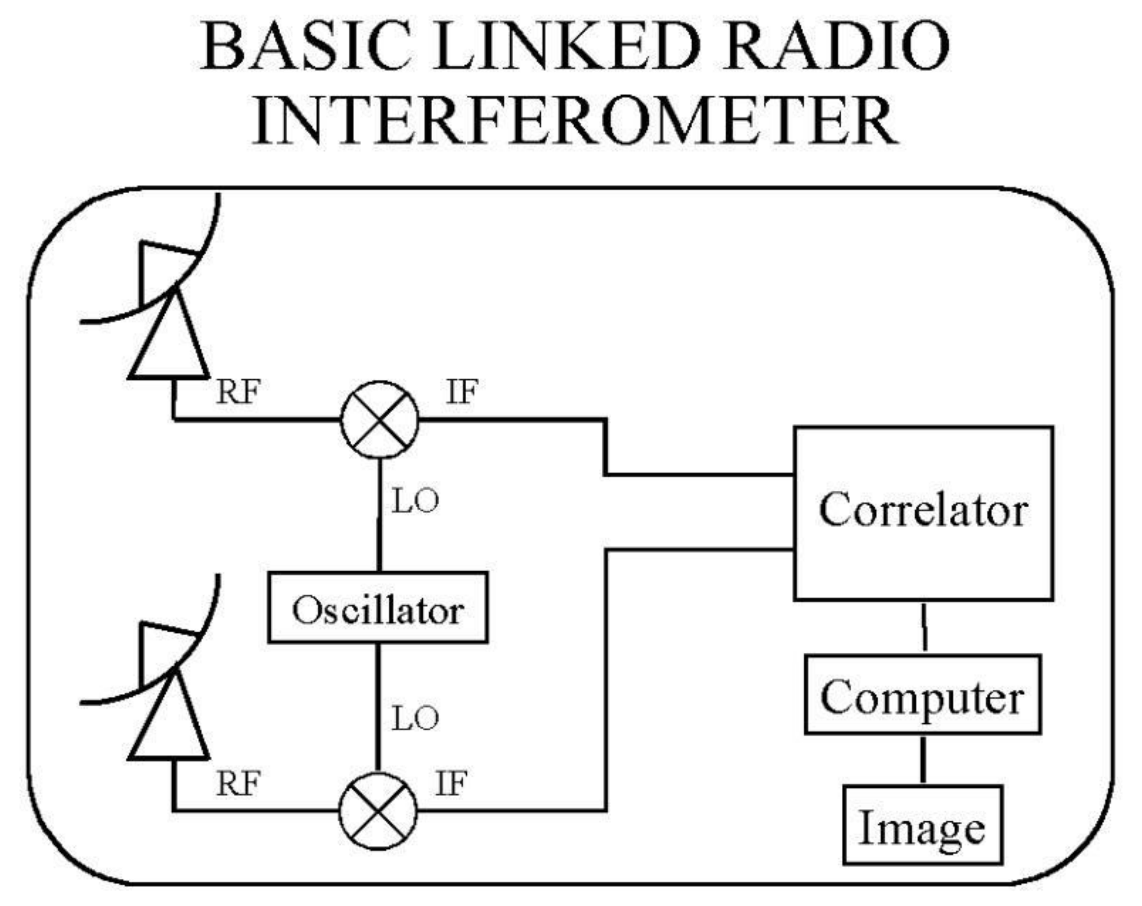
- Redundant spacing interferometers
  - Redundant spacings measure the same Fourier component
  - If corrupted by the atmosphere they will not be equal and the peak image intensity must decrease
  - S/N is maximised when the atmosphere is correct
- A broad band wireless link is corrupted by multiple reflections
  - Individual narrow Fourier components are not corrupted because delays are small
  - Each has a redundant low time resolution copy of the modulated signal
  - The S/N on the link is maximised when all these Fourier components are correctly aligned.
- A 1D real time Fourier transform has to be implemented in the communication chip

