High dynamic range and high fidelity imaging

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The Basic Measurement Equation of Interferometry

- The data we collect are related to what we want to know via a measurement equation

- For small fields of view, the monochromatic visibility is the 2D Fourier transform of the sky

$$V(u,v) = \int I(l,m) e^{2\pi j(ul+vm)} dl dm$$

- Straightforward to invert to obtain an image

$$I^D = \sum_i w_i V(u_i, v_i) e^{2\pi j(u_i l + v_i m)}$$

Fidelity

- Accuracy of representation of source structure
- ~ on source signal to noise
- Not directly measurable or easily quantifiable
- Requires simulation
- Related to measurement strategy

Deconvolution

$$I^D = \sum_i w_i V(u_i, v_i) e^{2\pi j(u_i l + v_i m)}$$

- Dirty image = true sky convolved with dirty beam
- Solve iteratively for sky using a deconvolution algorithm
- CLEAN, MEM, compressive sampling

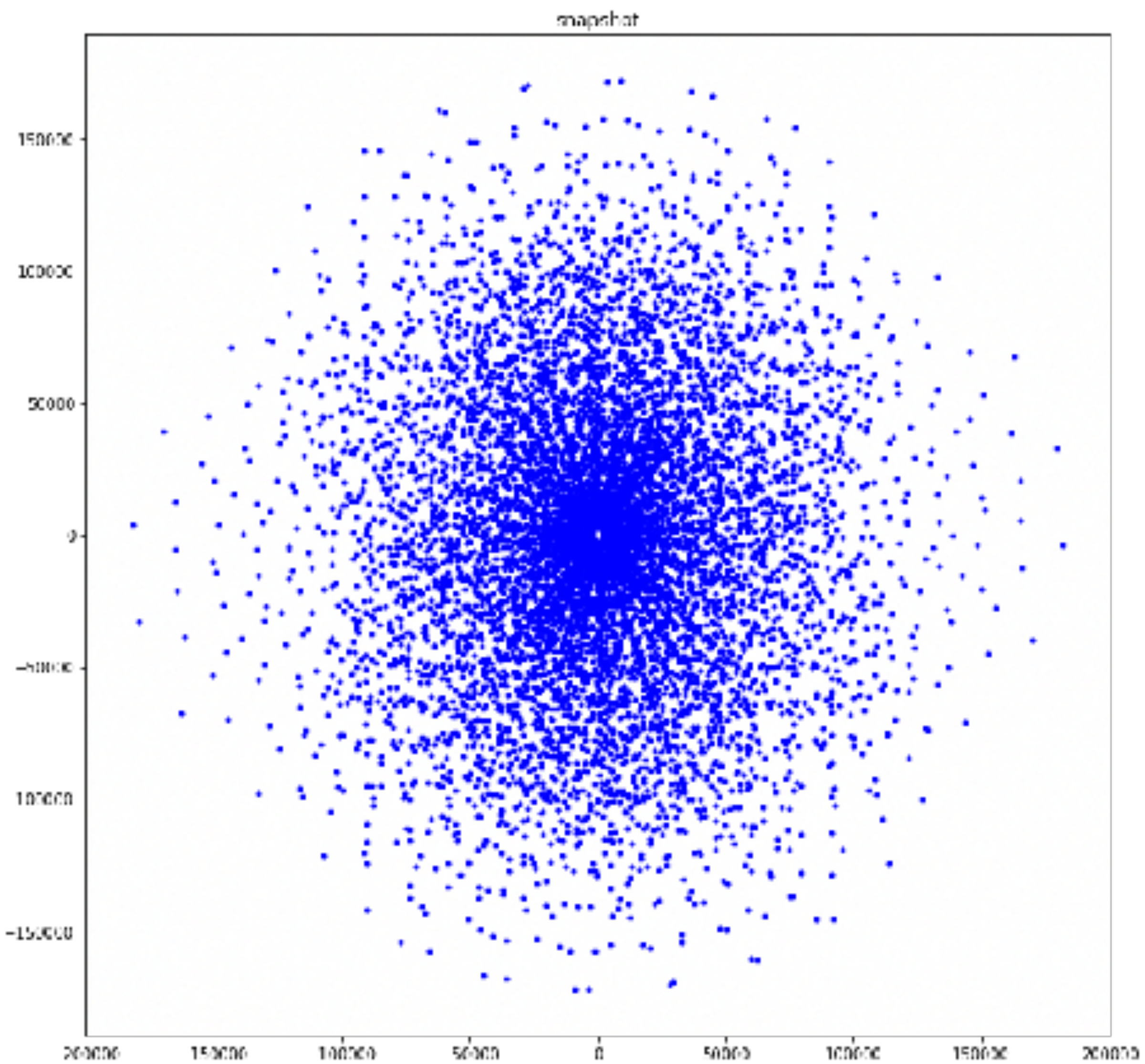
$$B^D = \sum_i w_i e^{2\pi j(u_i l + v_i m)}$$

$$I^D = B^D \otimes I^{sky}$$

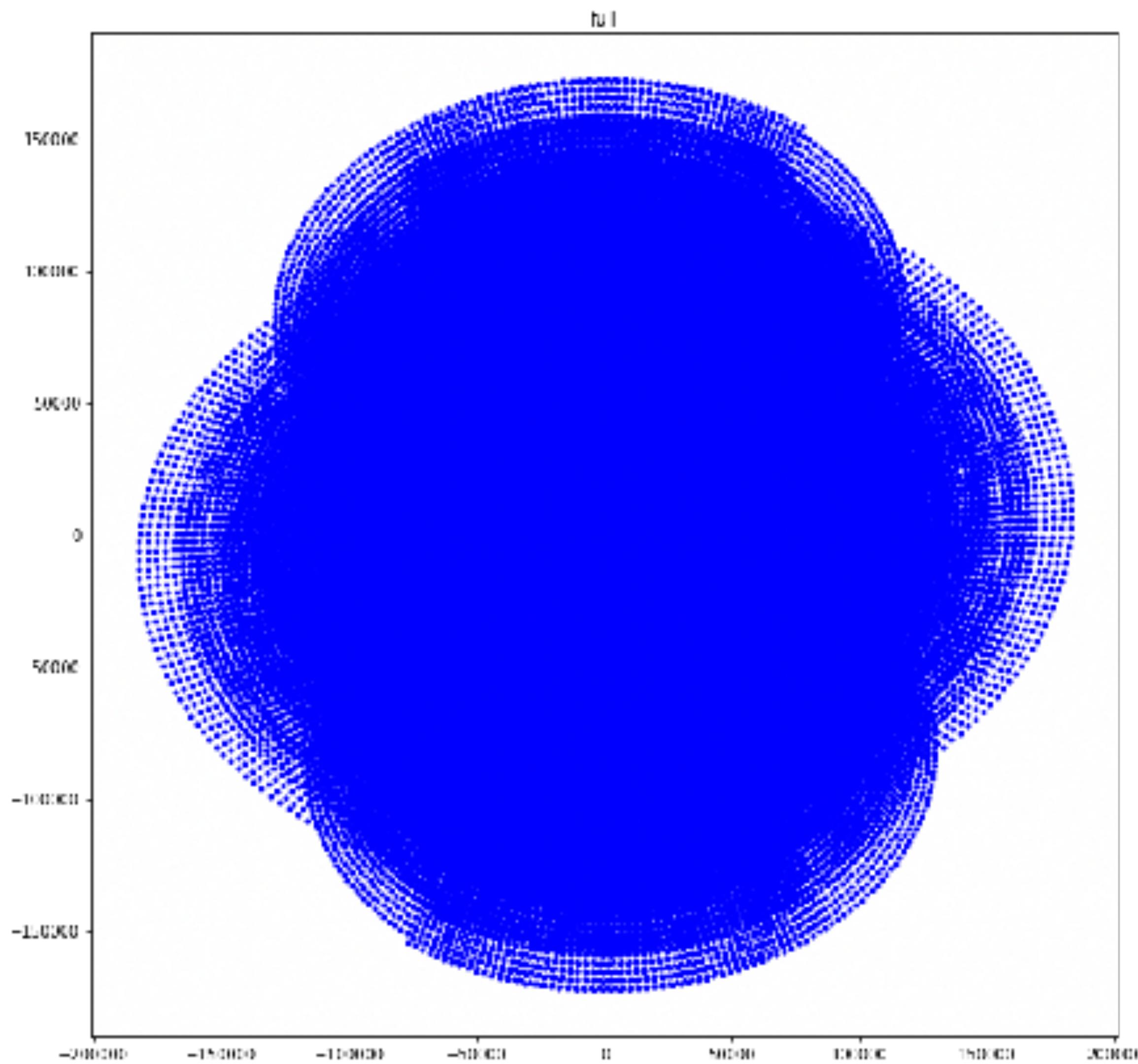
Problems in deconvolution

- Invisible distributions $B^D \otimes Z \approx 0$
- Cannot work miracles: complex field + poor uv coverage
- CLEAN models extended distributions as collection of point sources
- CLEAN is iterative and may not have converged
- Standard CLEAN emphasises full resolution
- Multi-scale CLEAN works for a range of scale sizes

