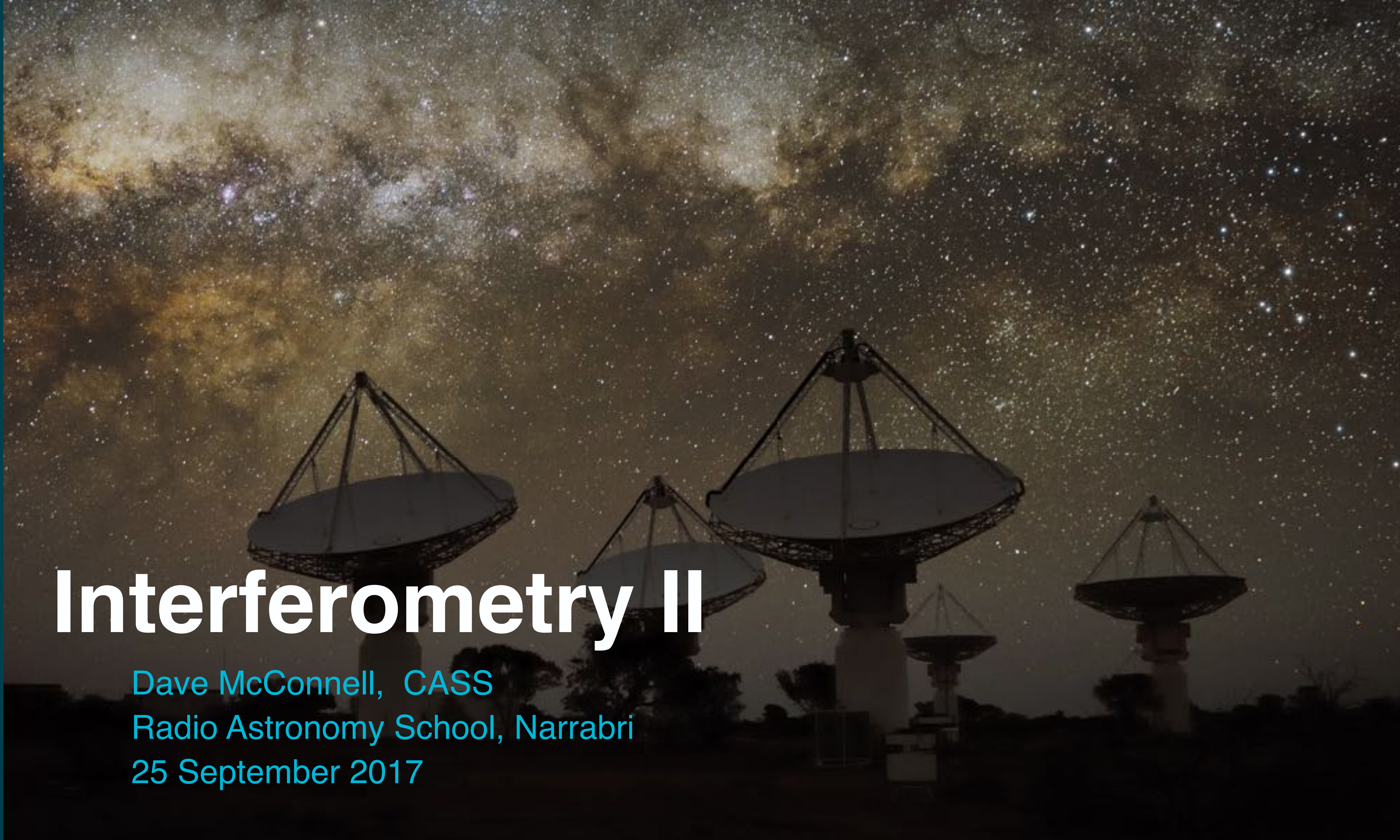


Interferometry II

Dave McConnell, CASS
Radio Astronomy School, Narrabri
25 September 2017

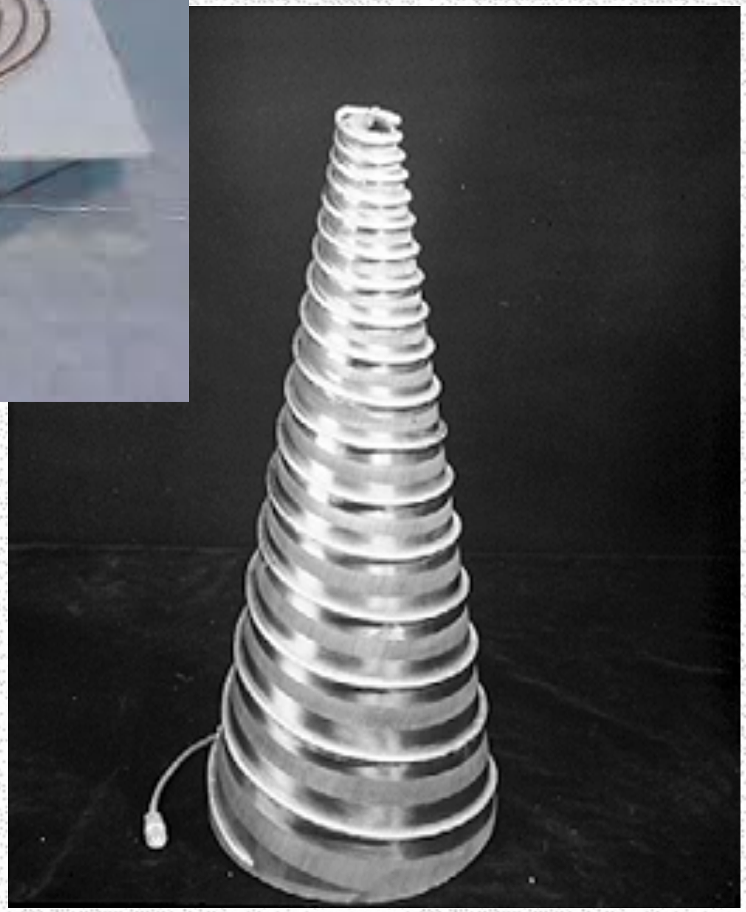
Outline of lecture

- Relationship between source structure and signal coherence
- How to measure the coherence (correlation)
- The complex visibility

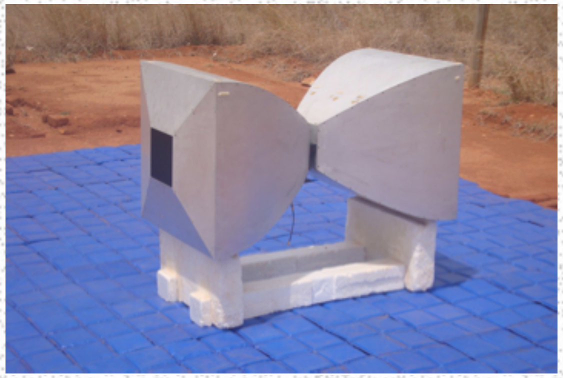
- Aperture synthesis
- The measurement equation

Ravi Subrahmanyan
Raman Research Institute
India

Radio antennas come in different shapes and sizes



A radio antenna is a ‘sensor’ of the EM field at its location in space

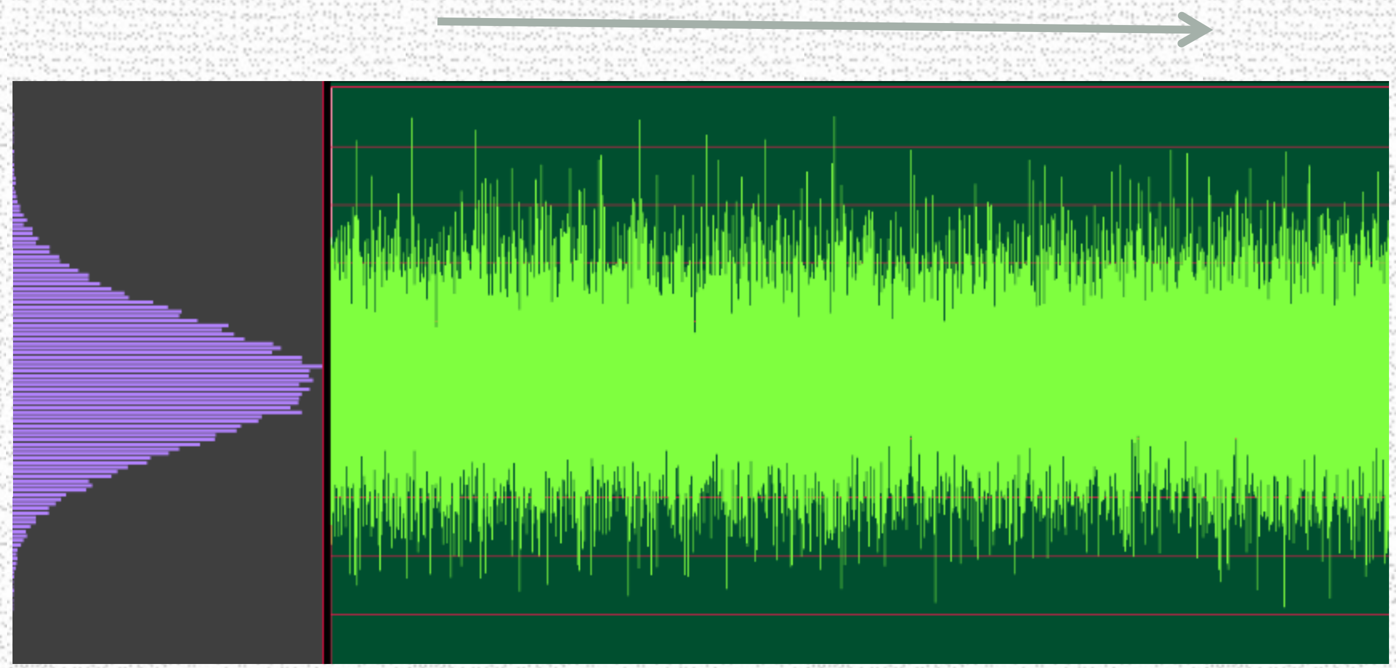


- The electric field is created by electromagnetic radiation arriving at that location in space from the sky and ground.
- The field is caused by a large number of independent atomic processes in the sky and ground
- The electric field is not a constant in time.
  It is time varying.
  It is not a sine wave.
  The field is a random variate.
- The field has a Normal (or Gaussian) probability distribution.
- Aside from RFI and ETI.

A radio antenna is a ‘sensor’ of the EM field at its location in space

- A radio antenna has metal conductors in which the fluctuating electric field causes time-varying currents.
- A radio antenna converts the fluctuating electric field into a fluctuating voltage on a transmission line = cable.

Histogram of the voltage from the antenna



Voltage from antenna vs Time

Radio antennas may be arrayed and the voltages summed to yield the summed field over an aperture.

These are called aperture arrays.

