

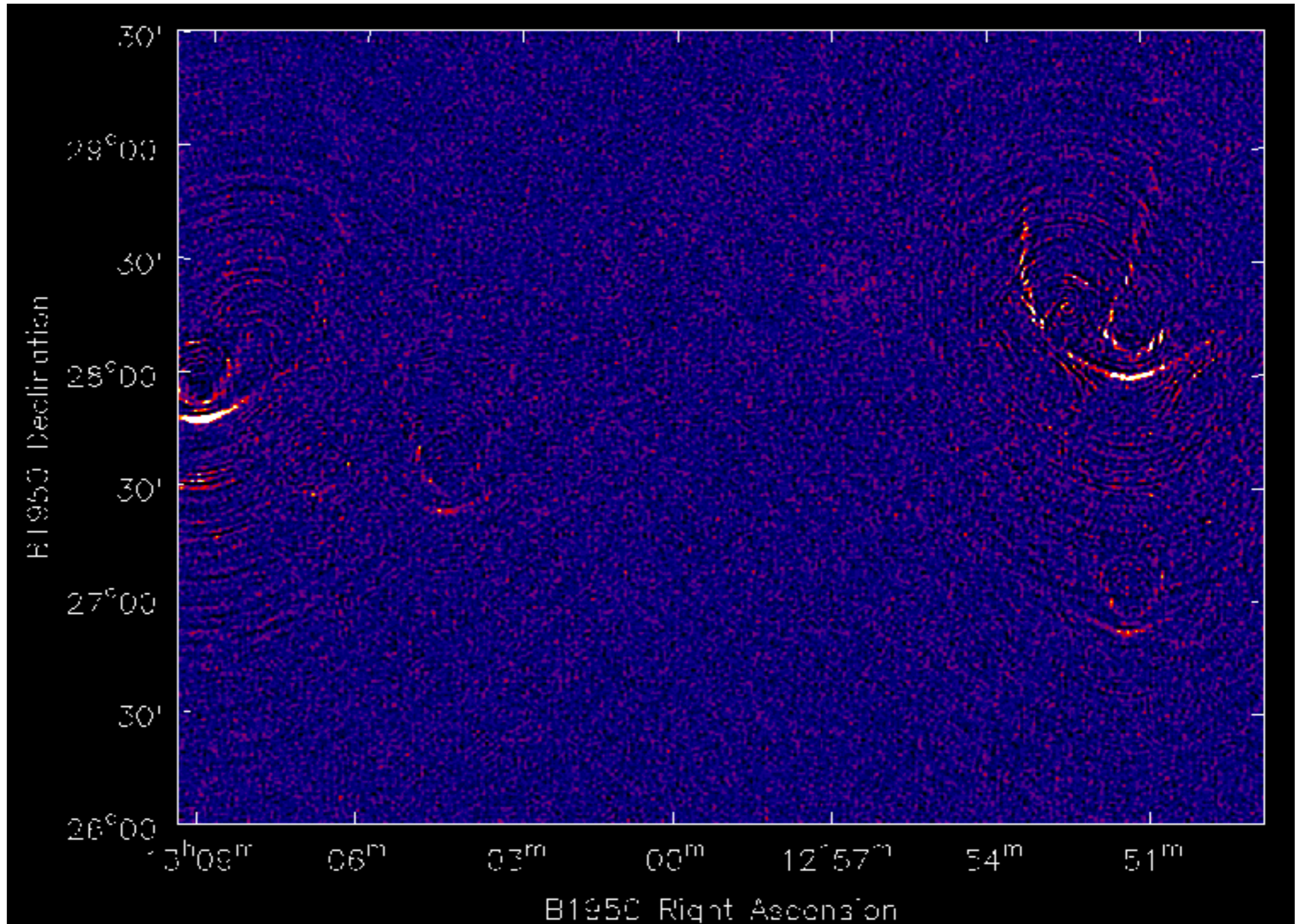
# Wide field and wide band imaging

Tim Cornwell

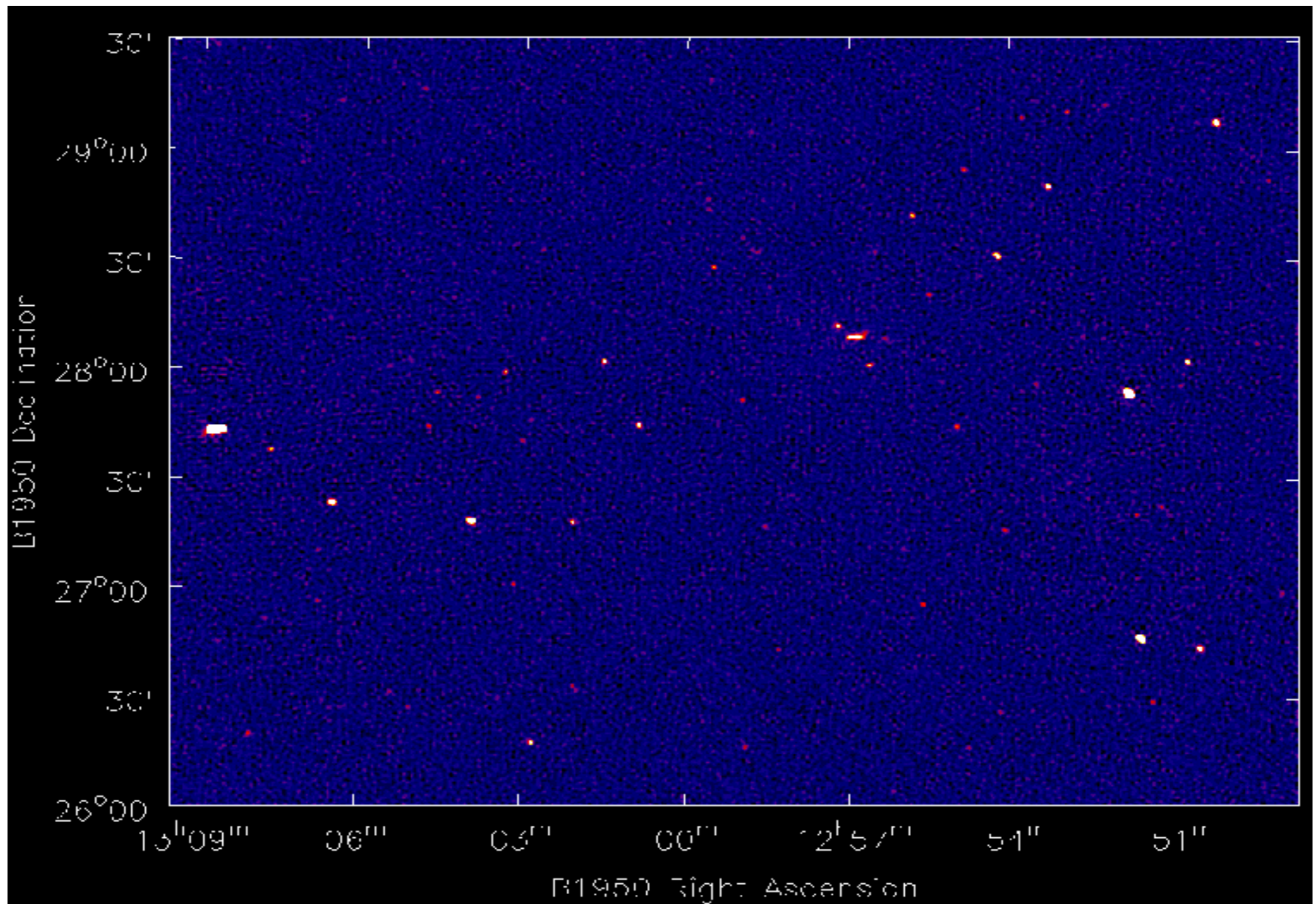
# Overview

- Wide field effects in imaging
- Wide band effects in imaging (largely defer to Urvashi's talks)

# Standard 2D imaging



# Faceted imaging

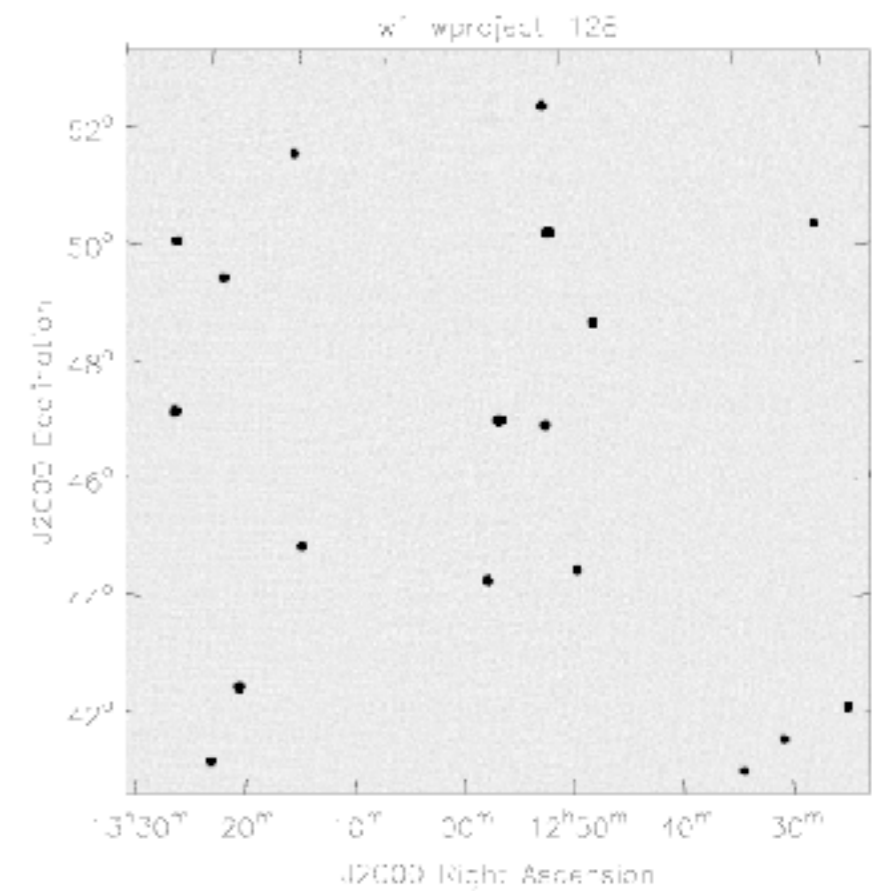
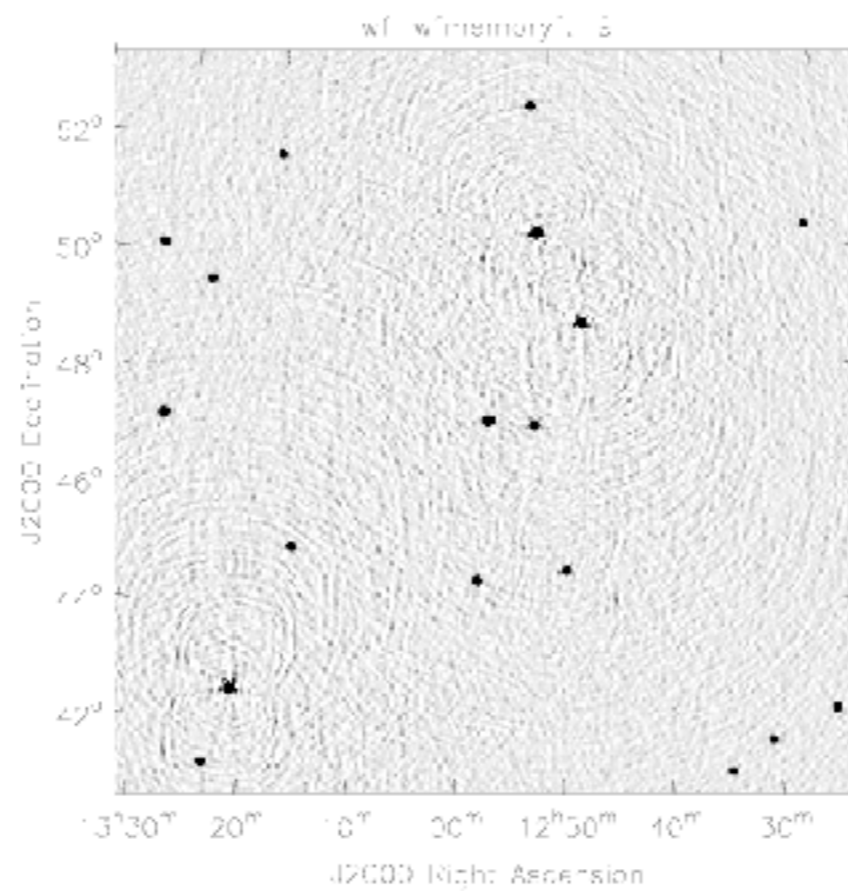
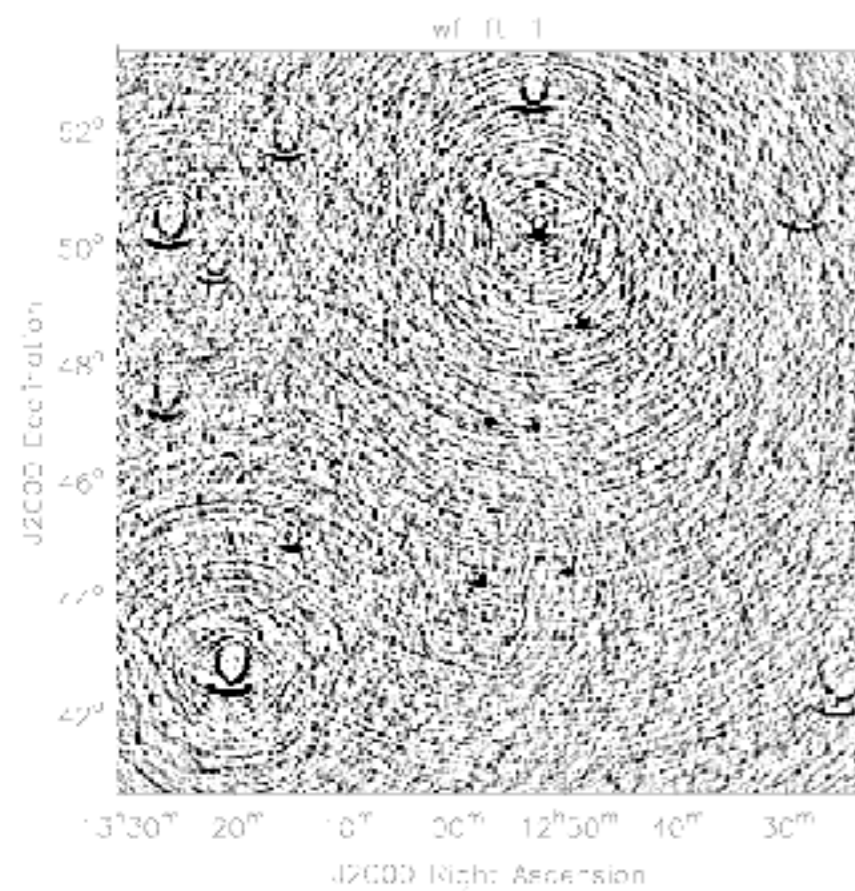


