Signal Processing for Astronomy

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+ some slides from John Tuthill

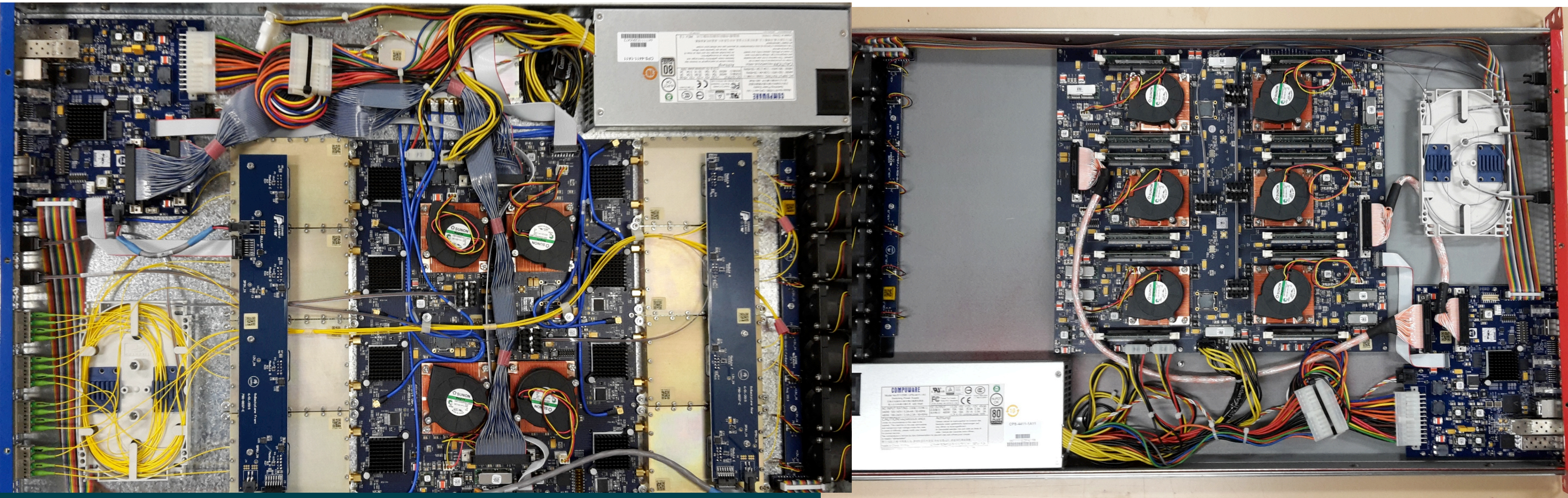
Context

Digital Signal Processing – manipulations of discrete samples approximating some continuous function

- Beamformer
- Tied-array unit
- Fringe rotator
- Correlator

In astronomy:
this is about backends

- well-defined core calculations
- often highly optimised
- often implemented in hardware



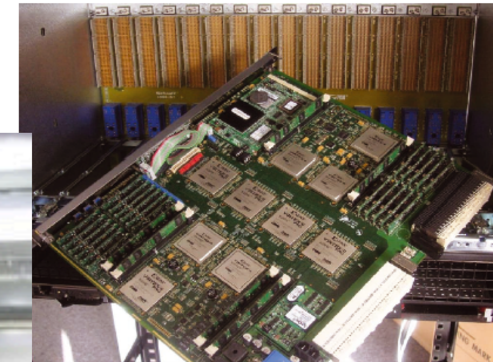
Correlator is the brain
of radio-interferometer

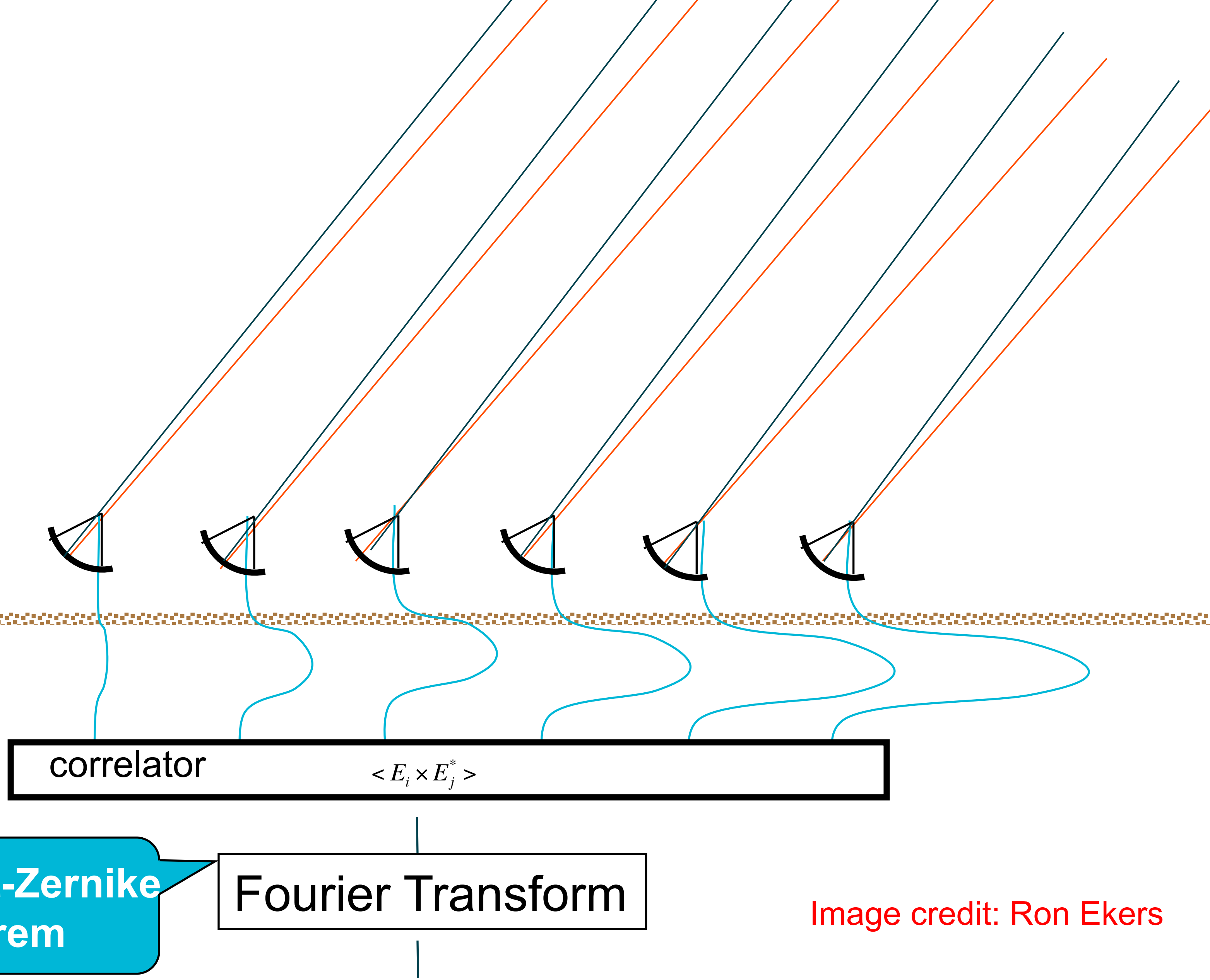
(as it computes visibilities)

”black box” for most science work

May need to know details,

if you start pushing the limits



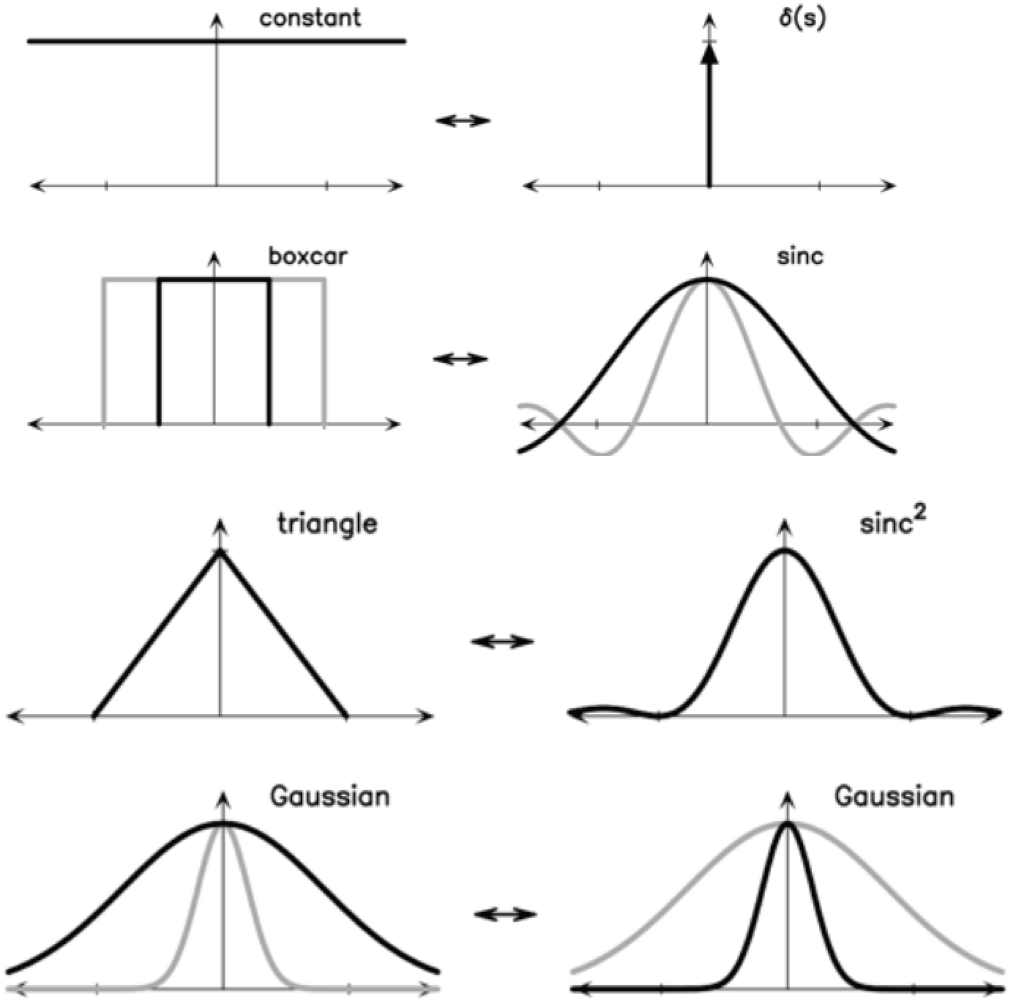


van Cittert-Zernike theorem

Image credit: Ron Ekers

I(r)