VLA simulation

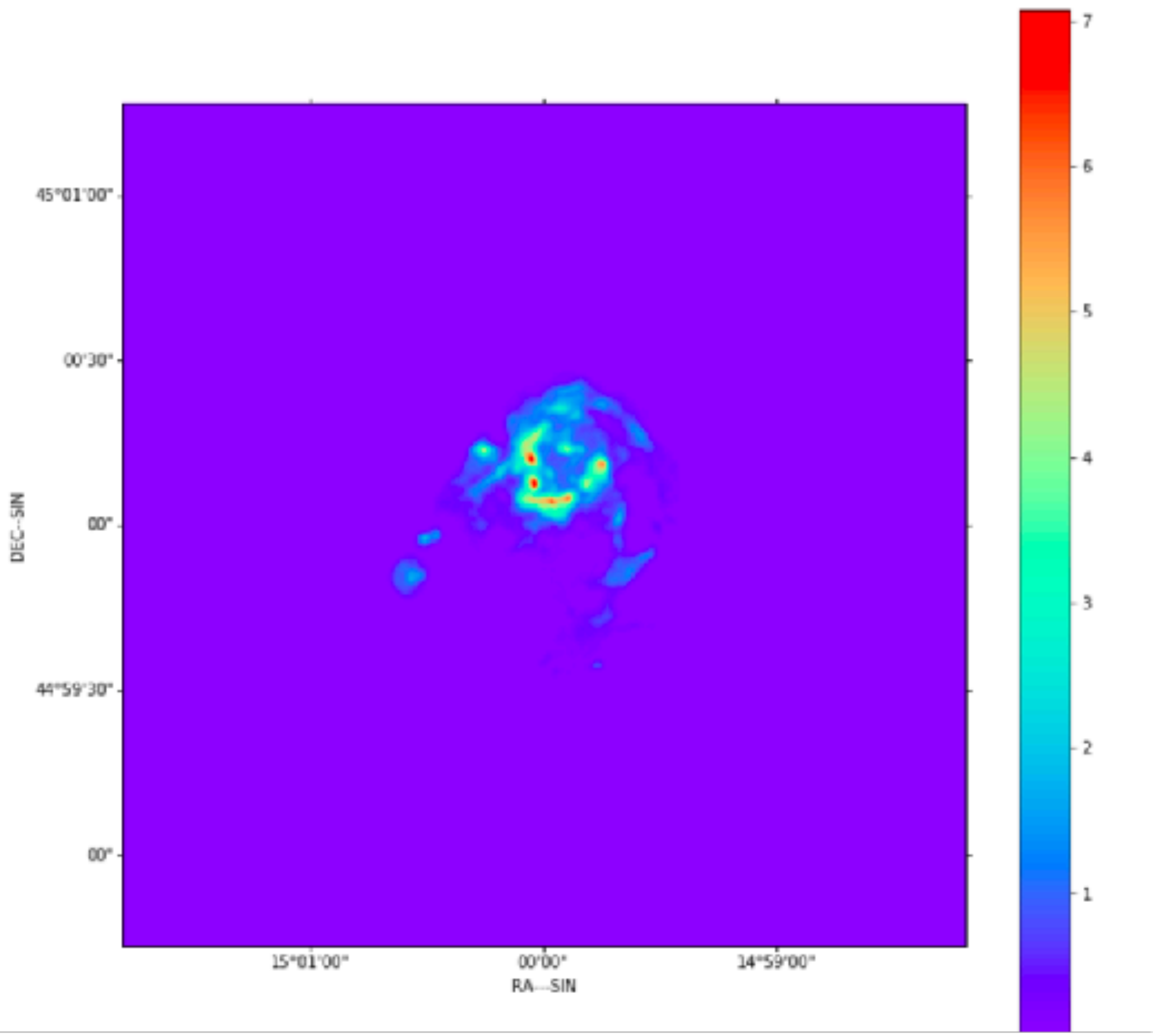


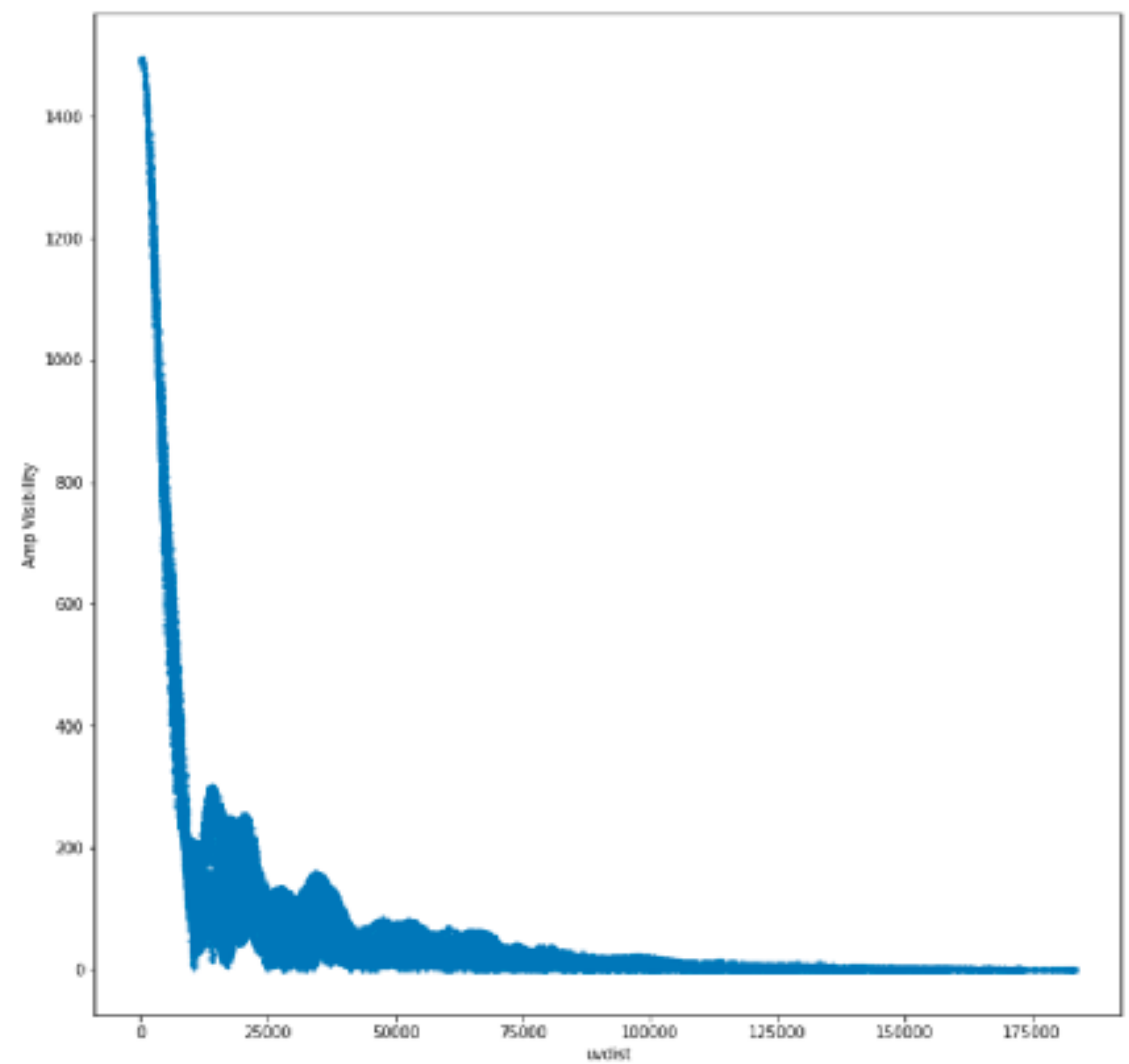
12 hour angles at one frequency



120 hour angles at eight frequencies

Model and visibilities

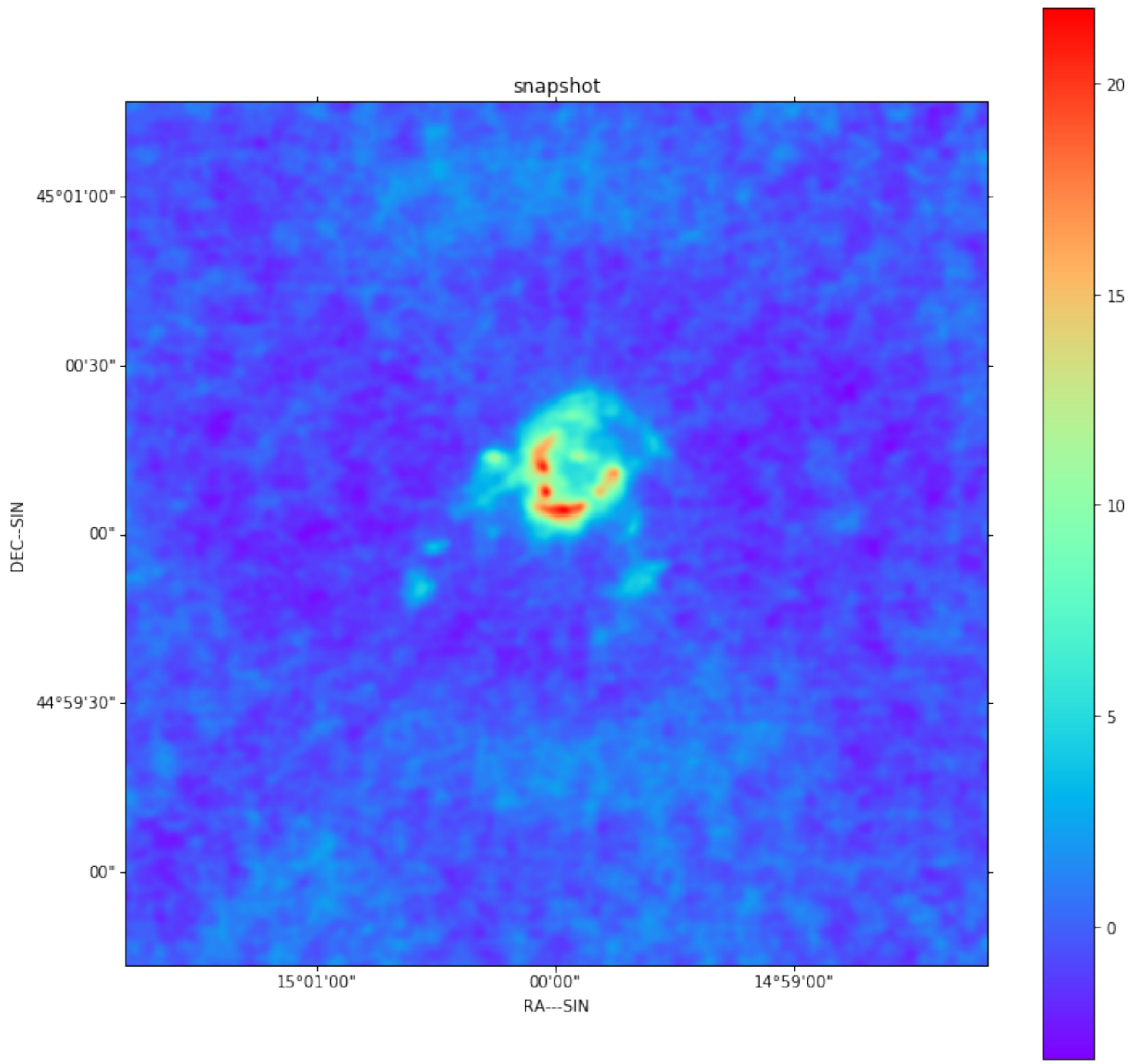




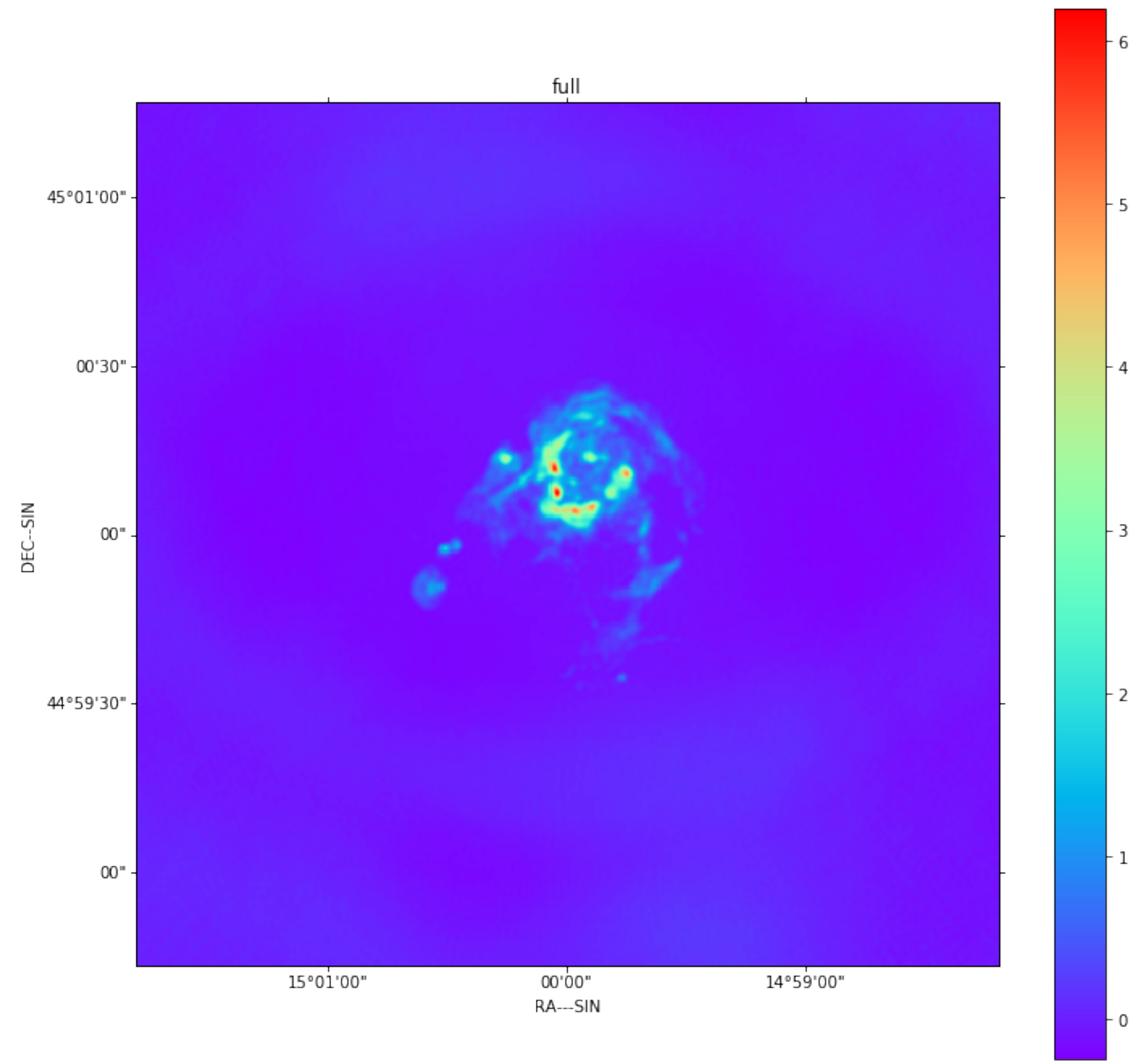
Model

Visibility amplitude

Dirty images



12 hour angles at one frequency



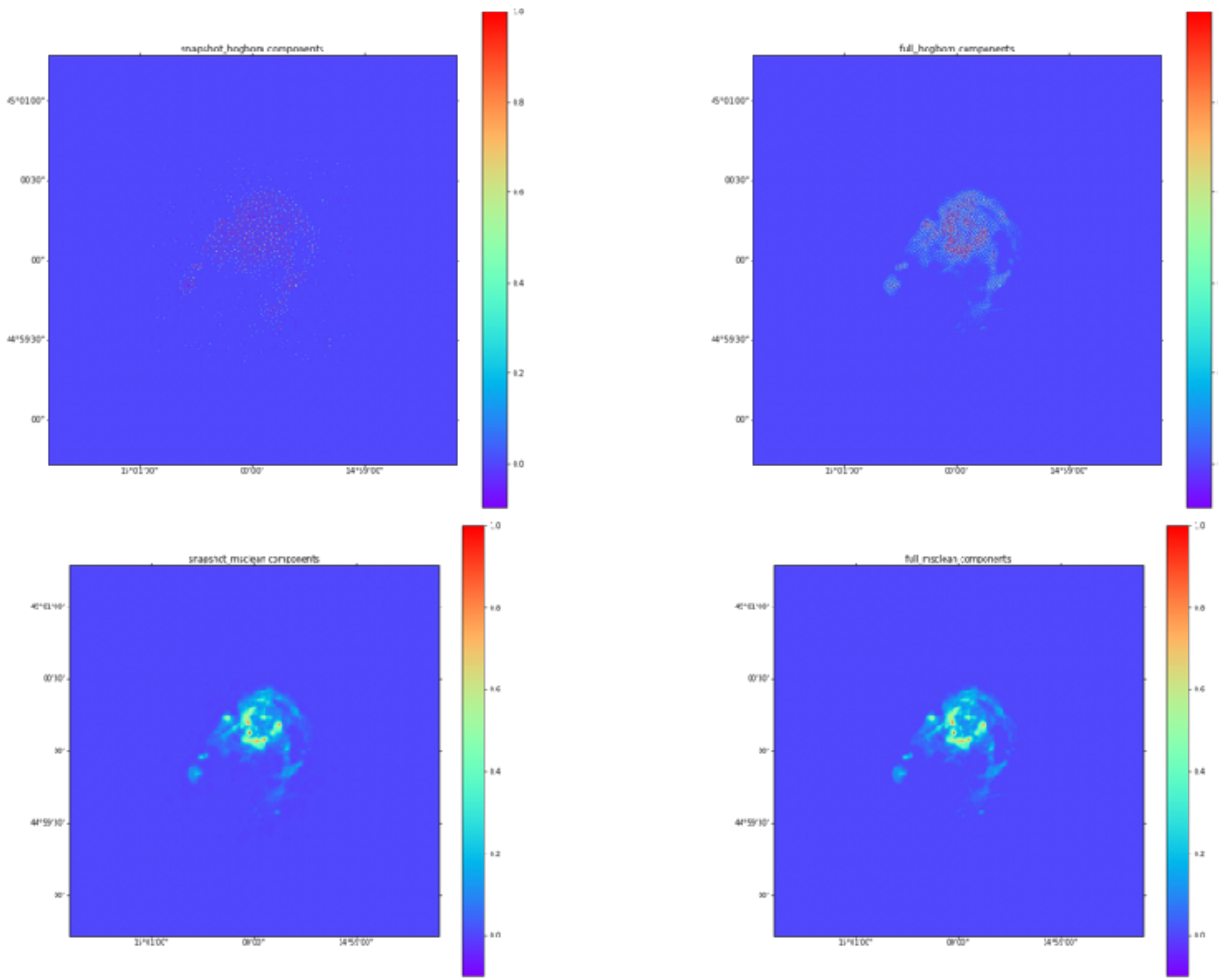
120 hour angles at eight frequencies

Recovered models

Multiple snapshots | Full observation

Hogbom CLEAN

Multiscale CLEAN



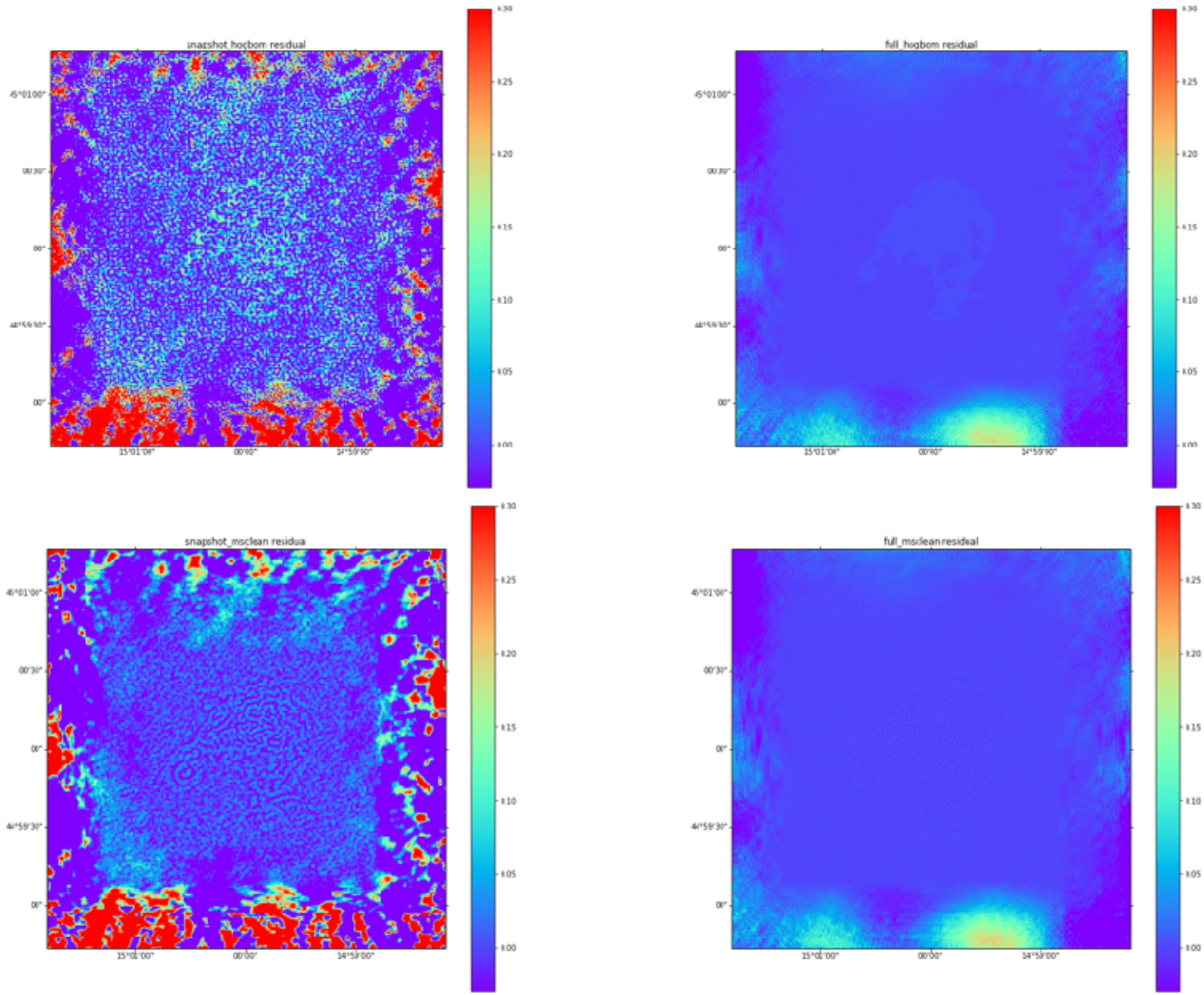
CLEAN residual images

Multiple snapshots

Full observation

Hogbom CLEAN

Multiscale CLEAN



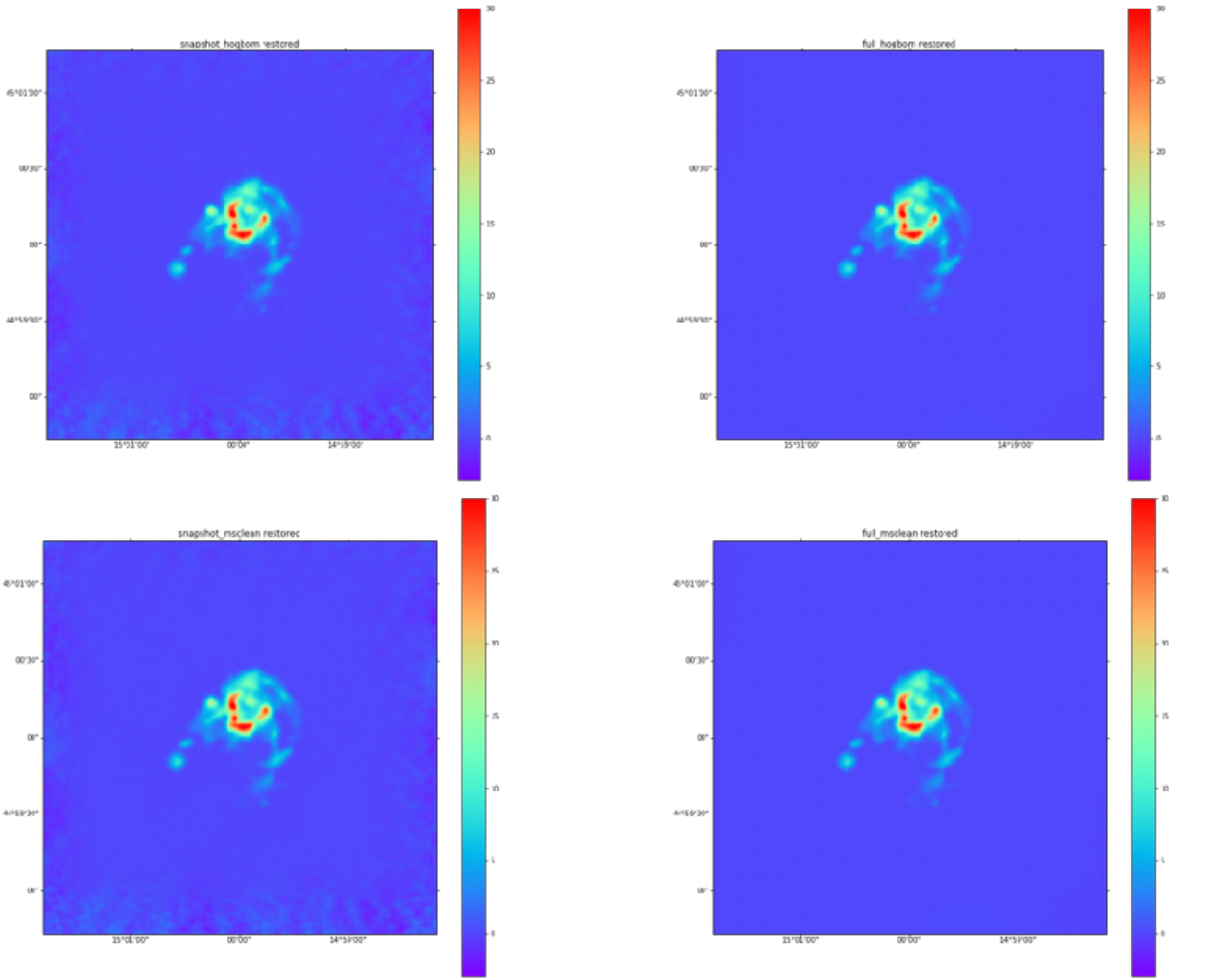
CLEAN restored images

Multiple snapshots

Full observation

Hogbom CLEAN

Multiscale CLEAN

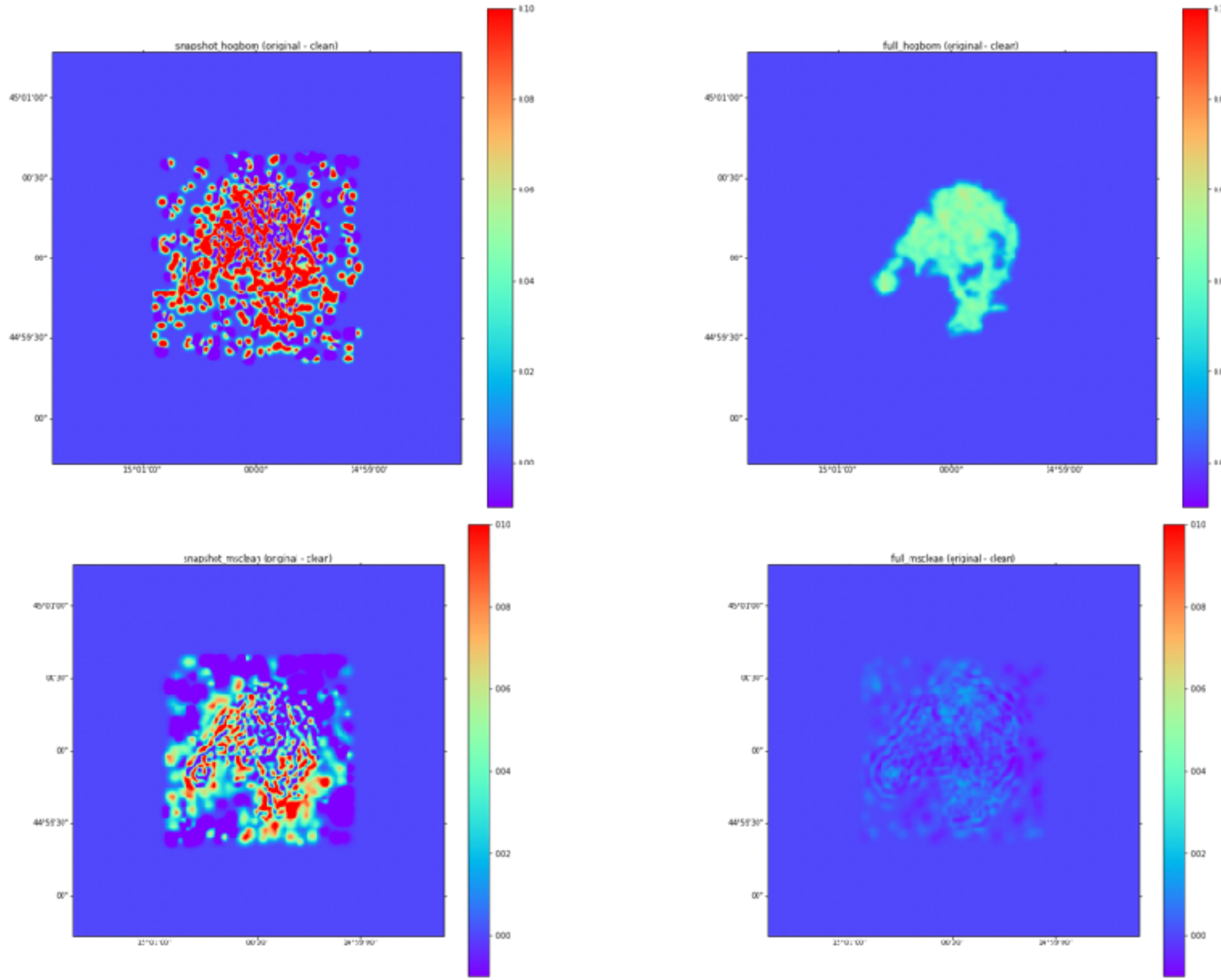


Model - original

Multiple snapshots | Full observation

Hogbom CLEAN

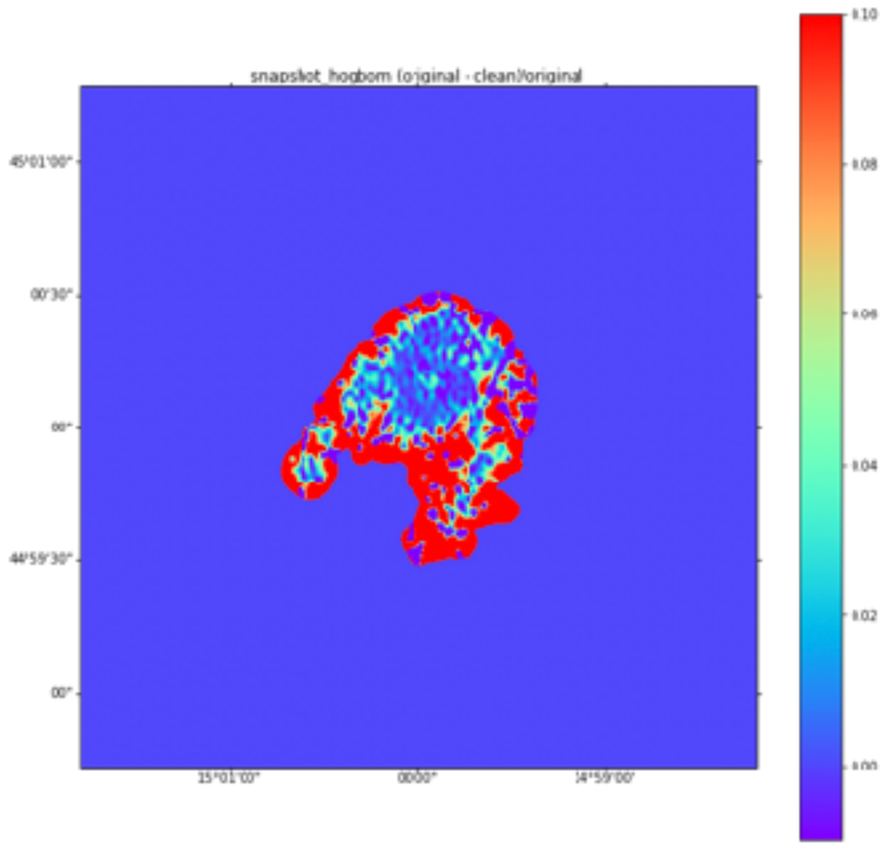
Multiscale CLEAN

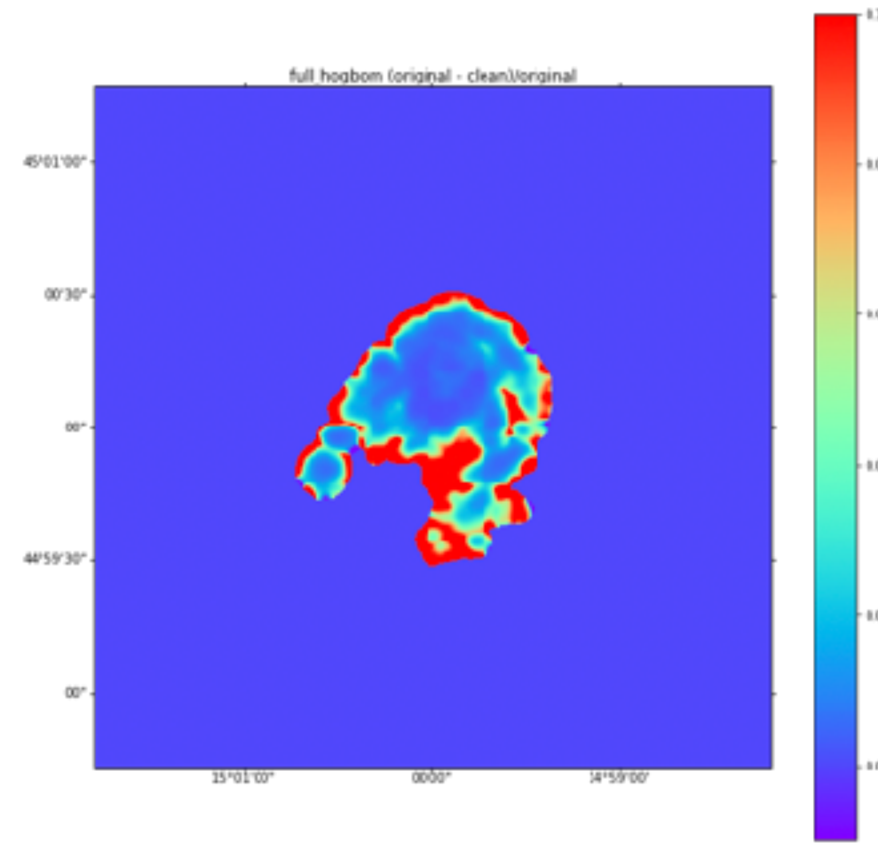


(Model - original)/original

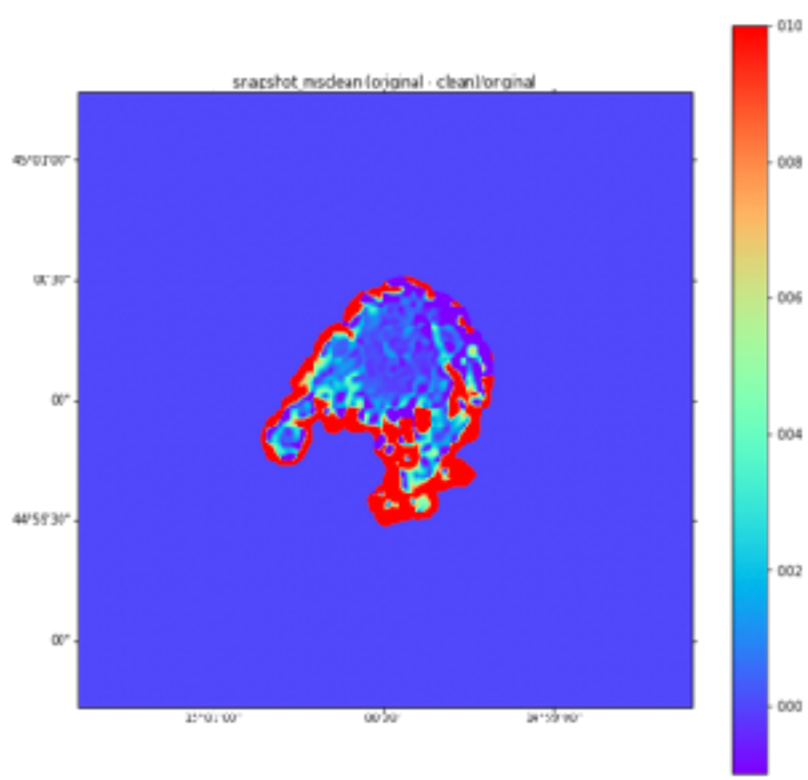
Multiple snapshots | Full observation