# Comparison of 2D, faceted, w projection algorithms

2D Fourier transform

Faceted Fourier transforms

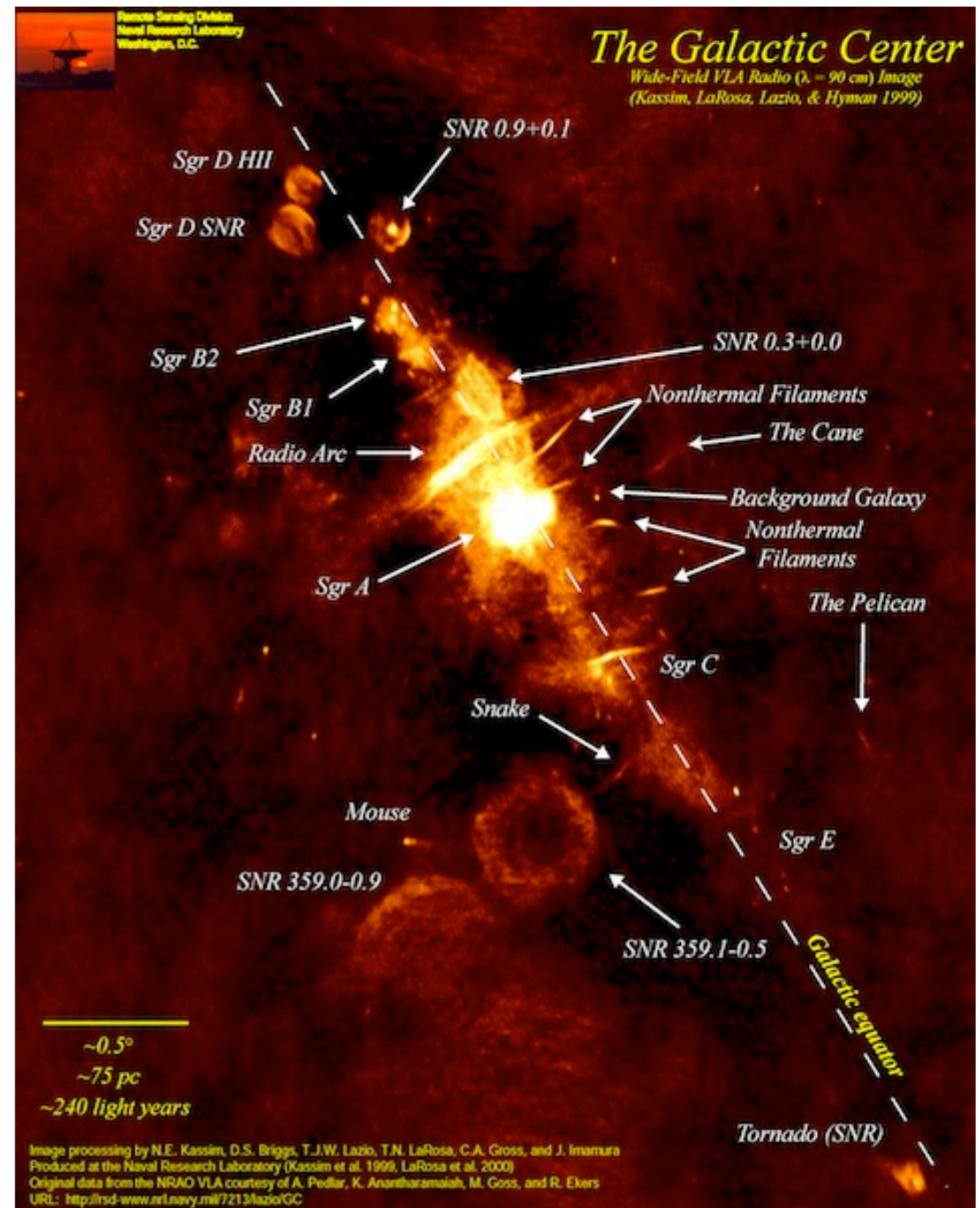
W projection



Simulation of VLA 74 MHz long integration

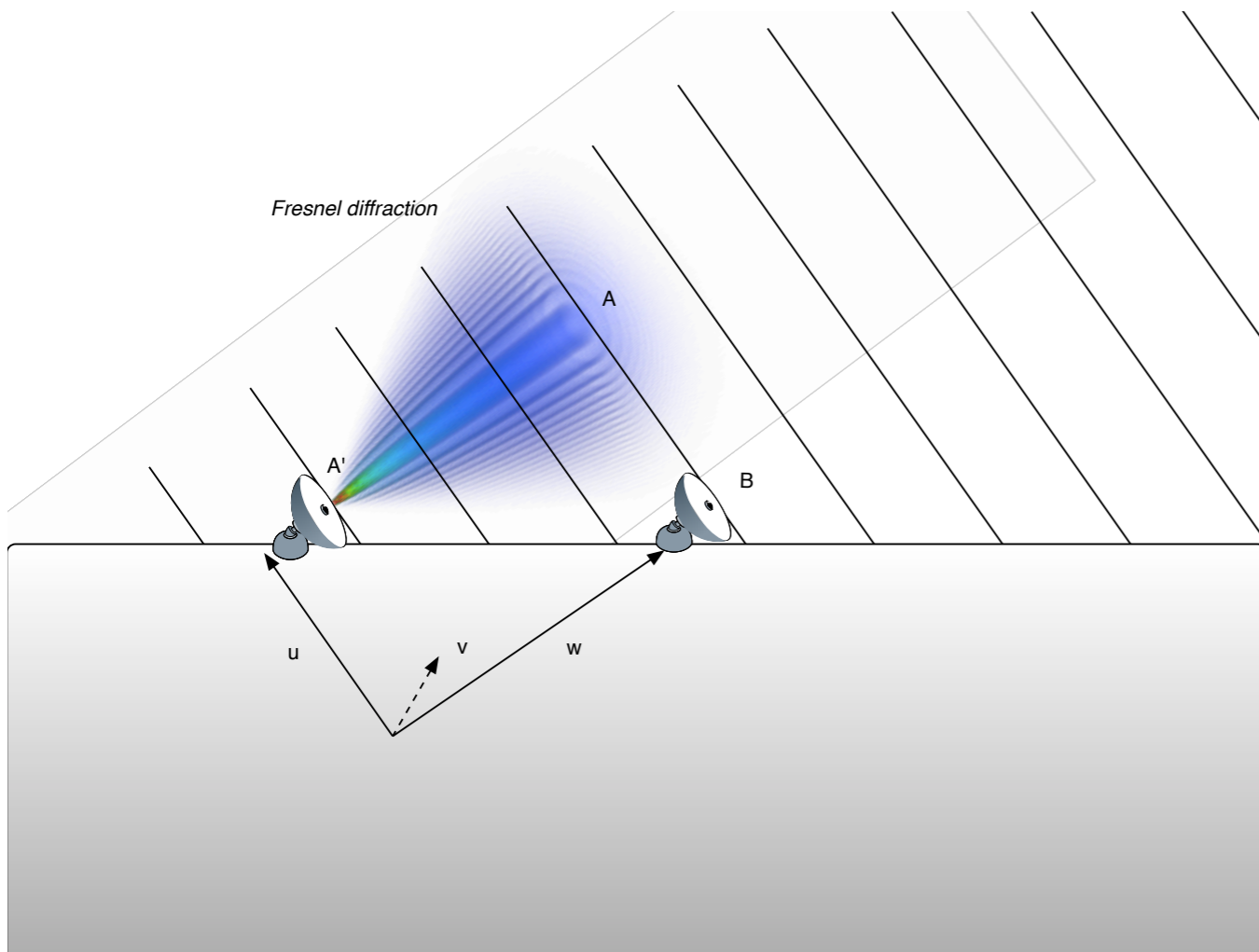
# Galactic centre at 327 MHz

- VLA 330MHz image
- Only possible when w term could be corrected
- Early example of parallel processing
- Used Fortran + PVM on networked IBM RS-6000s
- Dan Briggs did this processing



# Origin of w term effect

$$V(u, v, w) = \int \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{2\pi j \left( ul + vm - w \left( \sqrt{1 - l^2 - m^2} - 1 \right) \right)} dl dm$$



- Ron showed how to decompose single dish into antennas + delay lines
- Delay is a function of direction on sky
- First consequence: need to sample sufficiently in delay
- Second consequence: need to propagate to same wavefront

# Can telescope design help?

- Coplanar array e.g. WSRT, ATCA, MWA
- Large antennas/stations

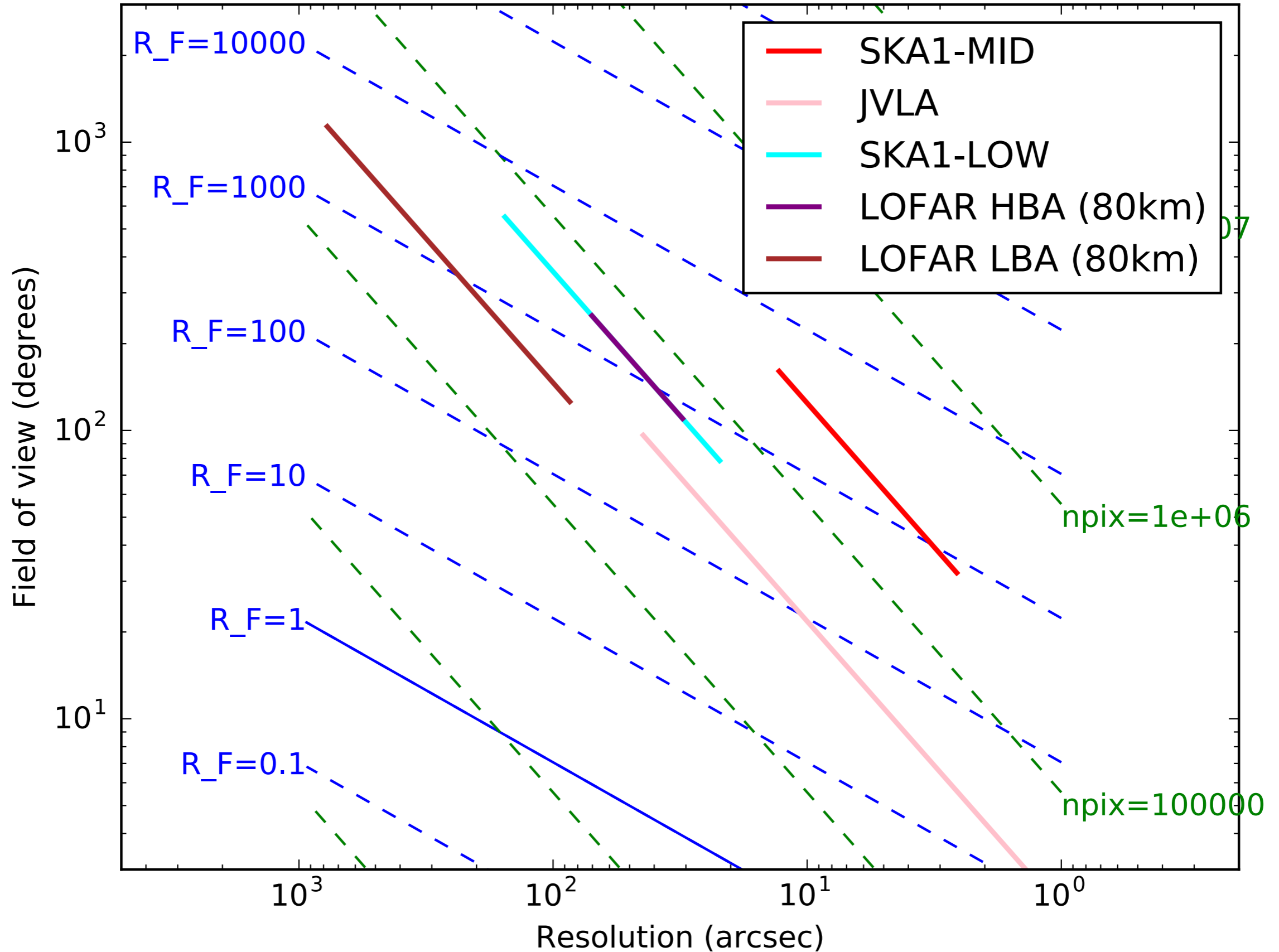
# Fresnel number

- Strength of phase term is measured by Fresnel number
- Worst at lower frequencies

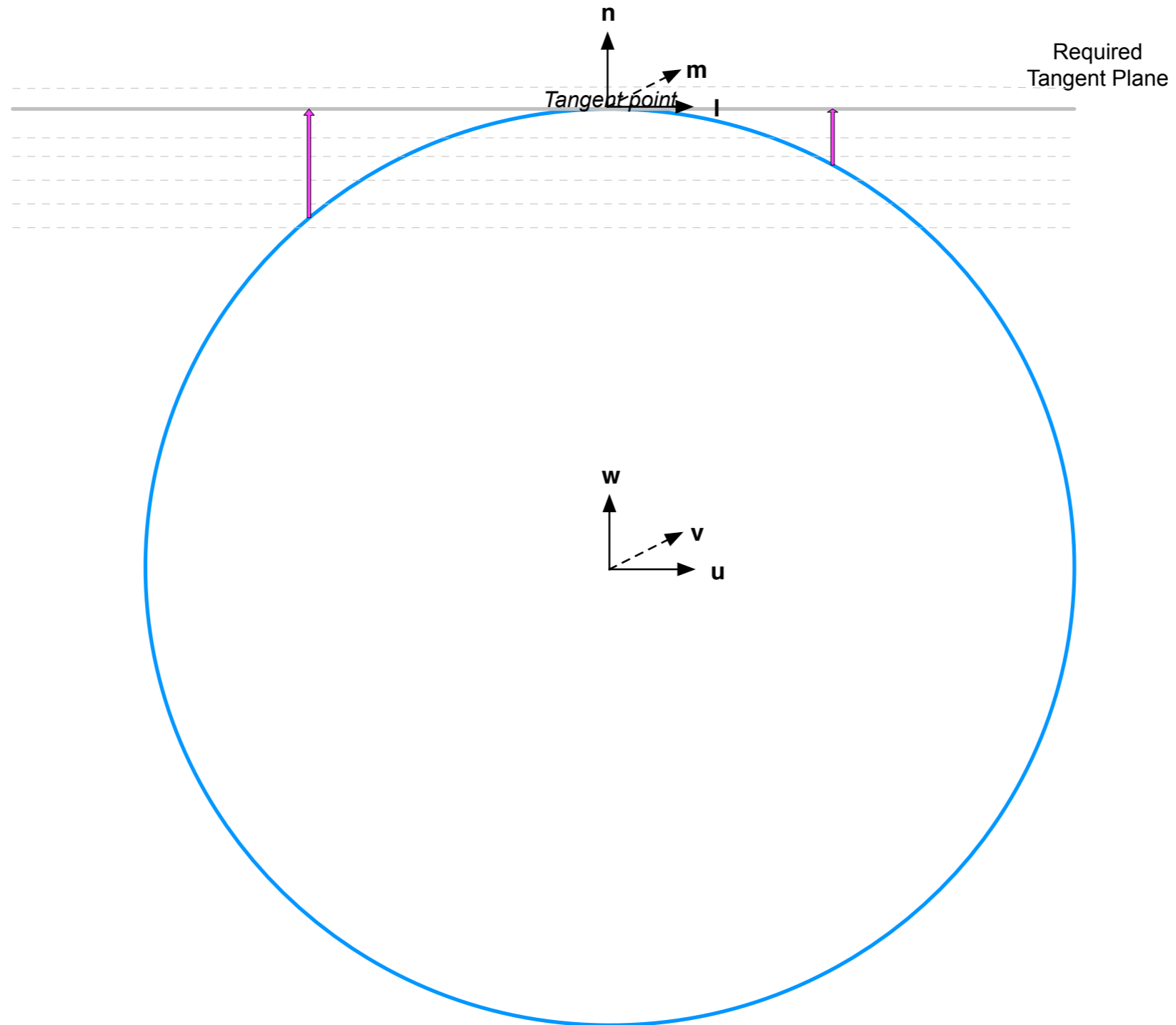
$$R_F = \frac{\theta_{FOV}^2}{\theta_{synth}}$$

$$R_F \approx \frac{\lambda B}{D^2}$$

Fresnel number (blue). Number of pixels per axis (green)



# Coordinate systems



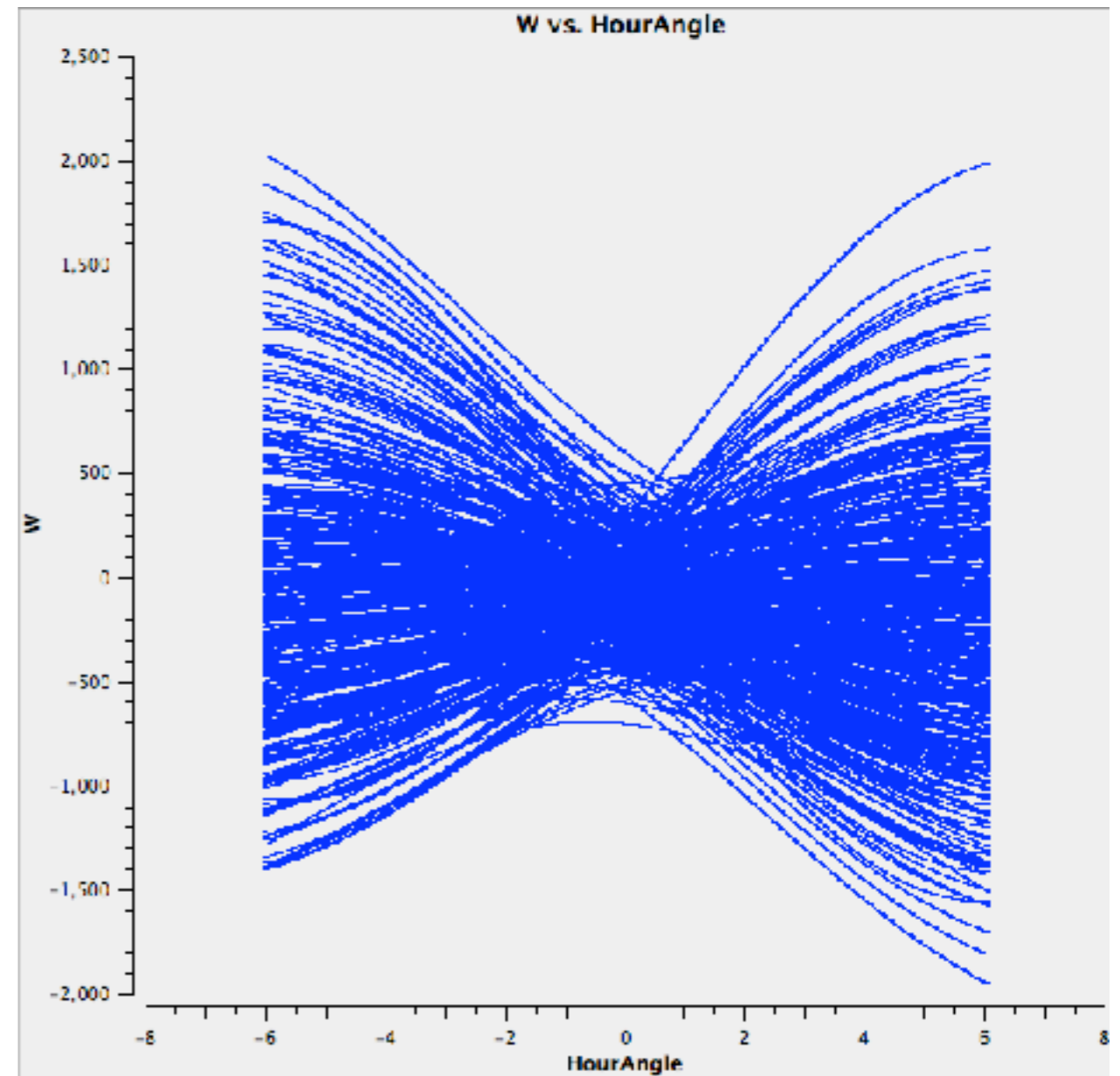
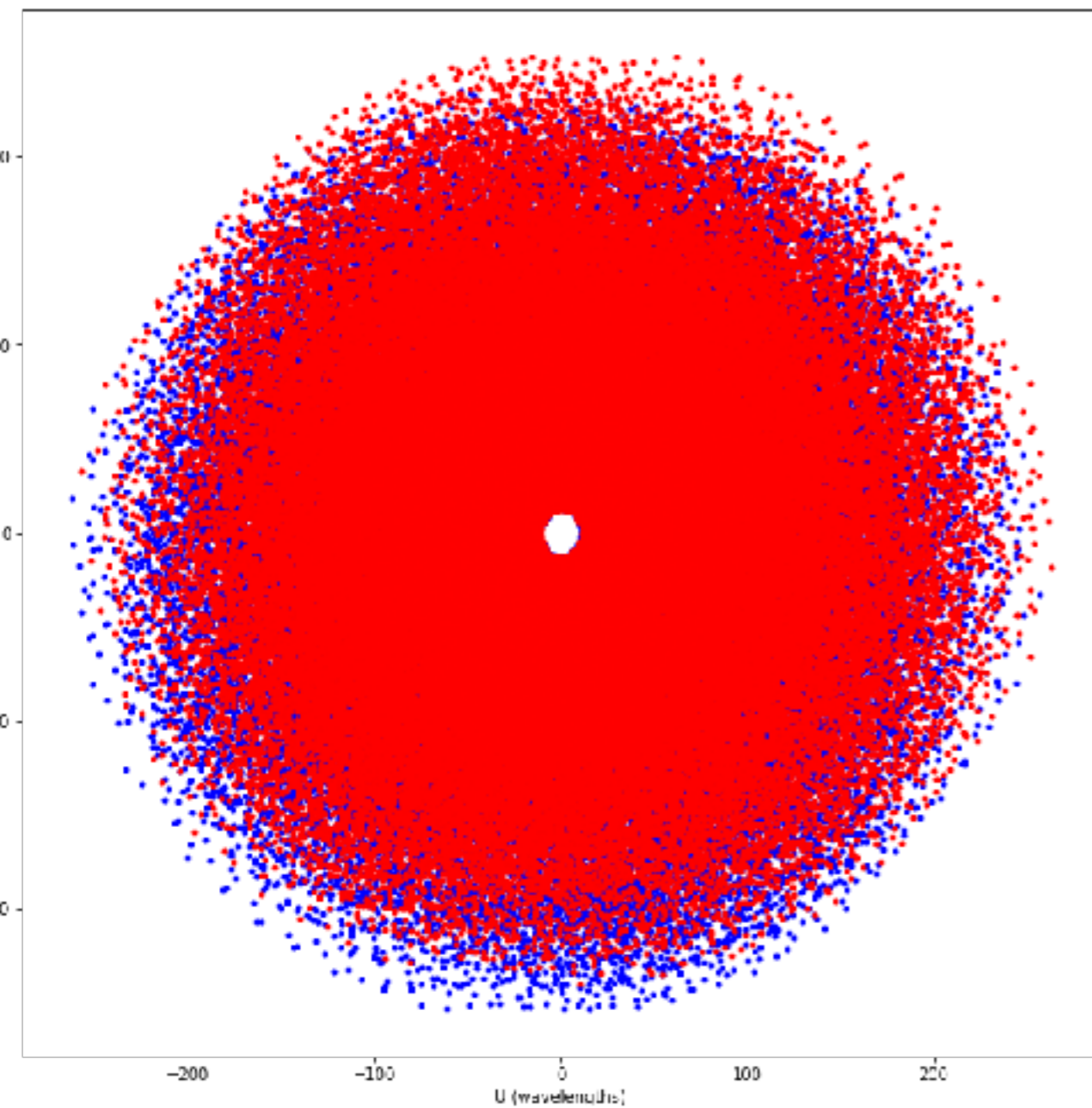
# 3D imaging