Fourier transforms – intro



Forward:

$$F(s) \equiv \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i s x}\, dx$$

Reverse:

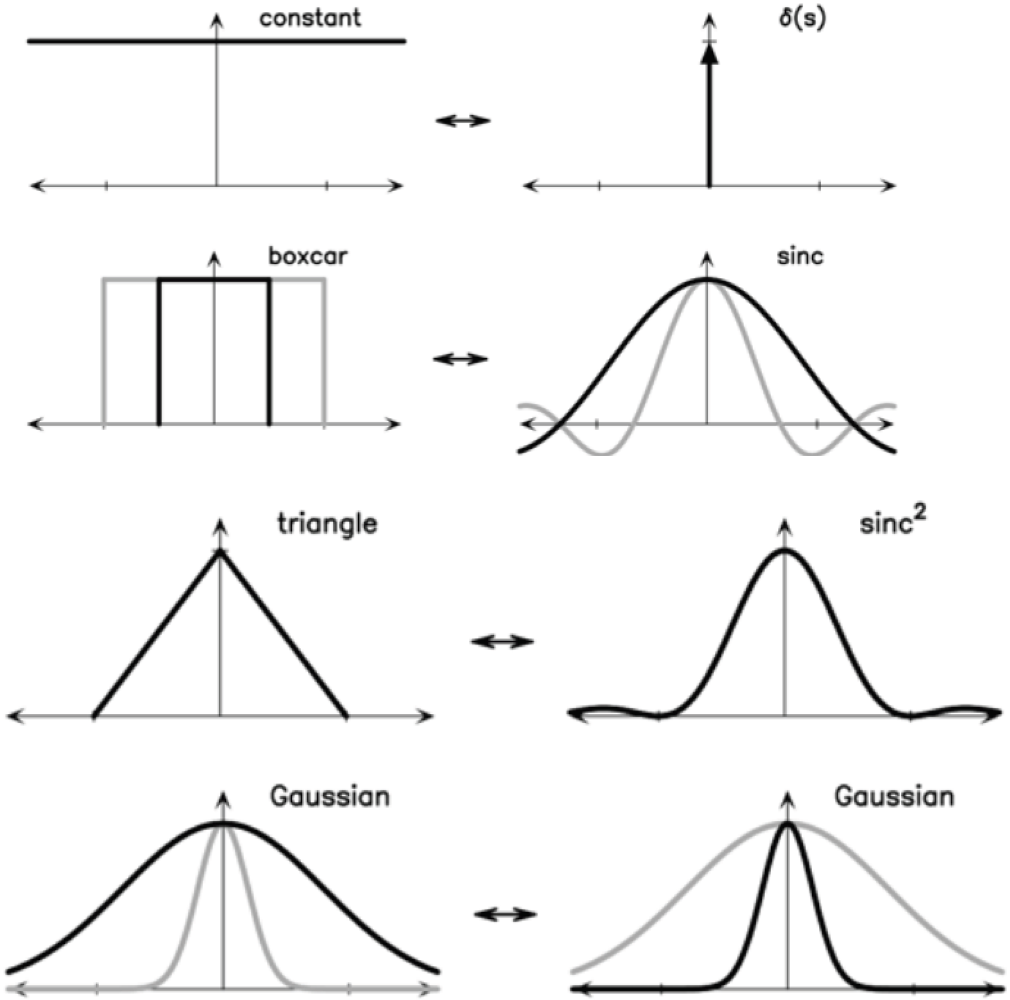
$$f(x) \equiv \int_{-\infty}^{\infty} F(s)\, e^{2\pi i s x}\, ds$$

Small ↔ Large

Convolution ↔ Product

Fourier transforms – intro

$j = \sqrt{-1}$



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Small ↔ Large

Convolution ↔ Product

Image credit: Josh Marvil

Sampling

The Sampling Theorem: A band-limited signal having no frequency components above f_{max} can be determined uniquely by values sampled at uniform intervals of T_s satisfying:

$$T_S \le \frac{1}{2f_{\max}}$$

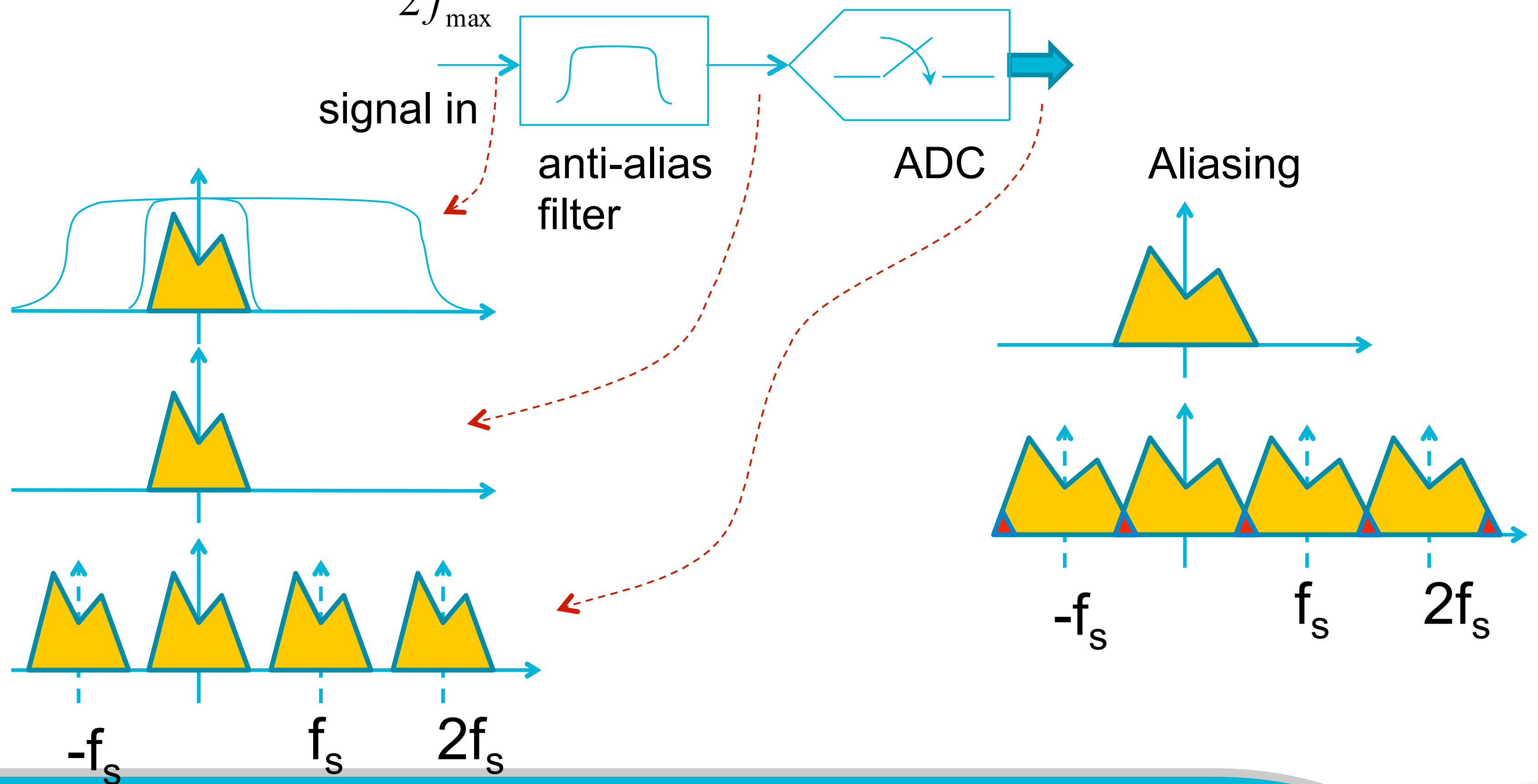


Image credit: John Tuthill

Simplest possible case

Assuming bandwidth is small enough & we don't care about frequency structure

$$V_{ij} = \frac{1}{N}\sum_{k=1}^{N} E_i(k) \times E_j^*(k-\tau)$$

Compute correlation directly (e.g. in software)

Code snippet from BETA-3 (3-baseline) software correlator:

```
IndexType offset1 = itsDelay1;
IndexType offset2 = itsDelay2;
IndexType offset3 = itsDelay3;

for (; (offset1 < size)&&(offset2 < size)&&(offset3 < size); ++offset1, ++offset2, ++offset3) {
    itsVis12 += *(stream1 + offset1) * conj(*(stream2+offset2));
    itsVis13 += *(stream1 + offset1) * conj(*(stream3+offset3));
    itsVis23 += *(stream2 + offset2) * conj(*(stream3+offset3));
}
```

Non-monochromatic input signal

More intuitive description via continuous formalism:

spectral representation

$$E(t) = \int s(\nu) \exp\{2\pi j \nu t\} d\nu$$

Correlation between antenna 1 and 2 data streams for a given lag:

$$\gamma(\tau) = \int E_1(t) E_2^*(t-\tau) dt = \int \underbrace{s_1(\nu) s_2^*(\nu)}_{} \exp\{2\pi j \nu \tau\} d\nu$$