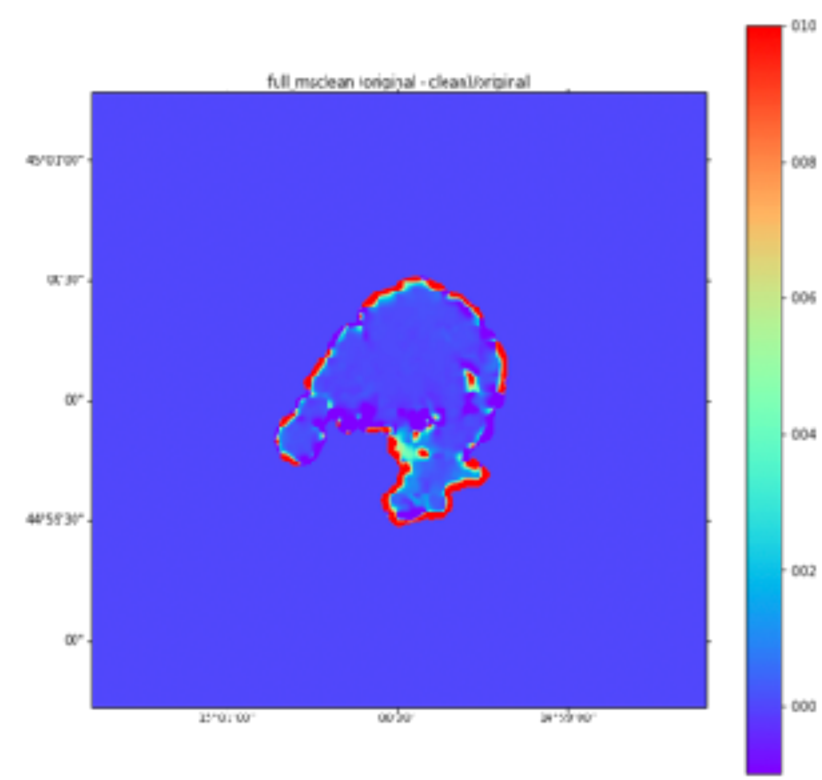
Hogbom CLEAN





Multiscale CLEAN





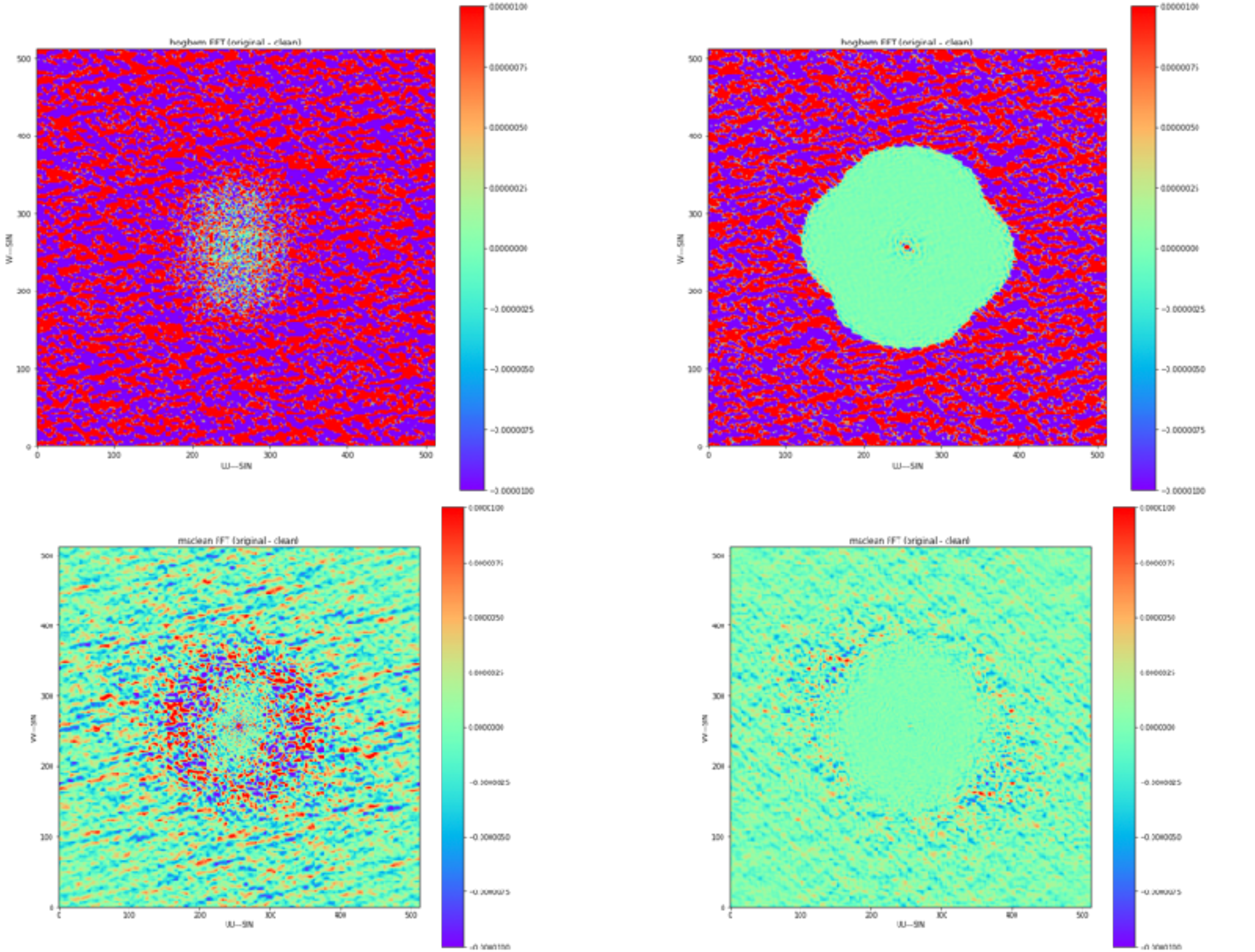
Fourier transform (model - original)

Multiple snapshots

Full observation

Hogbom CLEAN

Multiscale CLEAN



Fidelity lessons

- On source noise level >> off source noise level
- Residuals can be low but the image has on-source errors
- Different deconvolution algorithms give subtly answers
- Be cautious of invisible distributions
- See also Naomi's talk on zero spacing (broad scale structure)
- Take more data: more time, frequencies, different array, single dish, …

Dynamic range

- The ability to see a weak signal in the presence of a stronger signal
- Typically defined as ratio of peak source/rms rumble
- Measureable and quantifiable
- But varies with time, frequency, scale, polarisation,…
- Tests many aspects of the telescope
- Often directly related to science

Failure of normal calibration

- Effects depends on nature of errors
- e.g. Linear phase gradient across an array leads to position offset
- e.g. Time-variable phase gradients leads to source wandering