$$V(u, v, w) = \int \frac{I(l, m, n)}{n} \delta(l^2 + m^2 + n^2 - 1) e^{2\pi j(ul + vm + w(n-1))} dl dm dn$$

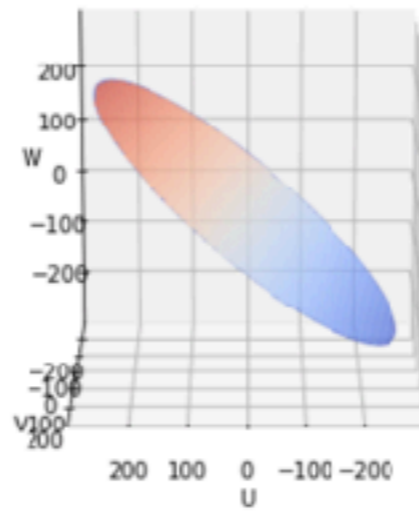
- Easy to do
- Obtain 3D convolution equation
- PSF in (l,m,n) space is shift-invariant
- Practical only for small cases
- Most of cube in (l,m,n) is empty

$$I^{D,3} = B^{D,3} \otimes I^{sky,3}$$

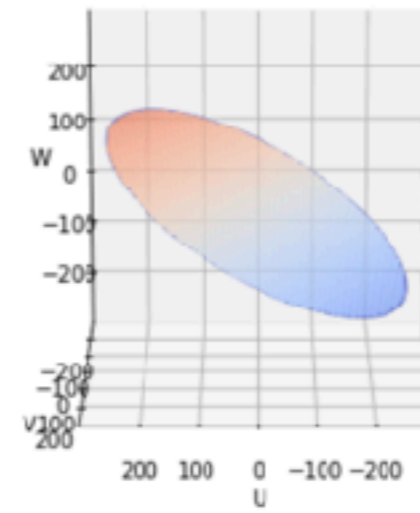
# Simulation of SKA1-LOW core long observation