Power (cross-correlation) spectrum

Option 1

Correlate streams for a number of lags and Fourier-transform

Lag or XF correlator

Option 2

Fourier-transform input streams and cross-multiply

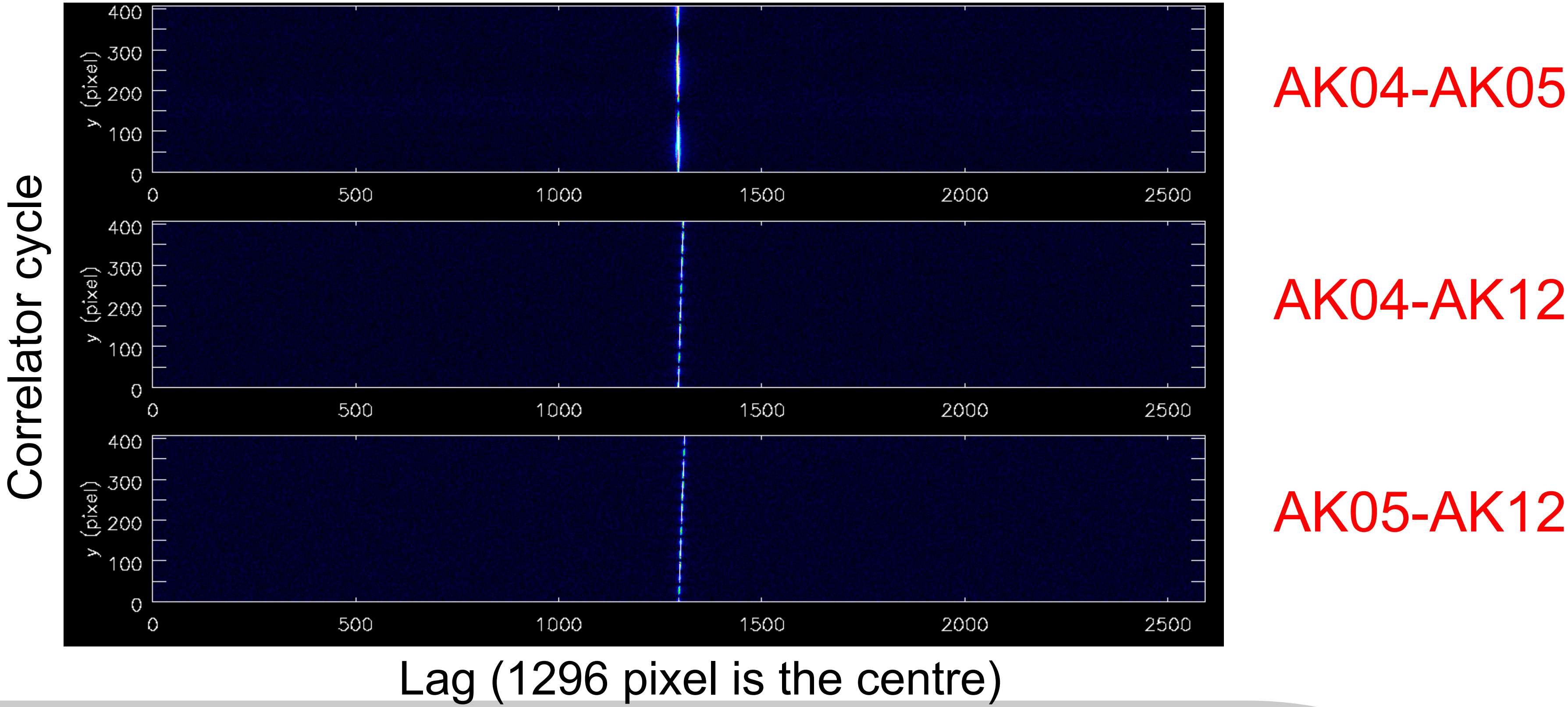
FX-correlator

More on lag domain

Regardless of the correlator architecture, lag spectrum

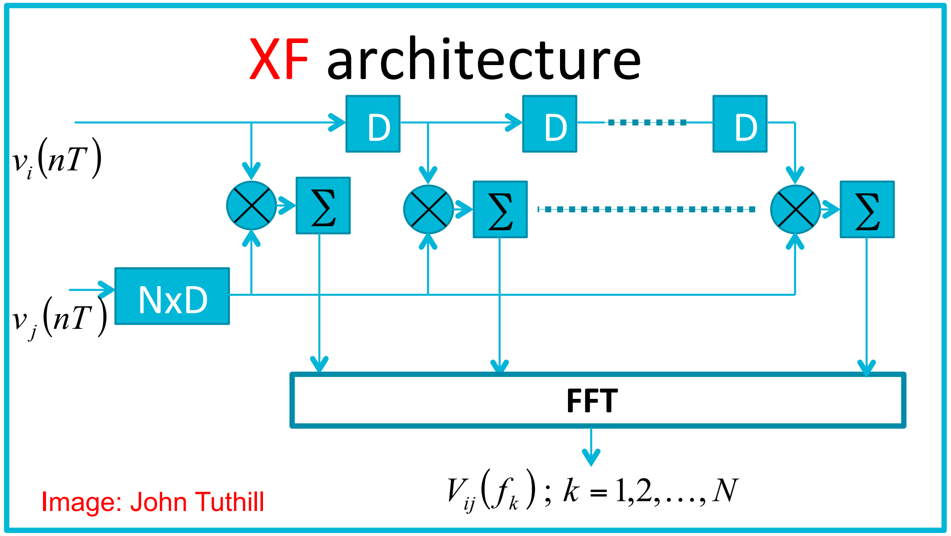
$$\gamma(\tau) = \int V_{12}(\nu)\exp\{2\pi j\nu\tau\}d\nu$$

is a useful diagnostic tool



Lag (XF) correlator

Old ATCA correlator (pre-CABB) is a good example

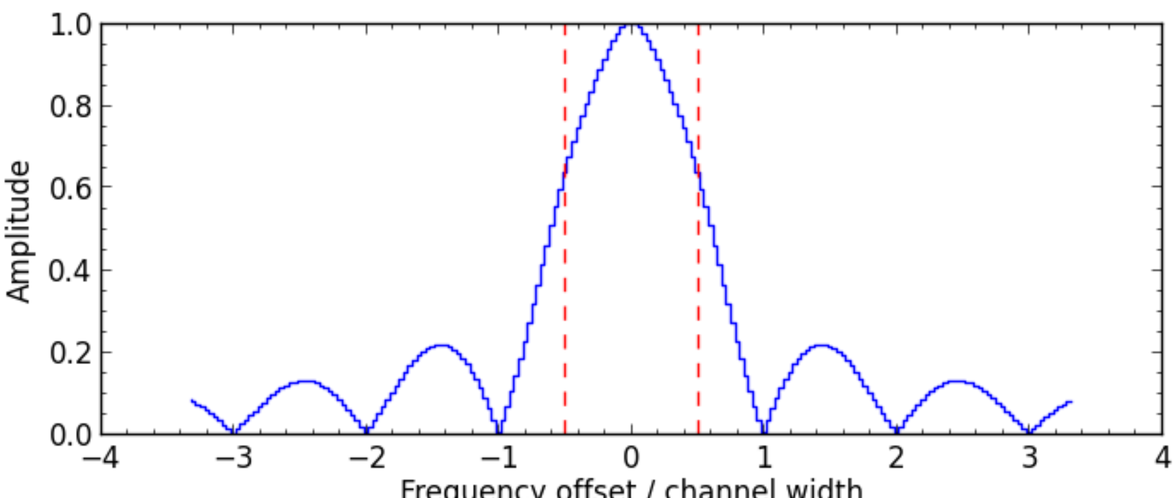


Fractional delays (< 1 sample) need to be corrected in some other way

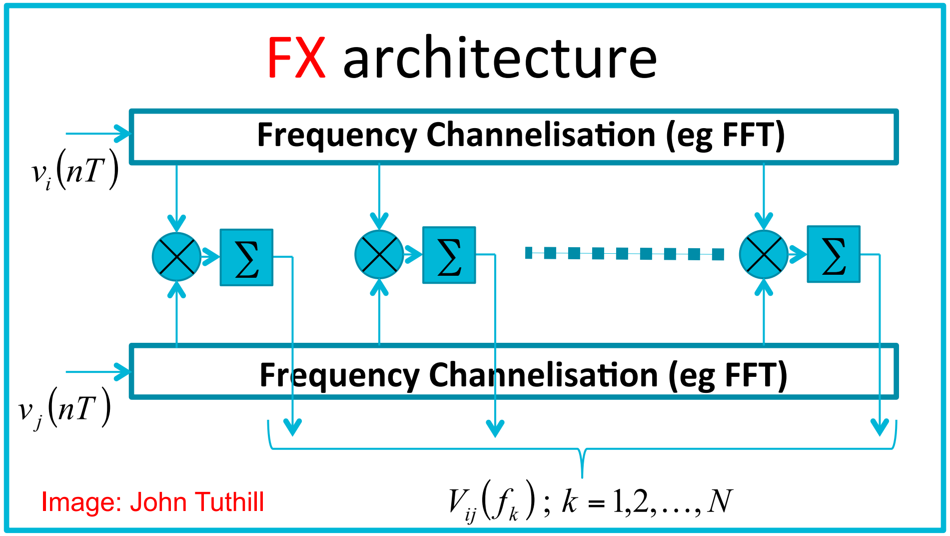
$$\gamma(\tau) = \int V_{12}(\nu)\exp\{2\pi j\nu\tau\}d\nu$$

Only finite number of lags can be measured

Convolution with *sinc* in the frequency domain

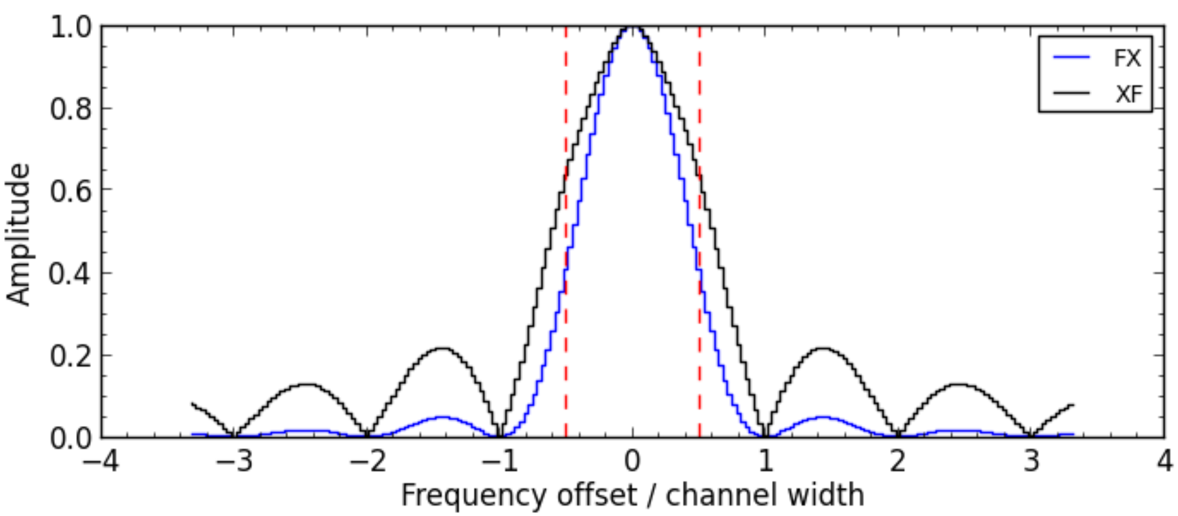


FX correlator



Fractional delays are easy to implement

Output is the product of two Fourier Transforms, each is presented with a finite chunk of data