Non Redundant

1unit 1x
2units 1x
3units 1x
4units 1x
5units 0x
6units 1x
7units 1x
etc



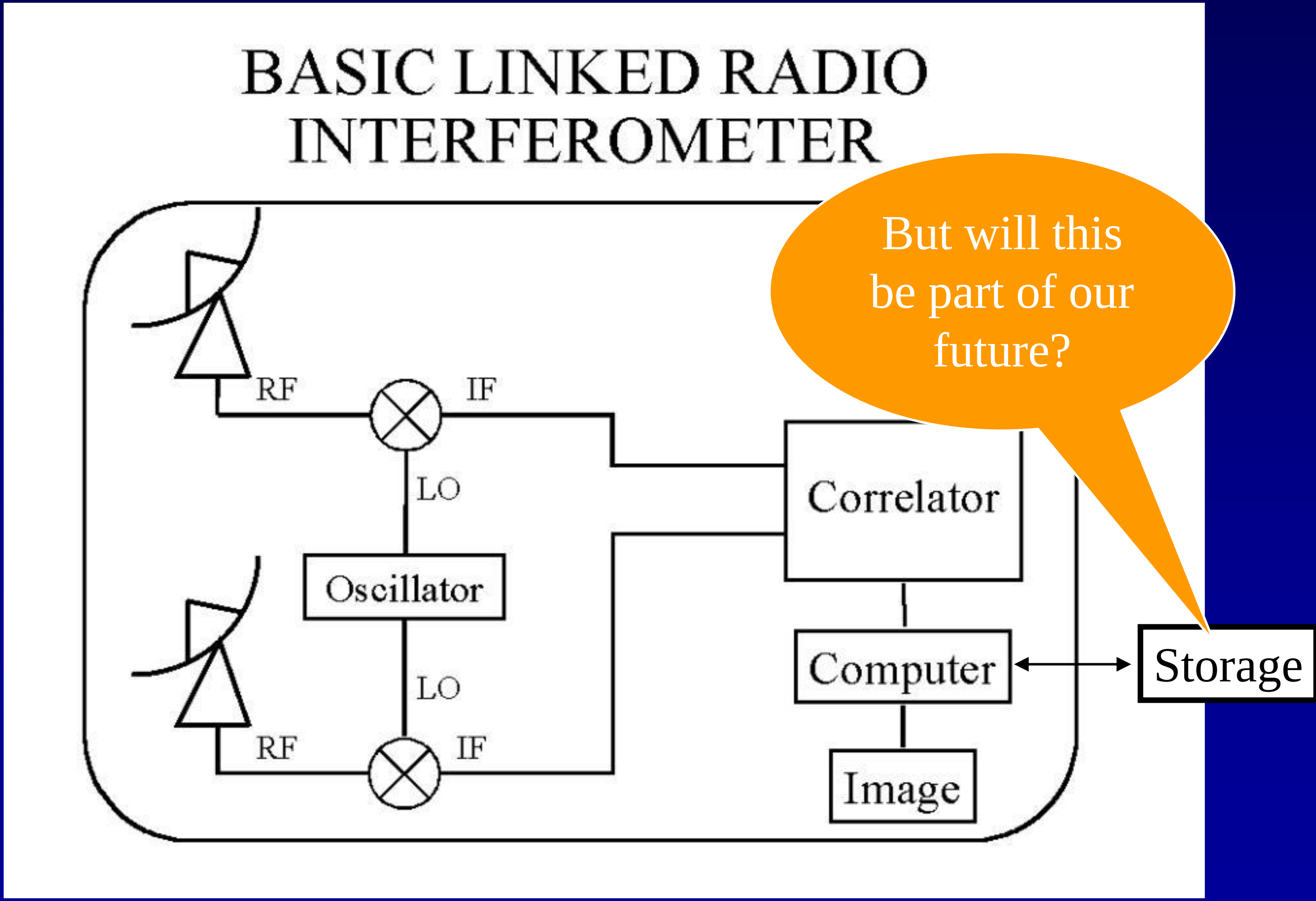
Basic Interferometer



Storing visibilities

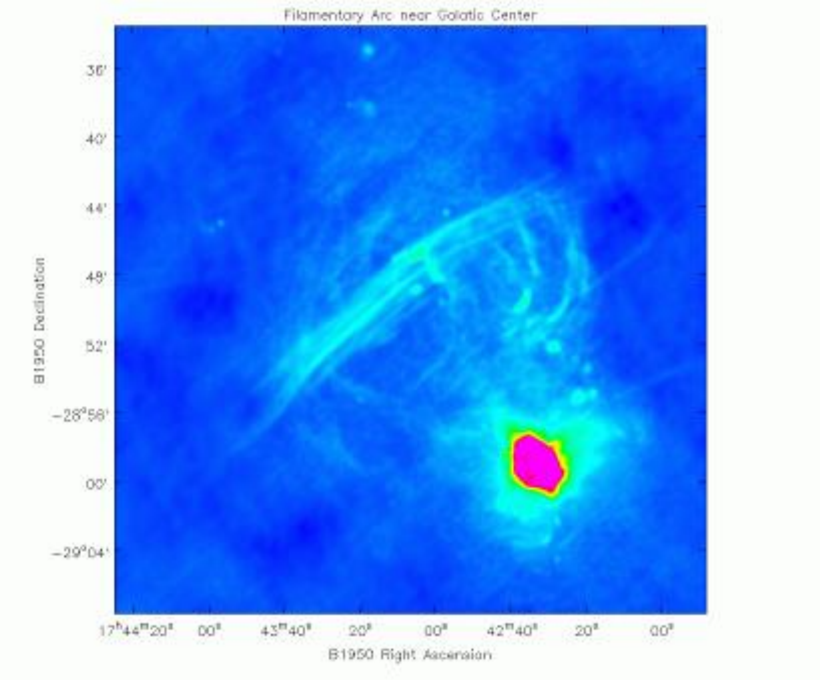
A powerful tool to manipulate the coherence function and re-image.

Not possible in most other domains

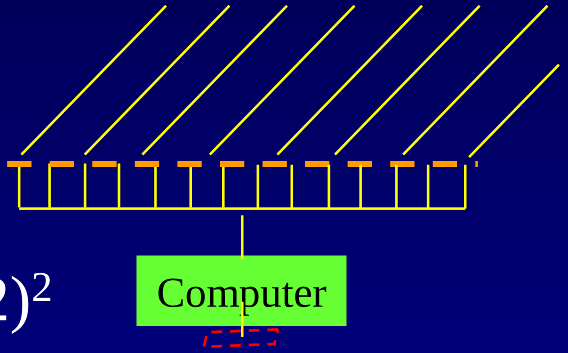


In practice…

1. Use many antennas (VLA has 27)
2. Amplify signals
3. Sample and digitize
4. Send to central location
5. Perform cross-correlation
6. Earth rotation fills the "aperture"
7. Inverse Fourier Transform gets image
8. Correct for limited number of antennas
9. Correct for imperfections in the "telescope" e.g. calibration errors
10. Make a beautiful image…