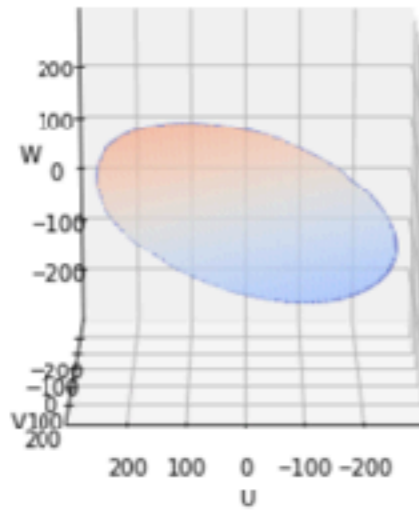
Hour angle -3.0 h



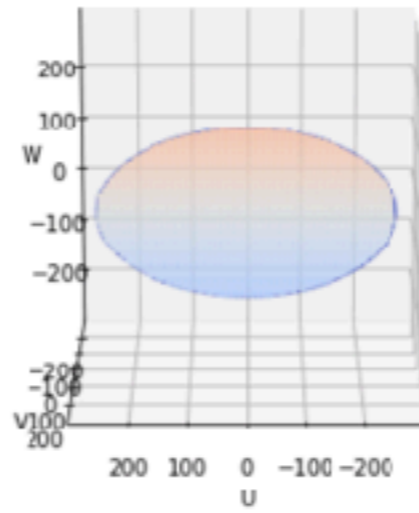
Hour angle -2.0 h



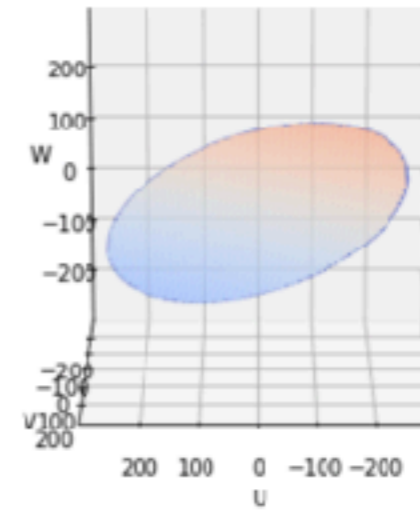
Hour angle -1.0 h



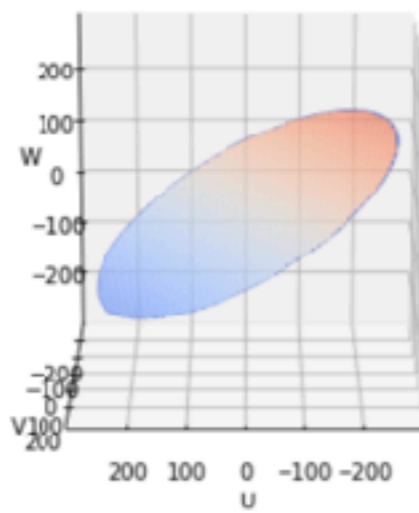
Hour angle 0.0 h



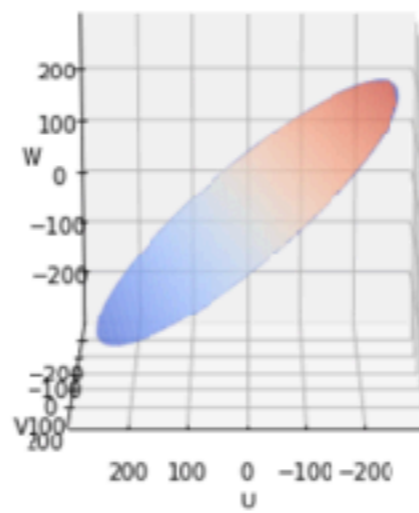
Hour angle 1.0 h

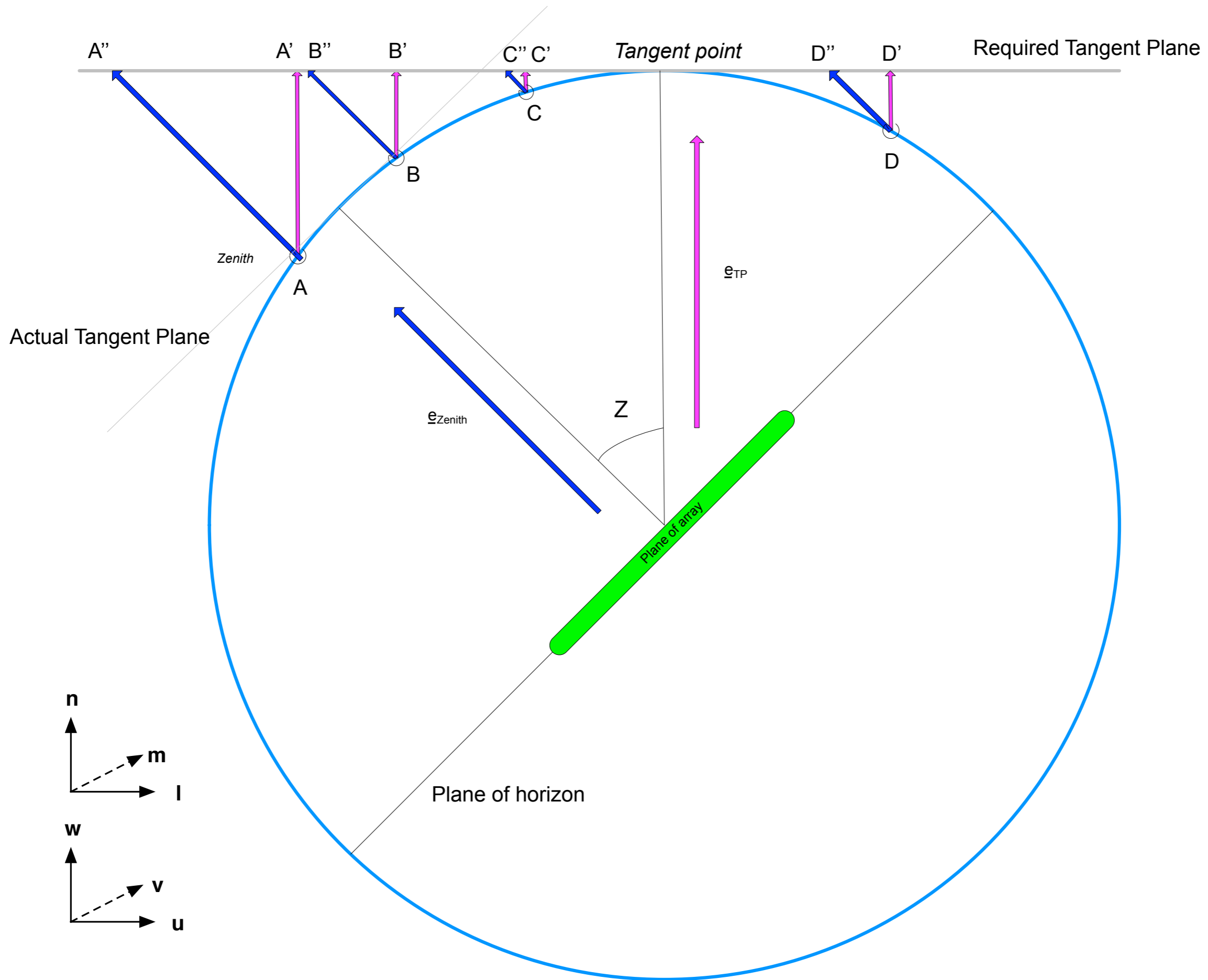


Hour angle 2.0 h



Hour angle 3.0 h

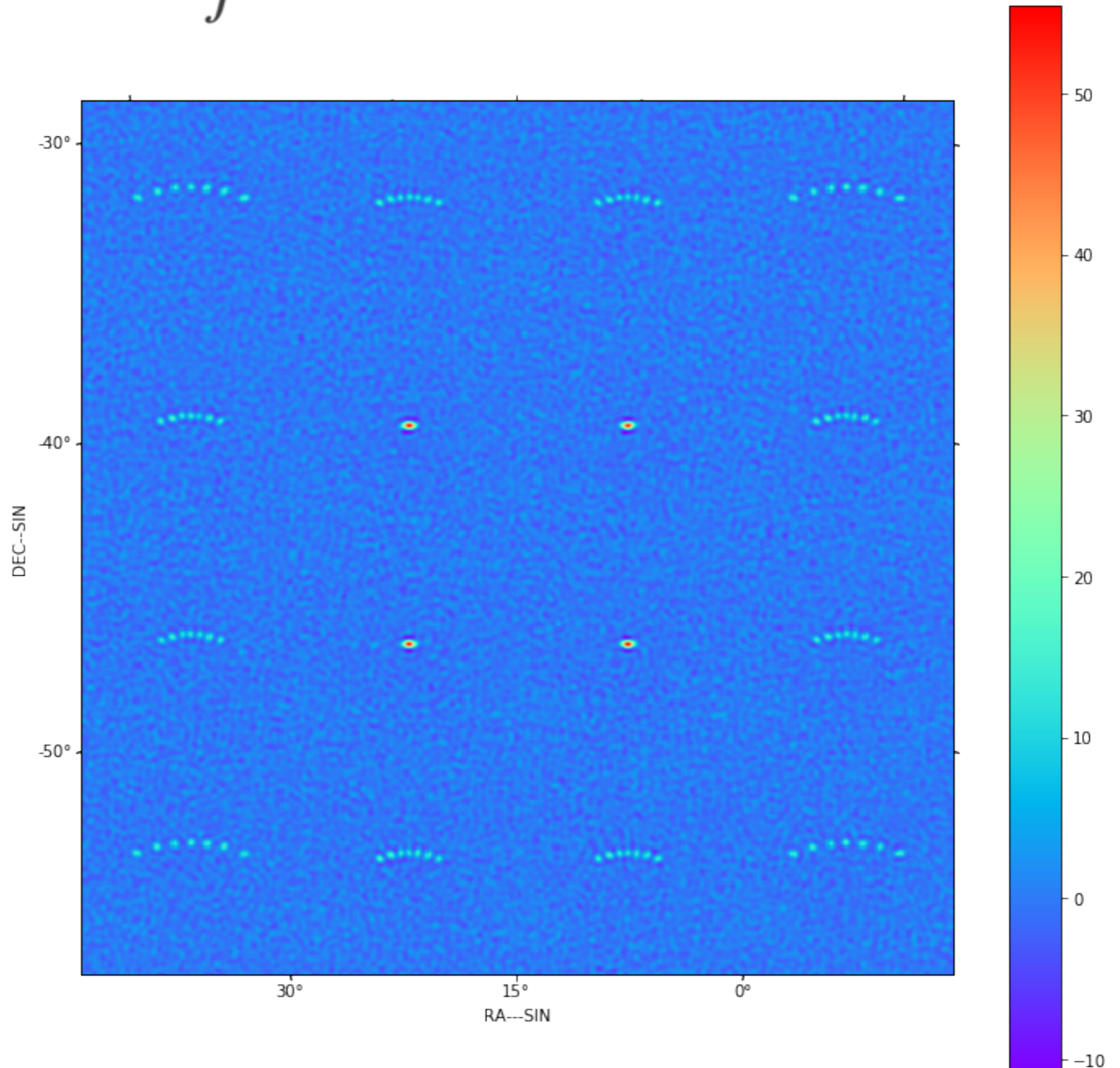


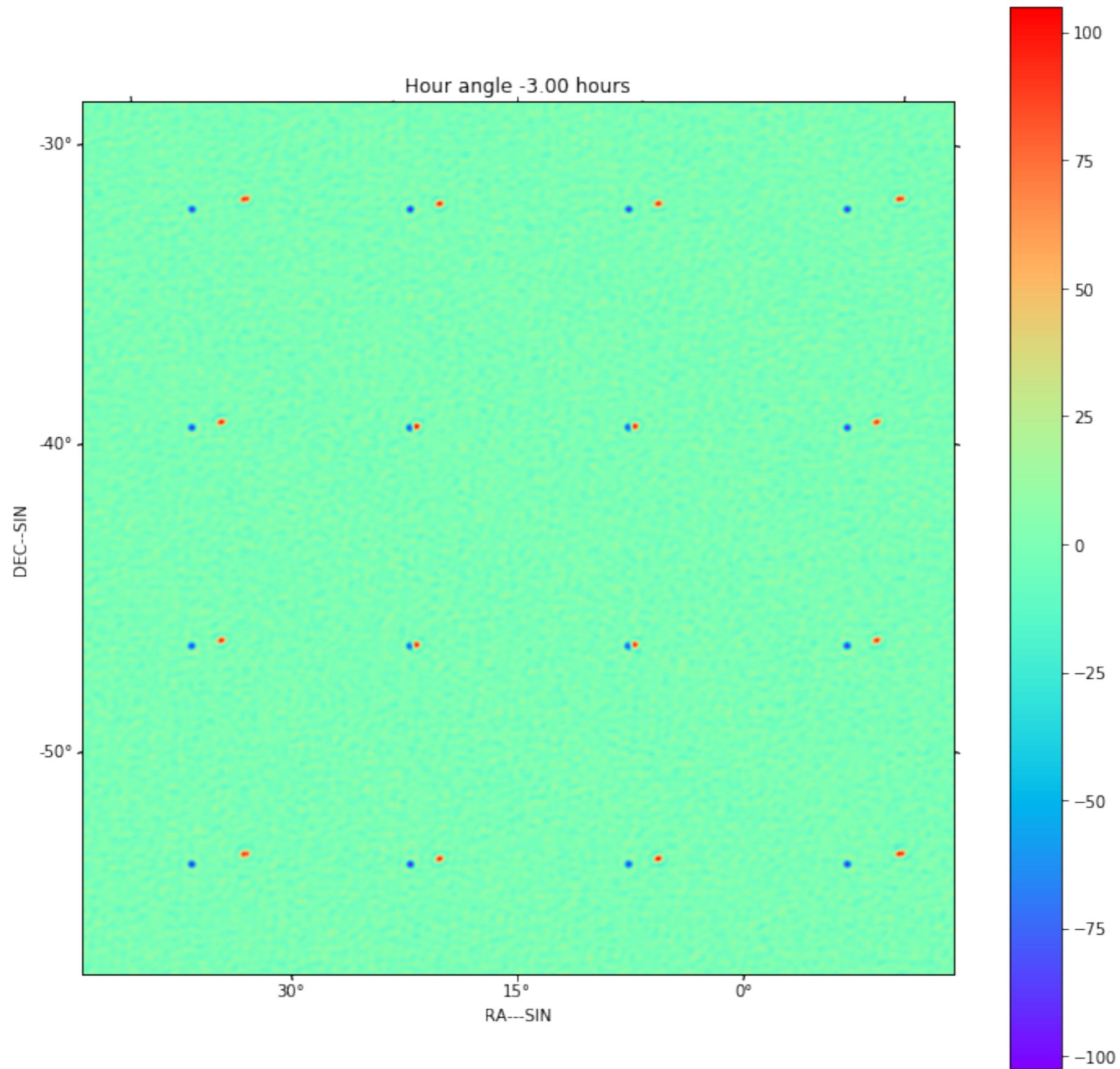


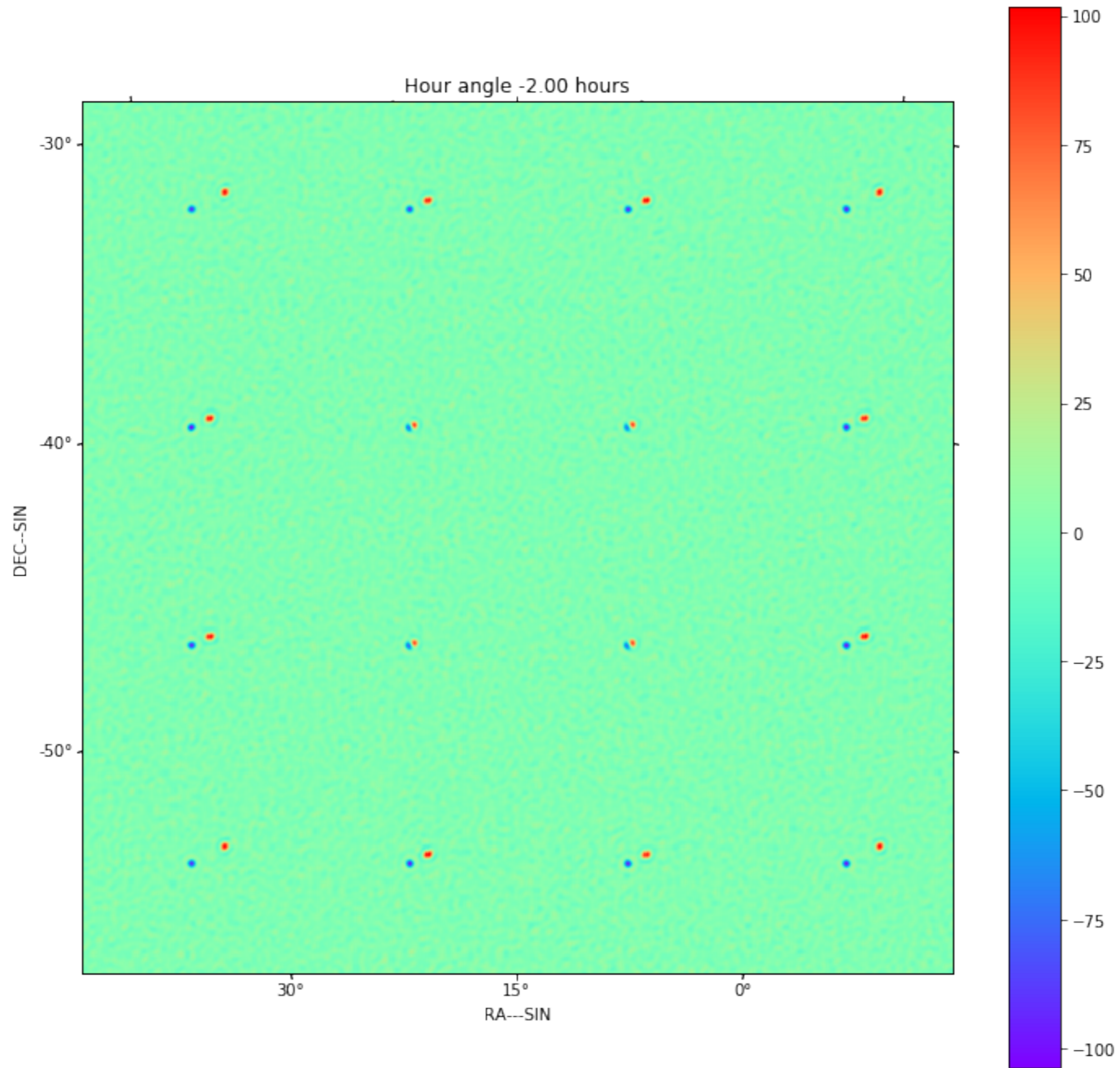
$$V(u, v, w) = \int I(l, m) e^{2\pi j(ul+vm)} dl dm$$

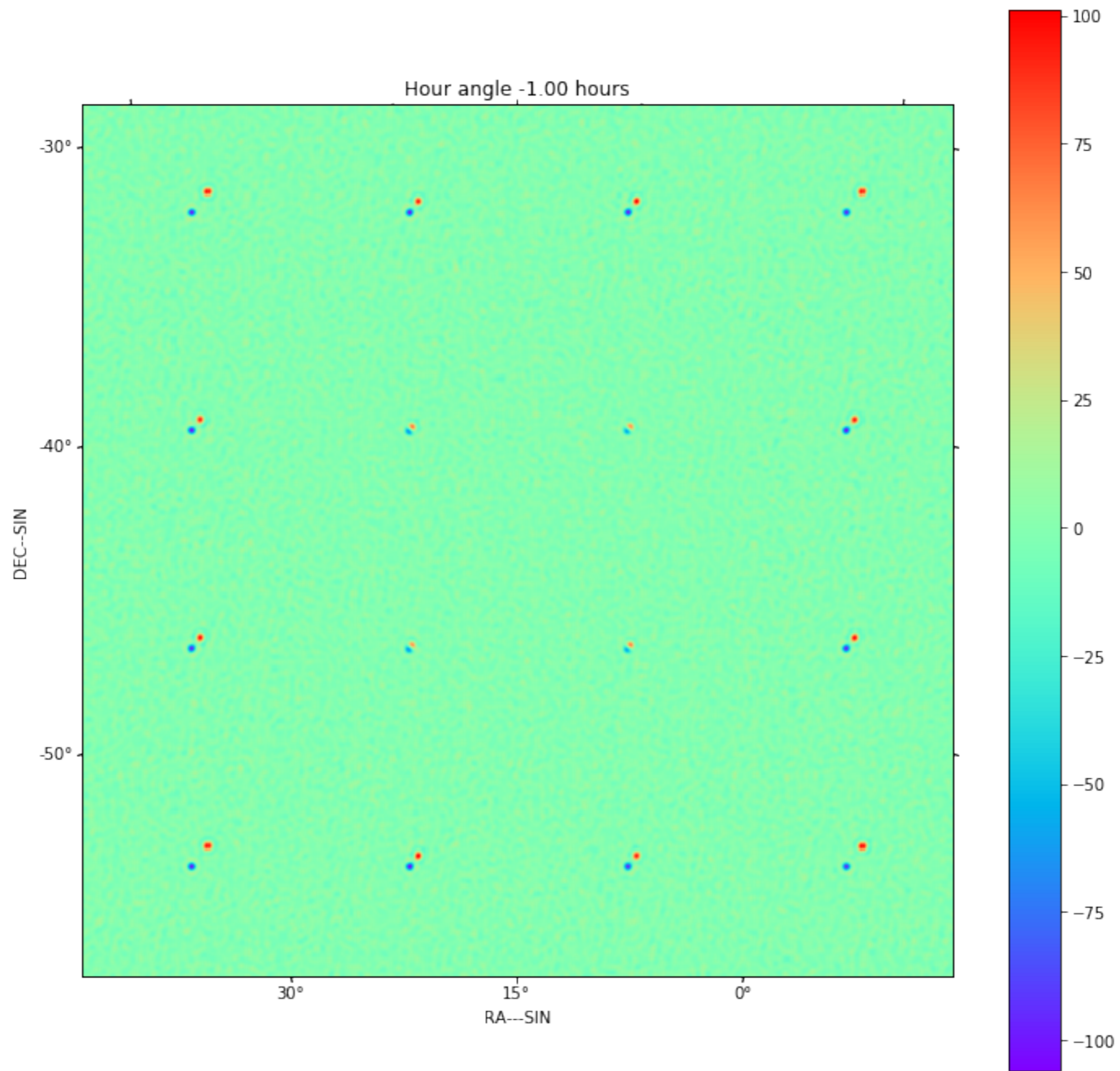
**2D transform  
for 7 hourangles**

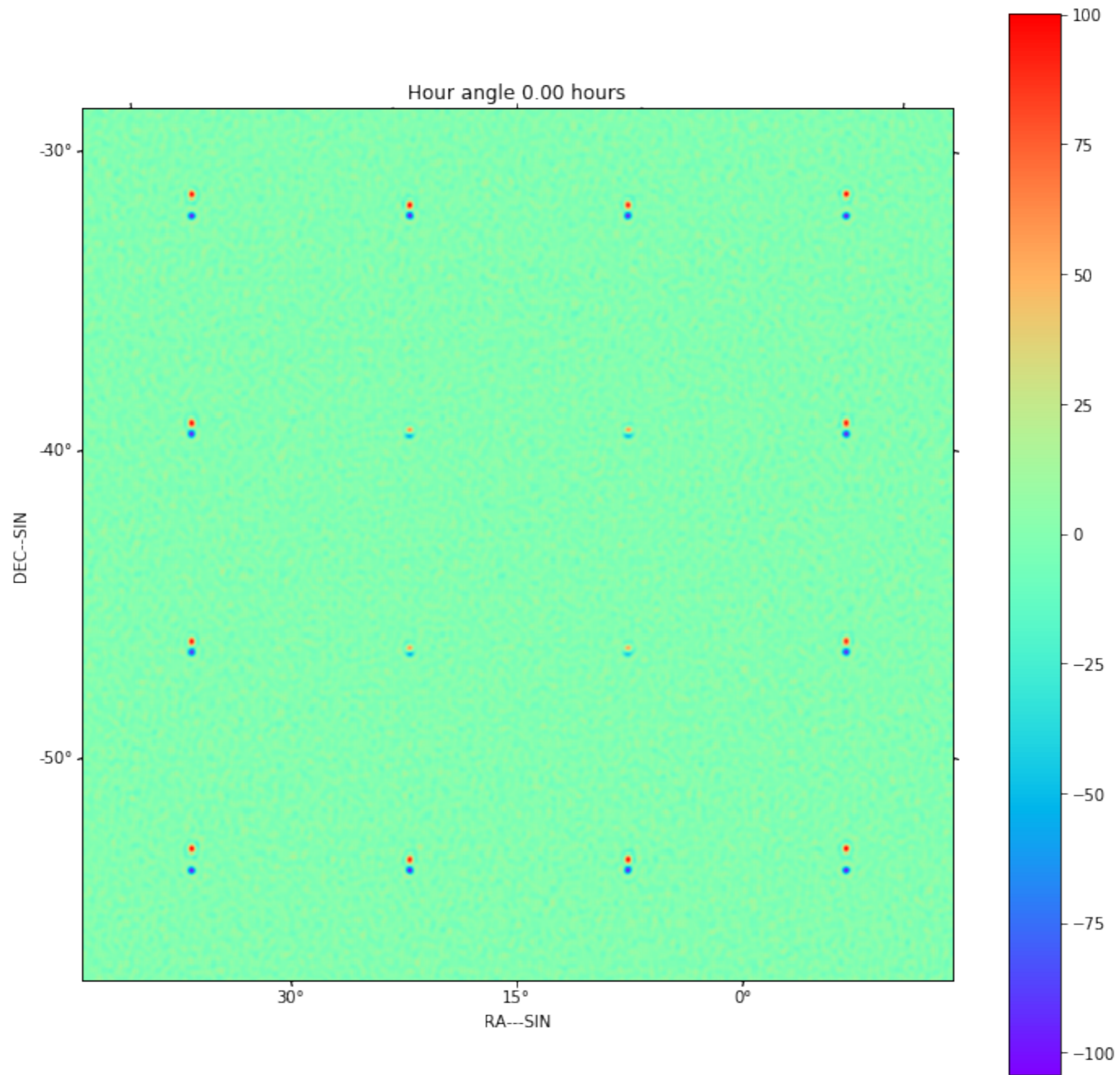
**Each hour angle causes  
different position error**