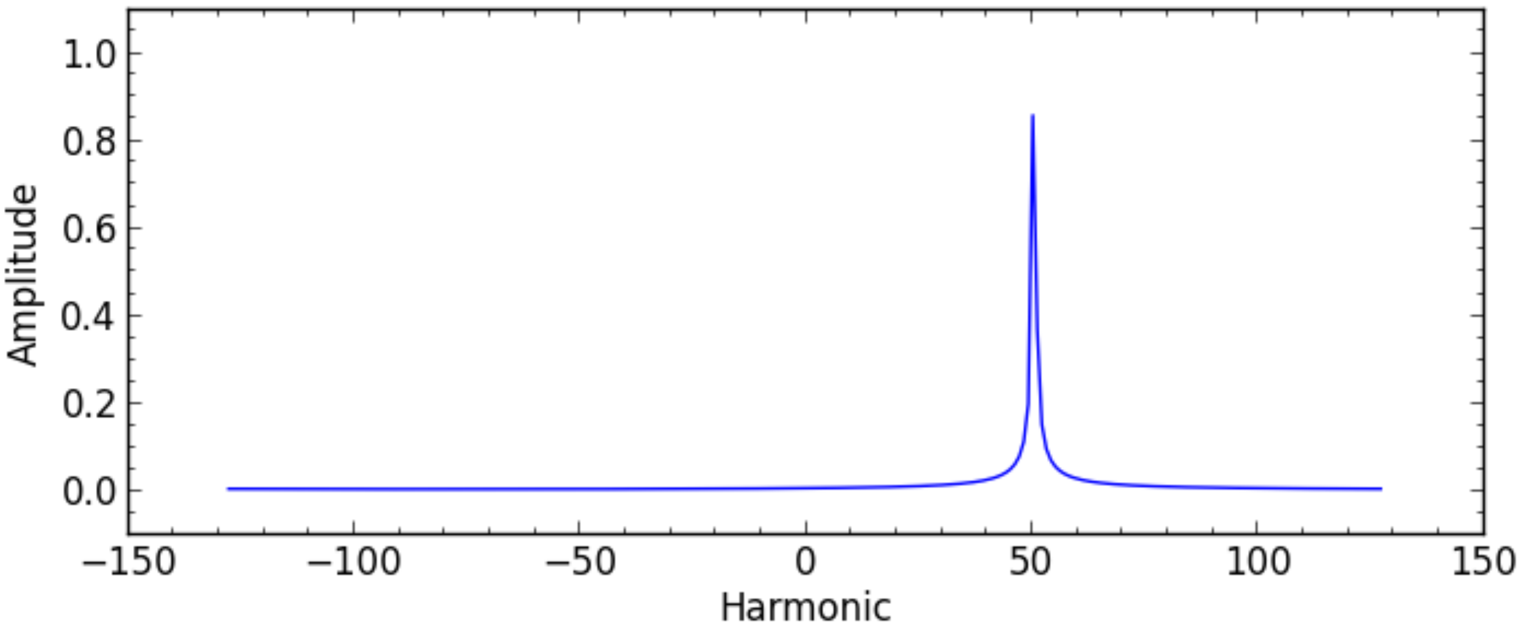


Convolution with $sinc^2$ in the frequency domain

Can we get a better channel response?

Let's consider FX architecture as a channeliser + cross-multiplyer (think of a simple correlator described earlier)

The channel response is determined by the channeliser performance and FFT is known to be quite bad



Power leakage into adjacent channels

FFT of the sine wave with frequency of 50.3 * f_s / 256

Filtering

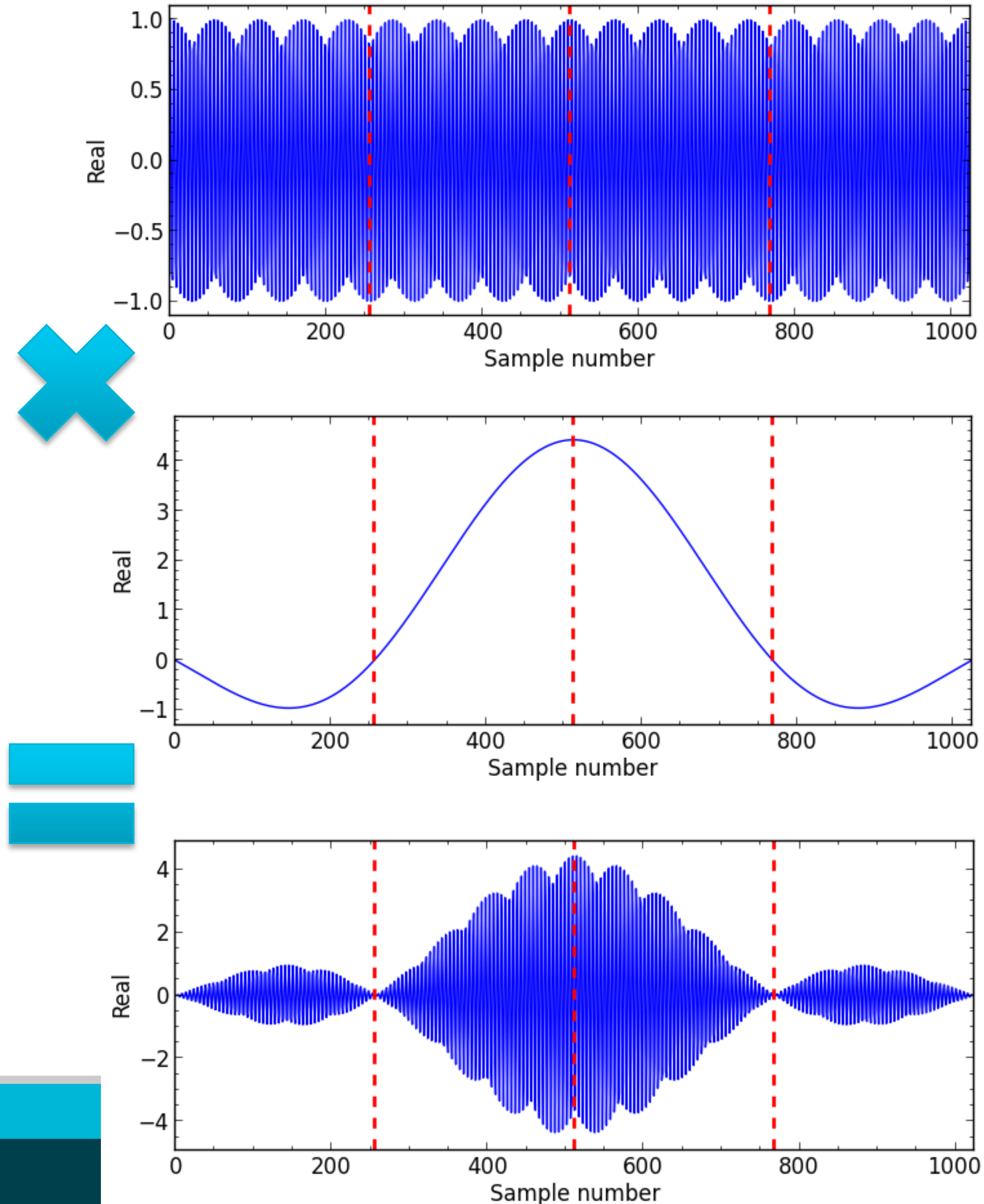
Input data samples

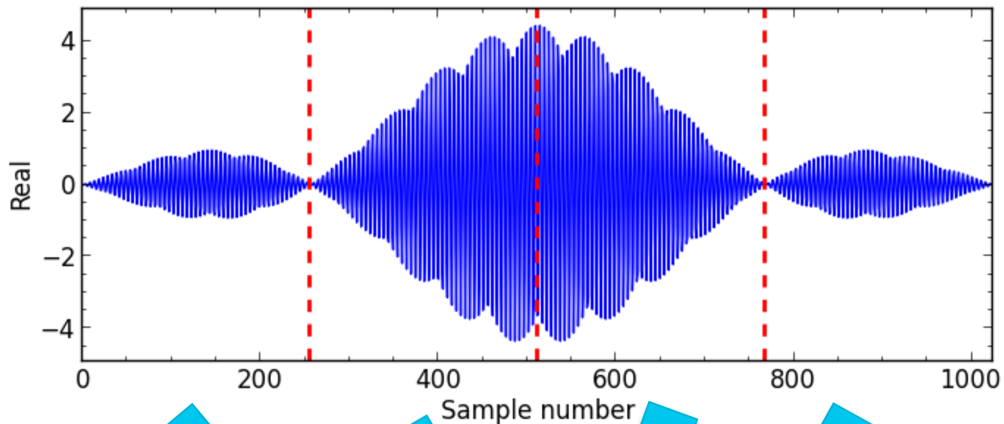
Sine wave with the frequency of $50.3\ f_s / 256$