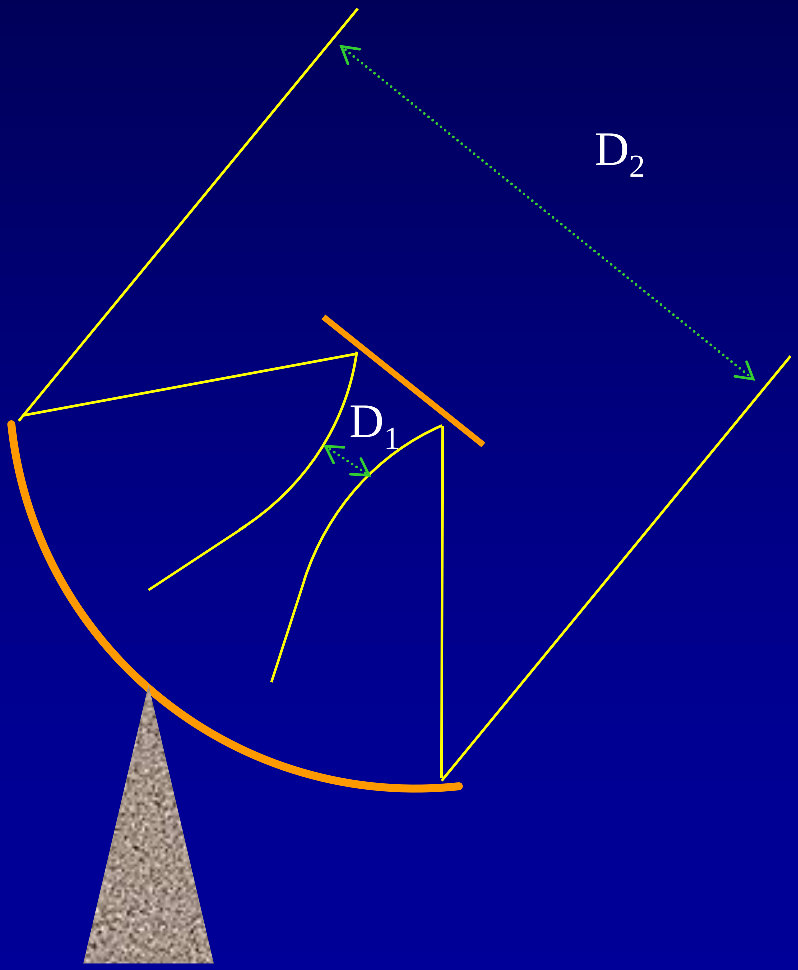


Aperture Array
or Focal Plane Array?



- Why have a dish at all?
  - Sample the whole wavefront
  - n elements needed: $n \propto \text{Area}/(\lambda/2)^2$
  - For 100m aperture and $\lambda = 20\text{cm}$, $n=10^4$