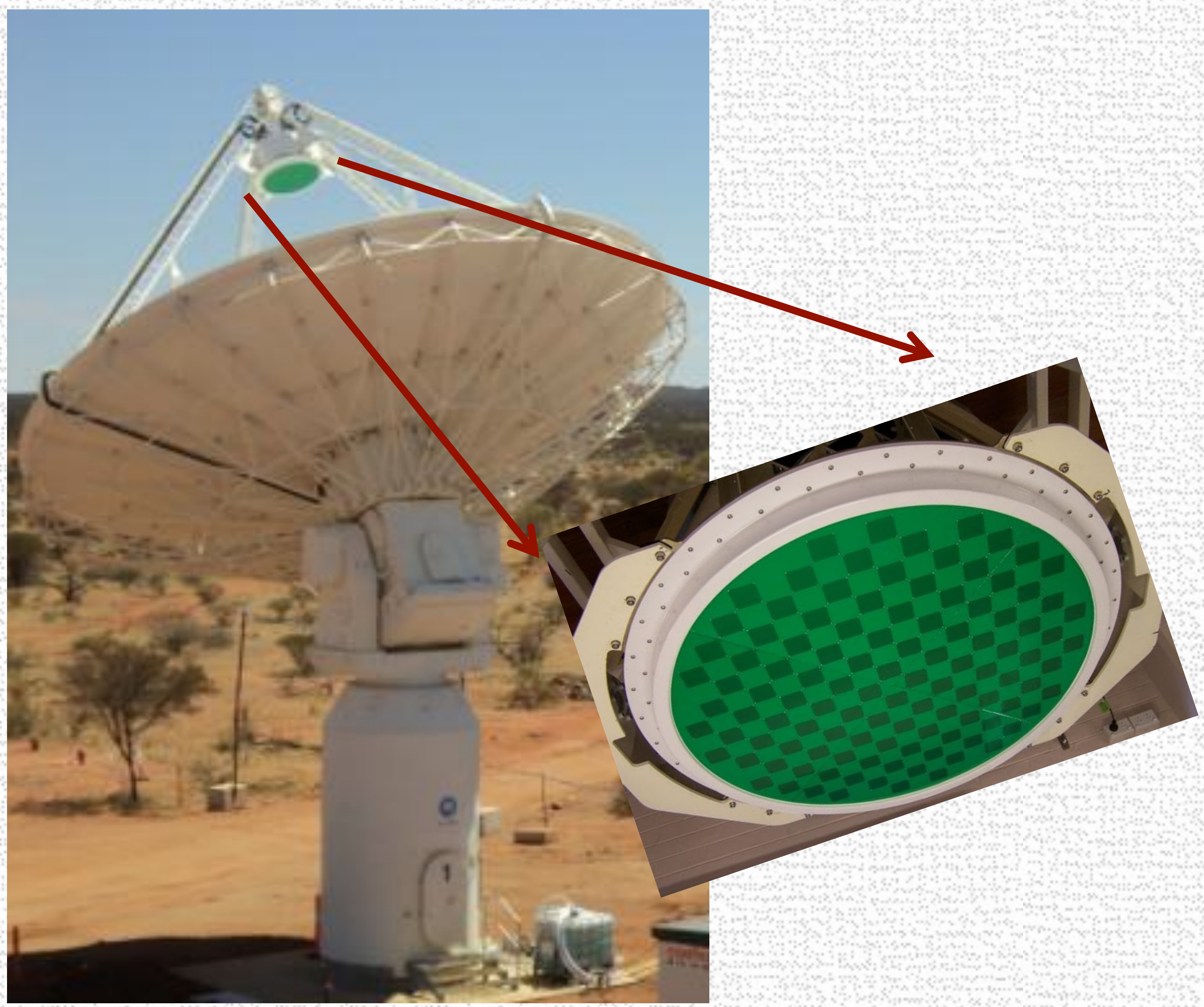
MWA tile

LOFAR low and high band stations

Apertures arrays may be phased to yield the EM field on the ground arising from selectively different patches of the sky.

Parabolic reflectors sum the field over the aperture of the dish.

Larger the aperture and higher the frequency – better will be the cancellation of fields owing to off-axis sources on the sky.



A 2D set of EM field sensors at the focal plane of a parabolic dish

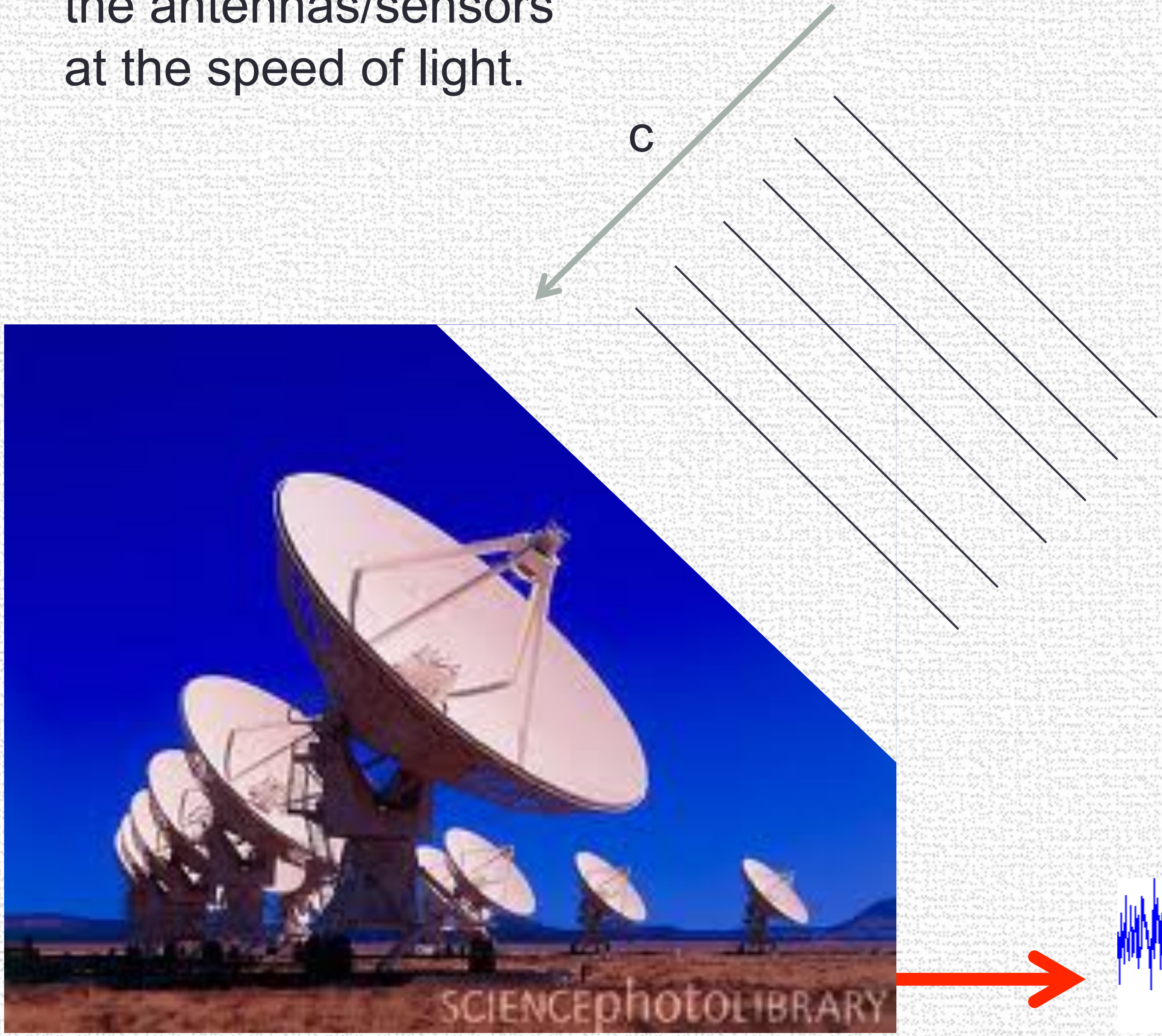
Focal Plane Array

measures the summed EM fields from sky regions that are offset in a 2D raster.

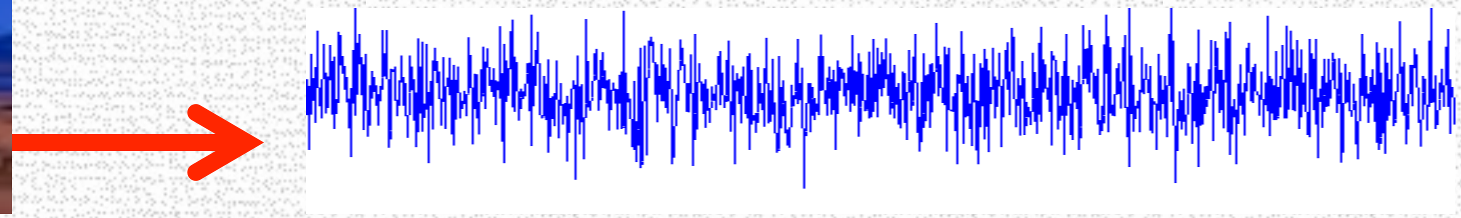
The radio sources in the sky result in a radiative electromagnetic field on the ground

Radio source on the sky

That rush past
the antennas/sensors
at the speed of light.



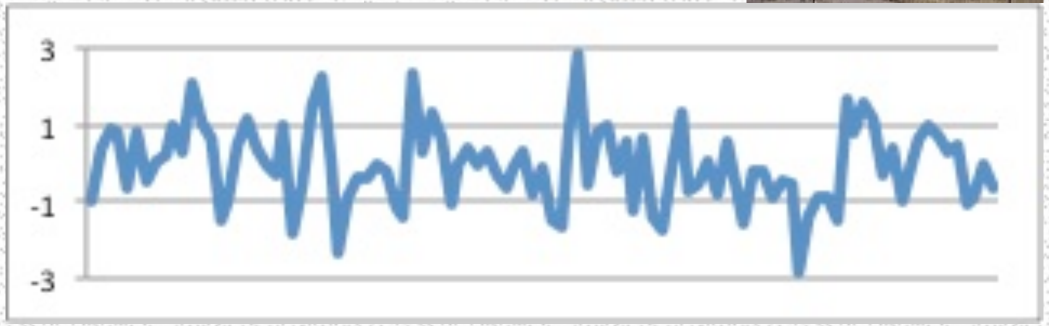
For a radio source that has a flat spectrum, and for a wideband antenna, the EM field produces a Gaussian random voltage signal at the terminals of antennas.