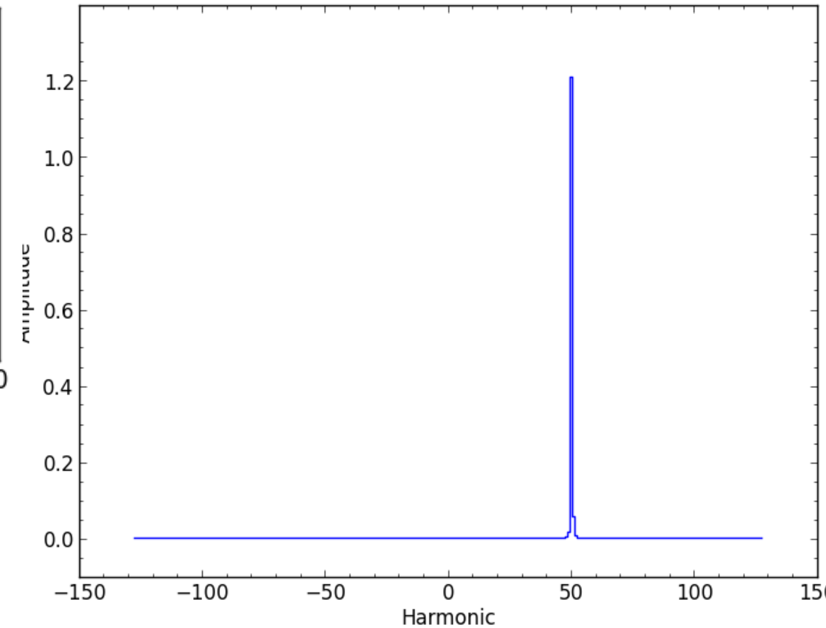
Filter

(Sinc in this particular example)

Filtered data samples







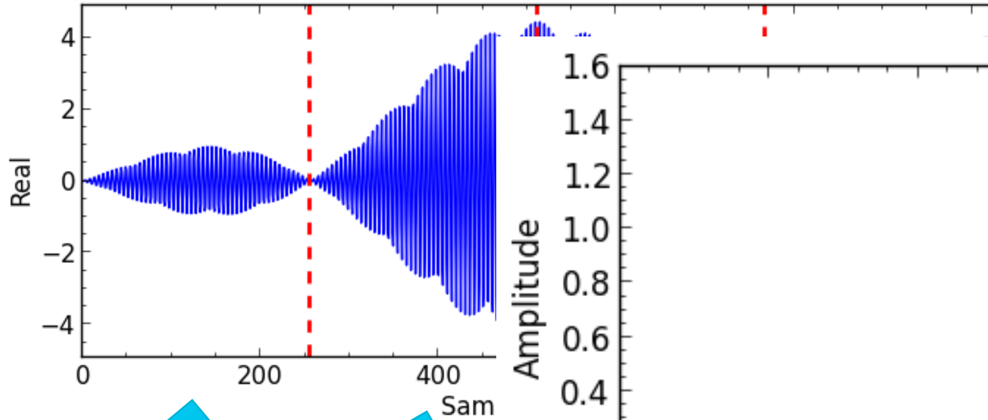
In this example:

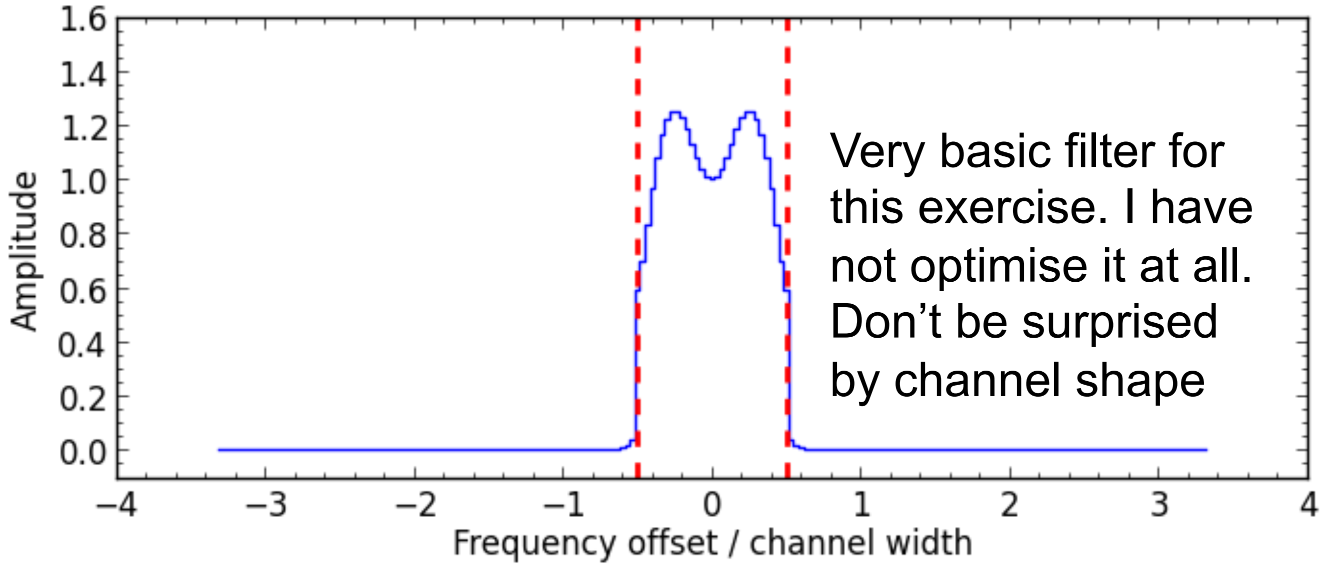
N = 256
M = 4

$$y(n) = \sum_{m=0}^{M-1} x(n+mN)h(n+mN)$$

FFT

polyphase sub-filters





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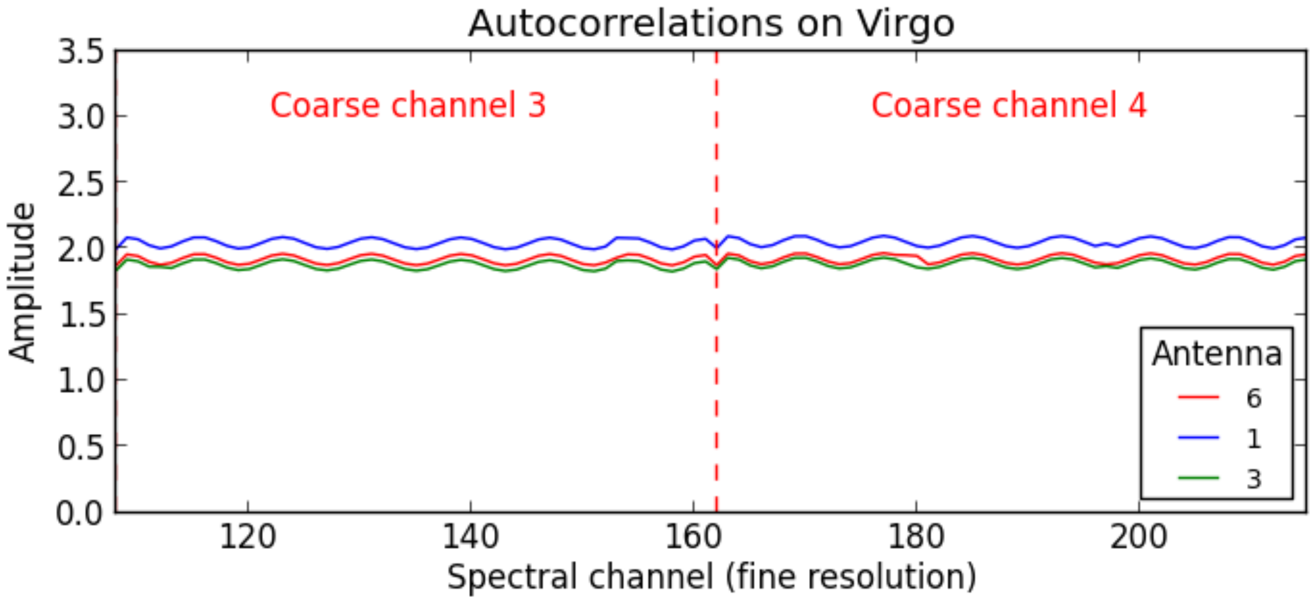
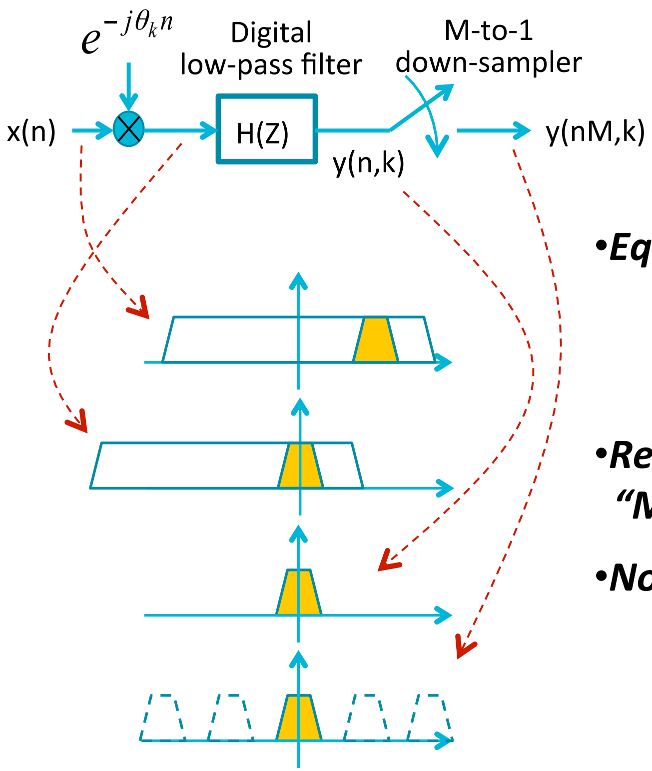


But filter does cause a ripple (can be corrected)