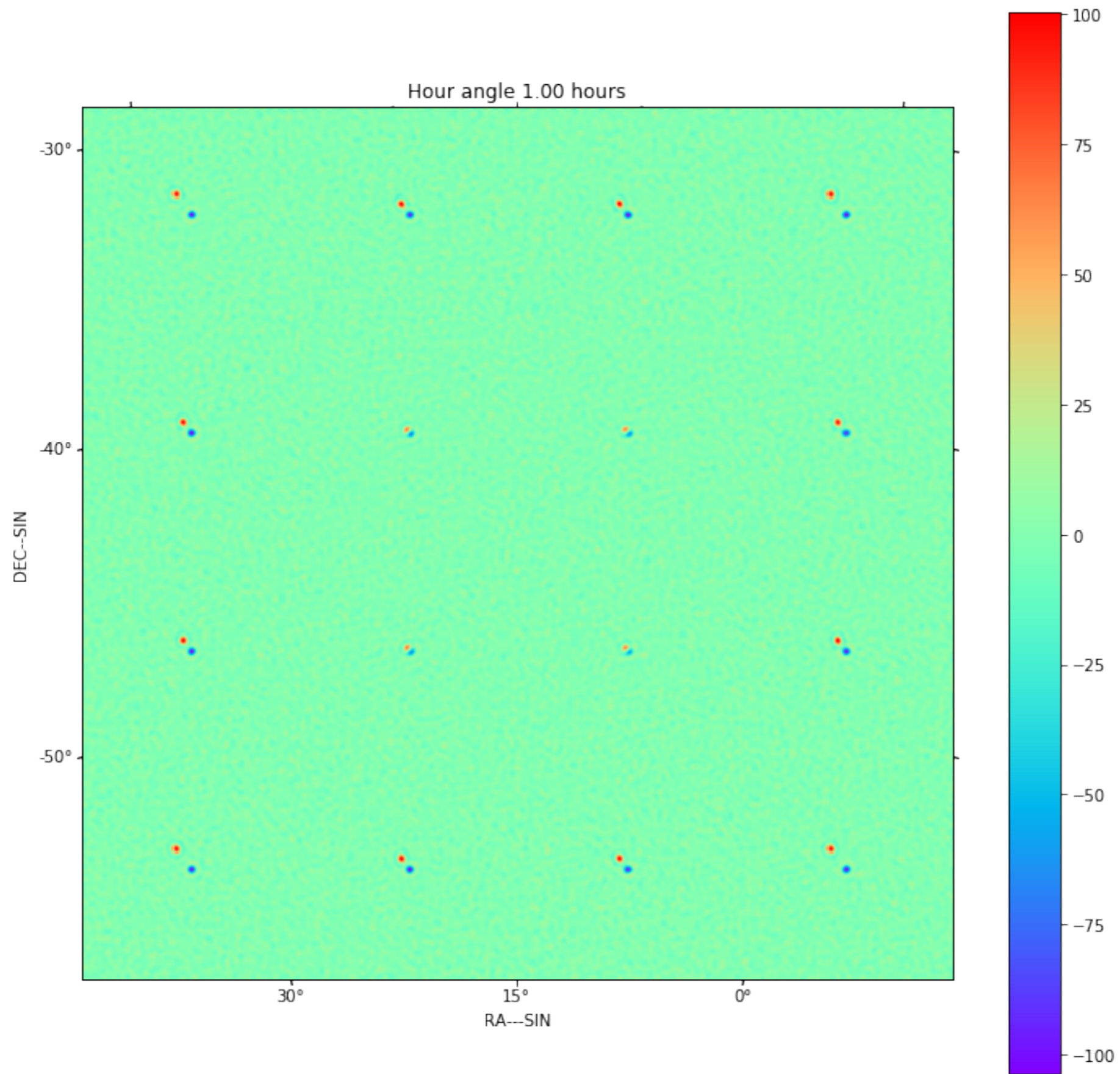


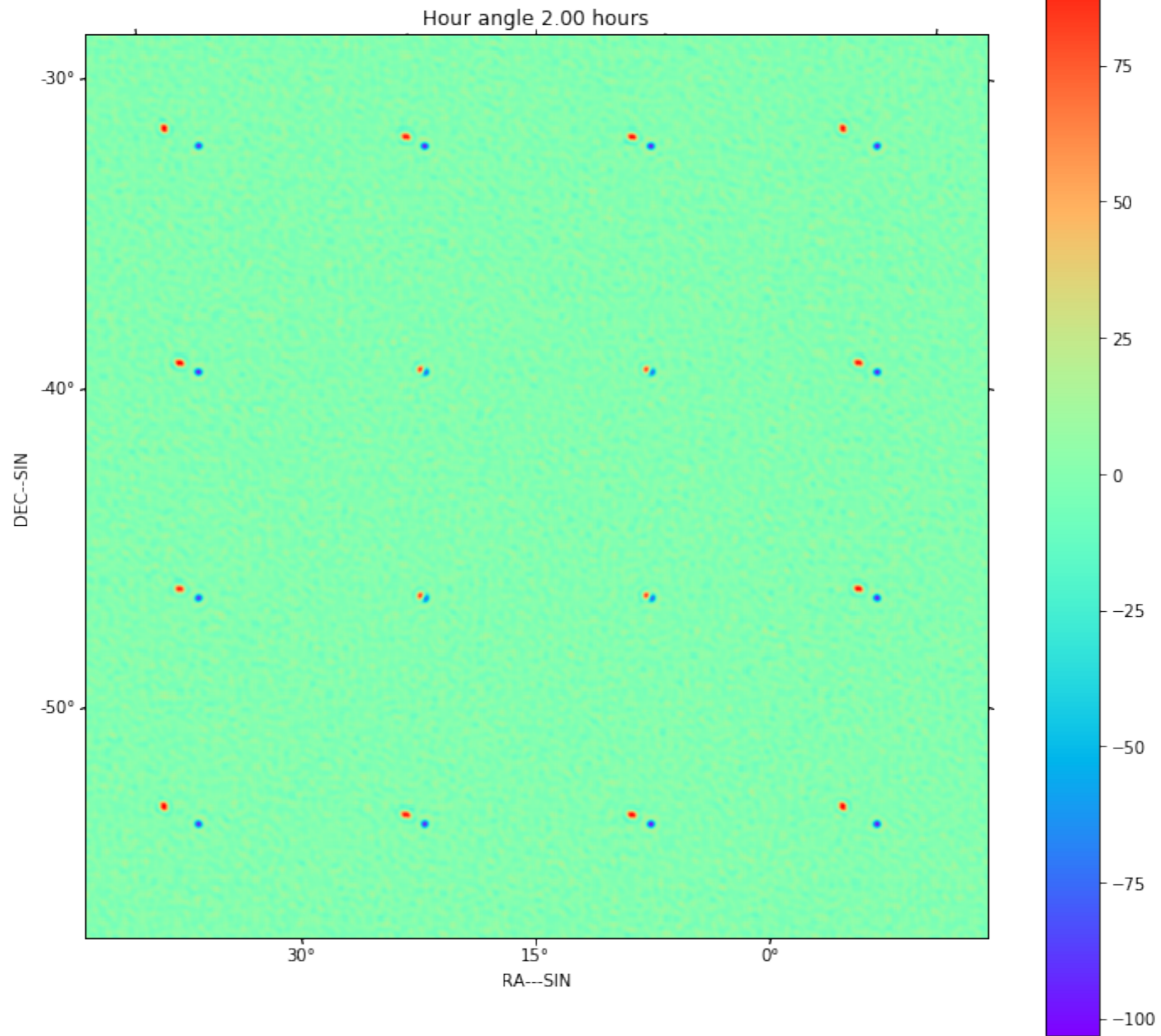


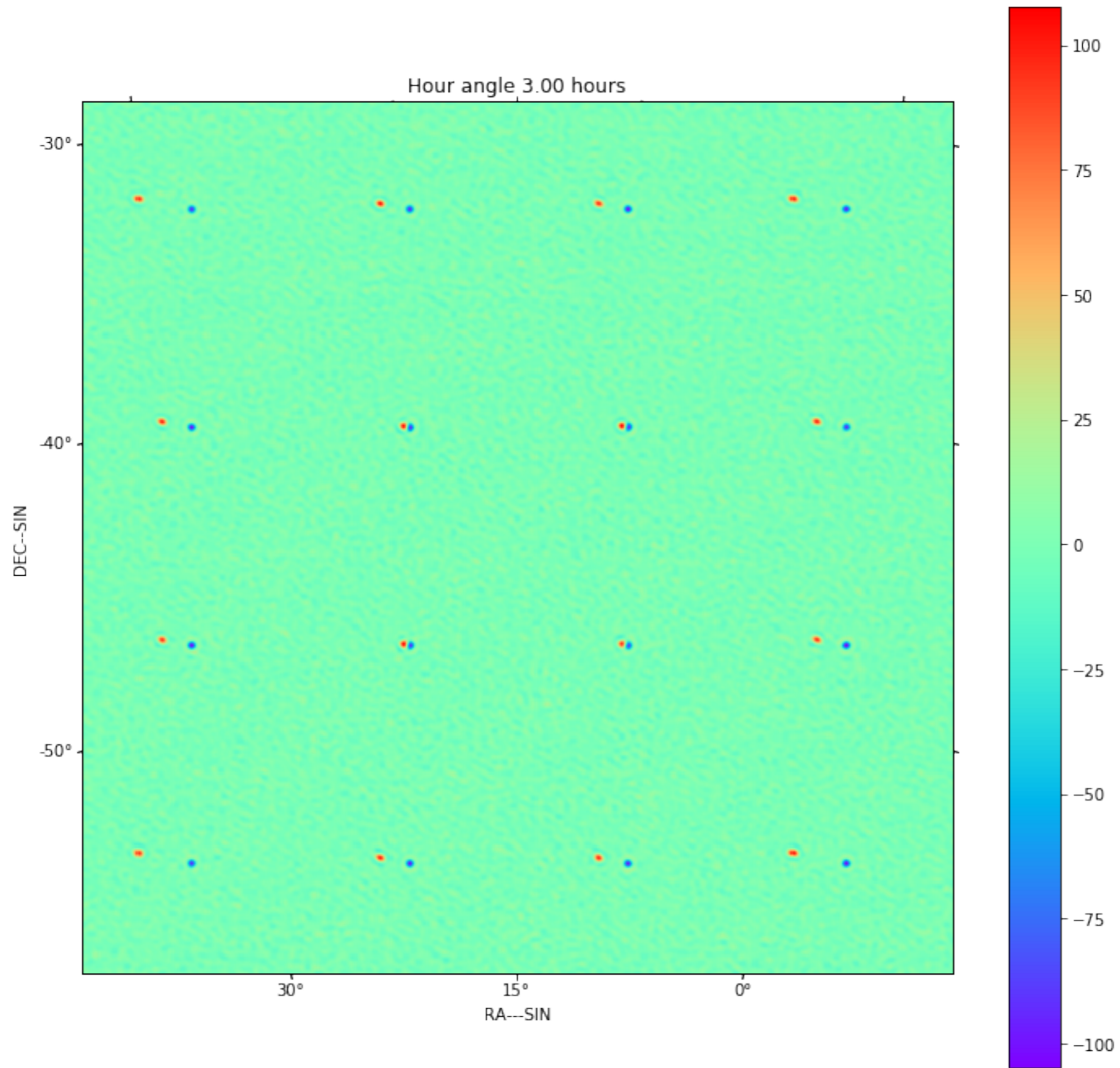








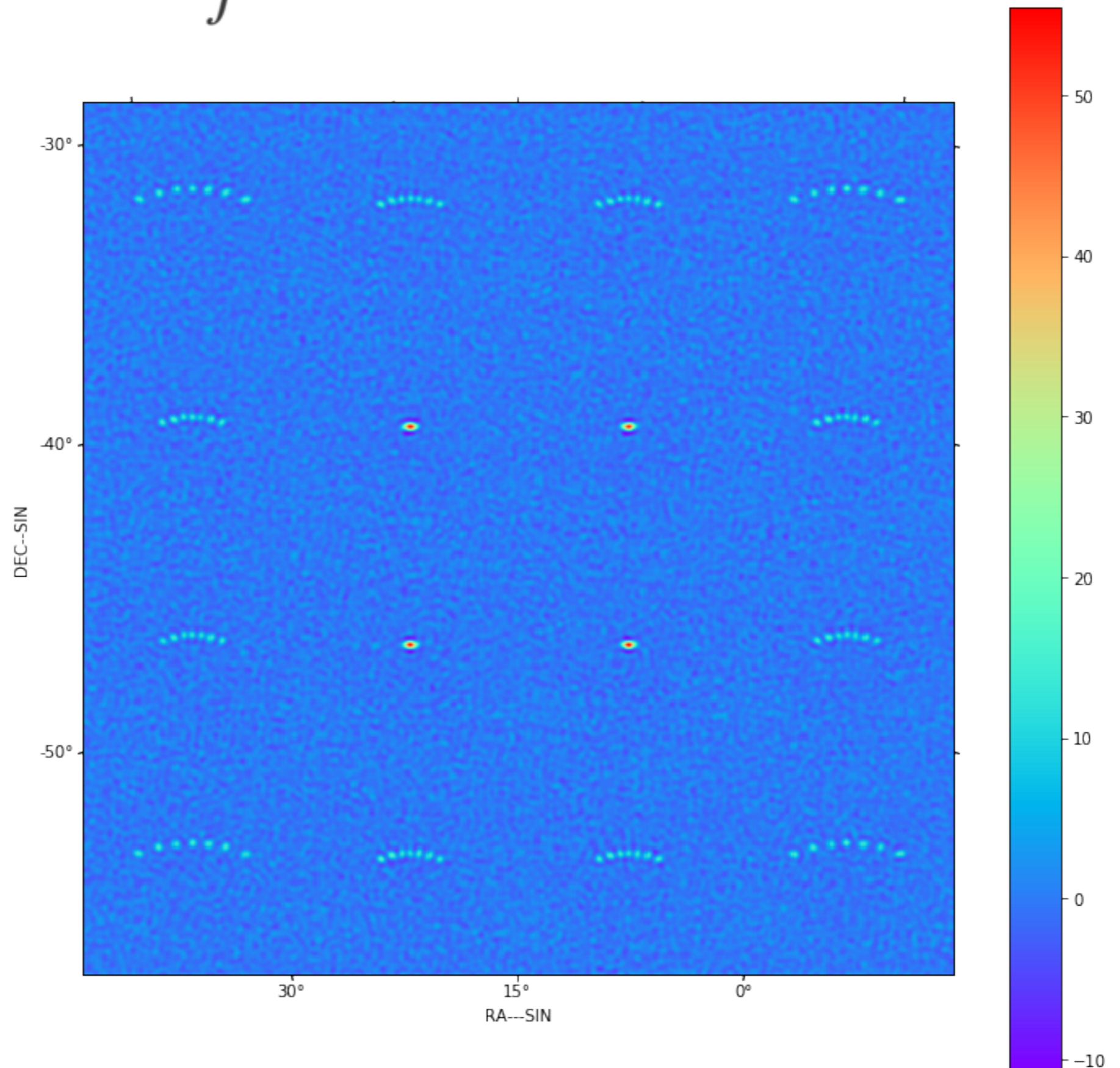




$$V(u, v, w) = \int I(l, m) e^{2\pi j(ul+vm)} dl dm$$

**2D transform  
for 7 hourangles**

**Each hour angle causes  
different position error**



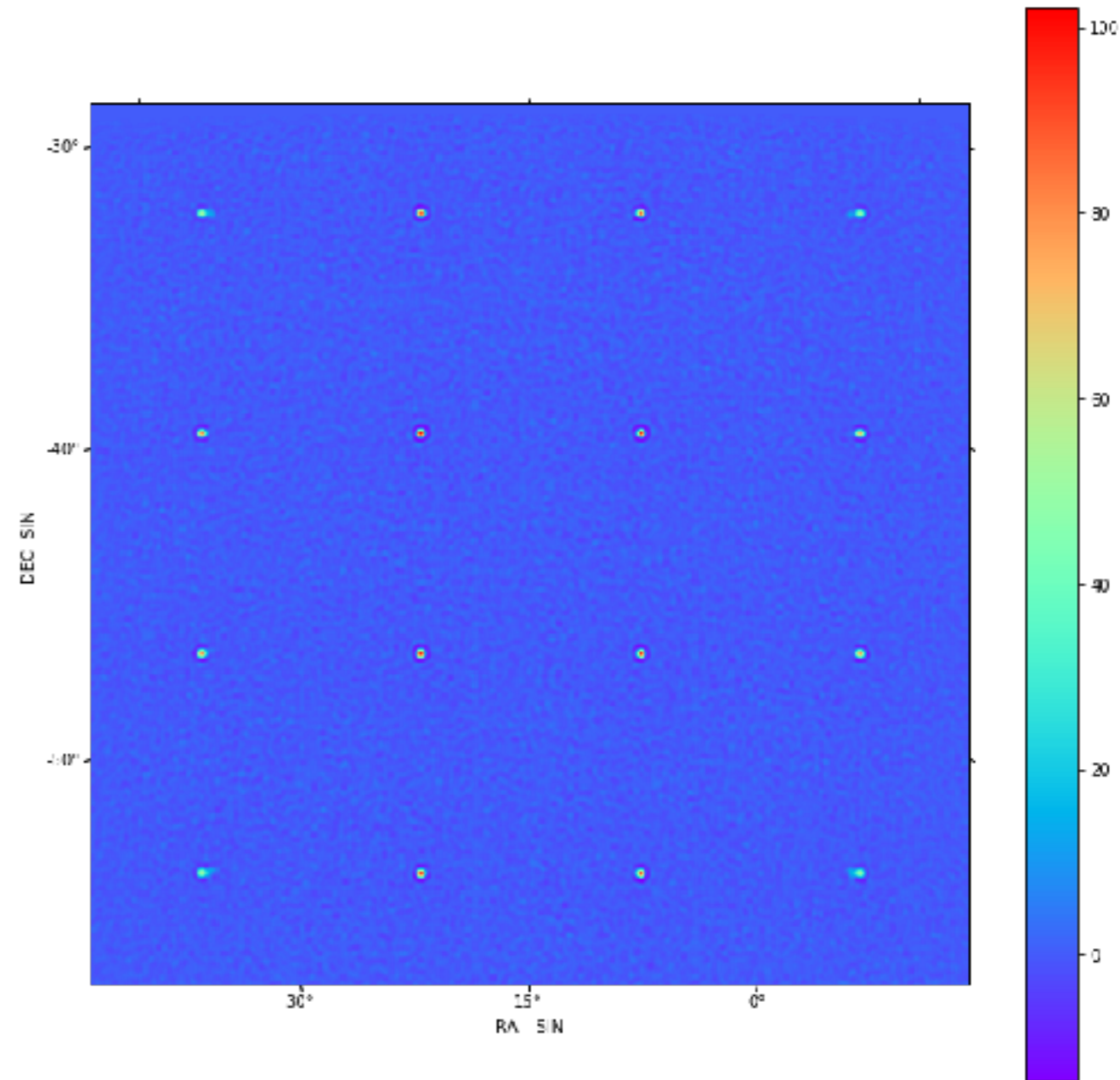
# Time slice imaging

$$w = au + bv$$

$$l' = l + a(\sqrt{1 - l^2 - m^2} - 1)$$

$$m' = m + b(\sqrt{1 - l^2 - m^2} - 1)$$

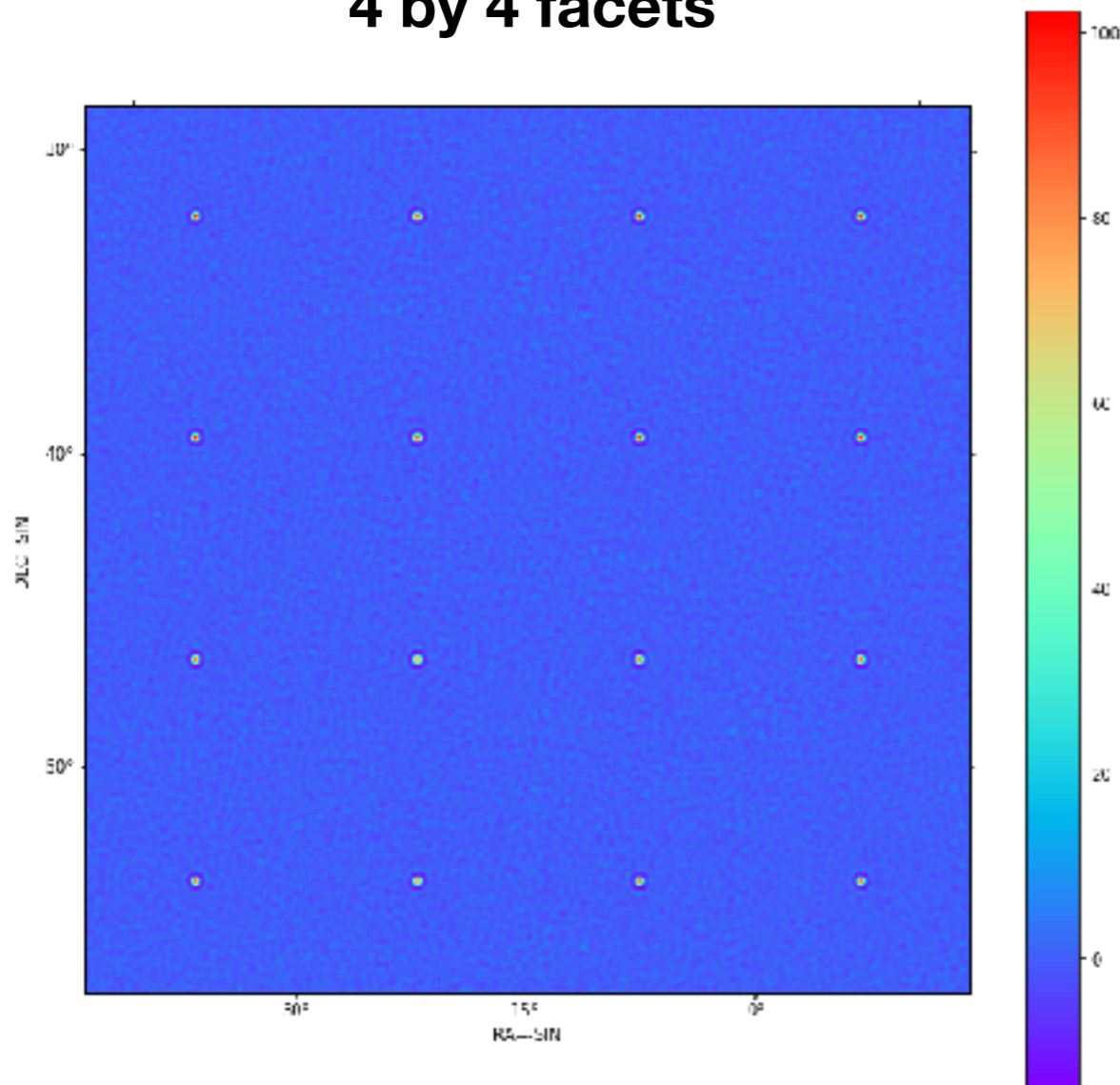
- Time-dependent coordinate distortion
- Known and easy to correct
- But accuracy for point sources is poor