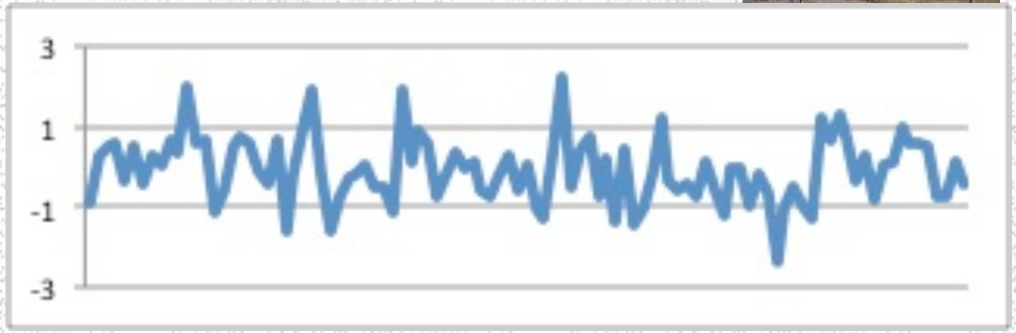
Principle of interferometers

Extended radio source of angular size θ radians

Spacing $< \lambda/\theta$

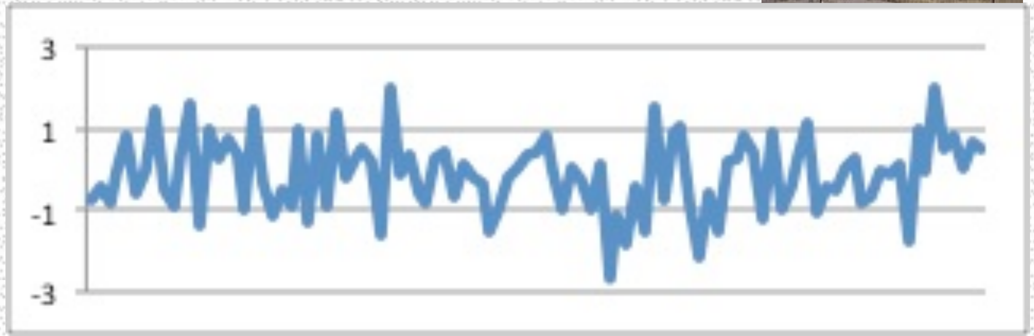


Voltage

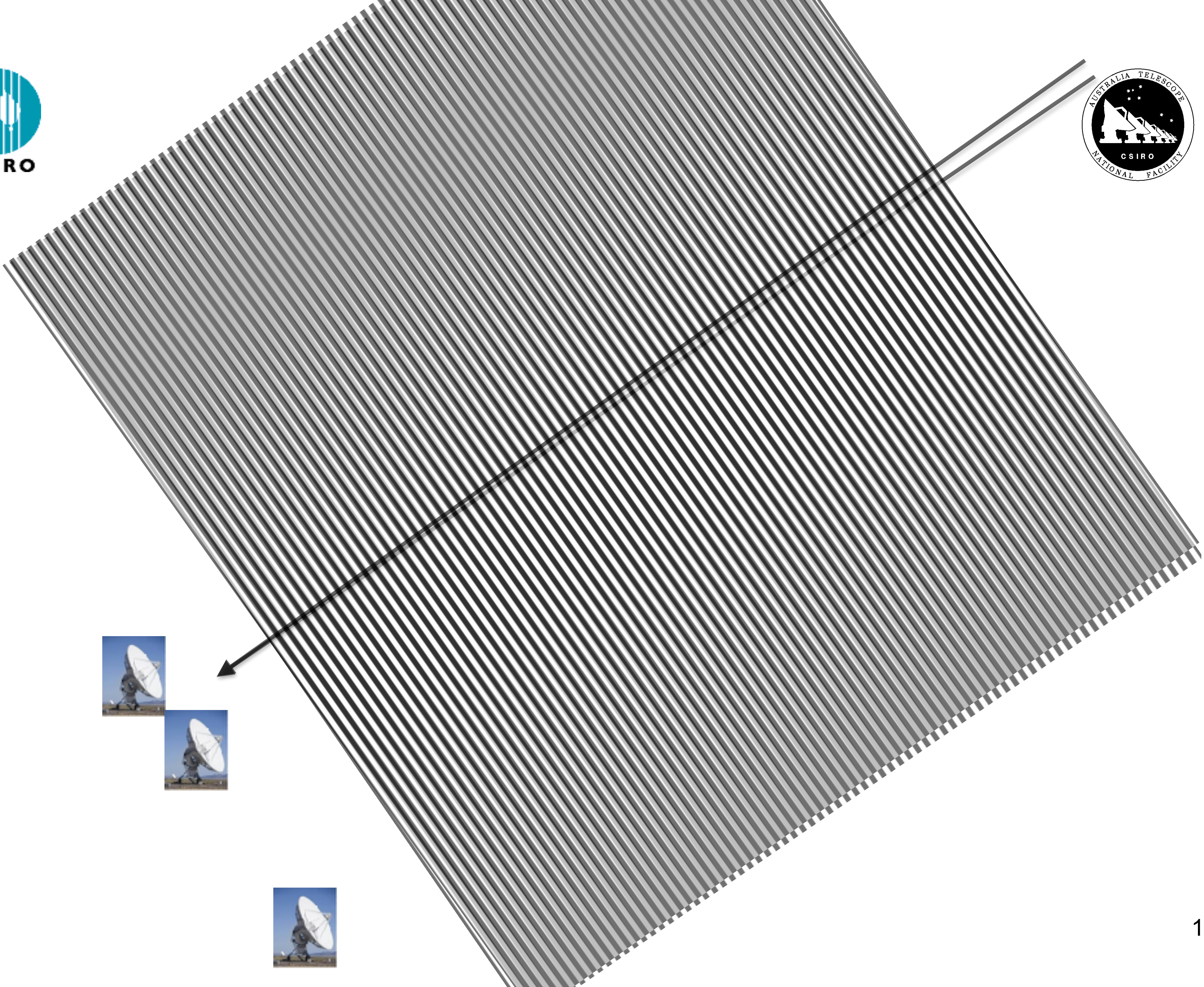


Time

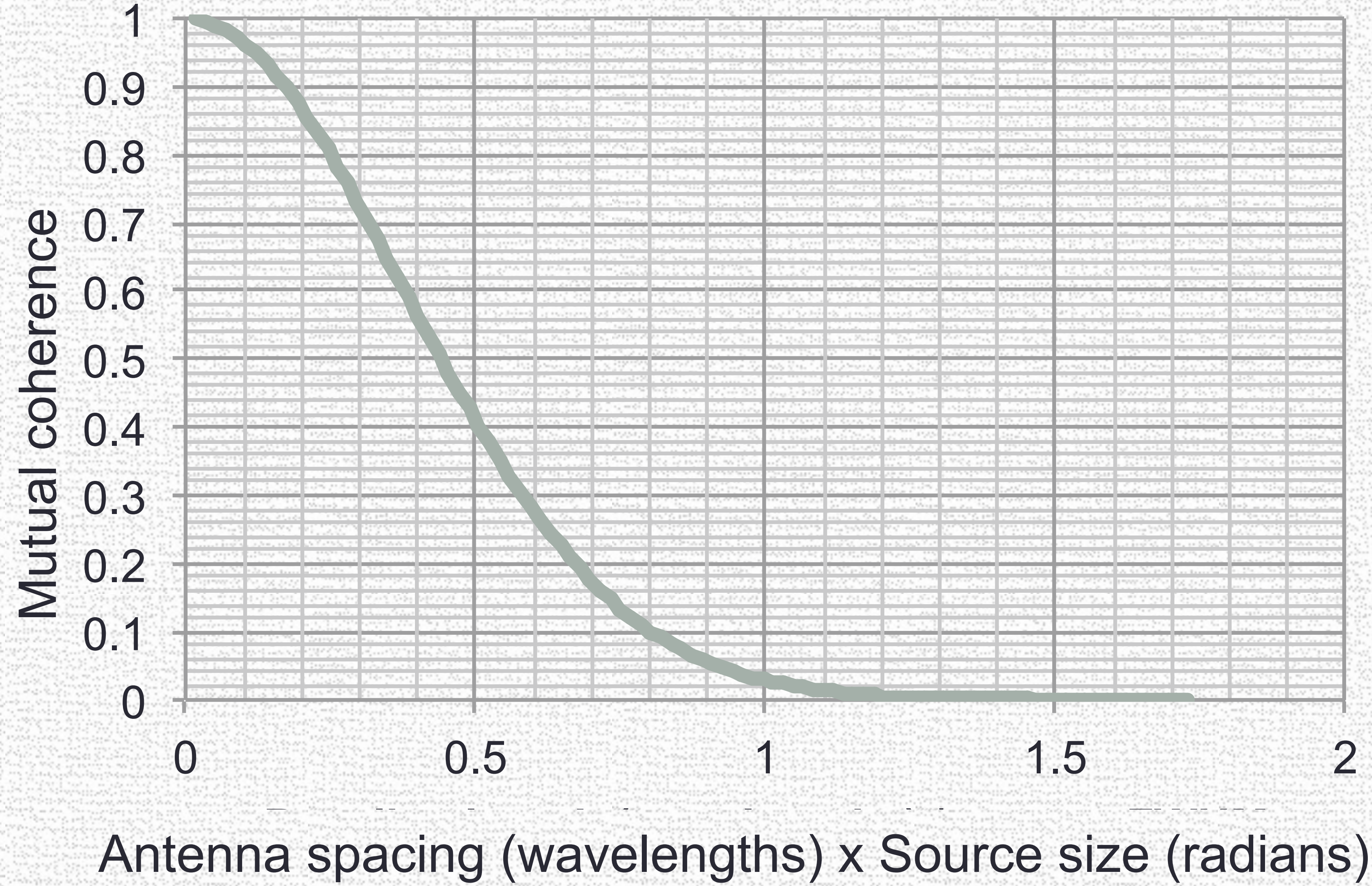
Spacing $> \lambda/\theta$



Mutual coherence of the EM fields

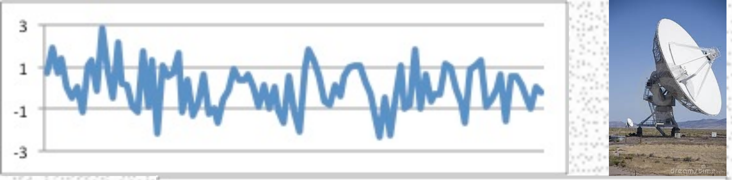


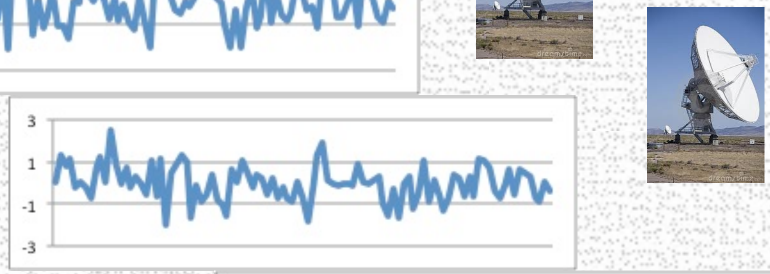
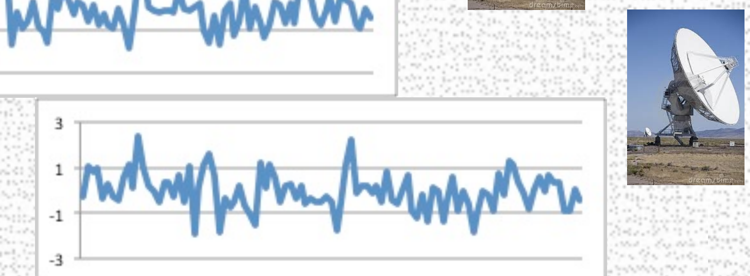
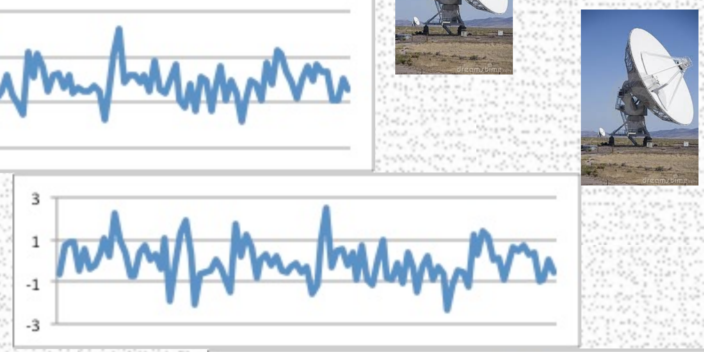
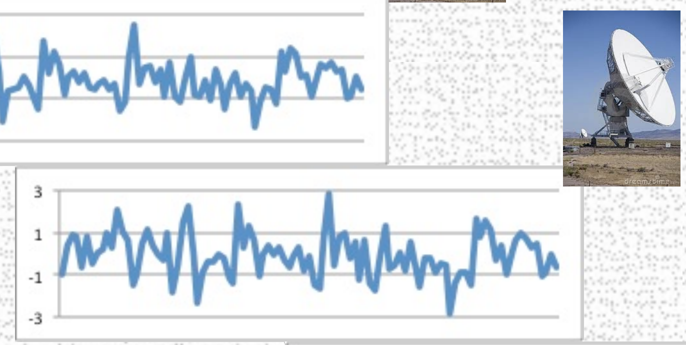
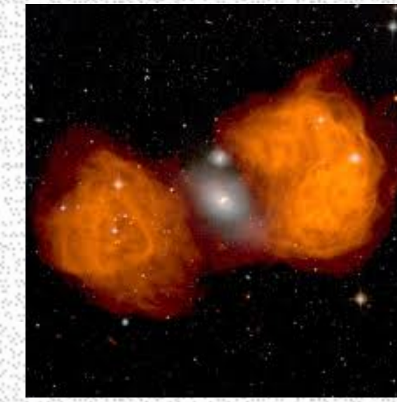
Mutual coherence between voltages from antenna elements drops off with separation



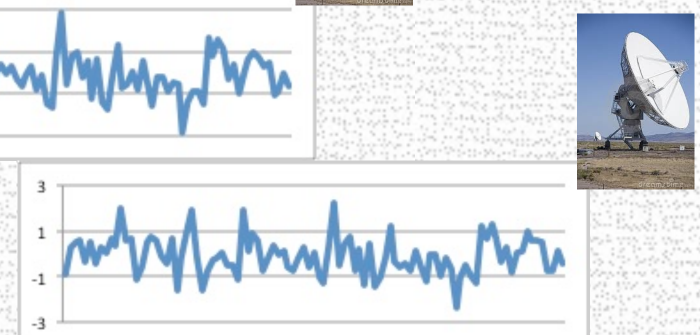
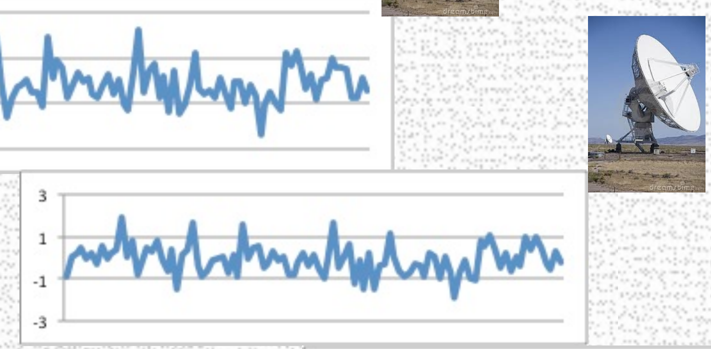
Mutual coherence drops off as baseline length exceeds inverse of source size