- e.g. Moderate statistical errors leads to decorrelation: seeing disk
- Large uncorrelated errors: cannot image!

Movie of point source at 22GHz

- Source moves [redacted]
- See anti-symm [redacted] errors: signatu [redacted] phase errors
- Three armed s [redacted] reflects VLA sh [redacted]

Effects of statistical errors in snapshot image

- N antennas, N(N-1)/2 baselines, snapshot imaging

One baseline with phase error ϕ $$DR \approx \frac{N^2}{\sqrt{2}\phi}$$

One baseline with amplitude error ε $$DR \approx \frac{N^2}{\sqrt{2}\varepsilon}$$

One antenna with random phase error $$DR \approx \frac{N^{3/2}}{\sqrt{2}\phi}$$

All antennas have random phase error $$DR \approx \frac{N}{\sqrt{2}\phi}$$

ME with calibration errors

- Antenna-based errors

$$V_{i,j}(u_{i,j},v_{i,j}) = g_i g_j^* \int I(l,m) e^{2\pi j\left(u_{i,j}l + v_{i,j}m\right)} dl dm$$

- Solve for the antenna gains from measurements with a point source of known strength and position

$$V_{i,j}(u_{i,j},v_{i,j}) = g_i g_j^* S$$

- Or from a known model

$$V_{i,j}(u_{i,j},v_{i,j}) = g_i g_j^* \sum_k S e^{2\pi j\left(u_{i,j}l_k + v_{i,j}m_k\right)}$$