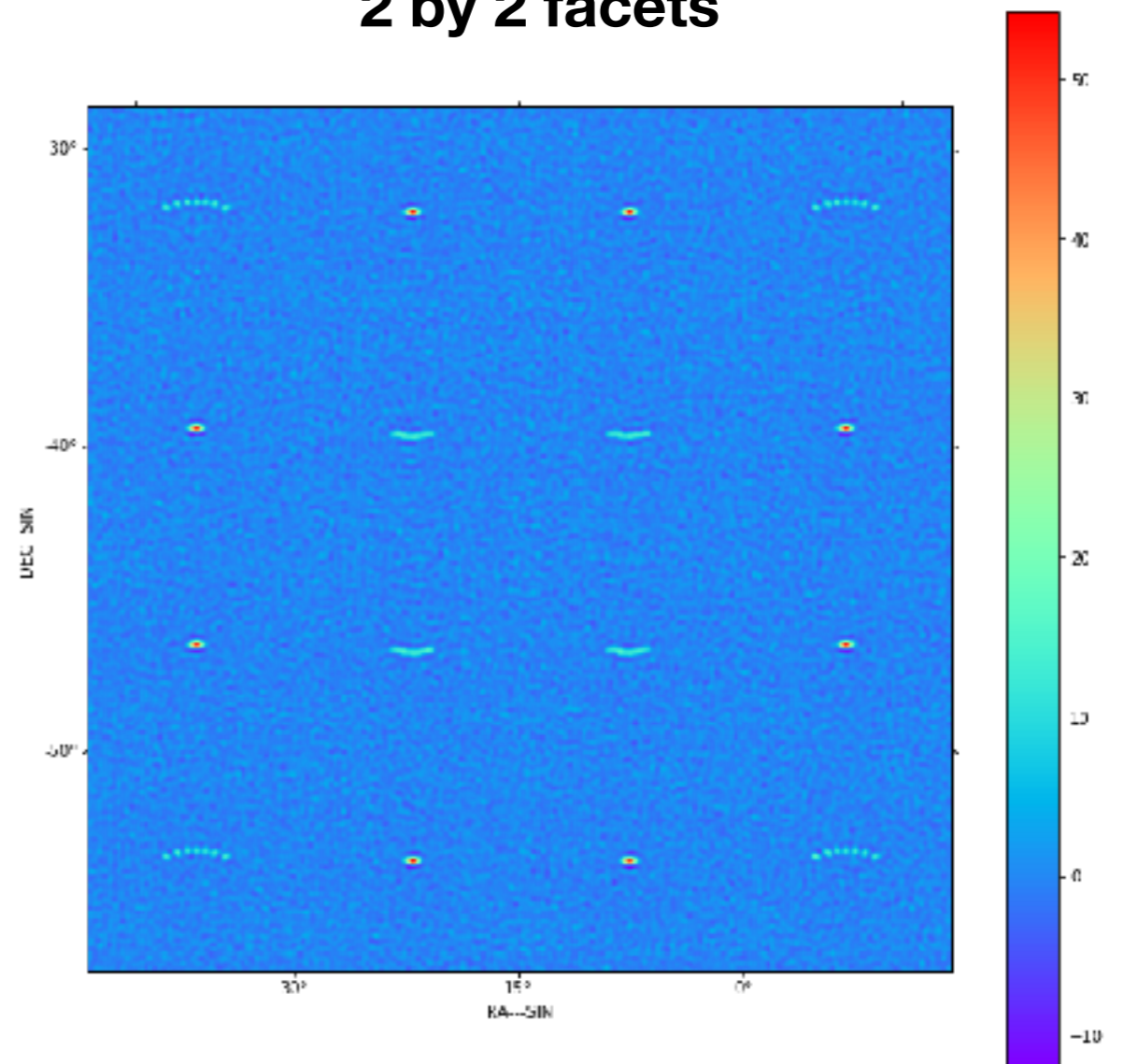
# Faceted imaging

$$V(u, v, w) = \sum_{ij} \frac{1}{\sqrt{1 - l_{ij}^2 - m_{ij}^2}} e^{-2\pi j(ul_{ij} + um_{ij} + w(\sqrt{1 - l_{ij}^2 - m_{ij}^2} - 1))} \int I(\Delta l, \Delta m) e^{-2\pi j(u\Delta l_{ij} + v\Delta m_{ij})} dl dm$$

4 by 4 facets



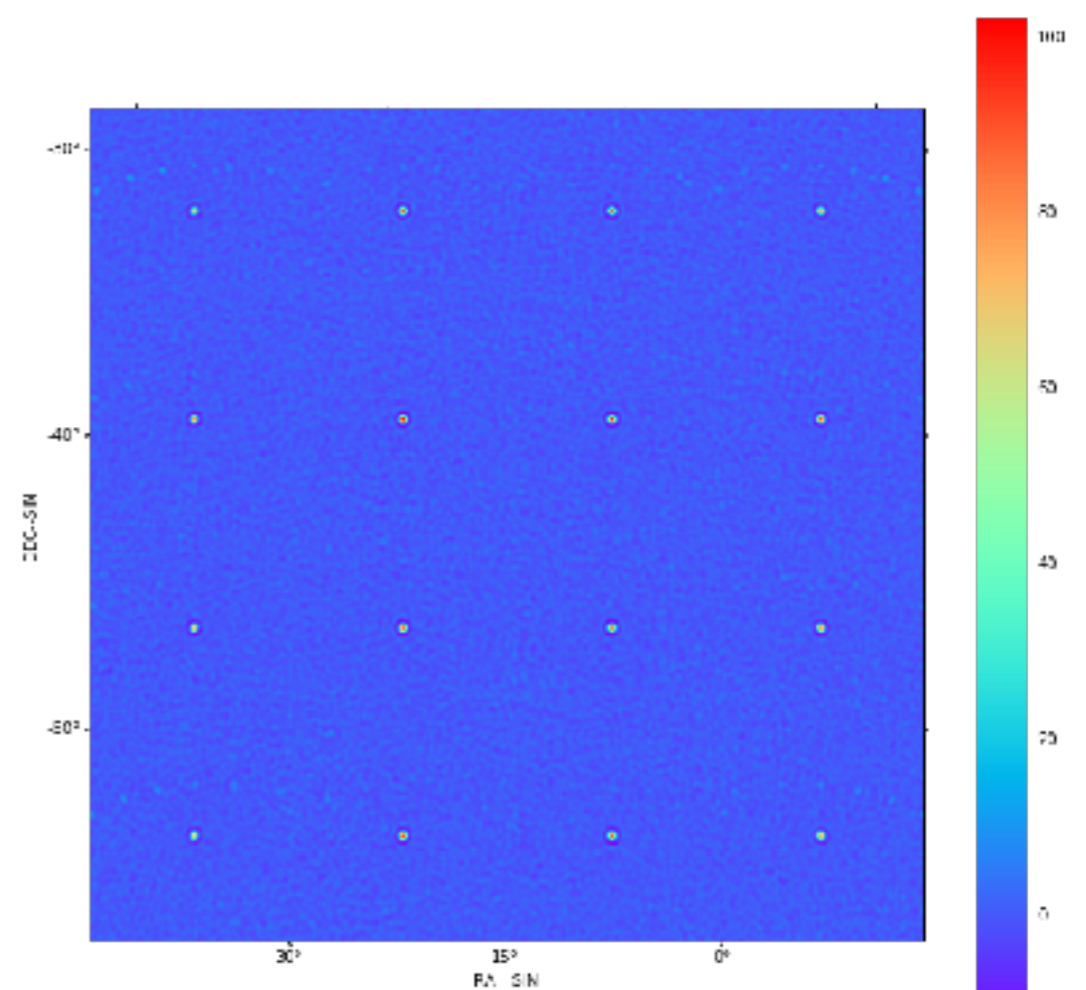
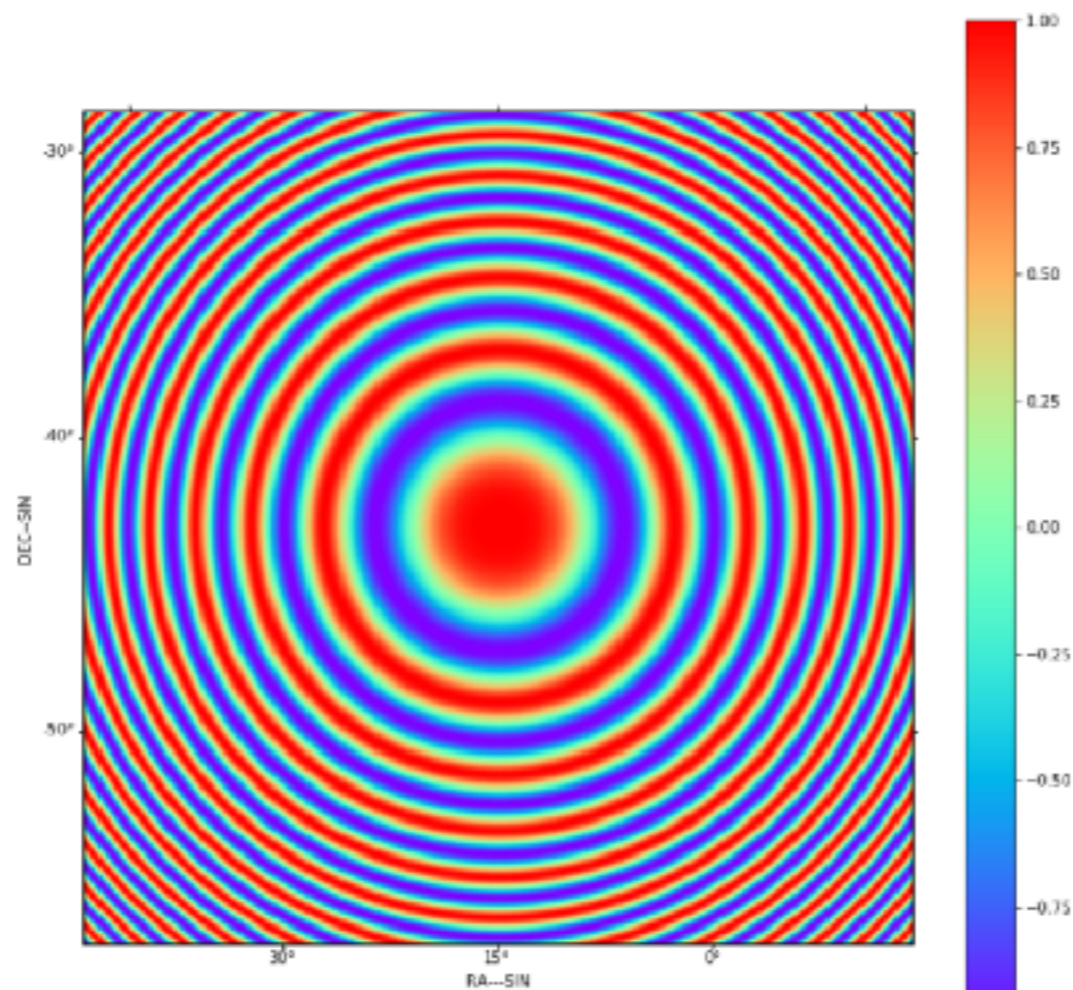
2 by 2 facets



# W stacking

$$V(u, v, w) = \sum_k \int G(l, m, w_k) I(l, m) e^{2\pi j(ul+vm)} dl dm$$

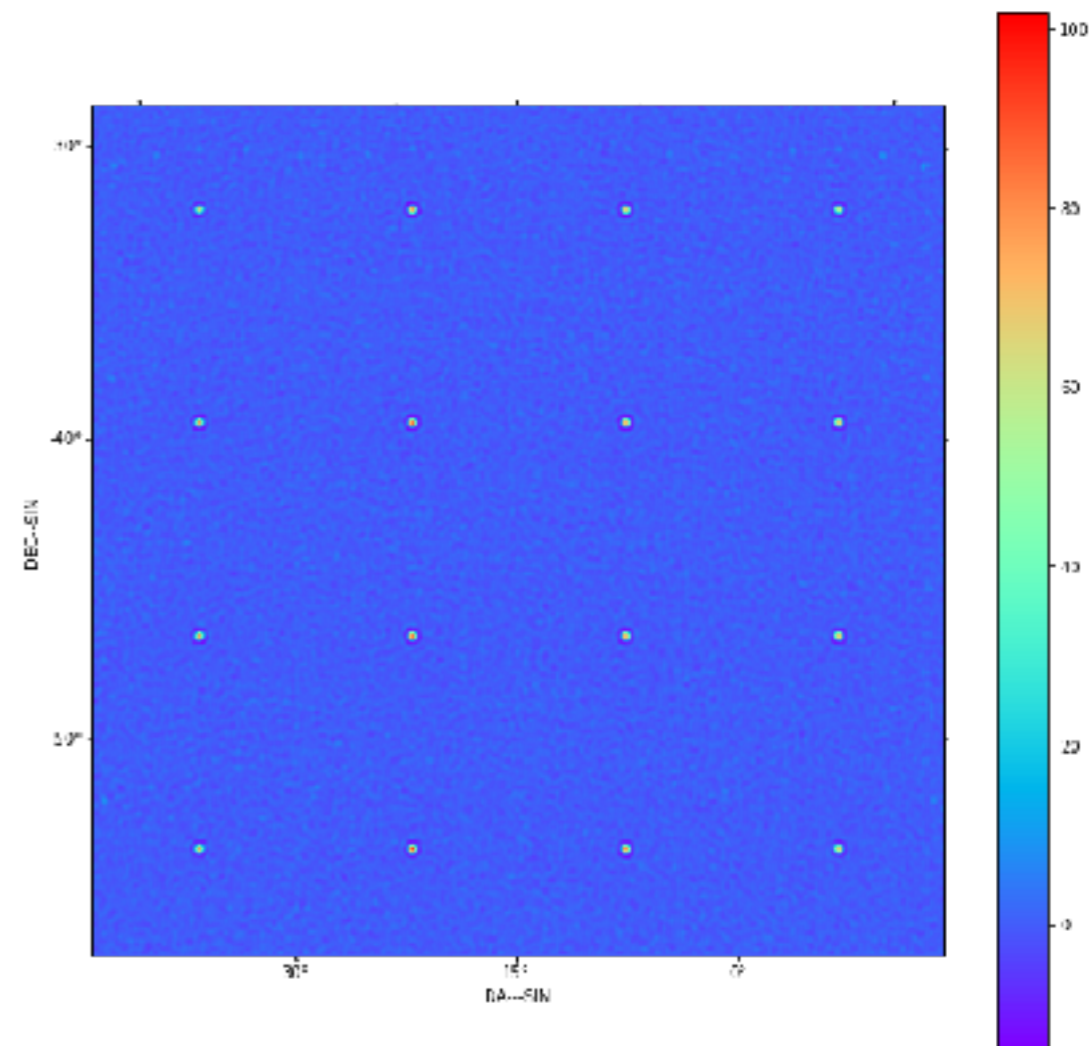
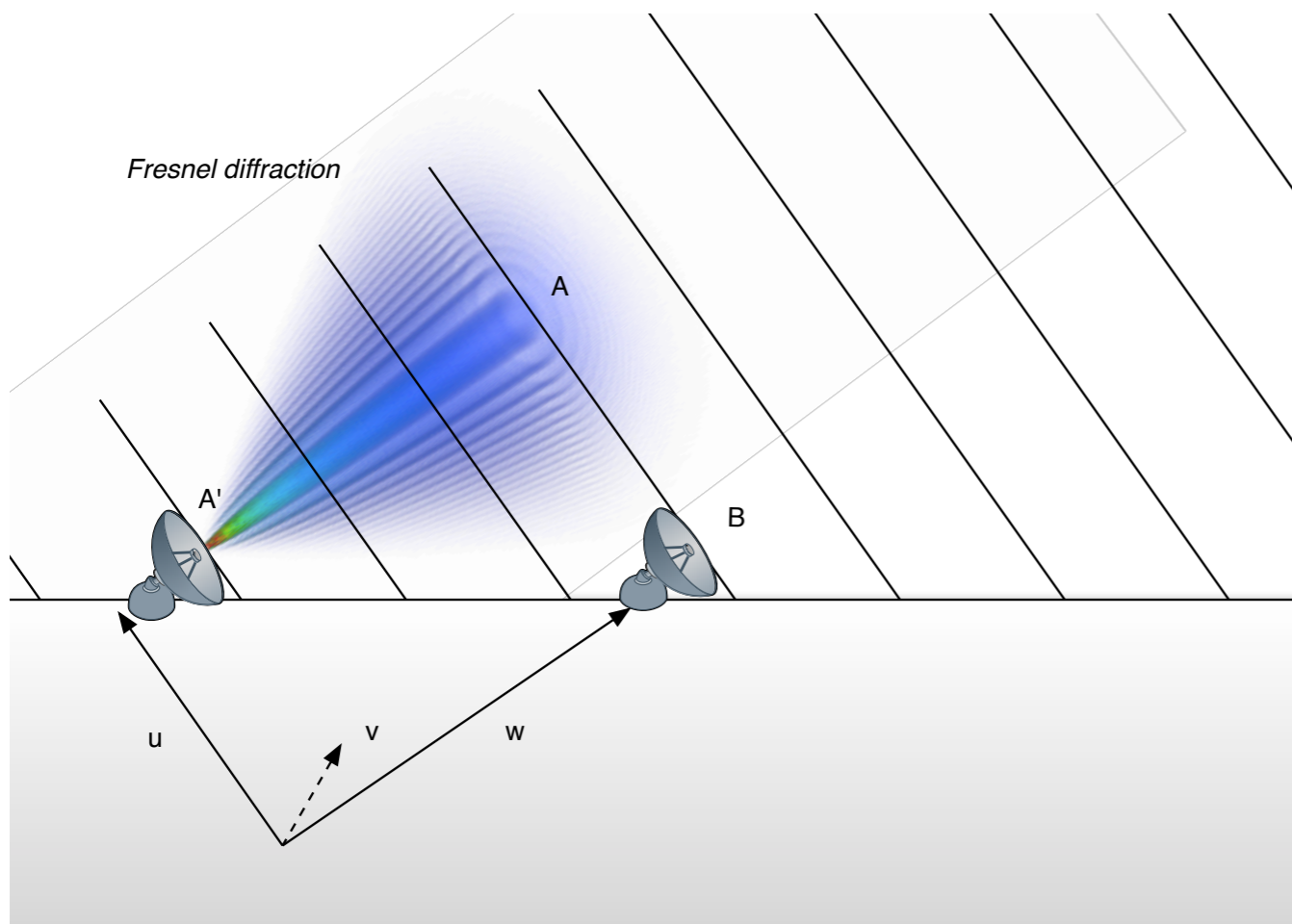
$$G(l, m, w_k) = \frac{1}{\sqrt{1-l^2-m^2}} e^{2\pi j w_k \left( \sqrt{1-l^2-m^2} - 1 \right)}$$