Early BETA tests

Polyphase decomposition: engineer's view

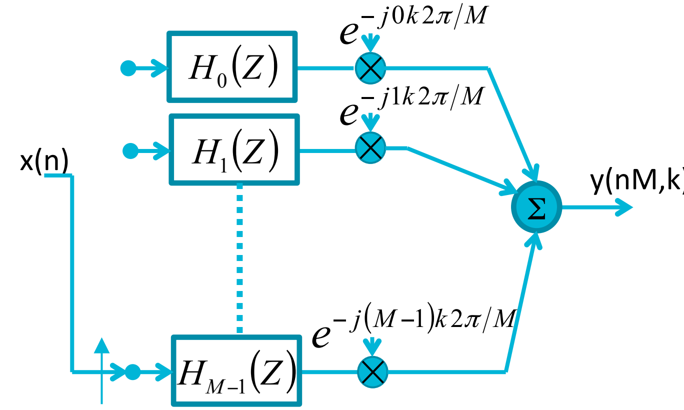
Standard single-channel down converter



- ***Equivalency Theorem***
  - Exchange mixer and low-pass filter with a band-pass filter and a mixer.
- ***Re-write the band-pass filter in "M-path form"***
- ***Noble Identity***
  - Move a down-sampler back through a digital filter

$$H\left(Z^M\right)\left(\downarrow M\right) = \left(\downarrow M\right)H\left(Z\right)$$

M-path Polyphase down converter



M-path Polyphase channeliser

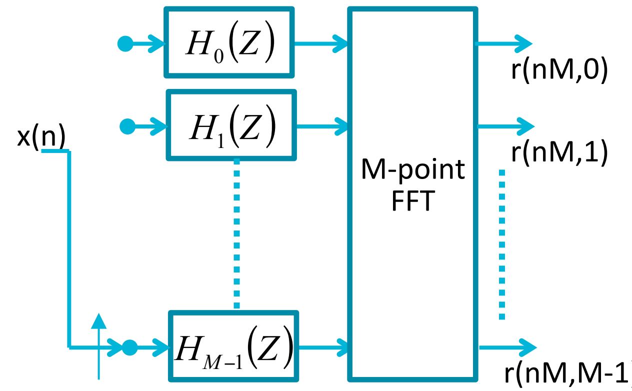
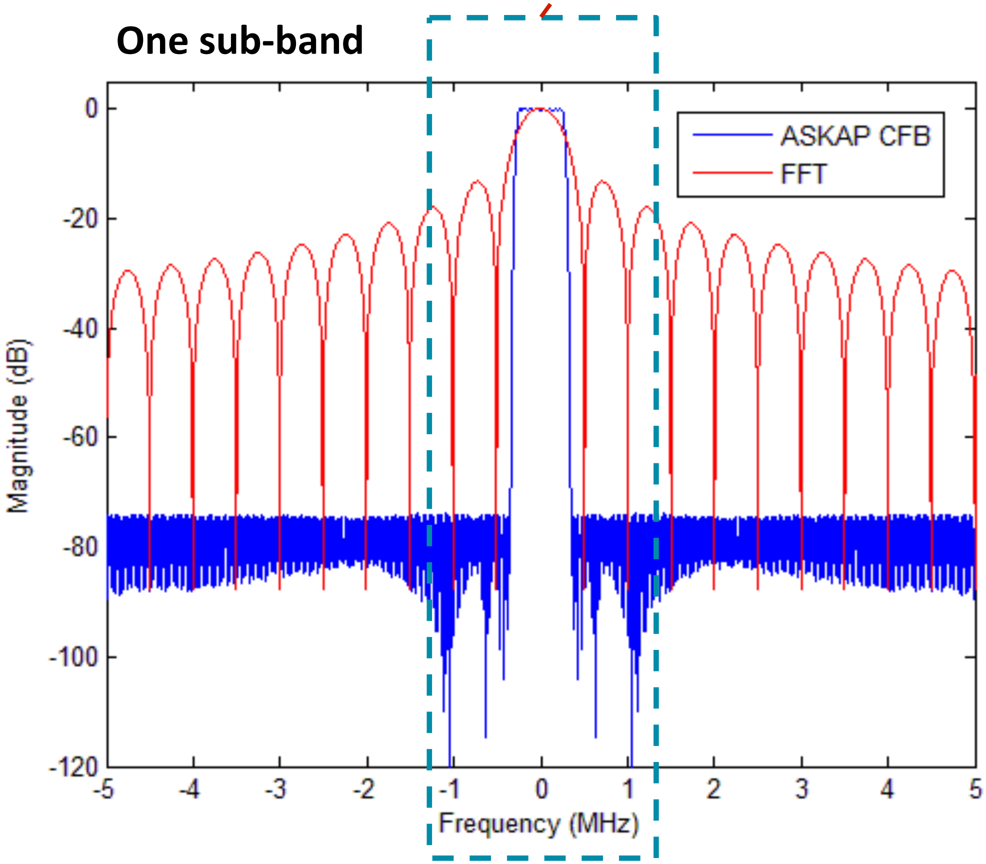


Image credit: John Tuthill

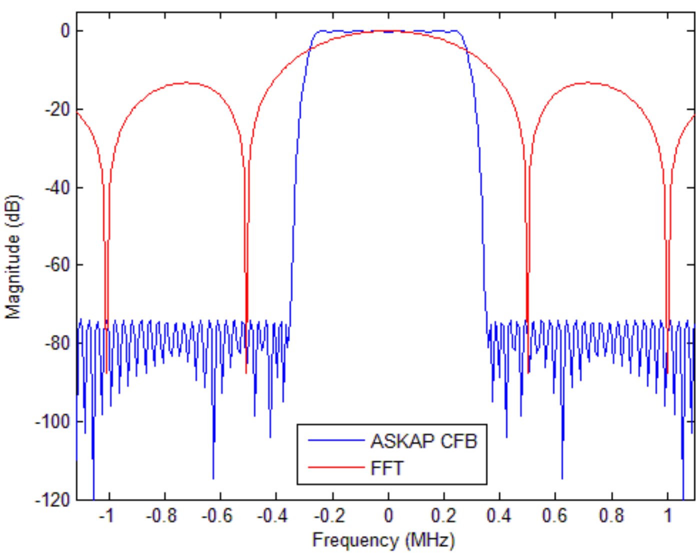
Filterbanks: FFT vs Polyphase Filters

One sub-band

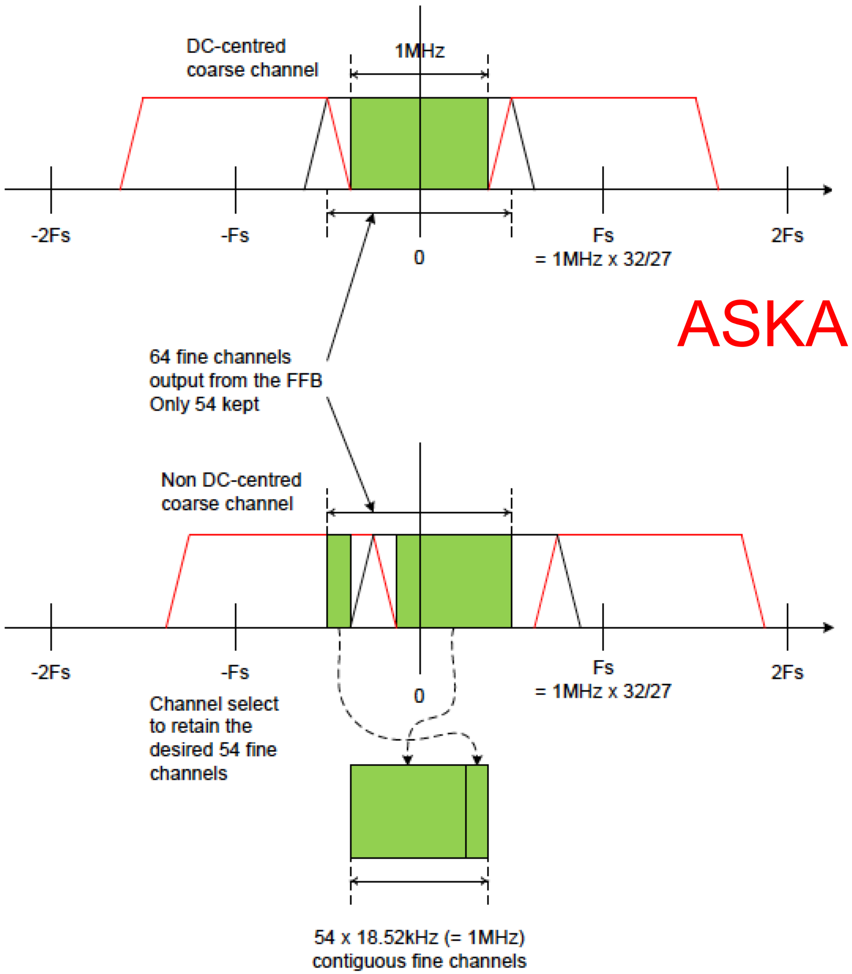


768-point FFT

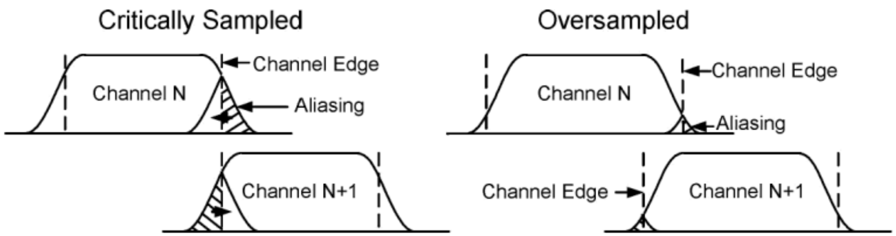
12,288-tap polyphase filter
+ 768-point FFT