- Or solve for both image and calibration

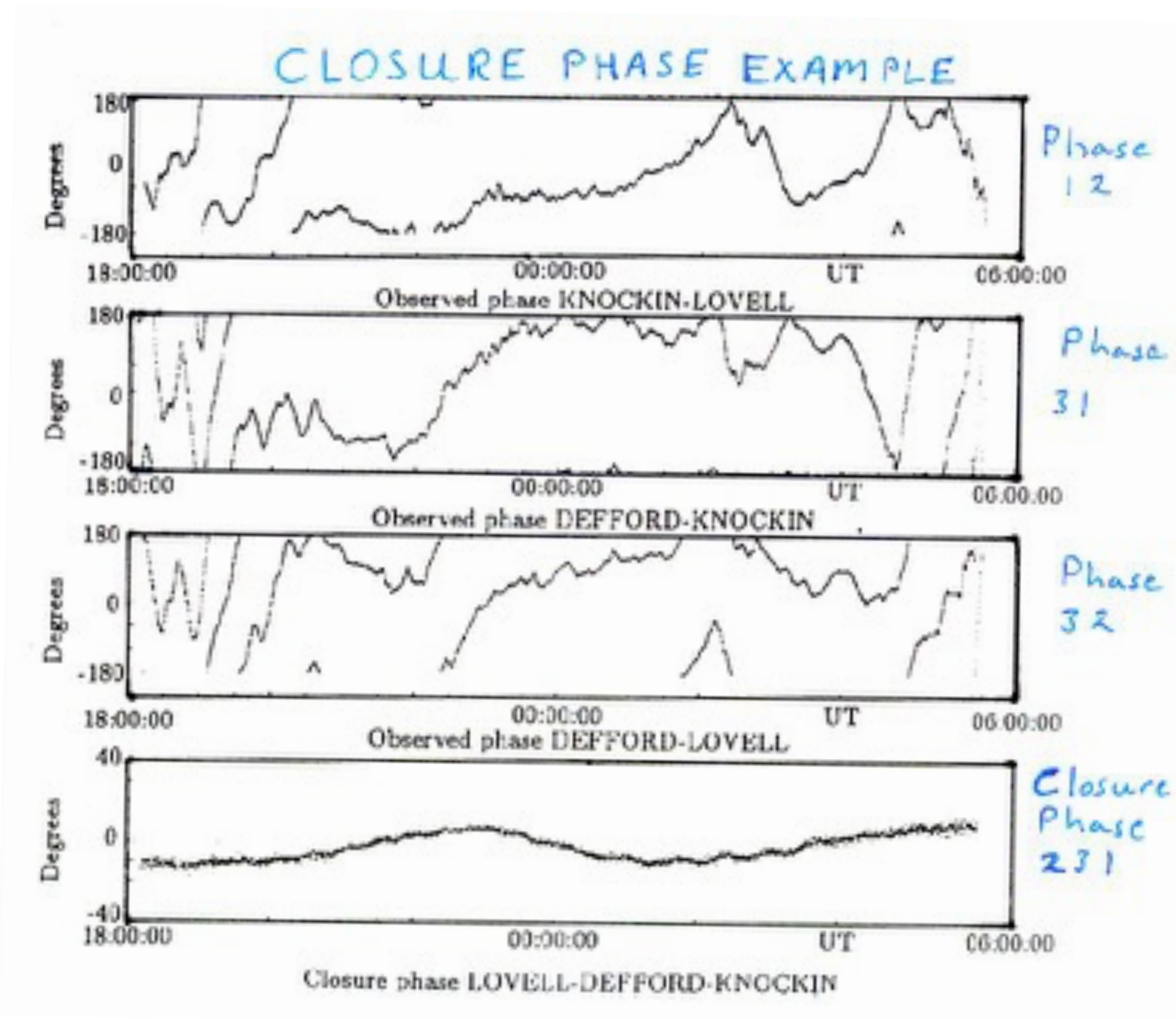
$$\sum_{i,j} w_{i,j} \left\| V_{i,j}(u_{i,j},v_{i,j}) - g_i g_j^* \sum_k S e^{2\pi j\left(u_{i,j}l_k + v_{i,j}m_k\right)} \right\|^2$$

Why does this work?

- Interferometric array measures $N(N-1)/2$ phases
- There are $N-1$ free antenna phases
- So we have $(N-1)(N-2)/2$ constraints - the closure phases

Closure phase

- Three antennas from 120km baselines in MERLIN at 408MHz
- Data taken in 1980!
- Top three lines are baseline phases
- Bottom is the closure phase - sum of phases around a loop
- Good observable even in present of strong antenna-based phase errors



ME with calibration errors

- Antenna-based errors

$$V_{i,j}(u_{i,j},v_{i,j}) = g_i g_j^* \int I(l,m) e^{2\pi j\left(u_{i,j}l + v_{i,j}m\right)} dldm$$

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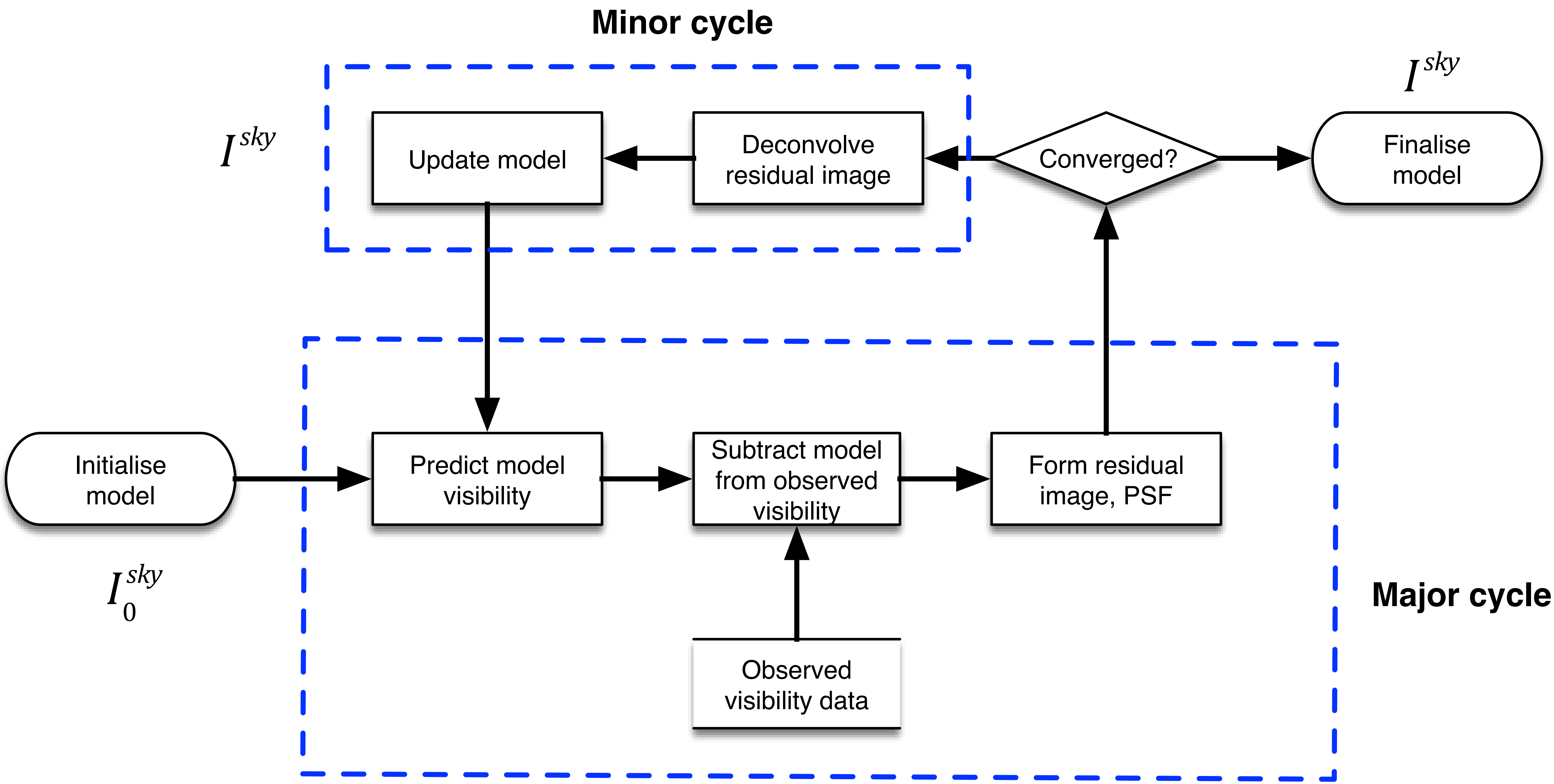
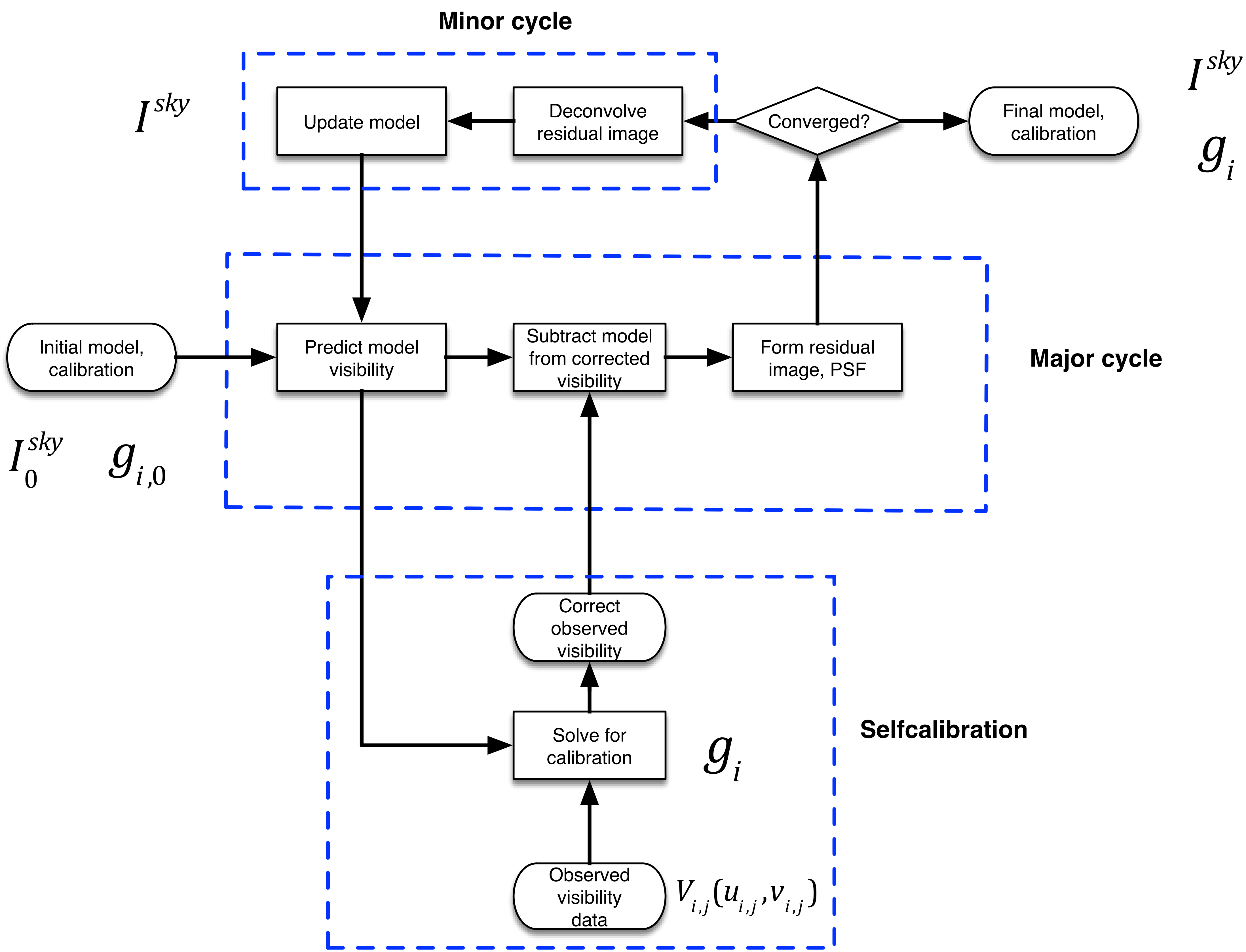
$$\sum_{i,j} w_{i,j} \left\| V_{i,j}(u_{i,j},v_{i,j}) - g_i g_j^* \sum_k S e^{2\pi j\left(u_{i,j}l_k + v_{i,j}m_k\right)} \right\|^2$$

Standard imaging



Selfcalibration



Selfcalibration signal to noise limits

- Self-calibration imposes consistency relationship for visibility phases
- SNR must be sufficient for phase measurement to be meaningful
- For quasi point source, error in phase part of gain is $\sigma_g^2 = \frac{\sigma_V^2}{\left(N_{ant} - 2\right) S^2}$
- Requires Signal to noise per antenna >> 1
- Beware bias! e.g. Selfcalibrating noise!

Closure errors

$$V_{i,j}(u_{i,j},v_{i,j}) = c_{i,j} g_i g_j^* \sum_k S e^{2\pi j \left(u_{i,j} l_k + v_{i,j} m_k\right)}$$

- What happens if the calibration errors do not factorise per antenna?
- Modern digital correlators should not have closure errors
- Can appear if delays or time standards have large errors
- Also pointing errors for well-filled field of view

Direction independent and direction dependent effects

$$V_{i,j}(u_{i,j},v_{i,j}) = g_i g_j^* \int I(l,m) e^{2\pi j\left(u_{i,j} l + v_{i,j} m\right)} dl dm$$

- In our formulation so far the errors are the same over the field of view

- Some effects are direction dependent

$$V_{i,j}(u_{i,j},v_{i,j}) = \int g_i(l,m) g_j^*(l,m) I(l,m) e^{2\pi j\left(u_{i,j} l + v_{i,j} m\right)} dl dm$$

Direction dependent effects

$$V_{i,j}(u_{i,j},v_{i,j})=\int g_i(l,m)g_j^*(l,m)I(l,m)e^{2\pi j\left(u_{i,j}l+v_{i,j}m\right)}dldm$$

- Some calibration errors can be direction dependent
- Antenna primary beams
- Ionospheric phase
- We know how to do the math
- But it can be very expensive to compute!