Mutual coherence drops off as Source size exceeds inverse of antenna spacing

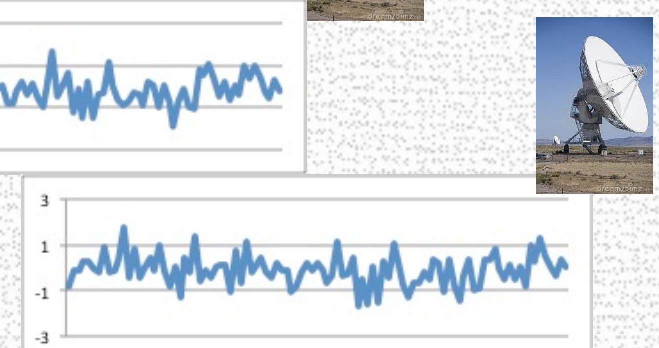
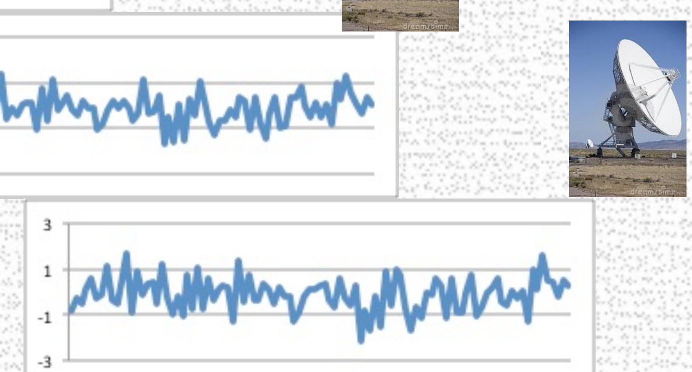


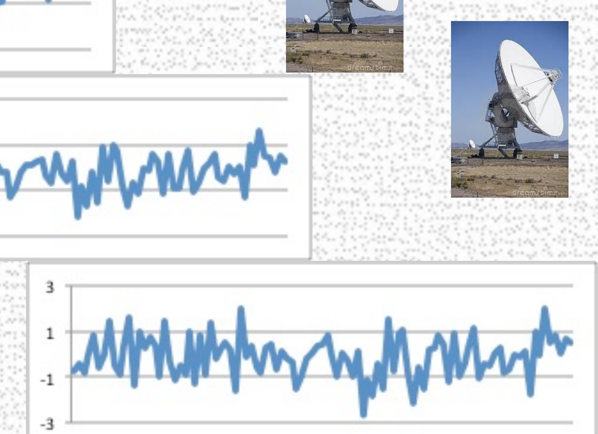
Voltage waveforms
are similar for antenna
pairs that are spaced less than λ/θ

Voltage waveforms are
progressively dissimilar for antenna
separations exceeding λ/θ

Mutual coherence of the EM field at the antenna
locations depends on source structure
[van Cittert-Zernike theorem]



Principle of synthesis imaging using radio interferometers

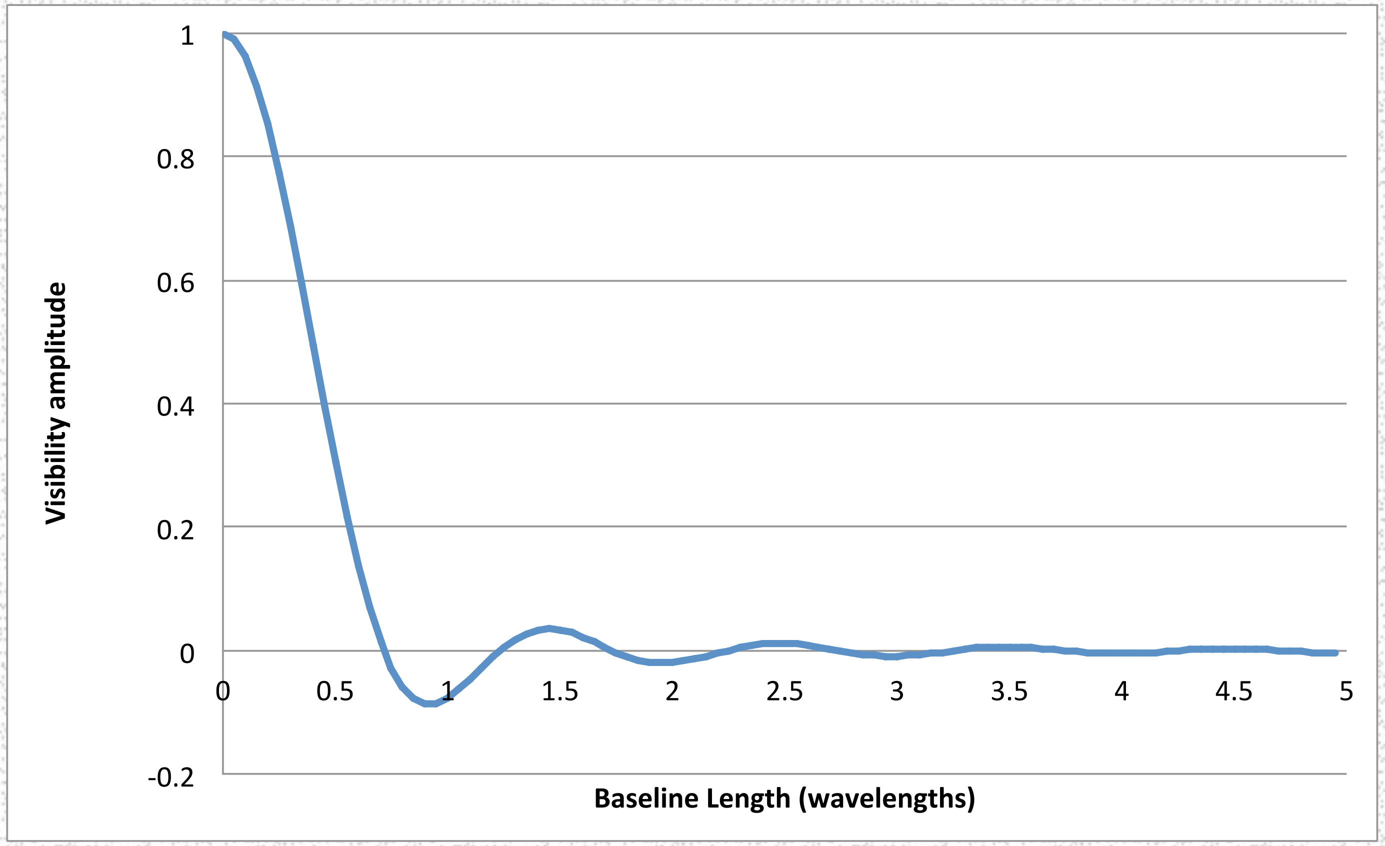
Structure in brightness distribution on the sky →
mutual coherence in the EM field on the ground

Mutual coherence between fields →
correlation properties of voltage waveforms sensed by antennas

Synthesis Imaging:

1. Measuring the cross-correlation between voltage waveforms sensed by pairs of antennas with a variety of separations = OBSERVING
2. To derive the mutual coherence properties of the EM field on the ground = CALIBRATION
3. Which may be inverted to solve for brightness distribution on the sky = IMAGING

Key advantage of using interferometers to measure the radio sky:



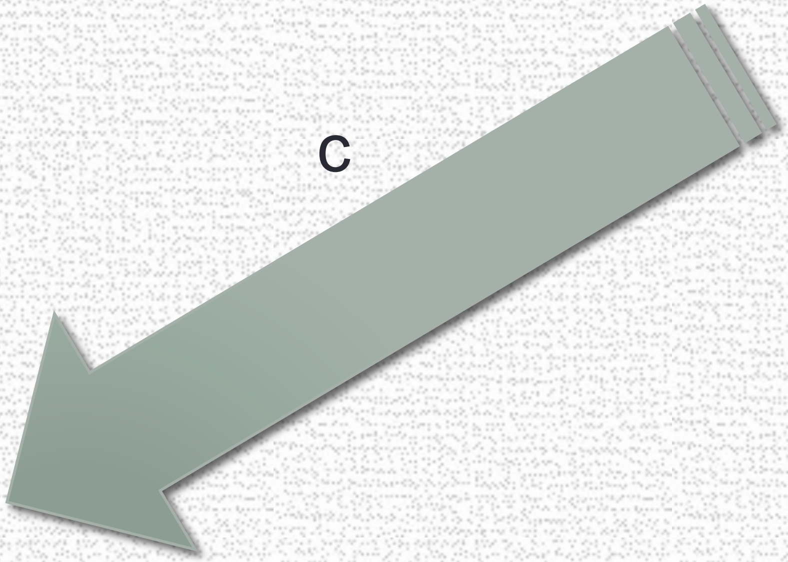
Interferometers see discrete sources and are blind to the uniform background

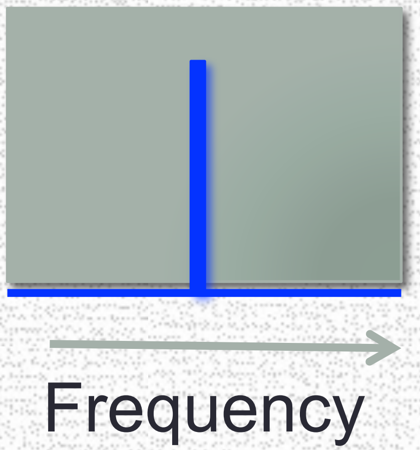
Interferometer response to mean sky brightness falls off on baselines exceeding a wavelength!

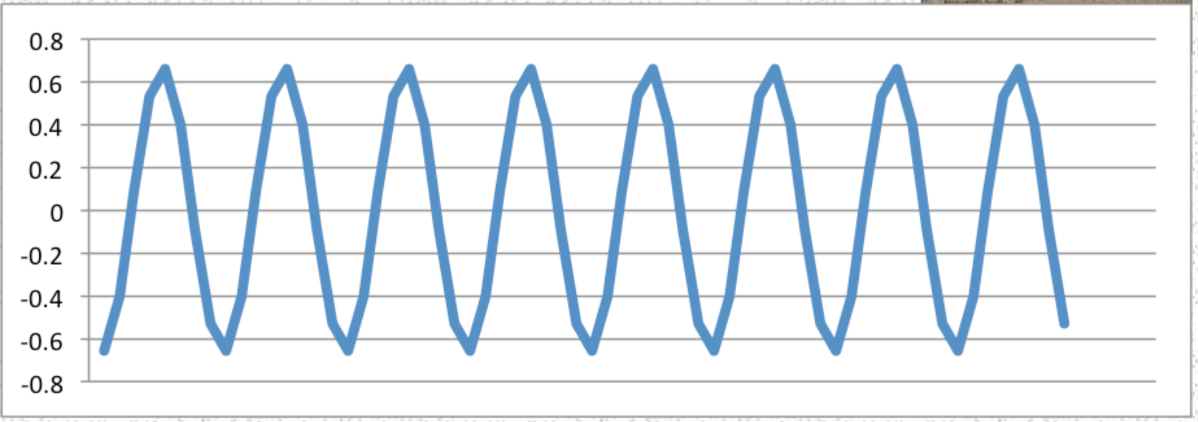
Interferometers are also blind to receiver noise, atmosphere radiation, ground radiation!