    - » Electronics costs too high!
- Phased Array Feeds
  - Any part of the complex wavefront can be used
  - Choose a region with a smaller waist
  - Need a concentrator

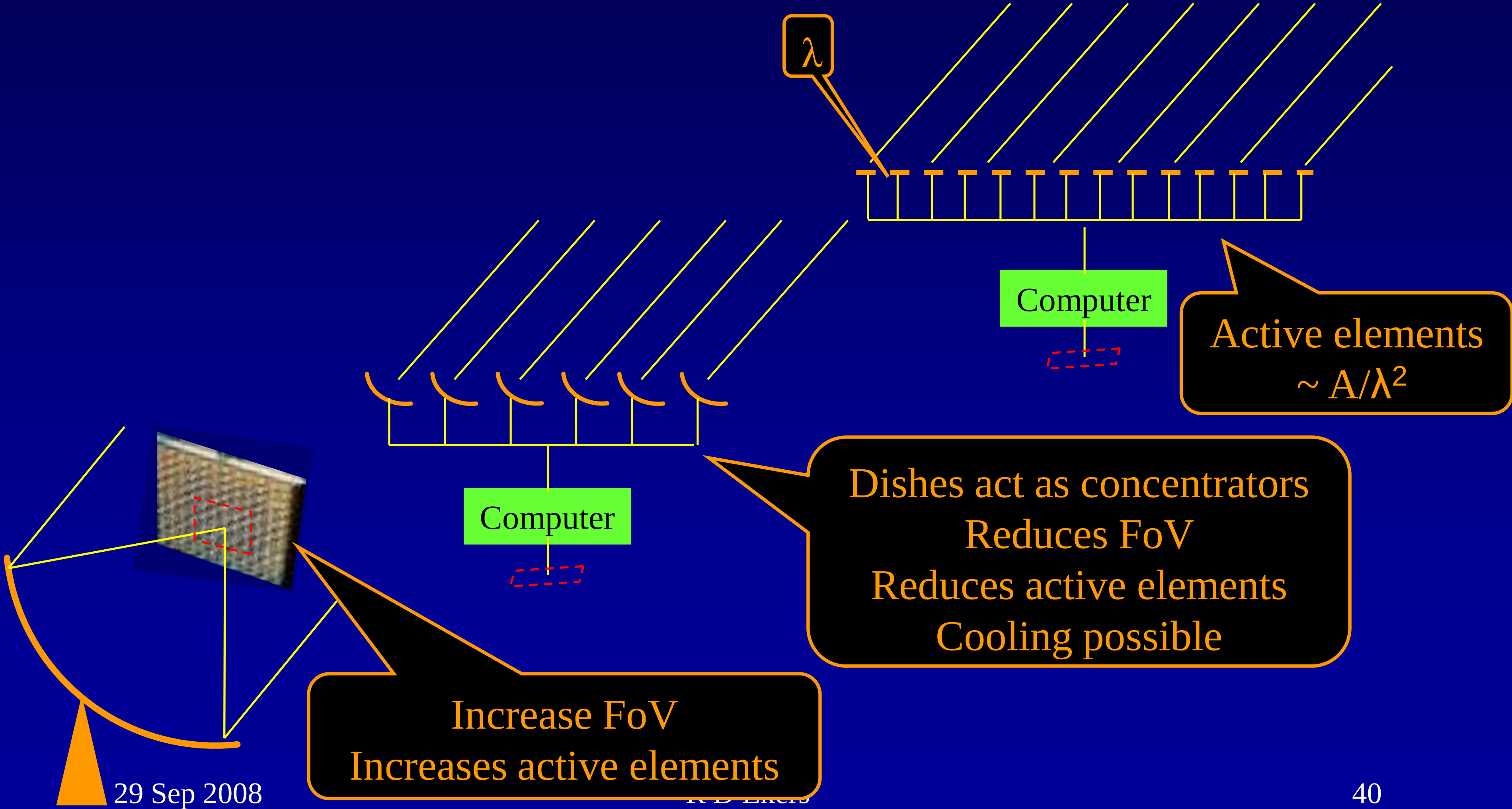
Find the Smallest Waist
use dish as a concentrator