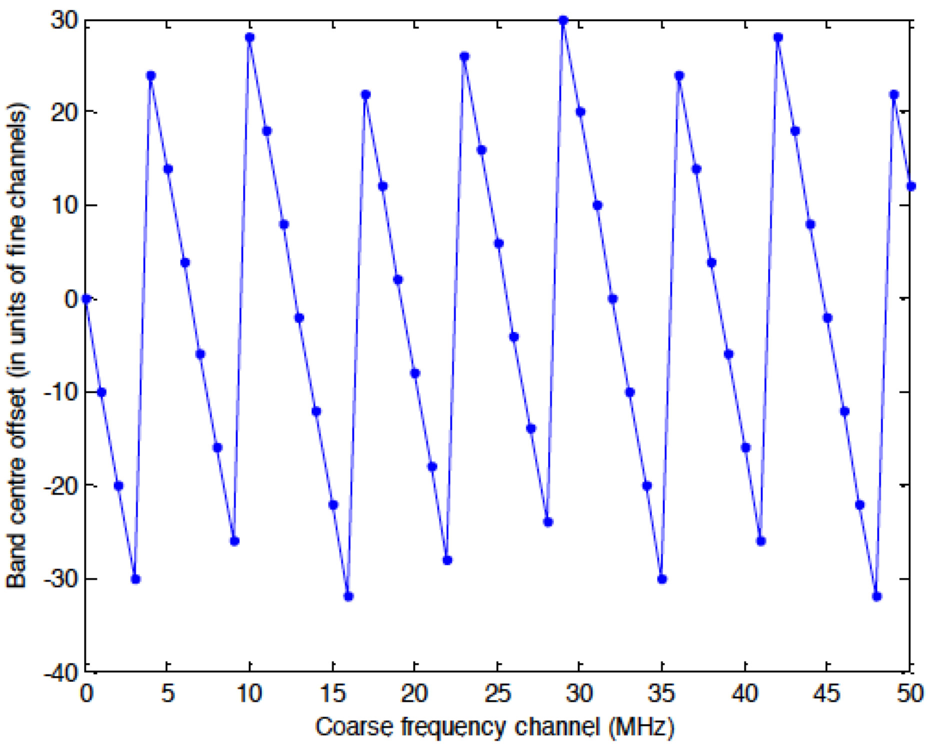


A few words on oversampled PFBs

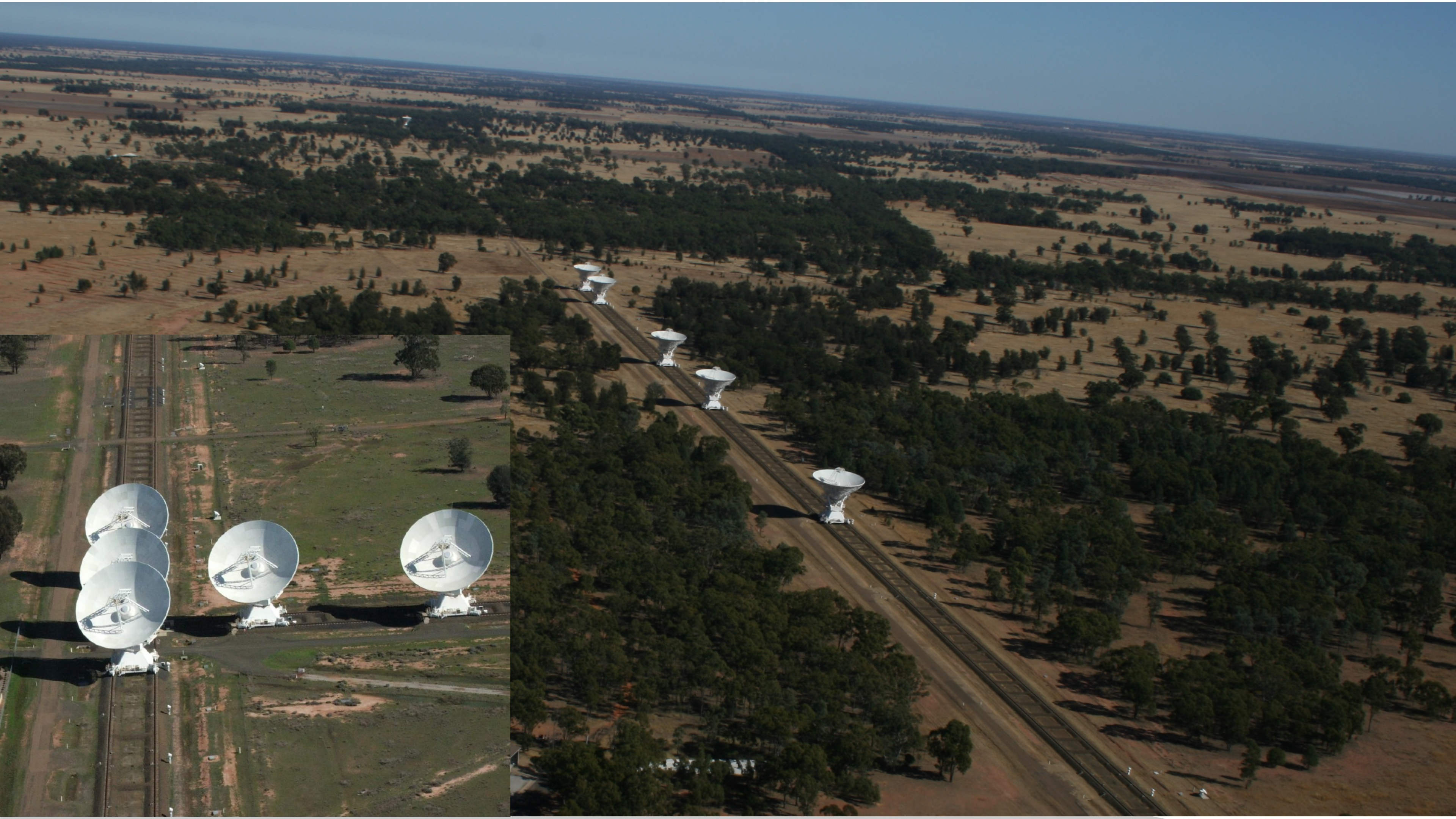


ASKAP





ATCA + CABB see Wilson et al. (2011, MNRAS, 416, 832)



Compact Array Broadband Backend (CABB)

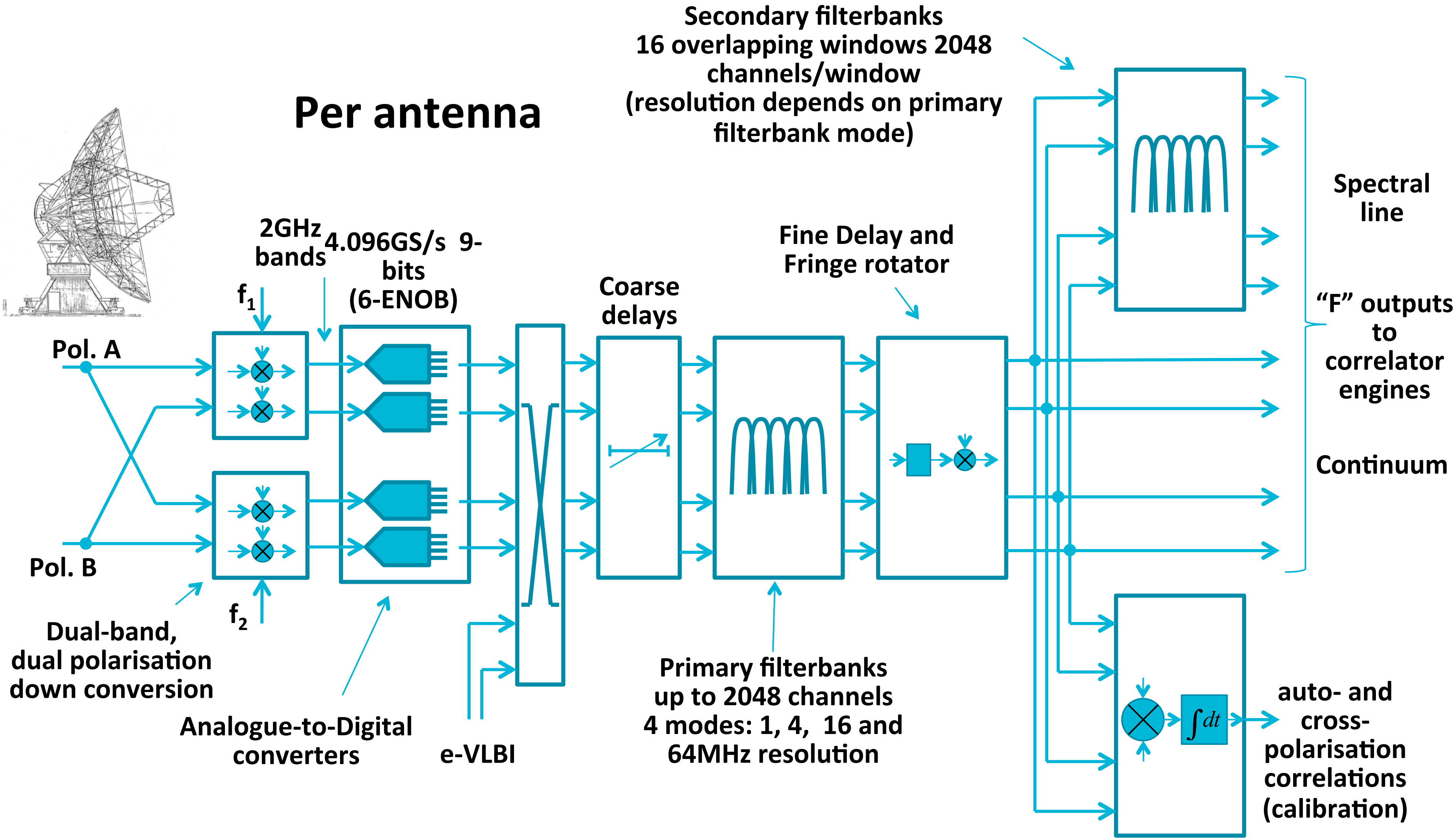
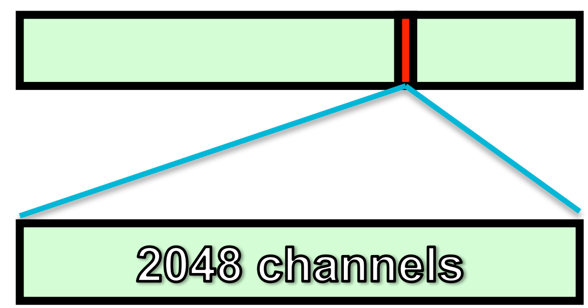