Primary Beam Correction : A-Projection

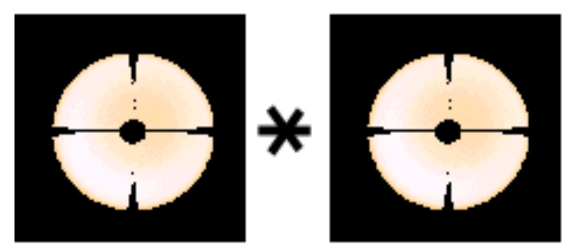
Bhatnagar et al, 2008

Apply PB correction in the UV-domain **before** visibilities are combined.

$$I_{ij}^{obs} = I_{ij}^{psf} * \left[P_{ij} . I^{sky} \right] \longleftrightarrow V_{ij}^{obs} = S_{ij} . \left[A_{ij} * V^{sky} \right]$$

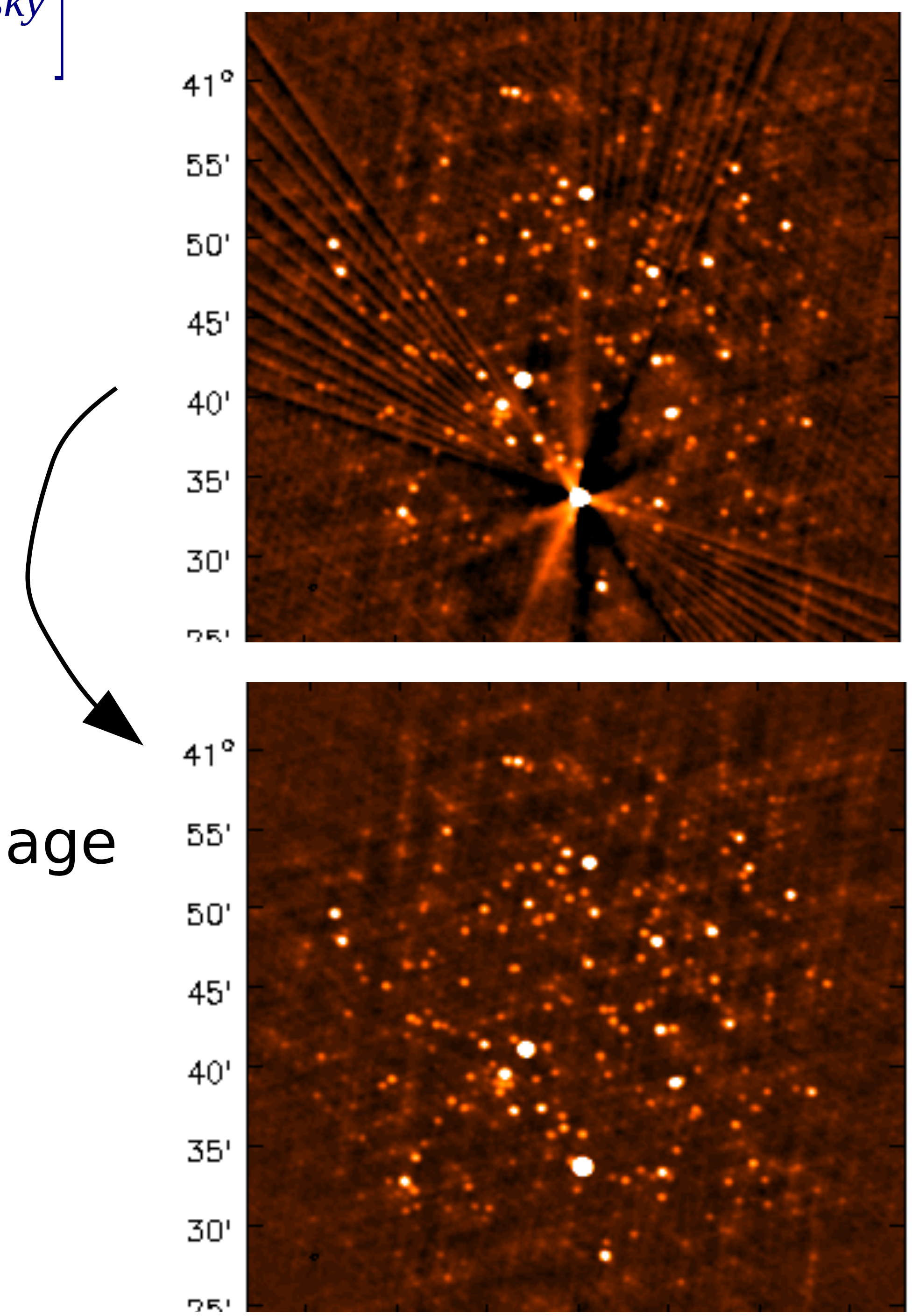
For each visibility, apply $A_{ij}^{-1} \approx \dfrac{A_{ij}^{T}}{A_{ij}^{T} * A_{ij}}$

(1) Use A_{ij}^{T} as the convolution function during gridding



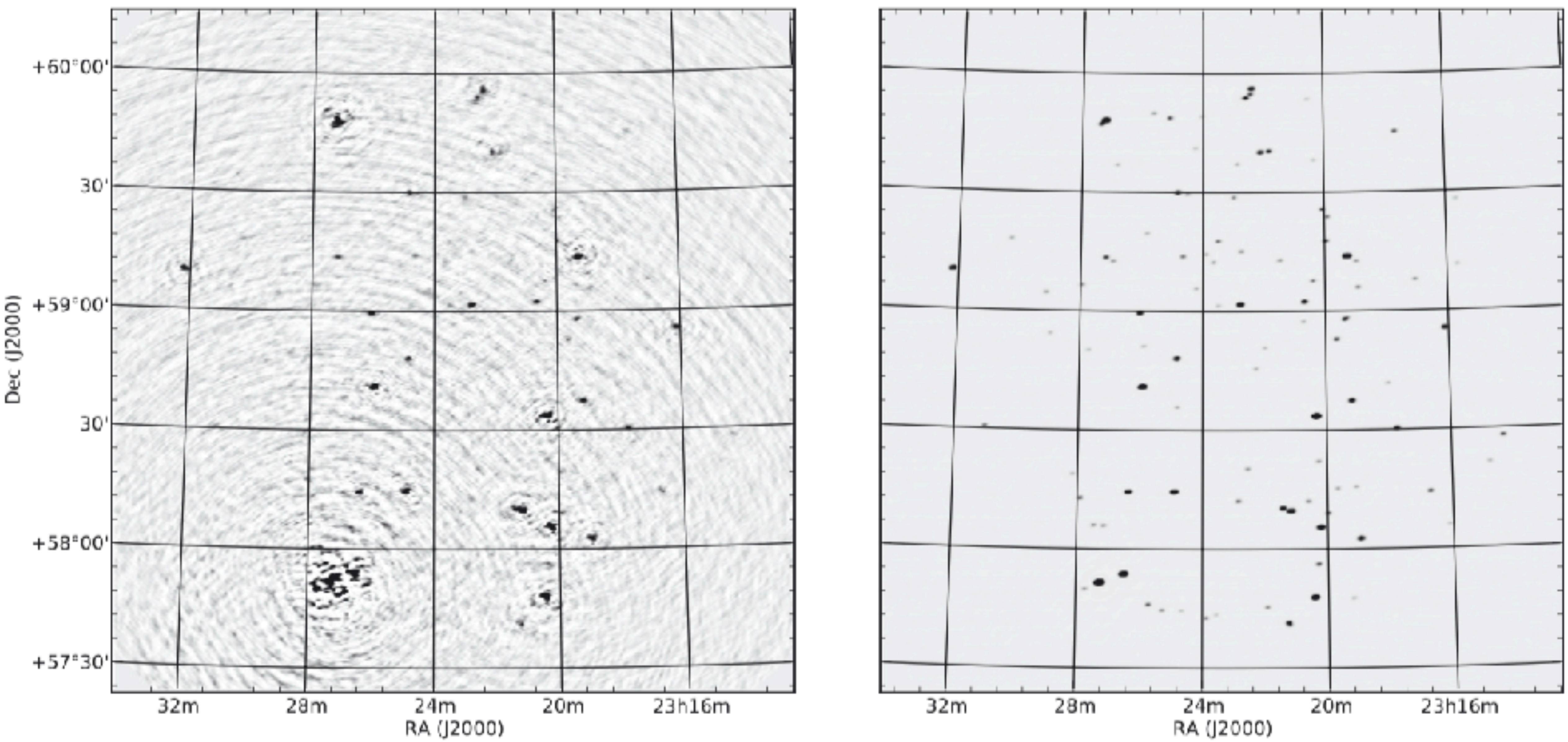
(2) Divide out $FT\left[\sum_{ij} A_{ij}^{T} * A_{ij} \right]$ from the image (in stages).

- Conjugate transpose corrects for known pointing offsets such as beam squint.
- An additional phase ramp is applied for different pointings to make a joint mosaic.



A projection

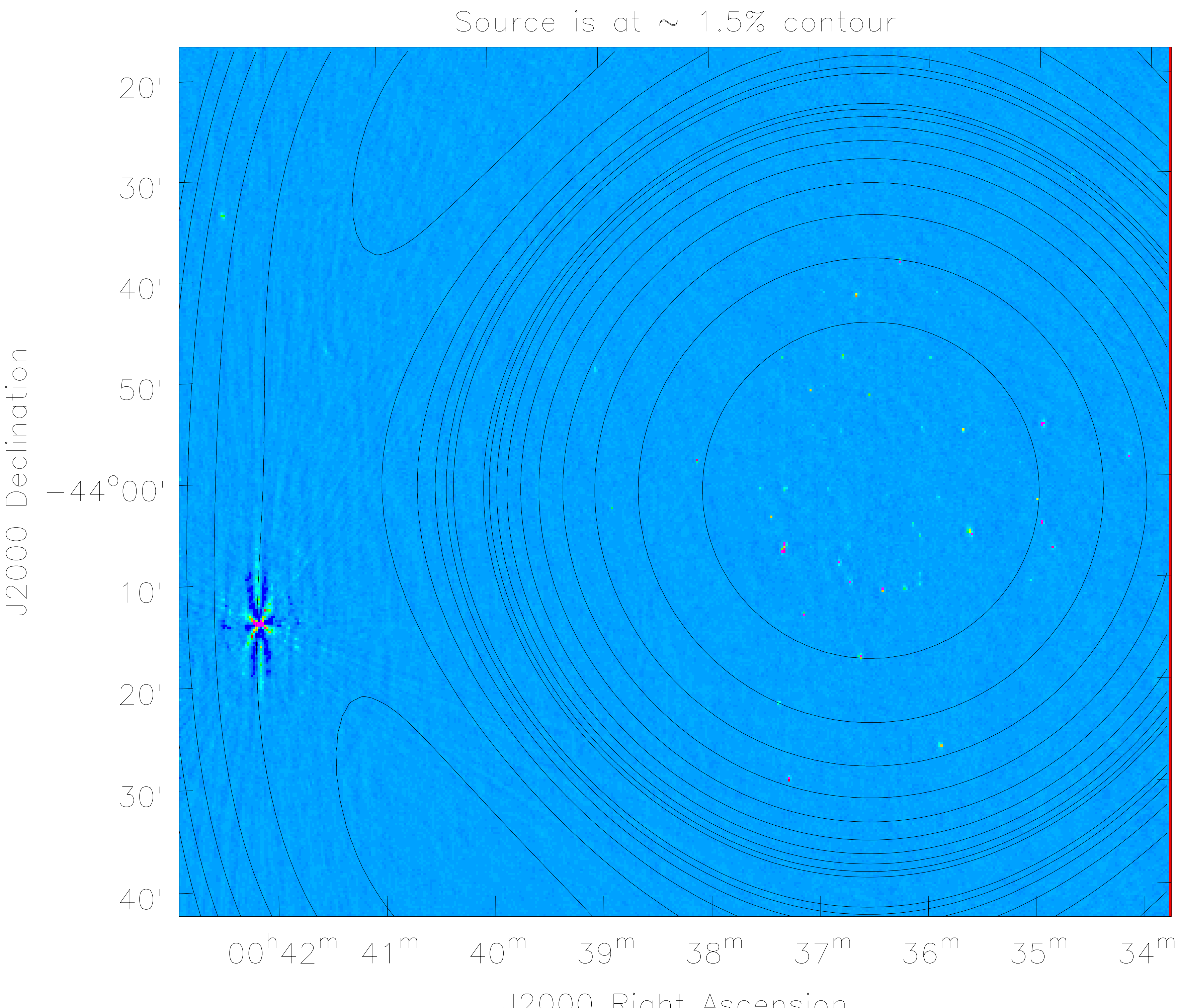
C. Tasse et al.: Applying full polarization A-Projection to very wide field of view instruments



Sidelobes from a single transient sources

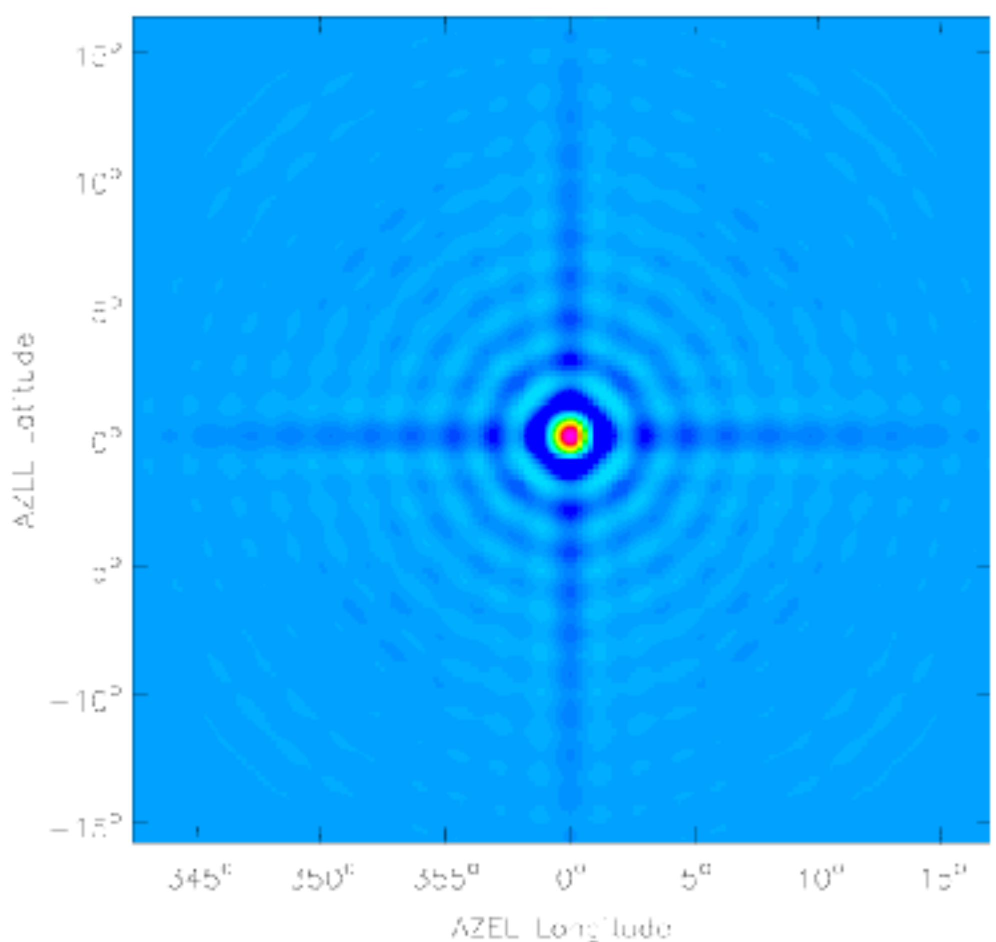
- Suppose a source at $\left(l_0, m_0\right)$ in the field increases flux by ΔS
- This causes a pattern $B^D_{snapshot}\left(l-l_0, m-m_0\right)\Delta S$
- This is weighted by the duty cycle
- e.g. 10% flux change for 6 min of a ten hour observation for an array with 1% rms sidelobes limits dynamic range to 100,000
- Can be much worse for if uv coverage is gathered over long time e.g. ATCA (Baerbel's talk)