# W Projection

$$V(u, v, w) = G(u, v, w) \otimes \int I(l, m) e^{2\pi j(ul+vm)} dl dm$$

$$G(u, v, w) = \int \frac{1}{\sqrt{1-l^2-m^2}} e^{2\pi j\left(ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right)} dl dm$$



# Processing Flops

- W projection

$$R_F^2$$

- Snapshots alone

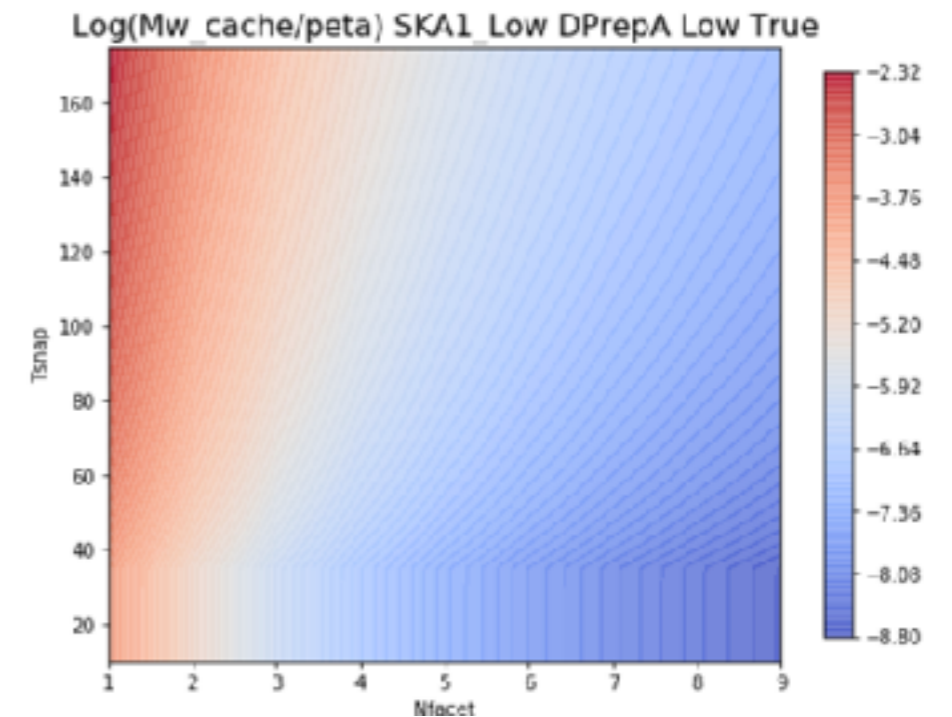
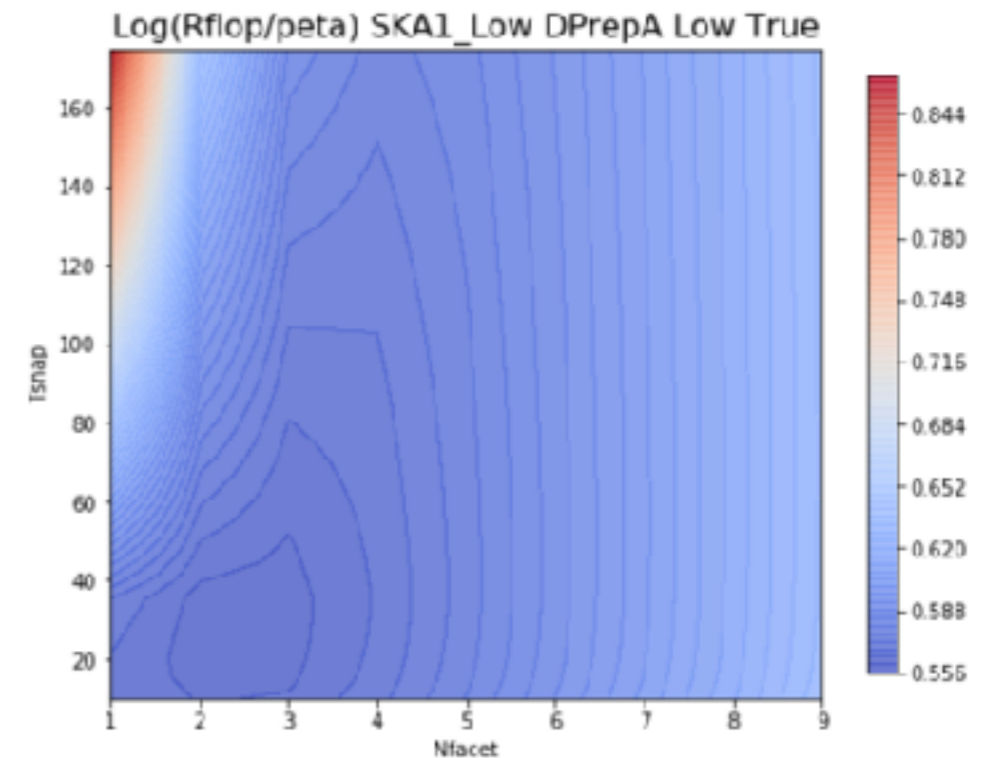
$$R_F$$

- W projection + snapshots

$$R_F^{2/3}$$

# Which algorithm?

- Many different algorithms available in different software
- Given sufficiently large resources all algorithms give the same answer
- Tradeoff between flops, memory, IO
- Optimum for SKA is wstacking + timeslice + facets
- Facets often useful as well for non-isoplanatic imaging



# W sampling

- W sampling for maximum amplitude error

$$\Delta w = \frac{\sqrt{2\Delta A}}{N_{\text{pixel}} \theta_{FOV}^2}$$

- Number of w planes

$$N_w = \frac{\pi}{\sqrt{2\Delta A}} R_F$$

- Fresnel number

$$R_F = \frac{\theta_{FOV}^2}{\theta_{\text{syn}}}$$

# The limit of wide field imaging

- Sampling in UV

$$\Delta uv = \frac{\sqrt{2\Delta A}}{2\pi\theta_{FOV}}$$

- Sampling in W

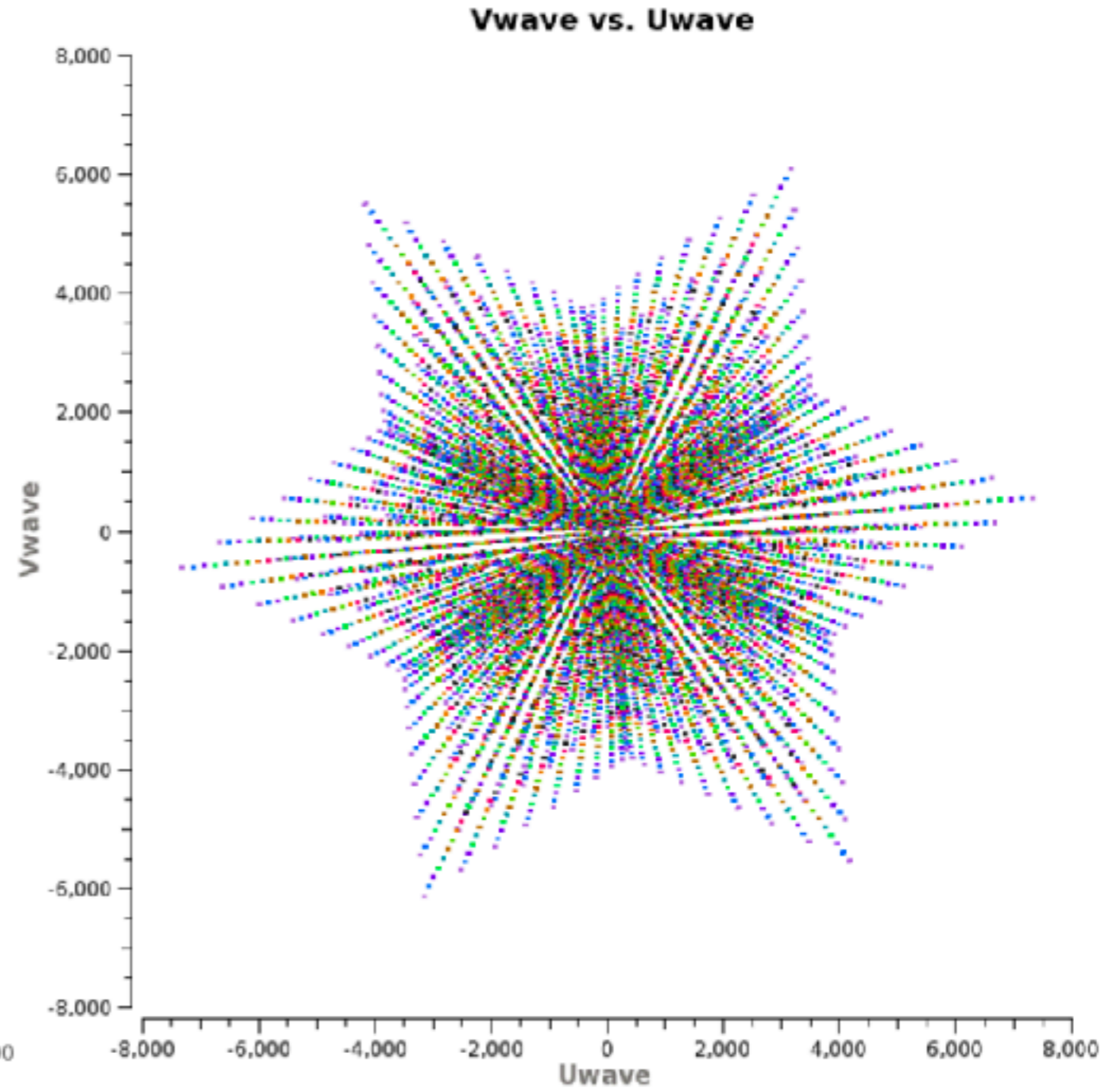
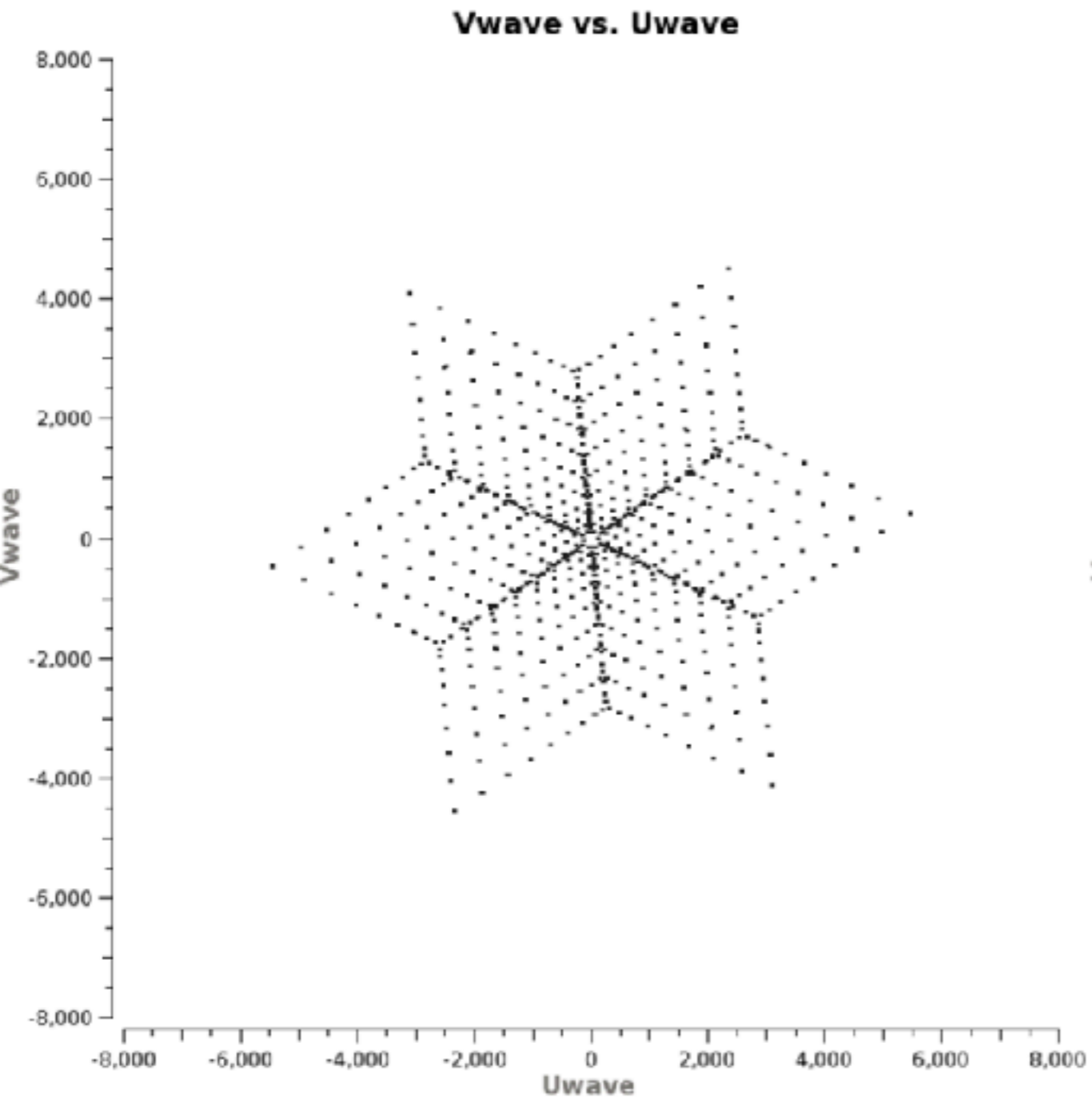
$$\Delta w = \frac{2\Delta uv}{\theta_{FOV}}$$

- As FOV expands, sampling must be same for U,V, and W
- Embed in 3D cube and use FFT's?

# Wide bandwidth imaging

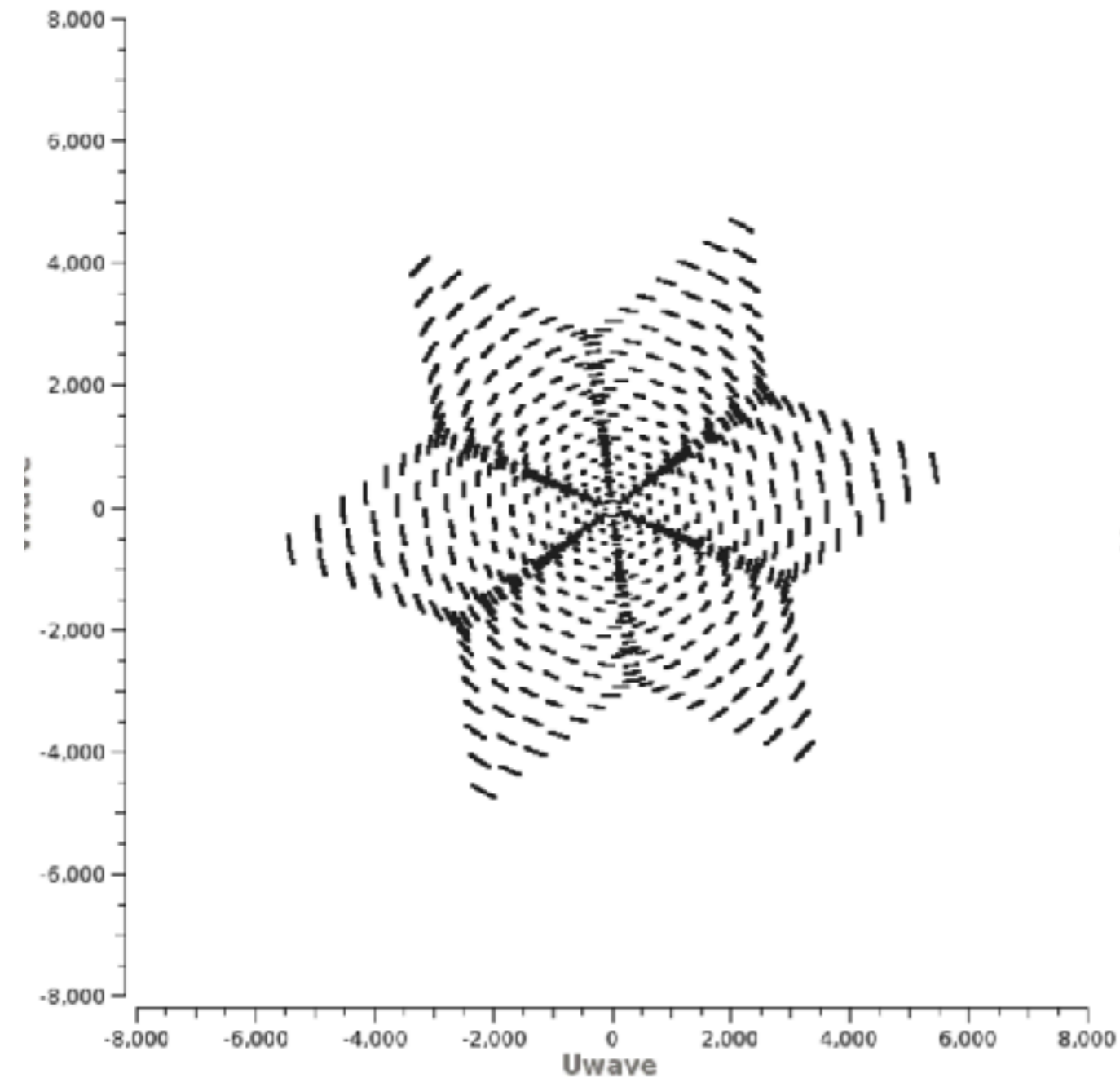
- Improving UV coverage improves the quality of imaging
- Both UV coverage and sensitivity scale with frequency
- But sources change with frequency
- And primary beam changes with frequency
- Need to solve for both image at some frequency plus change with frequency
- e.g. Taylor expansion in frequency