Time variation in the measured electric field

If the source in monochromatic
Emission is at a single frequency

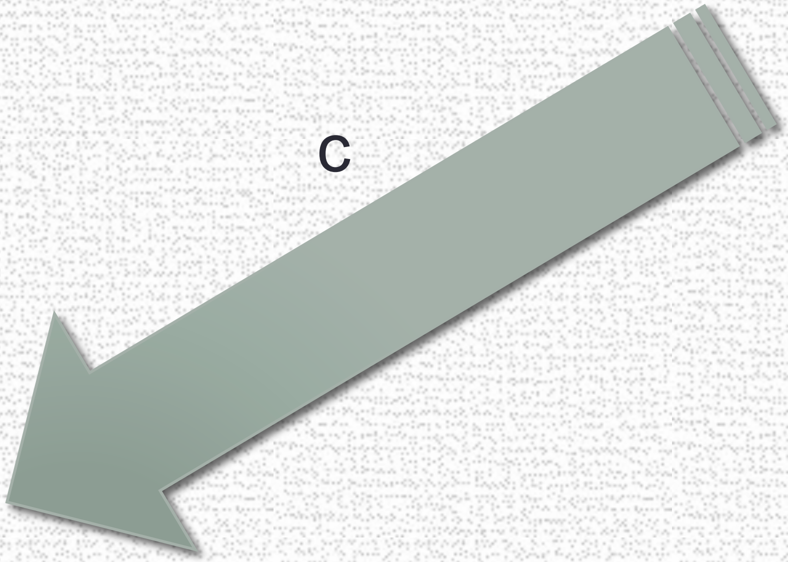


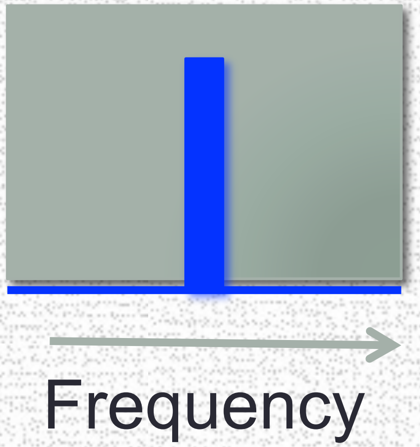


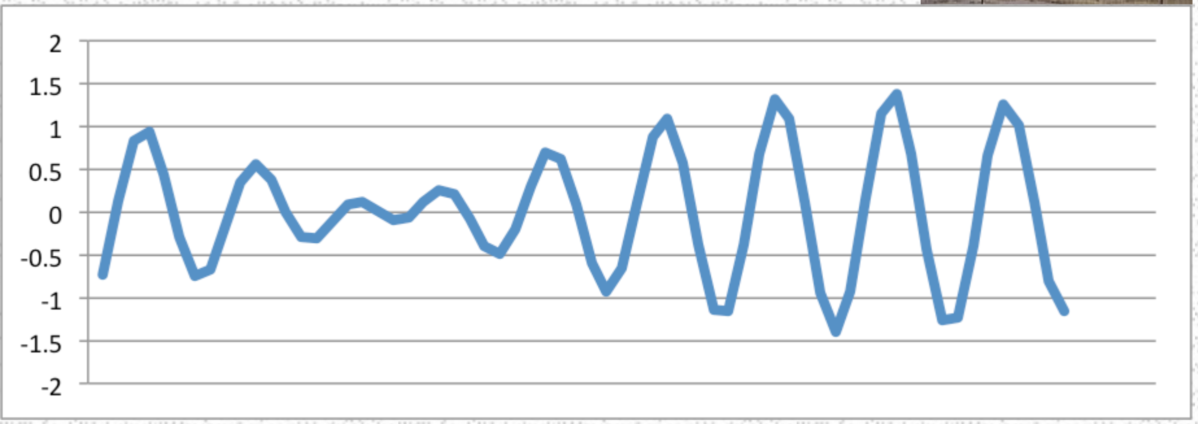


The measured electric field is a sine wave over time with constant amplitude.

If the source emission is over a narrow band

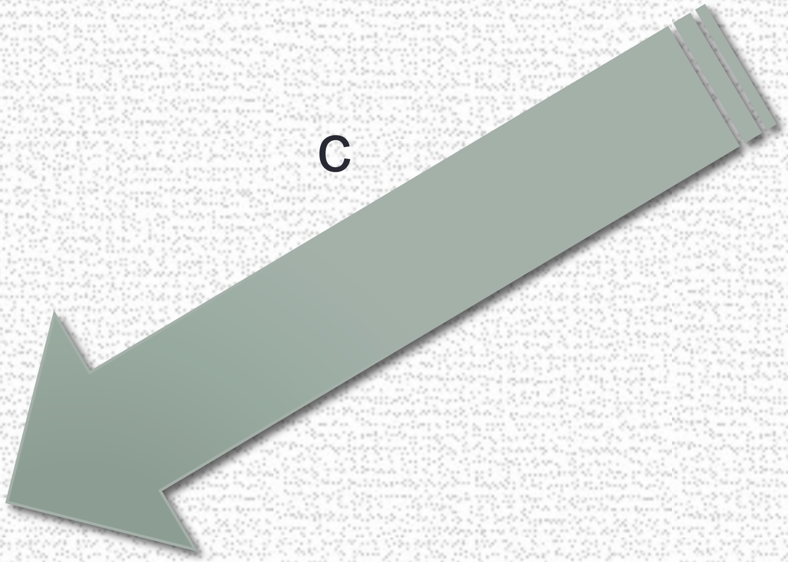


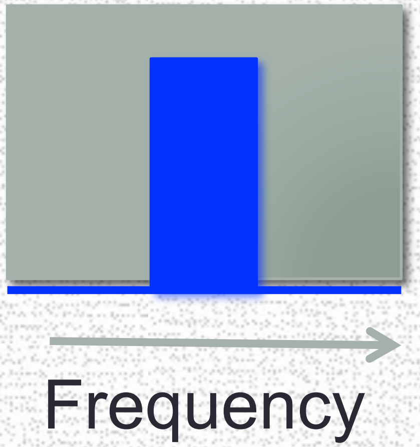


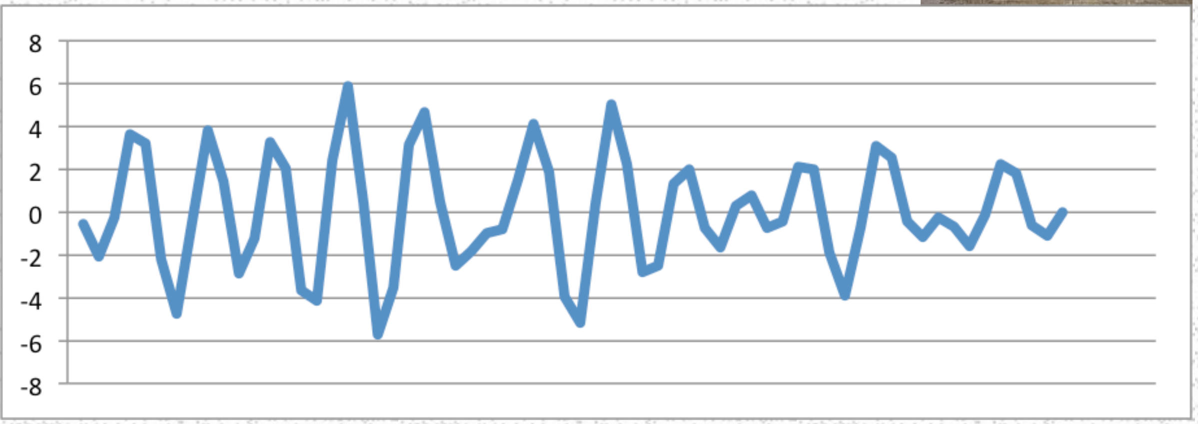


The measured electric field is a sine wave over time with slowly varying amplitude.

If the source emission is
over a wide band



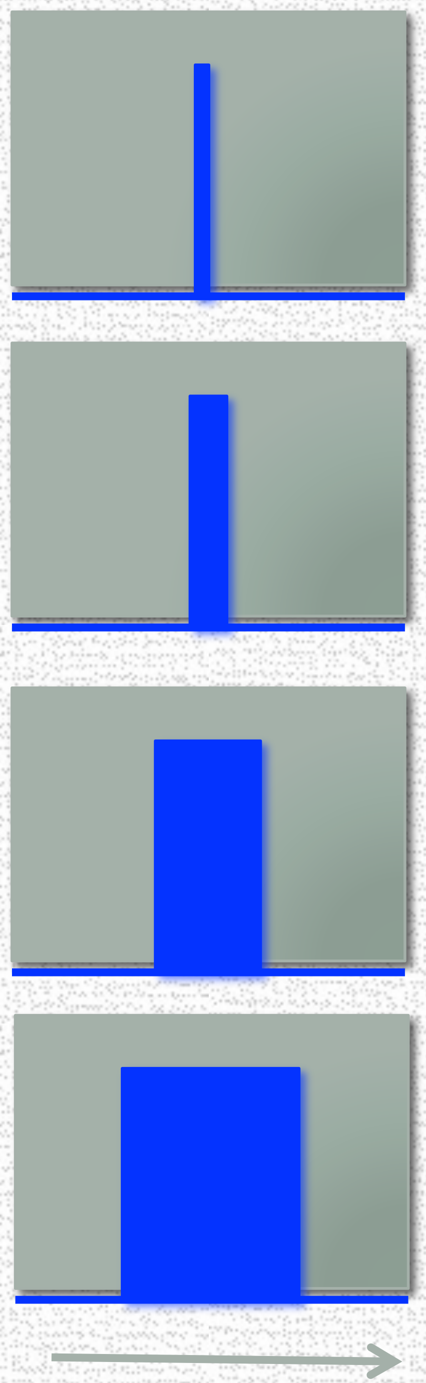




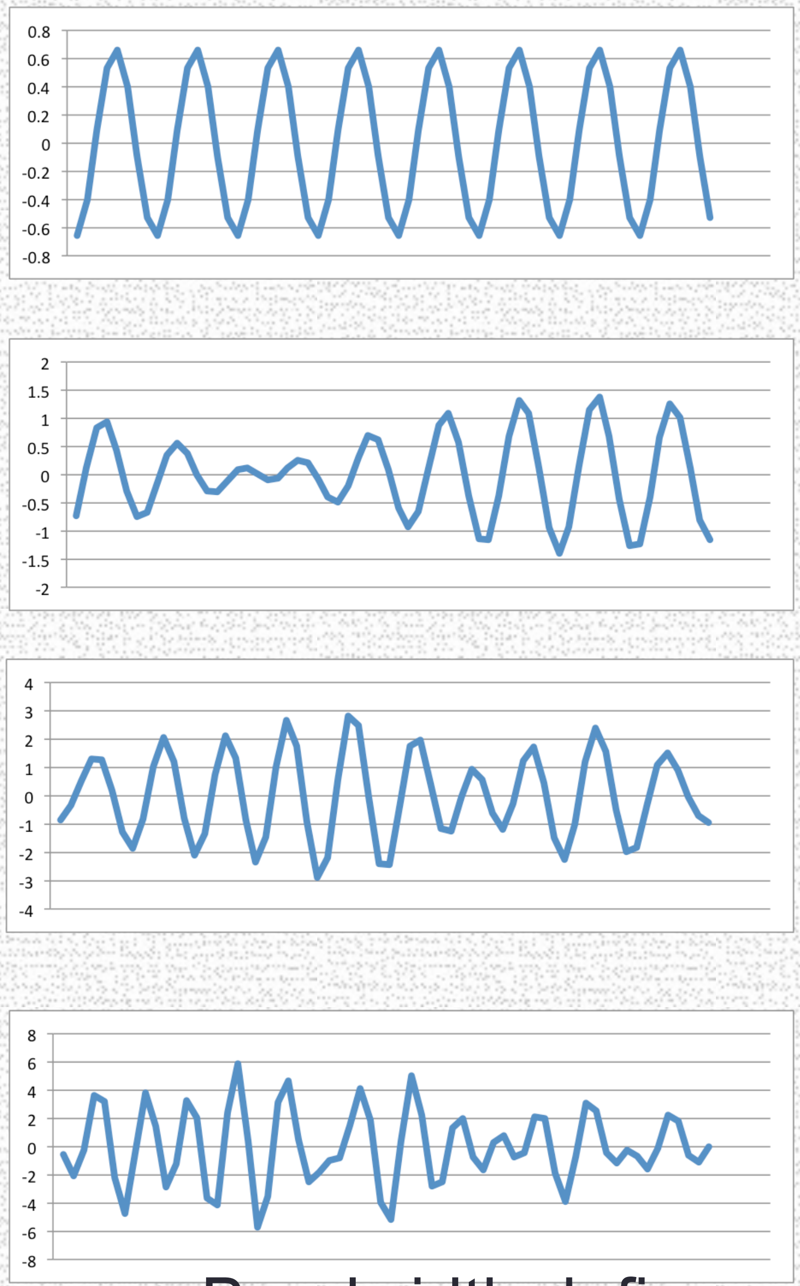
The measured electric field is
a sine wave over time with
rapidly varying amplitude.