Sidelobes from exterior sources in rotating primary beam



ATCA antenna

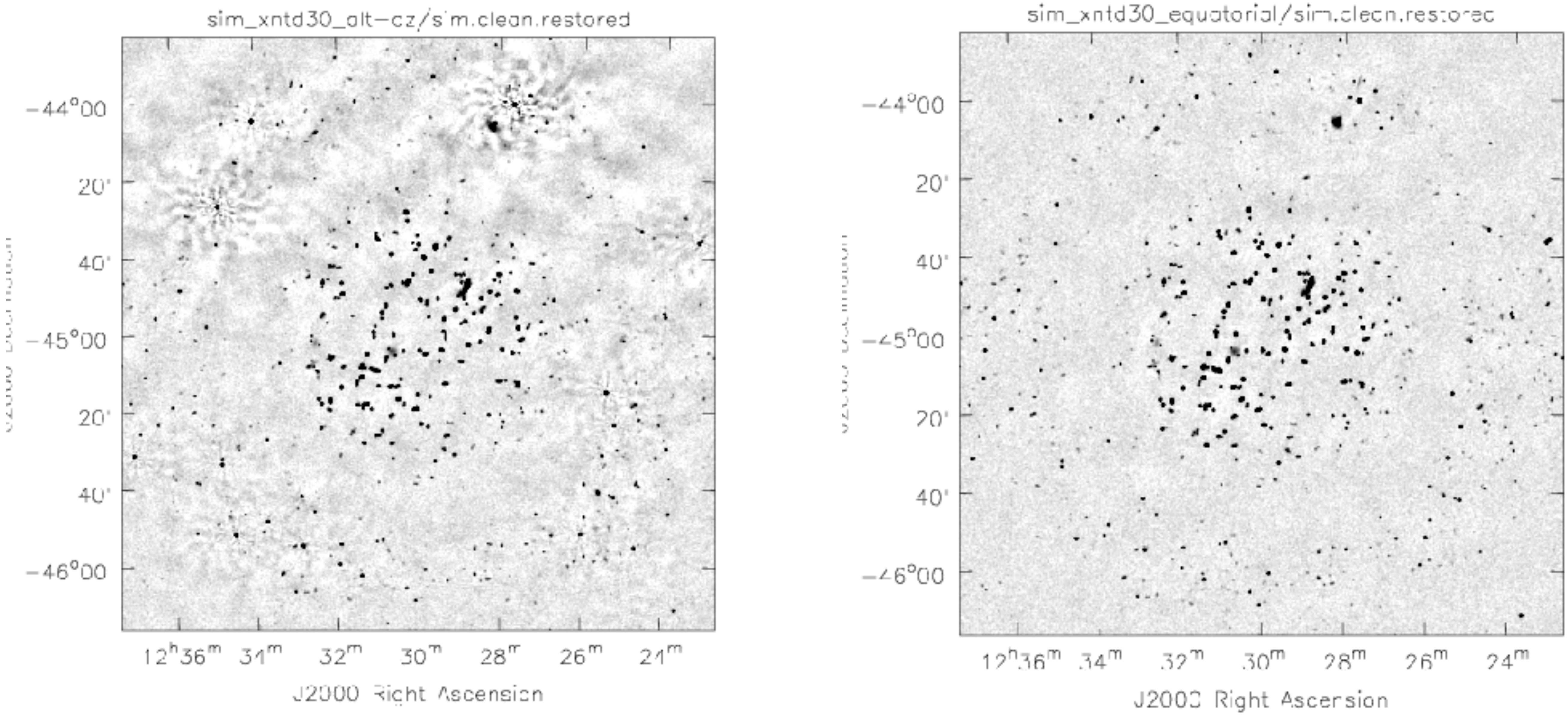
- Antenna coordinate system tied to Earth
- Twists over time with respect to sky
- See feed legs!



WSRT antenna

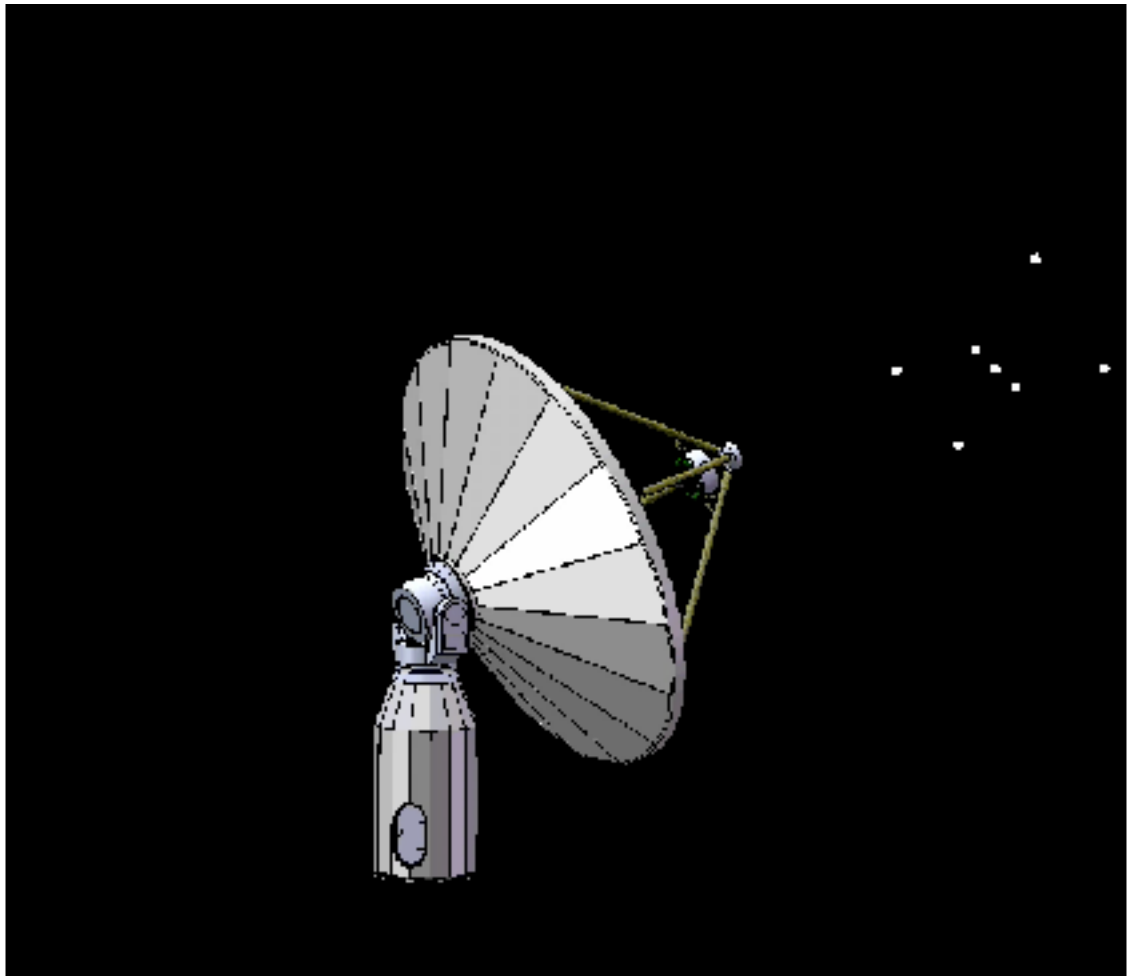
- Antenna coordinate system aligned to axis of Earth
- Antenna primary beam is fixed with respect to sky
- Allows very high dynamic range imaging

Sidelobes from sources in a rotating primary beam



ASKAP antennas

- Novel design of ASKAP antenna
- Surface and feed legs rotate independent of backing structure
- Small incremental cost
- Also simplifies focal plane array processing
- Improved science output!



https://youtu.be/gAgxY6QL5bI

“Peeling” exterior sources

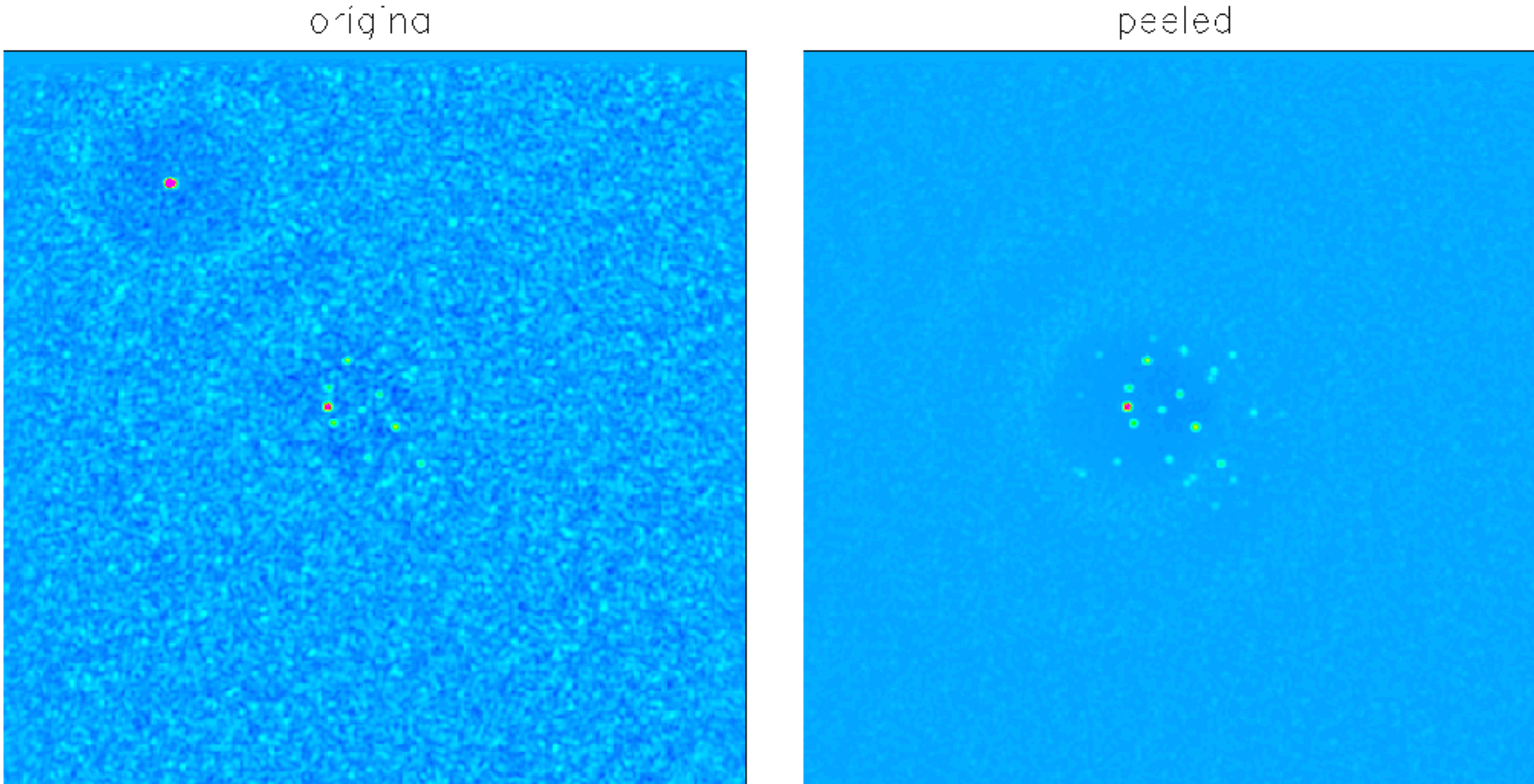
- Exterior source has different gain than main field of view

- Sidelobes and/or ionospheric phases

$$V_{i,j}(u_{i,j},v_{i,j}) = g_{i,peel} g^{*}_{j,peel} S_{peel} e^{2\pi j\left(u_{i,j} l_{k,peel} + v_{i,j} m_{k,peel}\right)} + \sum_{k} S e^{2\pi j\left(u_{i,j} l_{k} + v_{i,j} m_{k}\right)}$$

- Degrees of freedom can get out of hand if we peel too many sources!