$D_1 < D_2$

$n_1 < n_2$

Radio Telescope Imaging image v aperture plane

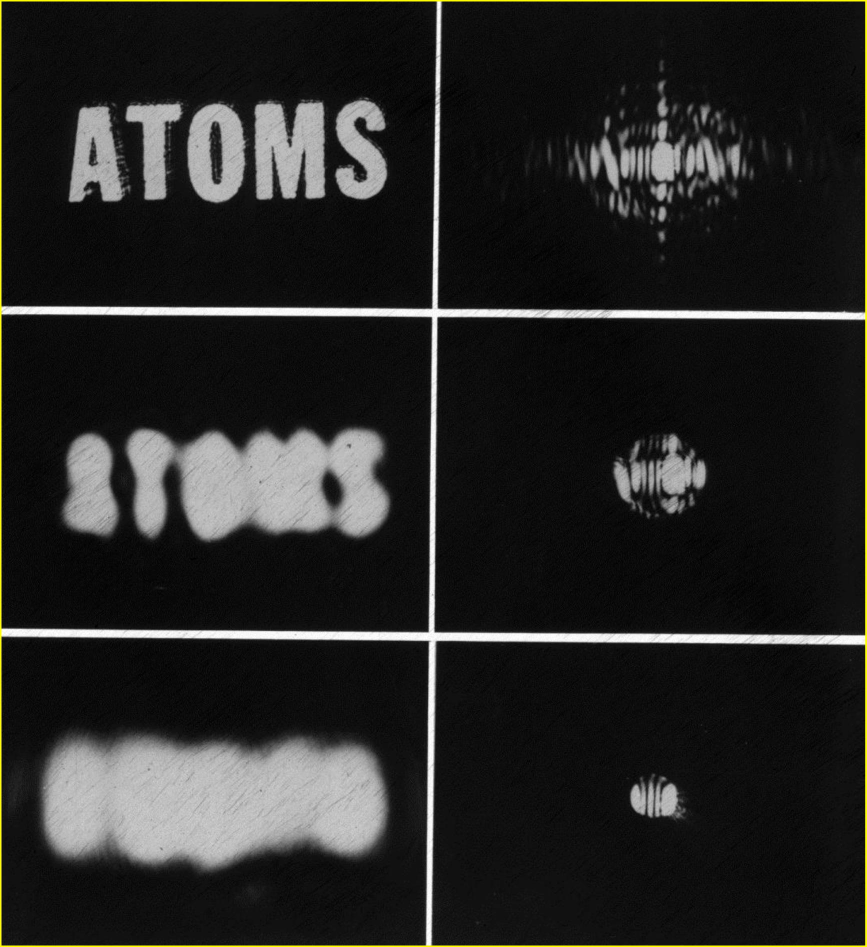


Active elements ~ A/λ^2

Dishes act as concentrators
Reduces FoV
Reduces active elements
Cooling possible

Increase FoV
Increases active elements

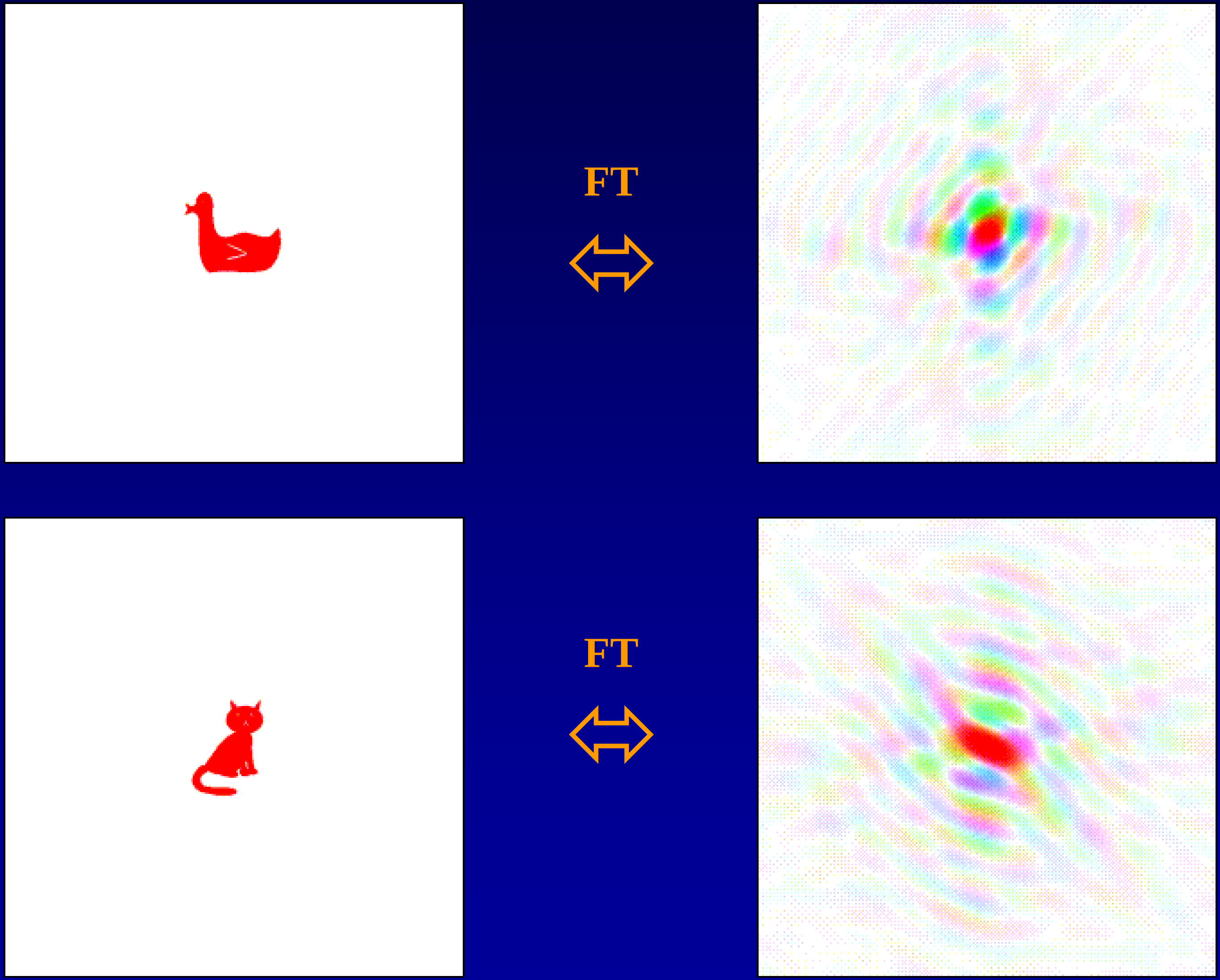
Fourier Transform and Resolution



- Large spacings
  - high resolution
- Small spacings
  - low resolution

Fourier Transform Properties

from Kevin Cowtan's Book of Fourier

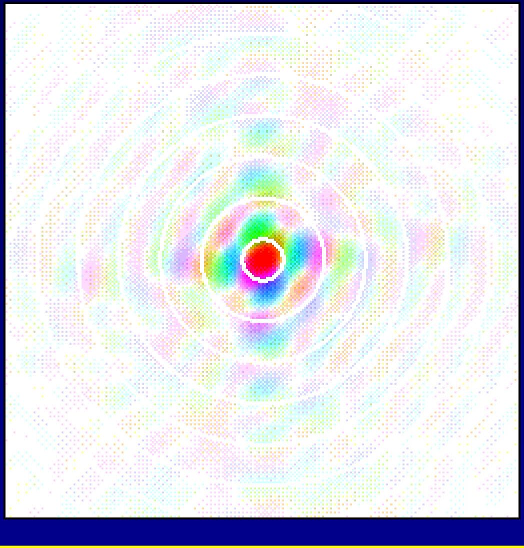


FT ⇔

FT ⇔

http://www.ysbl.york.ac.uk/~cowtan/fourier/fourier.html

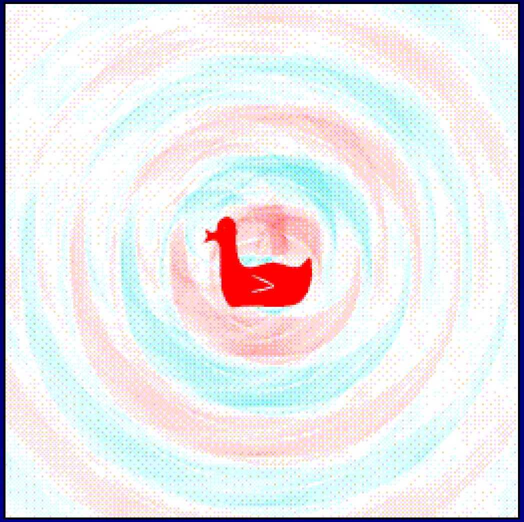