Wider the band, more quickly does the amplitude of the received field vary with time

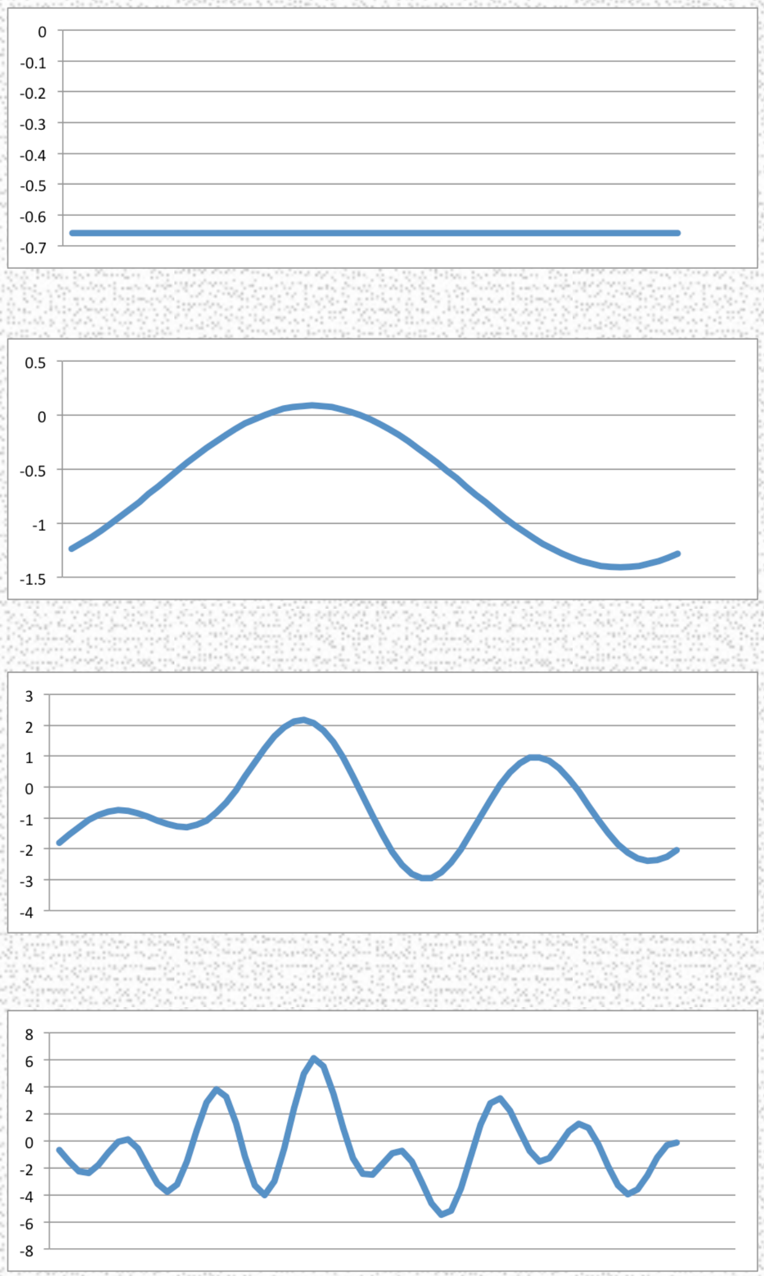
Emission Spectrum



Received waveform

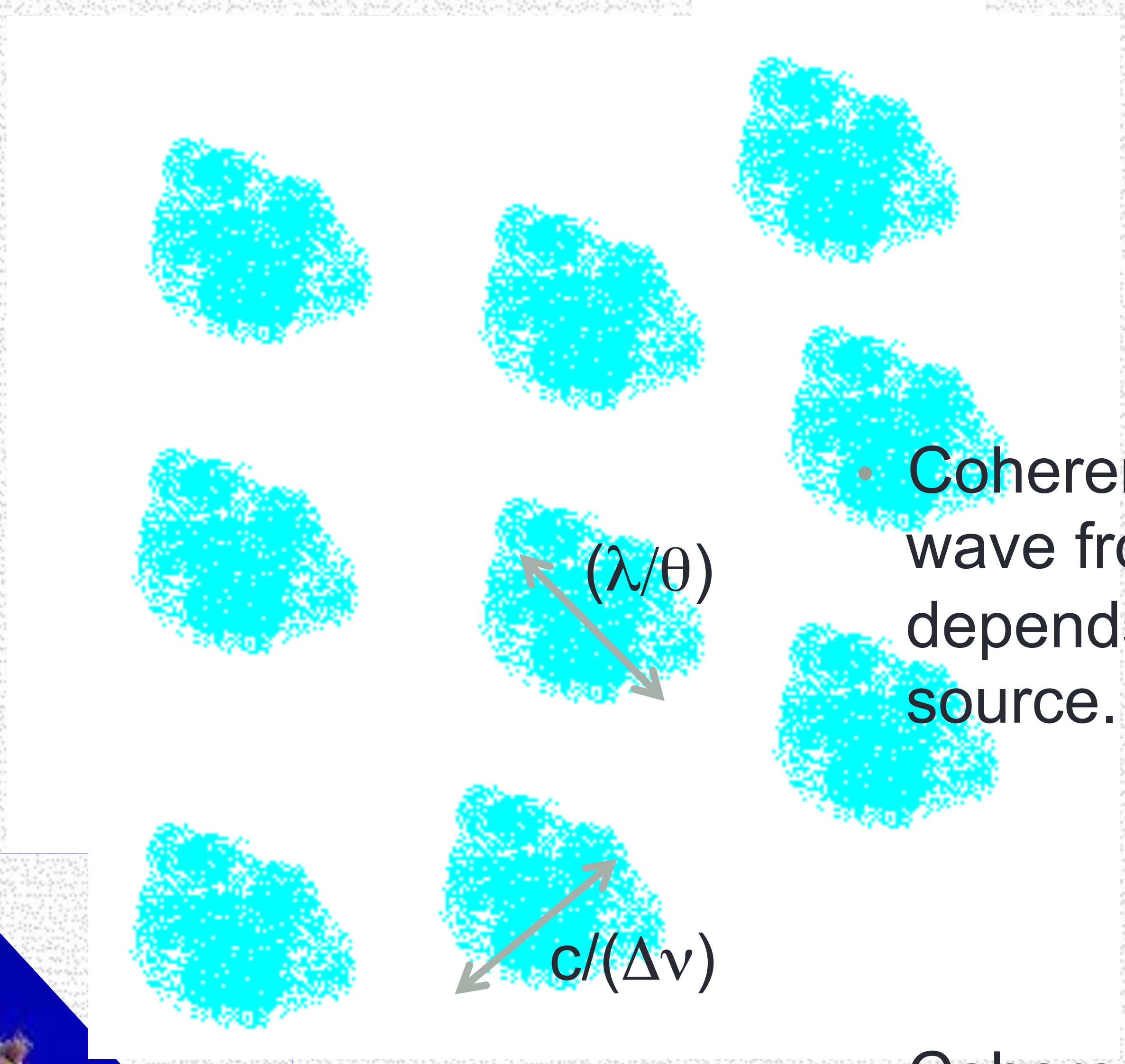


Received field amplitude



Bandwidth defines a coherence length $c\tau$ for the amplitude of the received field amplitude

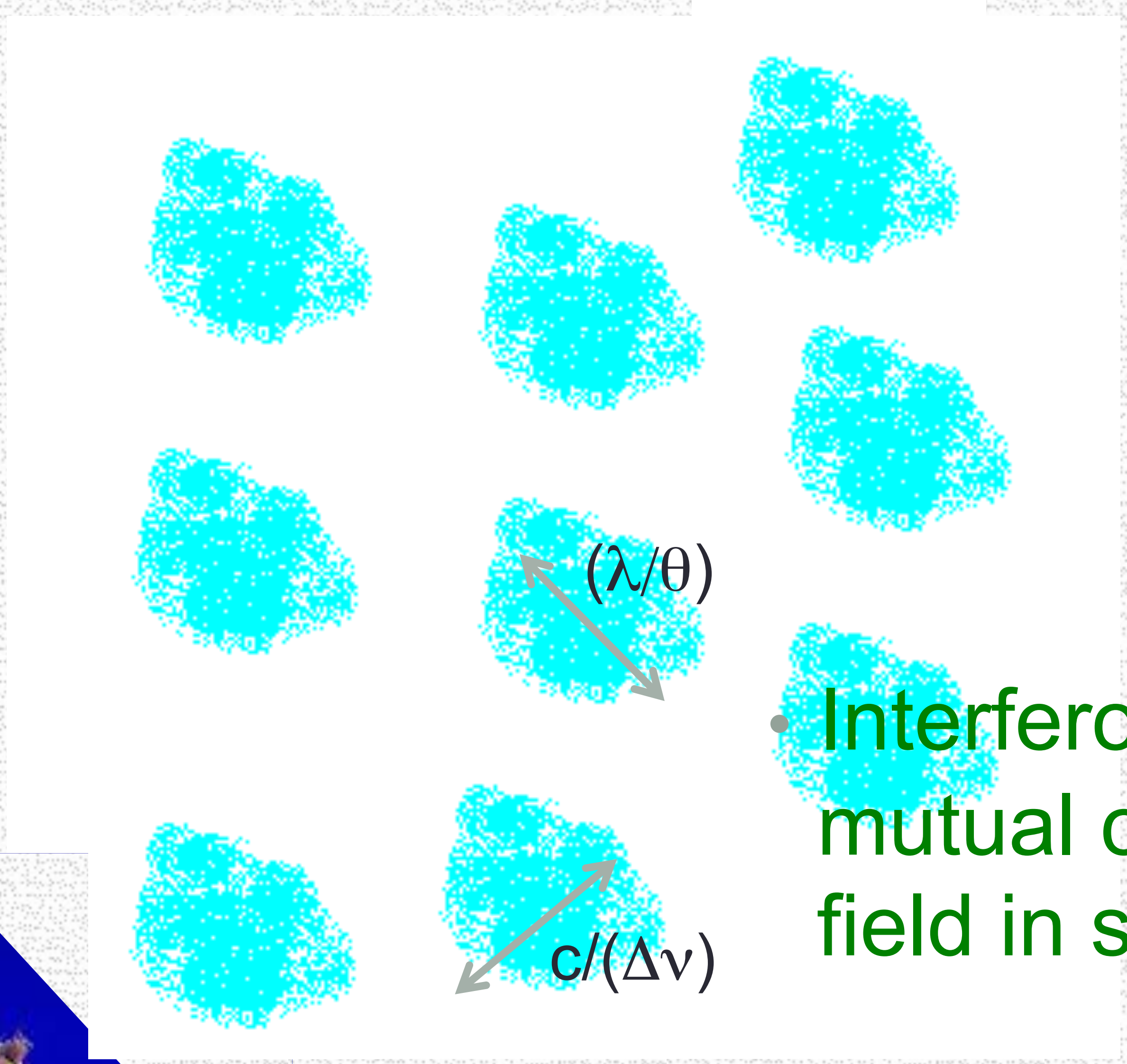
Depending on the source angular size and bandwidth: the EM field on the ground has a spatial-temporal coherence



- Coherence scale is (λ/θ) parallel to the wave front from the source, and depends on the angular size of the source.