# VLA snapshot

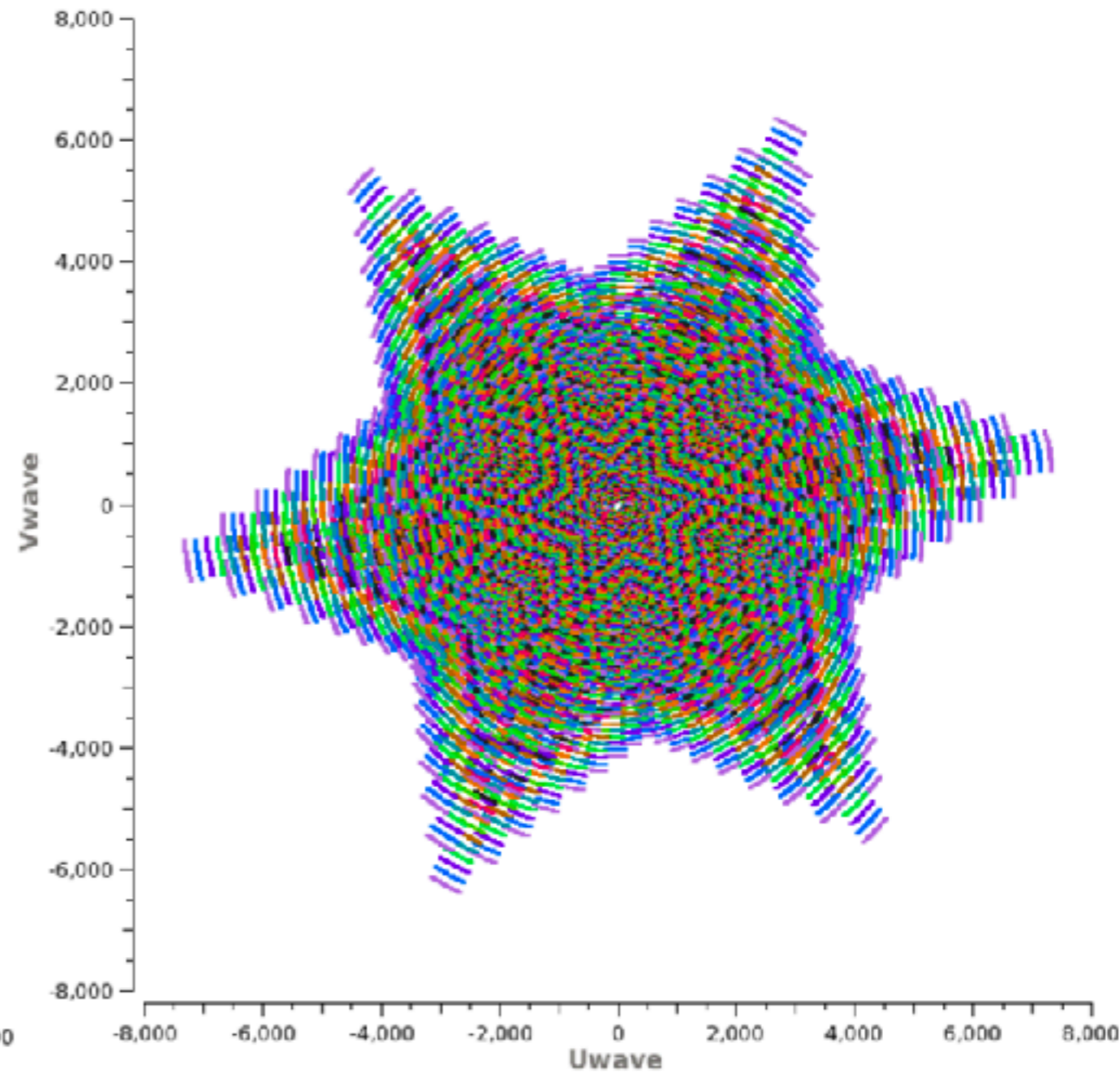


# VLA short integration

Vwave vs. Uwave



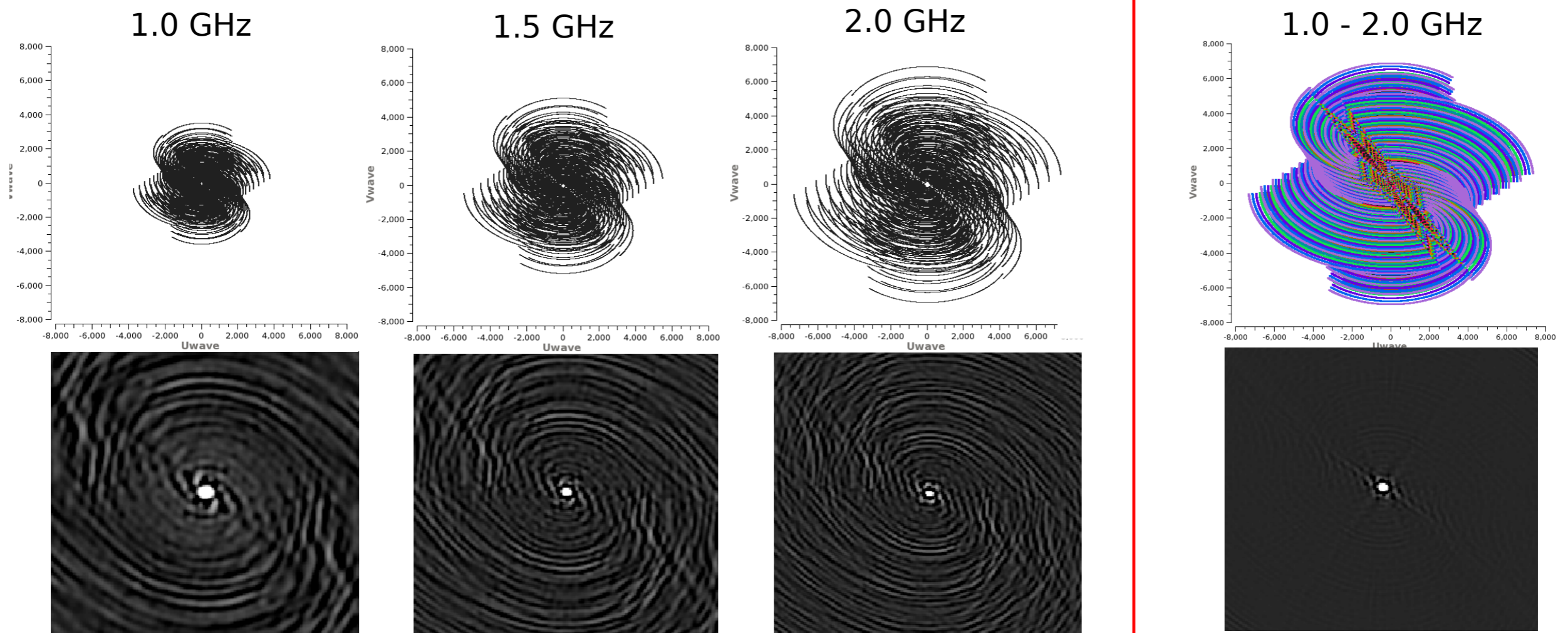
Vwave vs. Uwave



# Imaging across a wide frequency range

Large bandwidth => Increased 'instantaneous' imaging sensitivity  $\sigma_{cont} = \frac{\sigma_{chan}}{\sqrt{N_{chan}}}$

- Angular-resolution increases at higher frequencies
- Sensitivity to large scales decreases at higher frequencies
- Wideband UV-coverage has fewer gaps => lower PSF sidelobe levels



Observed image :  $I_{\nu}^{obs} = I_{\nu}^{sky} * PSF_{\nu}$

$$I_{wb}^{obs} = \sum_{\nu} \left[ I_{\nu}^{sky} * PSF_{\nu} \right]$$

# Algorithms

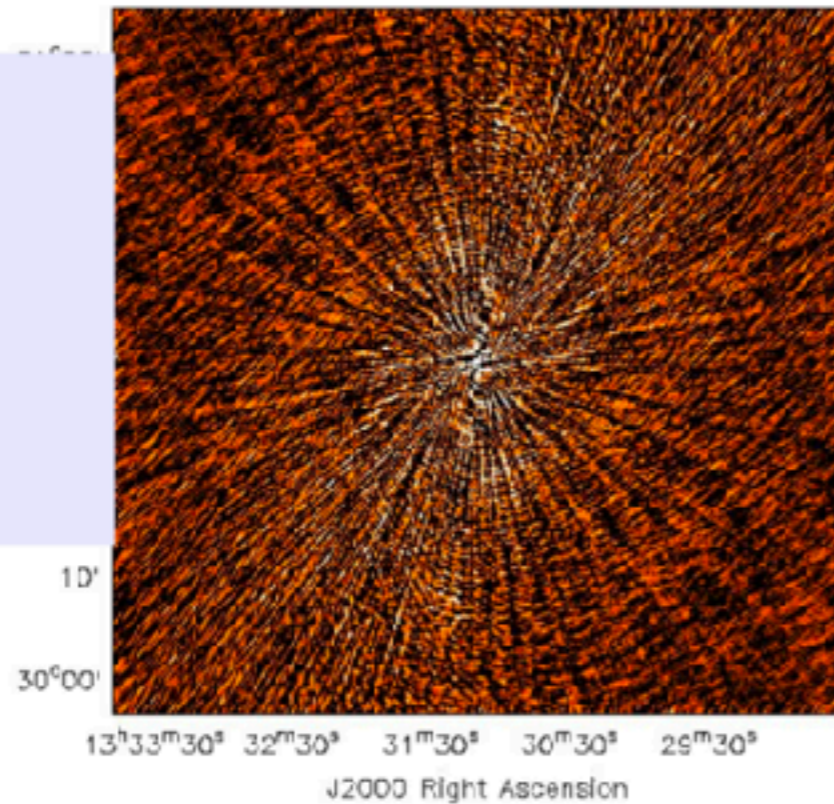
- Sault-Wieringa algorithm Clean, first order in frequency
- Rau algorithm, multiscale, n order expansion in frequency

# Dynamic-range with MS-MFS : 3C286 example : $N_t=1,2,3,4$

**NTERMS = 1**

Rms :  
9 mJy -- 1 mJy

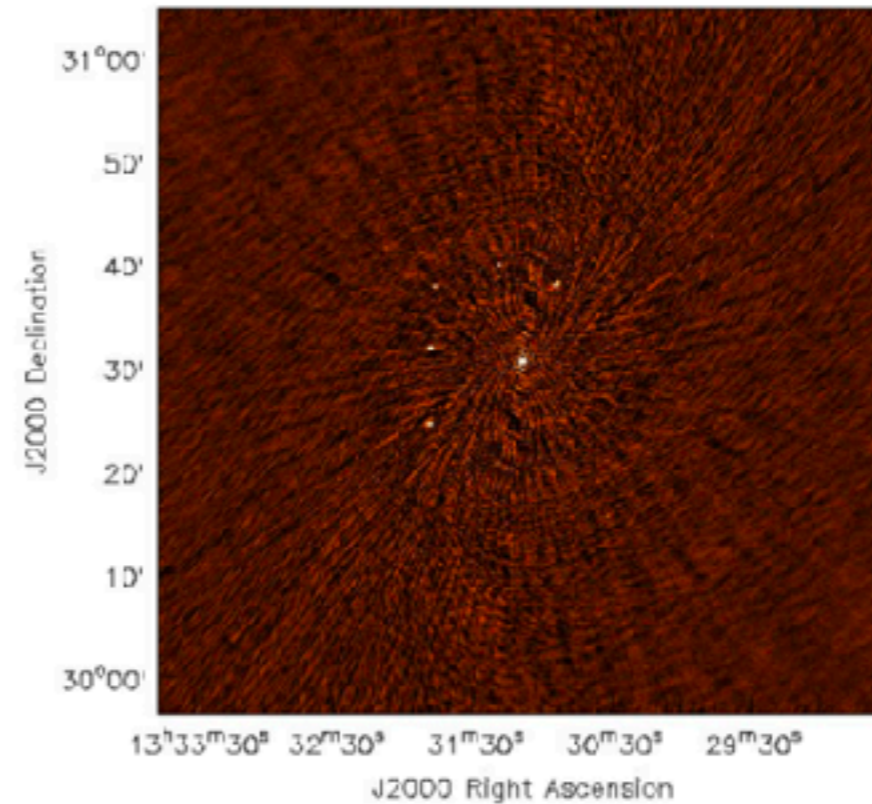
DR :  
1600 -- 13000



**NTERMS = 2**

Rms :  
1 mJy -- 0.2 mJy

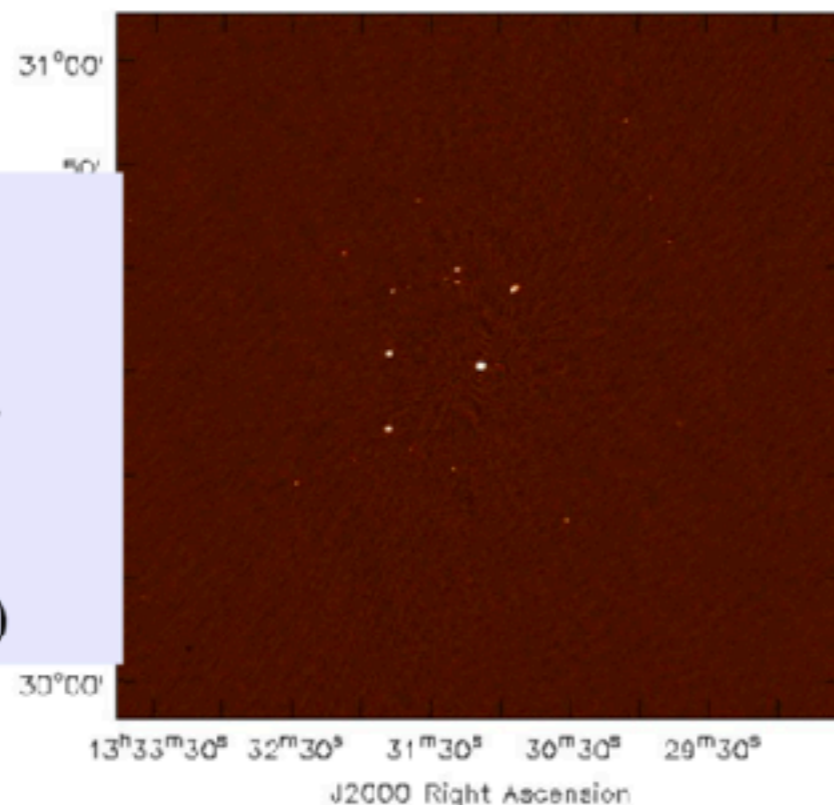
DR :  
10,000 -- 17,000



**NTERMS = 3**

Rms :  
0.2 mJy -- 85 uJy

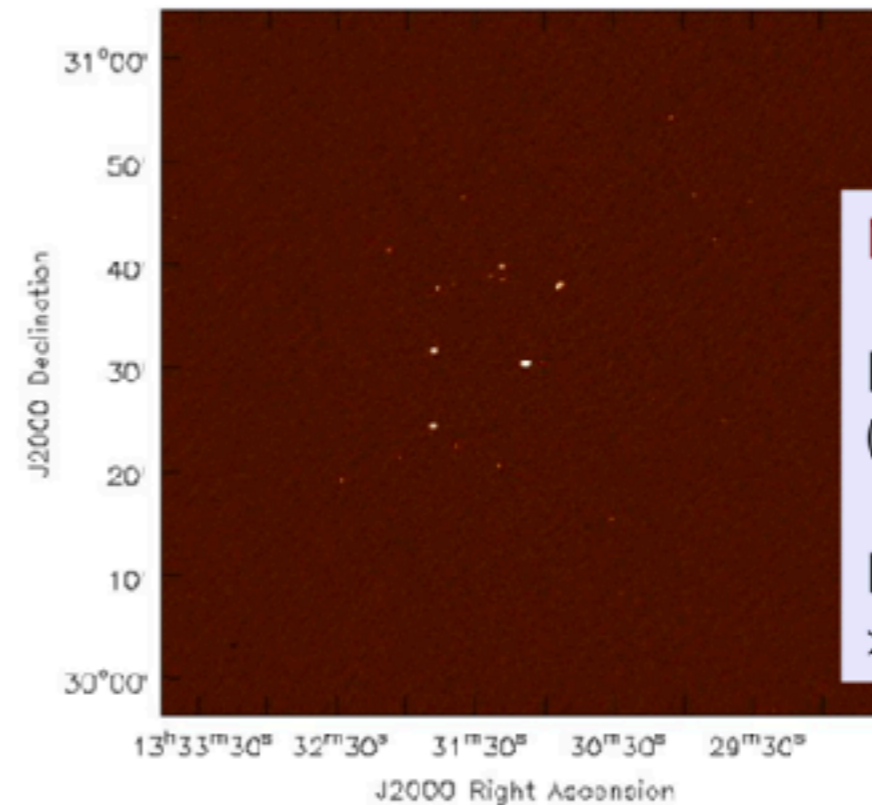
DR :  
65,000 -- 170,000



**NTERMS = 4**

Rms  
0.14 mJy -- 80 uJy

DR :  
>110,000 -- 180,000



# More information

- Urvashi Rau Ph.D. thesis
- Urvashi Rau RAS talk, 2012