“Peeling” exterior sources

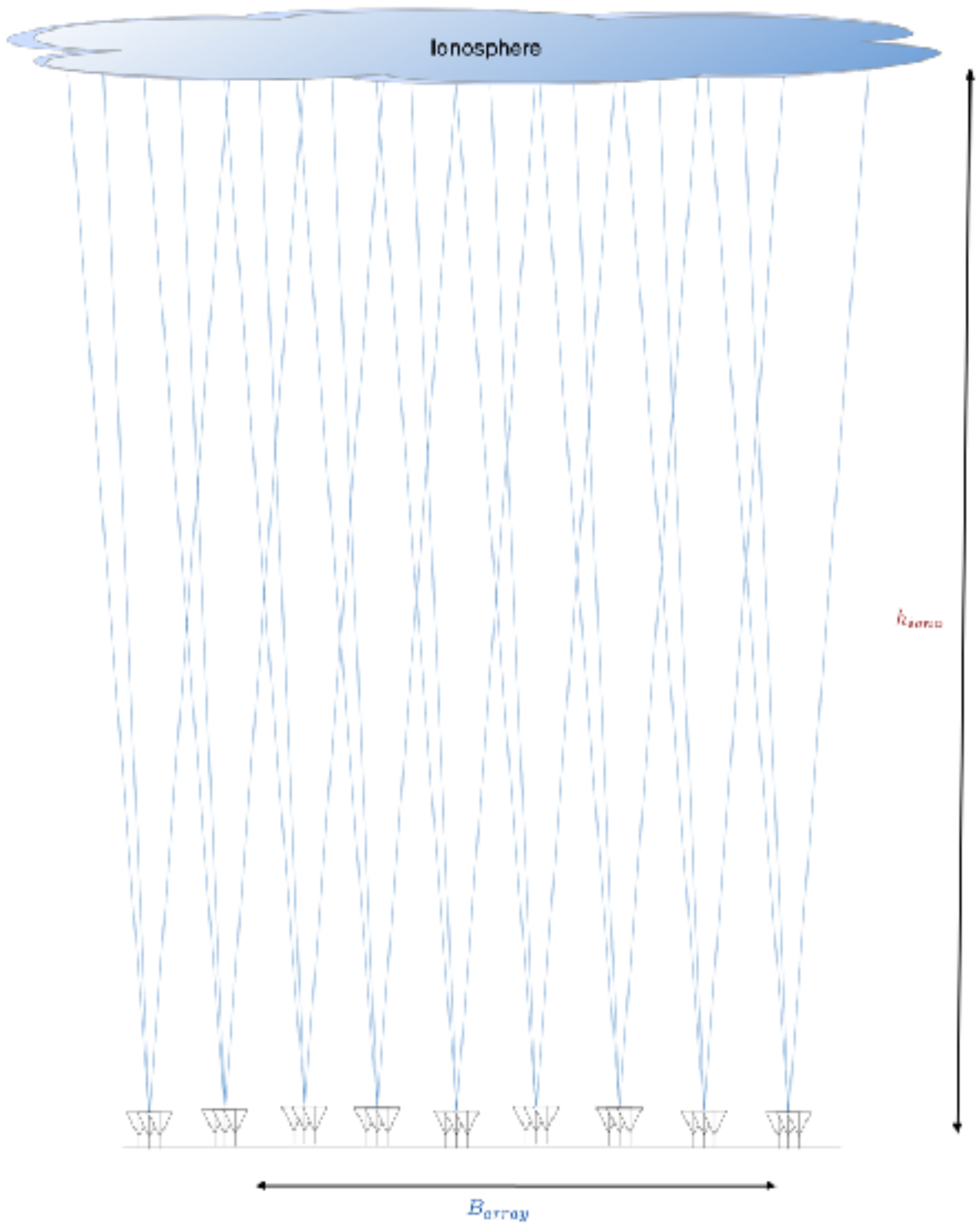


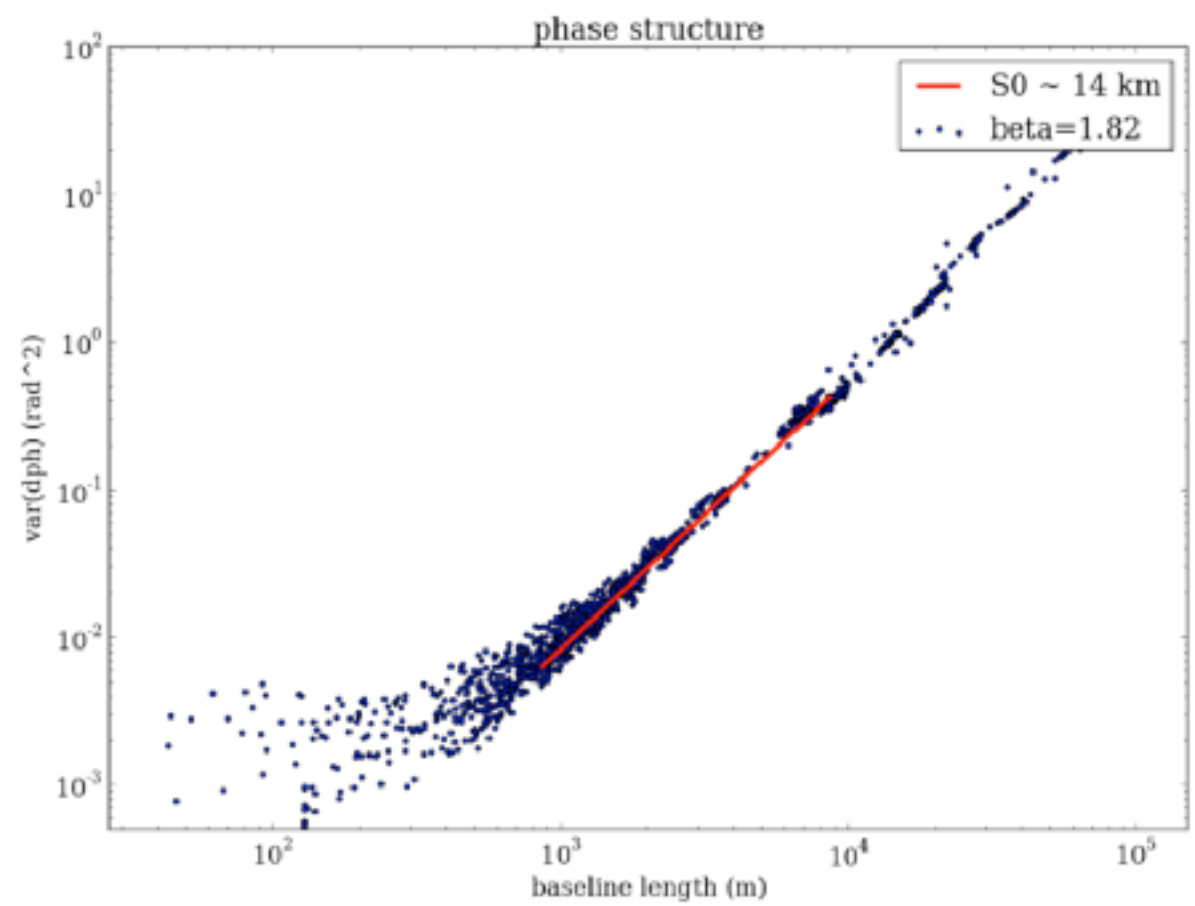
Non-isoplanatism

$$V_{i,j}(u_{i,j},v_{i,j}) = \sum_k g_i(l_k,m_k)\, g_j^*(l_k,m_k)\, S e^{2\pi j\left(u_{i,j} l_k + v_{i,j} m_k\right)}$$

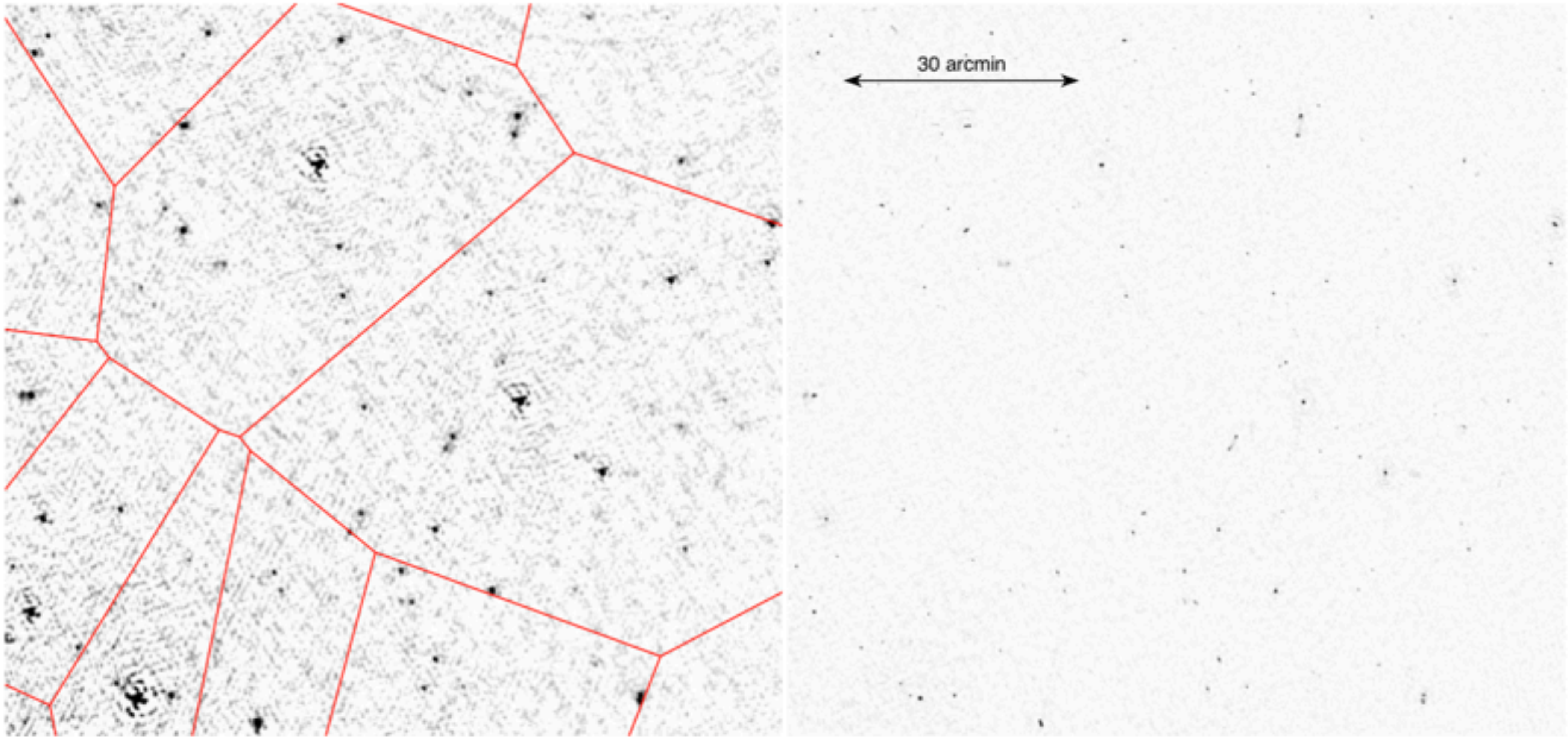
- All sources have a different complex gain
- Need some constraints to glue phases together
  - e.g. nearby sources have same phase error
  - e.g. phase screen or screens at height on ionosphere
- It is possible to calibrate if the phase screen (the ionosphere) is sufficiently well behaved
- Multiple competing approaches being developed

Origin of ionospheric non-isoplanatism

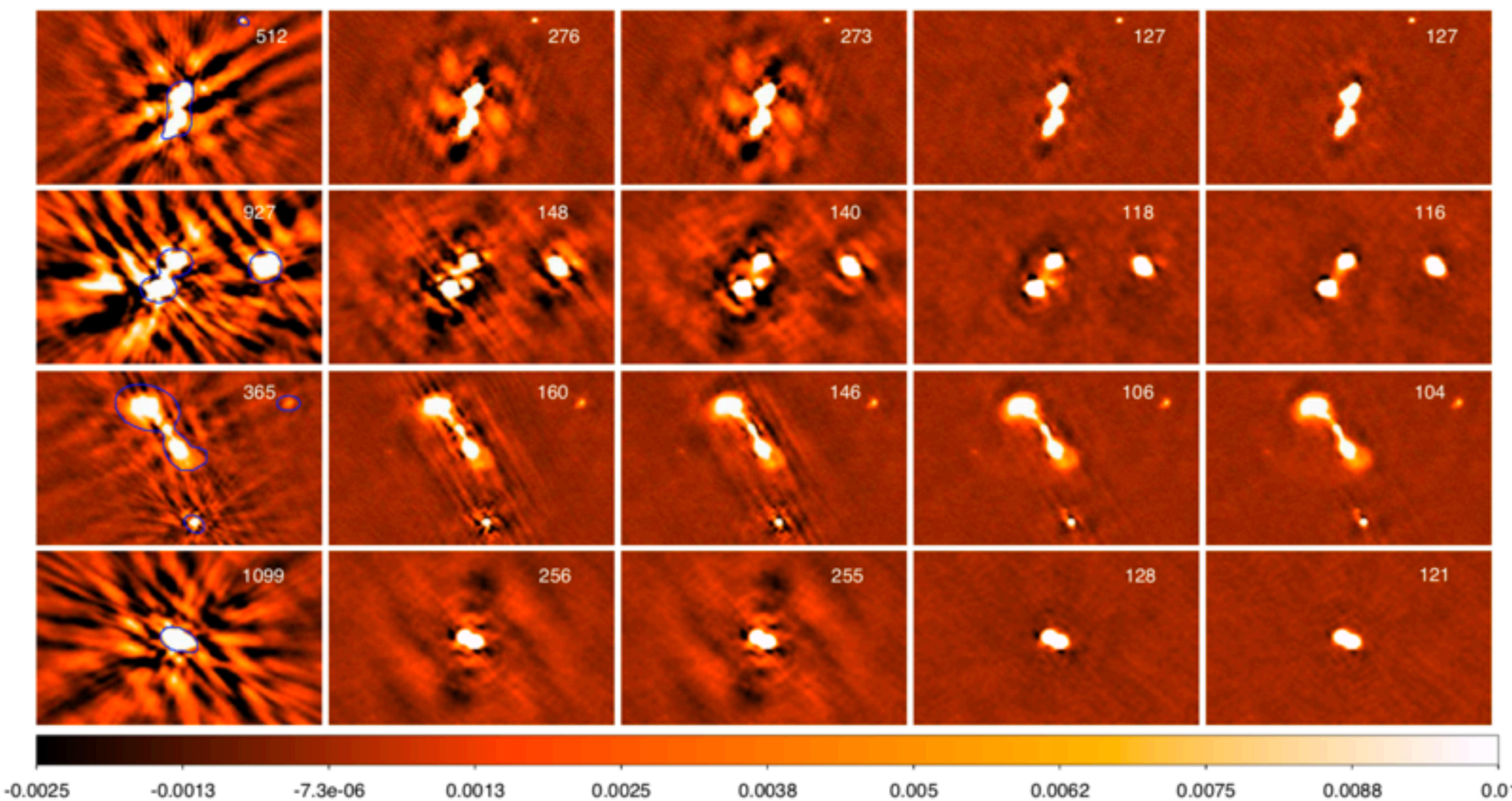




Facet-based calibration



Improvement during facet selfcalibration



Iteration

Summary

- The Measurement Equation formalism describes an idealised “non-ideal telescope”
- Provided the ME is sufficiently accurate and there is enough SNR then the family of self-calibration techniques can help with unknown time-variable effects
- The ME approach can deal with complicated effects such as time-variable but known primary beams
- Future is to apply to unknown effects such as pointing errors
- Computing complexity for direction dependent effects can be very high

High dynamic range lessons

- Can be estimated from results
- Calibration and imaging can both lead to dynamic range limitations
- High DR requires deep understanding of the telescope