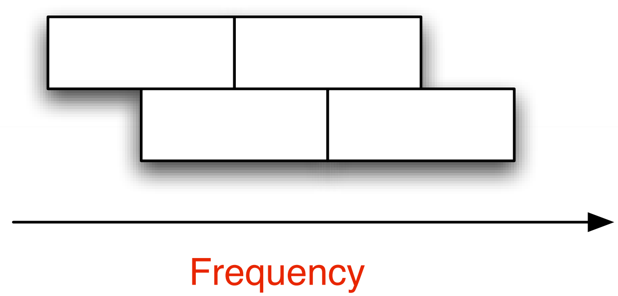
Image credit: John Tuthill

Zoom modes: high spectral resolution

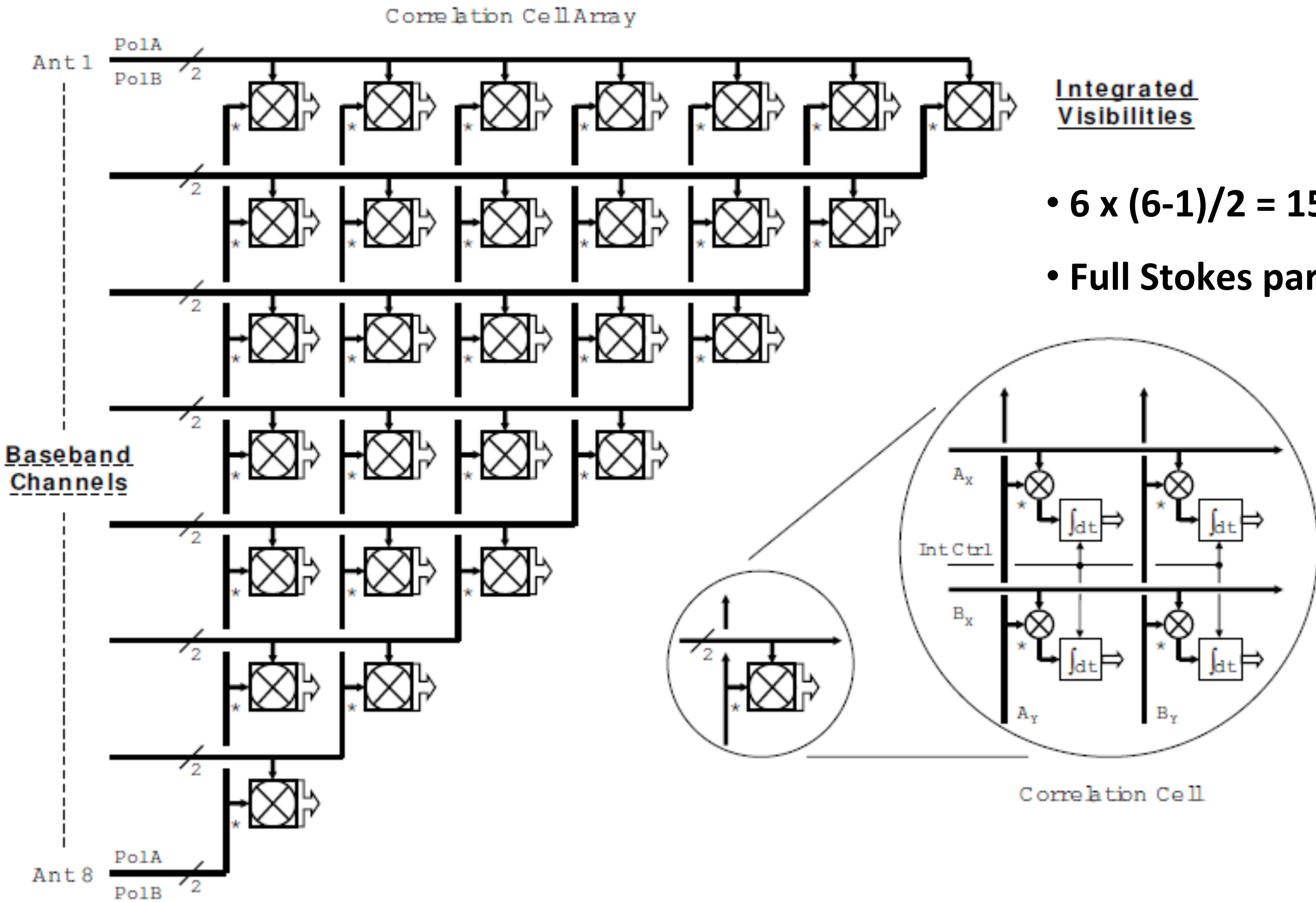
- Up to 16 “zoomed in” channels per each 2 GHz window (user selected)
- Positioned in steps equal to half of the wide-band spectral resolution (i.e. 0.5 MHz or 32 MHz)
- Each zoom window has 2048 spectral channels



Stitching zoom windows:



CABB Correlator – correlation engines



- 6 x (6-1)/2 = 15 baselines
- Full Stokes parameters

Image credit: John Tuthill

Australian Square Kilometre Array Pathfinder

- Has beamformer before the correlator (needs channelisation)

ASKAP (BETA) digital back-end

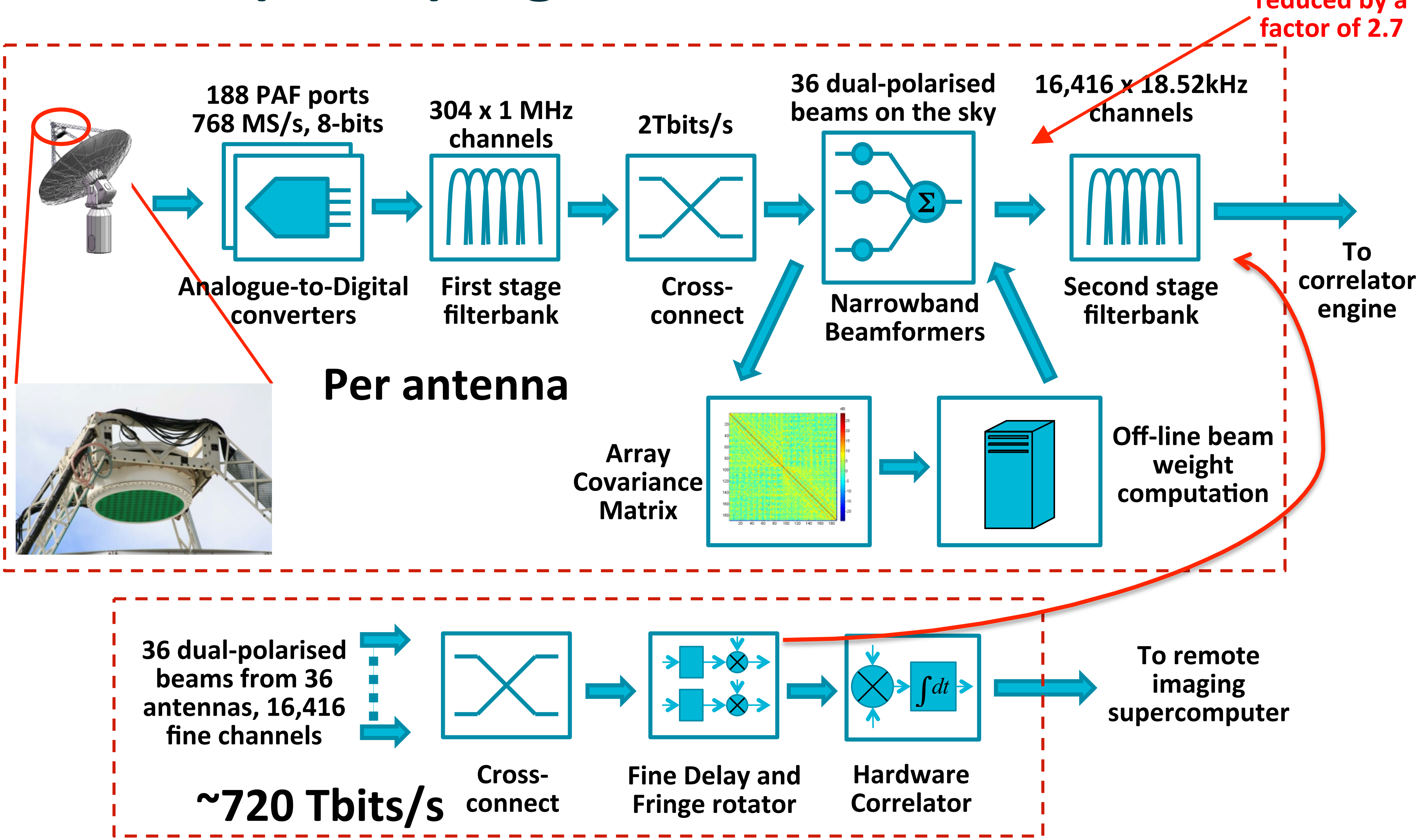


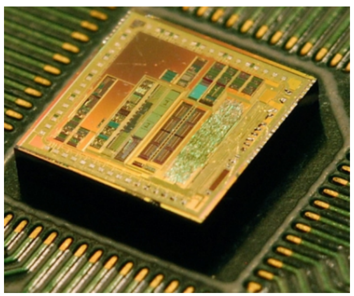
Image credit: John Tuthill

Calculation Engines: so many choices…

Hard-wired logic	Stored (programmed) logic

ASIC's

Application-**S**pecific **I**ntegrated **C**ircuit



•EVLA
•ALMA

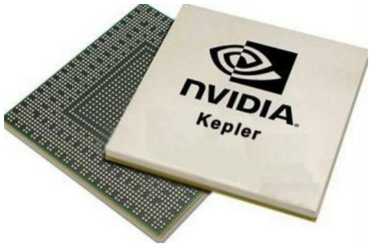
FPGA's

Field **P**rogrammable **G**ate **A**rray

•CABB
•ASKAP

GPU's

Graphics **P**rocessing **U**nit



•MWA
•MeerKAT

CPU's/DSP's

Central **P**rocessing **U**nit/ **D**igital **S**ignal **P**rocessor

•DiFX

•Less flexible
•Lower power/computation
•Higher initial development

↔

•More flexible
•Higher power/computation
•Lower initial development

Image credit: John Tuthill

Further Reading…

- Radio Astronomy:
  - H. C. Ko, "Coherence Theory in Radio-Astronomical Measurements," *IEEE Trans. Antennas & Propagation,* pp. 10-20, Vol. AP-15, No. 1, Jan. 1967.
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  - D. R. DeBoer, et.al, "Australian SKA Pathfinder: A High-Dynamic Range Wide-Field of View Survey Telescope," *Proc. IEEE*, 2009.
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Thank you

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