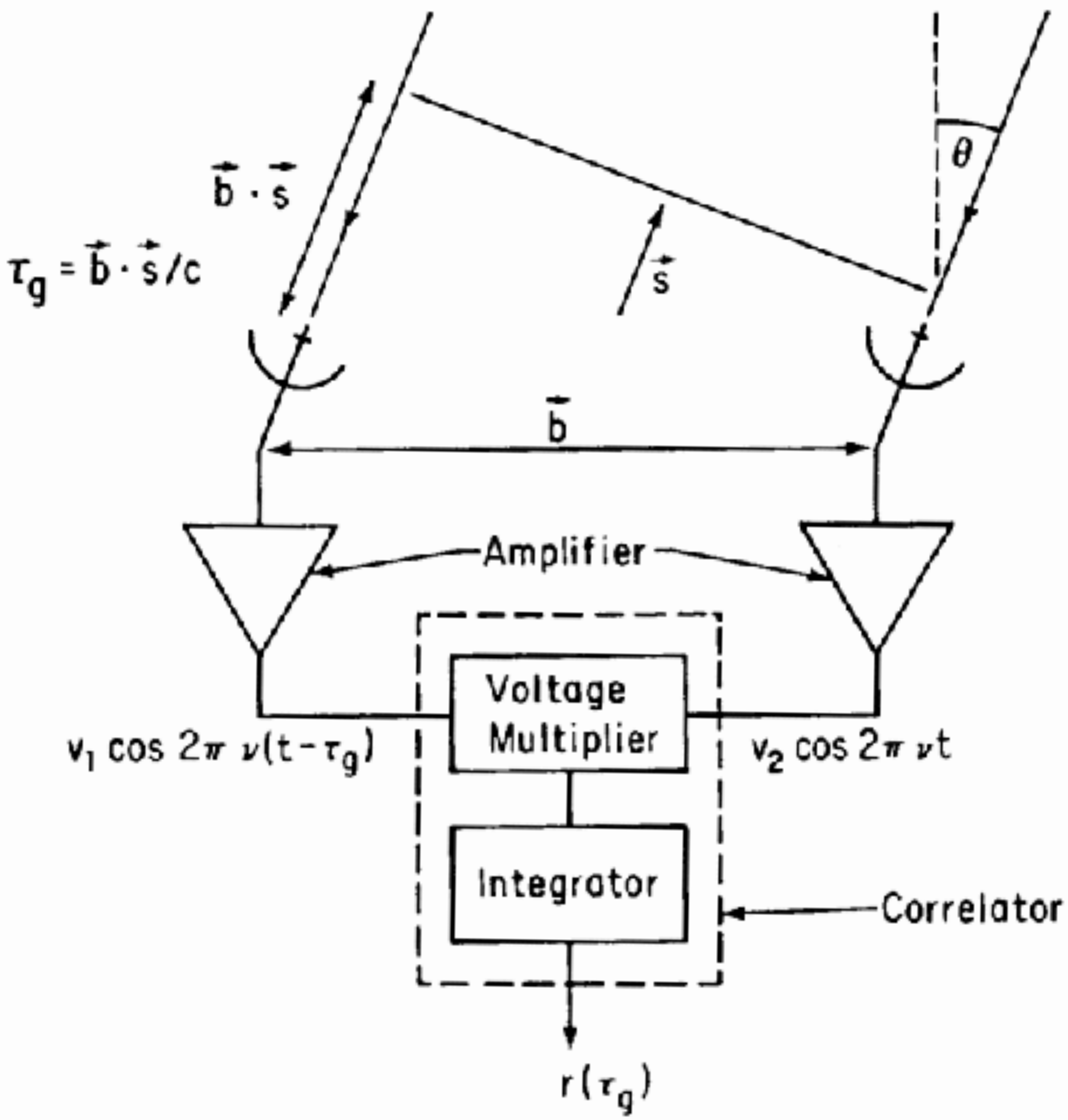
- Coherence scale is $c/(\Delta\nu)$ along the line of sight to the source = perpendicular to the wave front, and depends on the bandwidth of the source emission.

Depending on the source angular size and bandwidth: the EM field on the ground has a spatial-temporal coherence



- Interferometers measure the mutual coherence of the EM field in space and time.
- Which is a measure of the structure of the emission in angular scale and frequency.

An interferometer

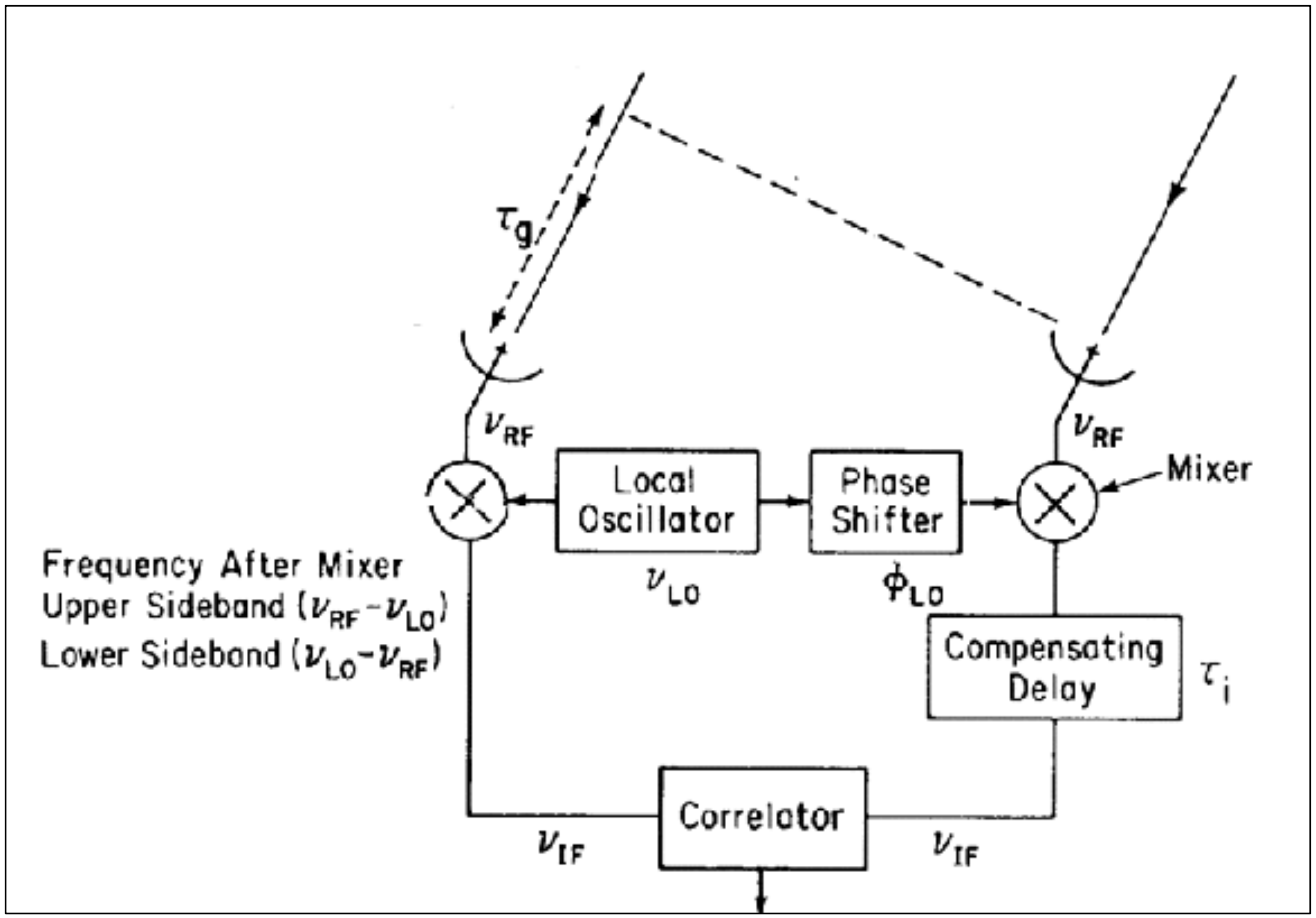


$$r(\tau_g) = [v_1 v_2 \cos(2\pi\nu\vec{b}\cdot\vec{s}/c)]/2$$

In fact, voltages are complex with amplitudes and relative phases
Use a complex correlator:

$$r(\tau_g) = v_1 v_2 e^{-i2\pi\nu\vec{b}\cdot\vec{s}/c}/2$$

An interferometer



In practice we "stop the fringes" by adjusting one arm of the interferometer, and define the point on the sky for which fringes are stationary as the "phase centre".

The visibility

$$r(\tau_g) = v_1 v_2 e^{-i2\pi\nu\vec{b}\cdot\vec{s}/c}/2$$

$$v \propto E$$

$$E^2 \propto I$$

The complex visibility is defined as

$$V(\vec{b}) = \int\int I_\nu(\vec{s}) e^{-i2\pi\nu\vec{b}\cdot\vec{s}/c} d\Omega$$

The visibility

$$V(\vec{b}) = \int\int I_\nu(\vec{s}) e^{-i2\pi\nu\vec{b}\cdot\vec{s}/c} d\Omega$$

We look at the simple case of a small field $\vec{s} = (l, m)$

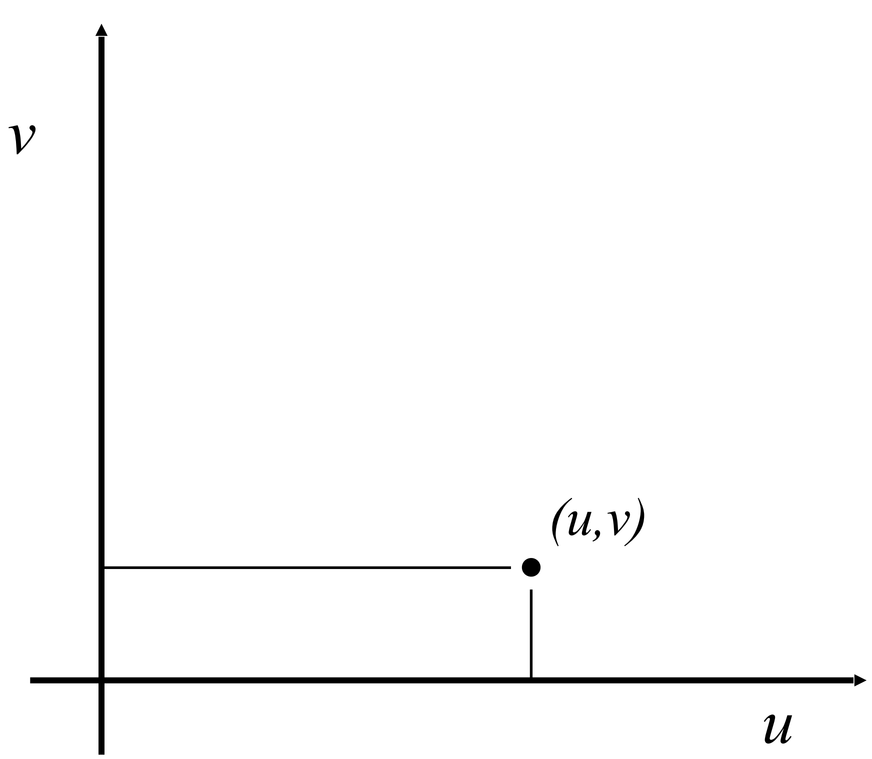
(we have stopped the fringes, so
$\vec{s}$ is now referred to the phase centre).

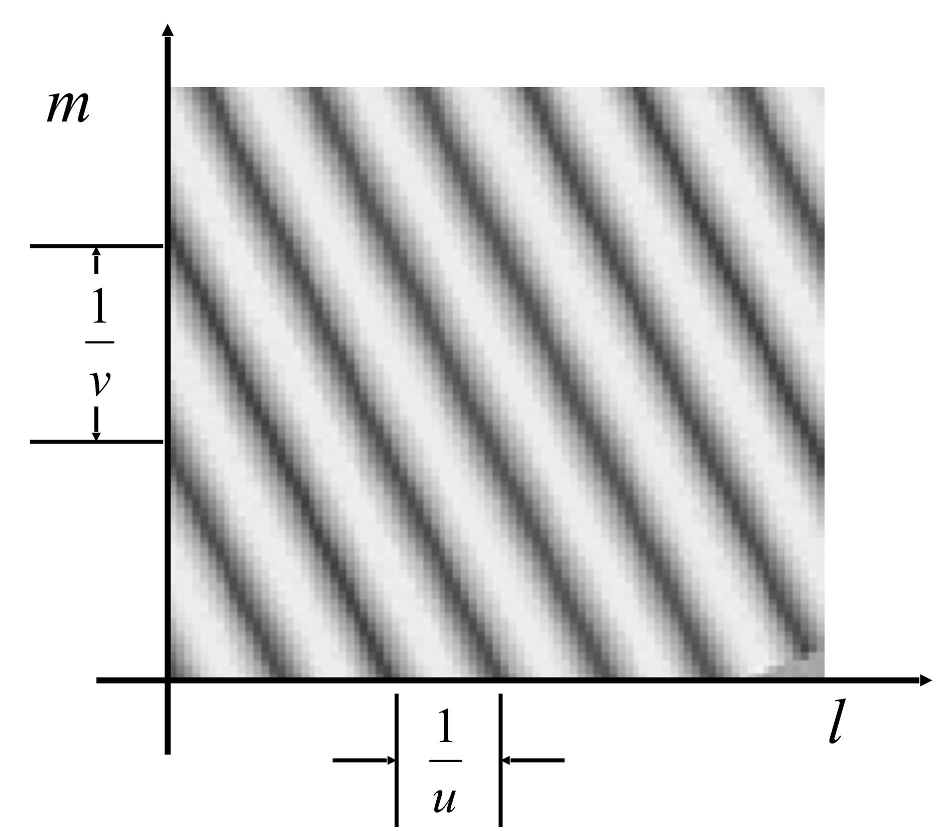
and project $\vec{b}$ onto a plane perpendicular to $\hat{s}$: $\dfrac{\vec{b}}{\lambda} = (u, v)$

$$V(u, v) = \int\int I_\nu(l, m) e^{-i2\pi(ul+vm)} dl dm$$

The visibility

$$V(u,v) = \int\int I_\nu(l,m)e^{-i2\pi(ul+vm)}dldm$$

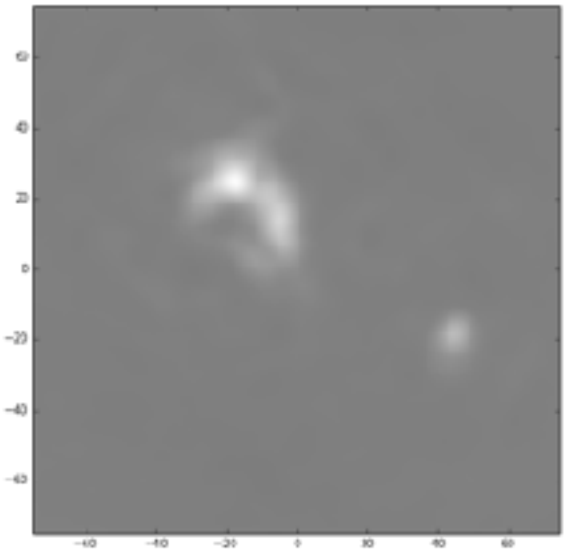




The visibility

$$V(u,v) = \int \int I_\nu(l,m) e^{-i2\pi(ul+vm)} dl dm$$

(real part)