Fourier Transform Properties



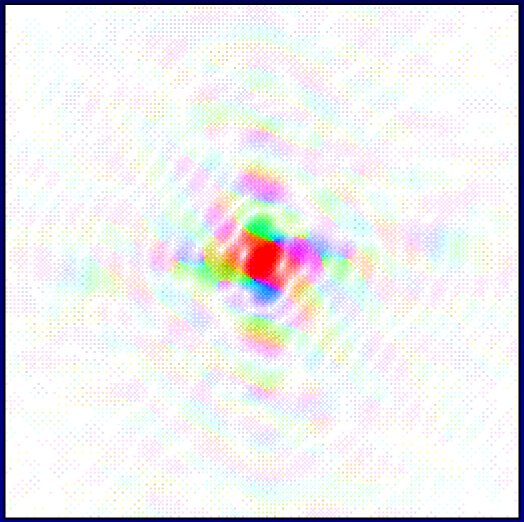
10% data omitted in rings

FT

⇔



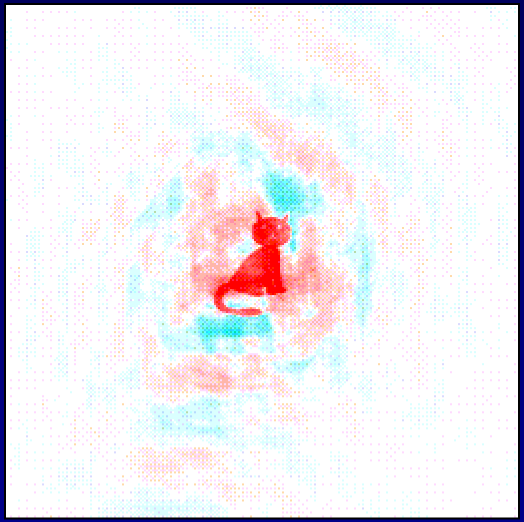
Fourier Transform Properties



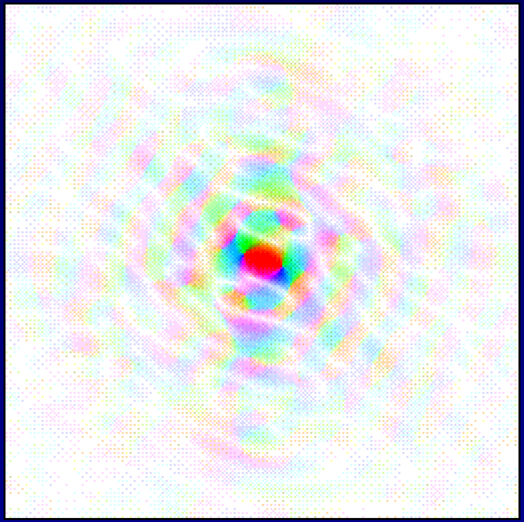
Amplitude of duck
Phase of cat

FT

⇔

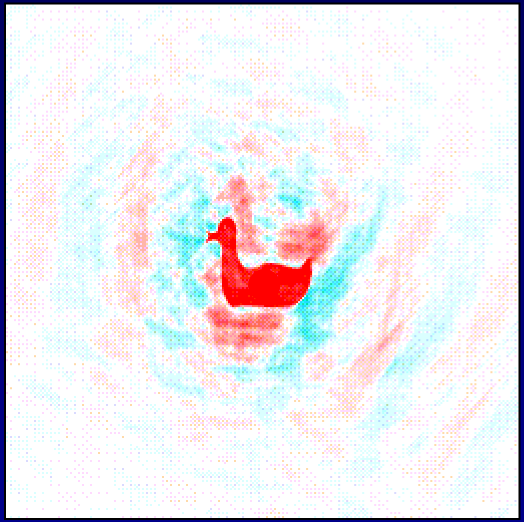


Fourier Transform Properties



FT

⇔



Amplitude of cat
Phase of duck

Analogies

RADIO		OPTICAL
grating responses	⇔	aliased orders
primary beam direction	⇔	grating blaze angle
UV (visibility) plane	⇔	hologram
bandwidth smearing	⇔	chromatic aberration
local oscillator	⇔	reference beam

Terminology

RADIO		OPTICAL
Antenna, dish	⇔	Telescope, element
Sidelobes	⇔	Diffraction pattern
Near sidelobes	⇔	Airy rings
Feed legs	⇔	Spider
Aperture blockage	⇔	Vignetting
Dirty beam	⇔	Point Spread Function (PSF)
Primary beam	⇔	Field of View

Terminology

RADIO		OPTICAL
Map	⇔	Image
Source	⇔	Object
Image plane	⇔	Image plane
Aperture plane	⇔	Pupil plane
UV plane	⇔	Fourier plan
Aperture	⇔	Entrance pupil
UV coverage	⇔	Modulation transfer function

Terminology

RADIO		OPTICAL
Dynamic range	⇔	Contrast
Phased array	⇔	Beam combiner
Correlator	⇔	*no analog*
no analog	⇔	Correlator
Receiver	⇔	Detector
Taper	⇔	Apodise
Self calibration	⇔	Wavefront sensing (Adaptive optics)