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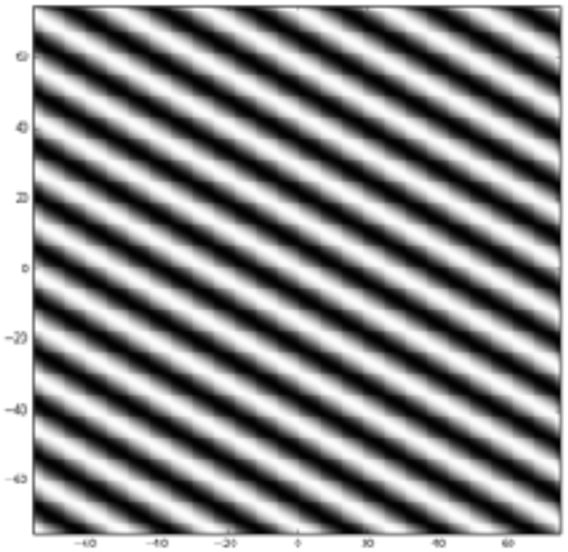

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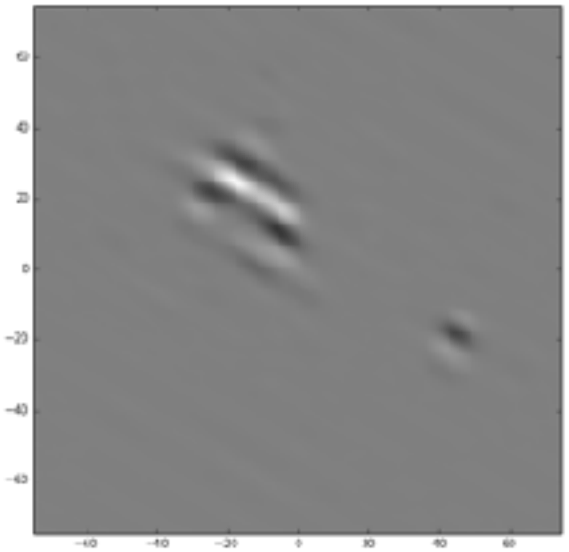

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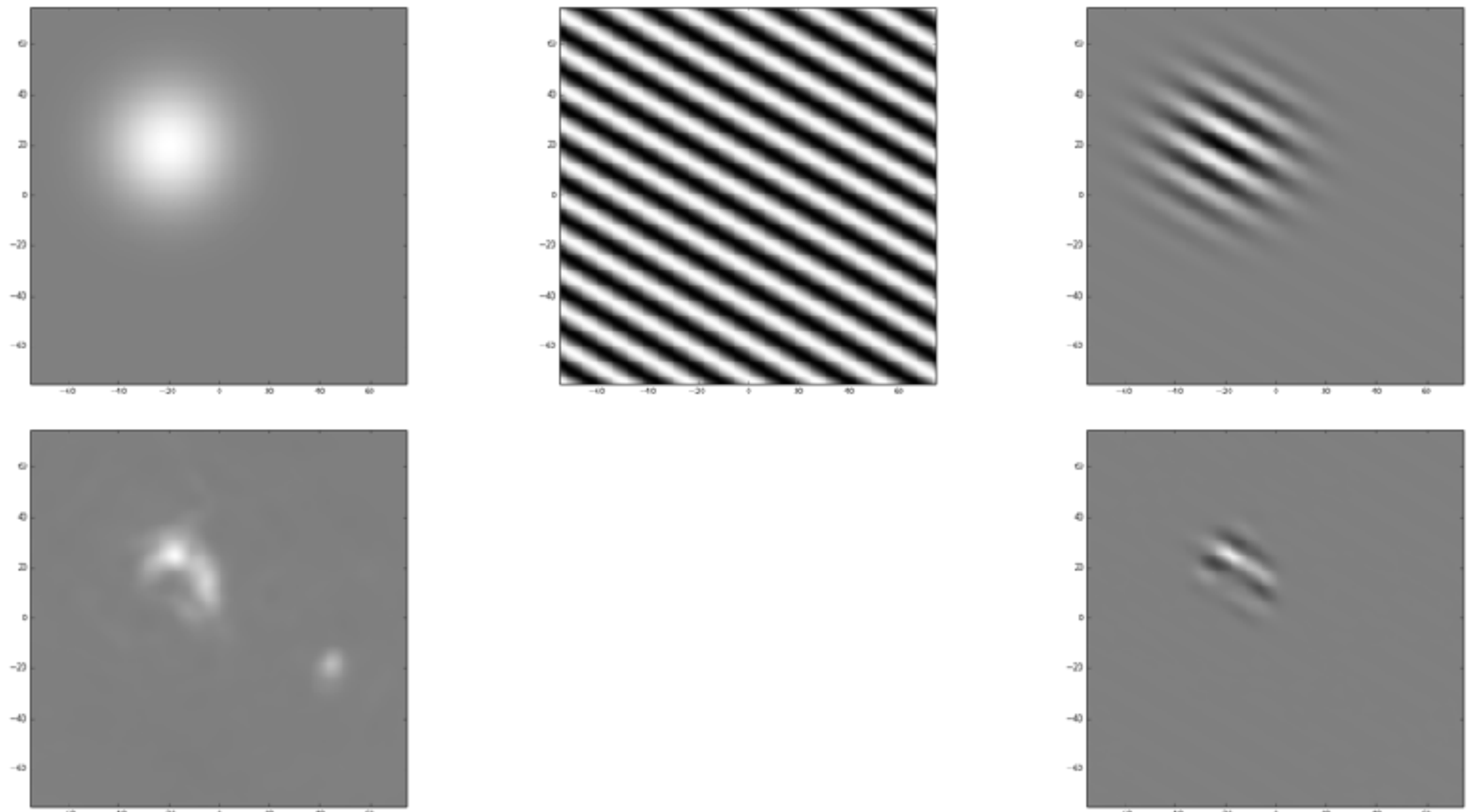

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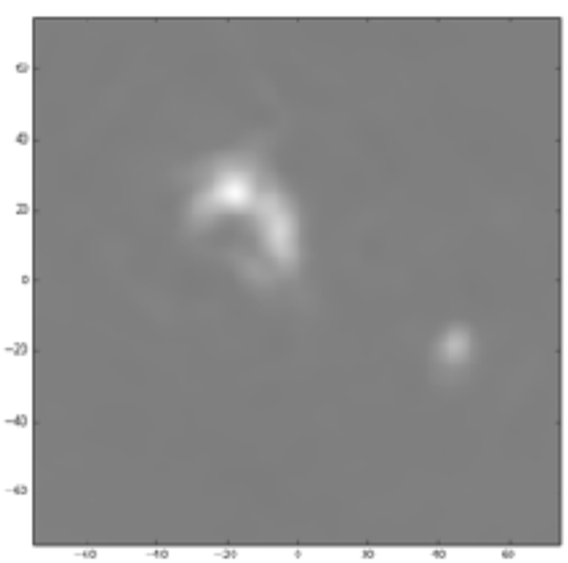
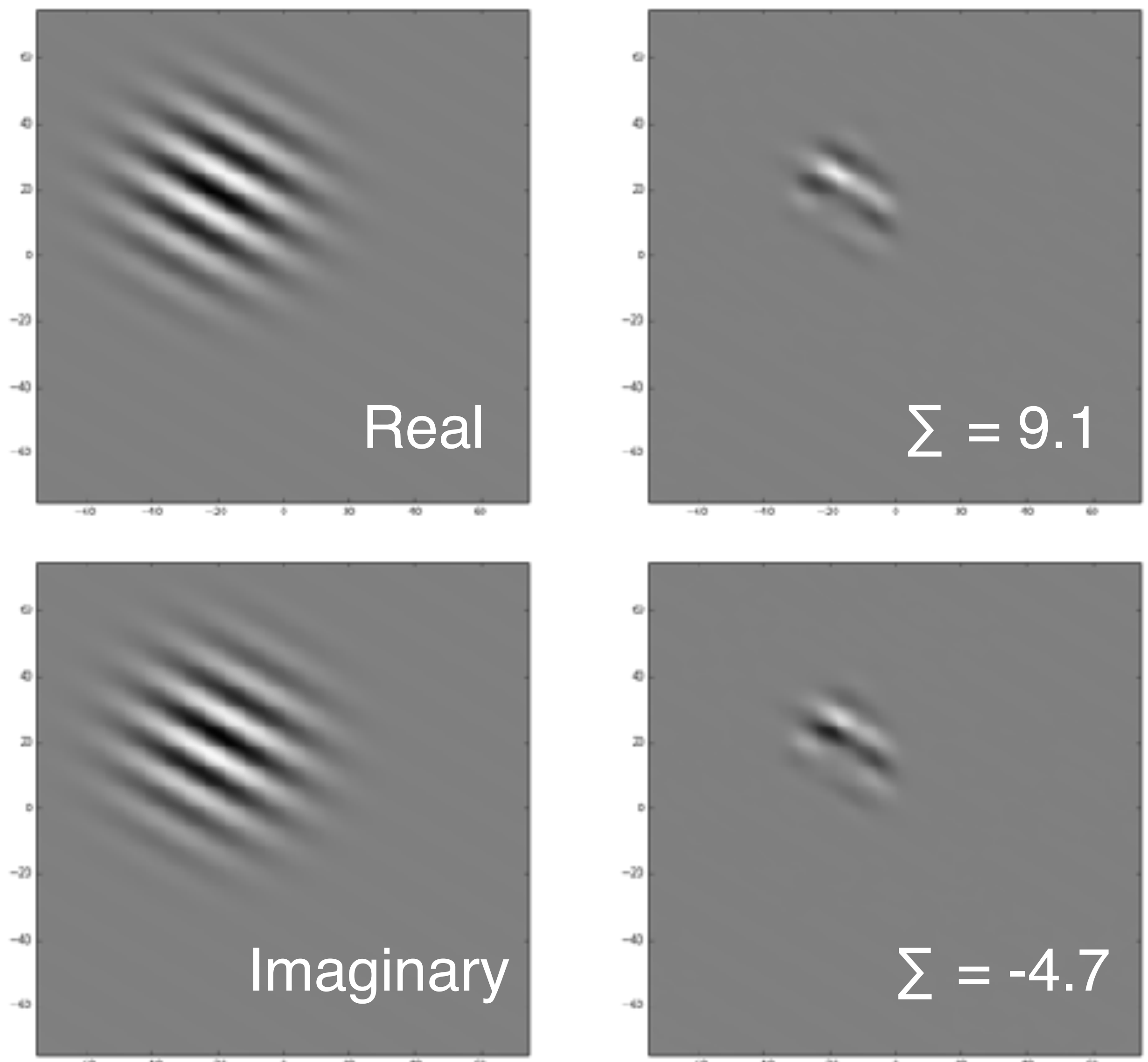
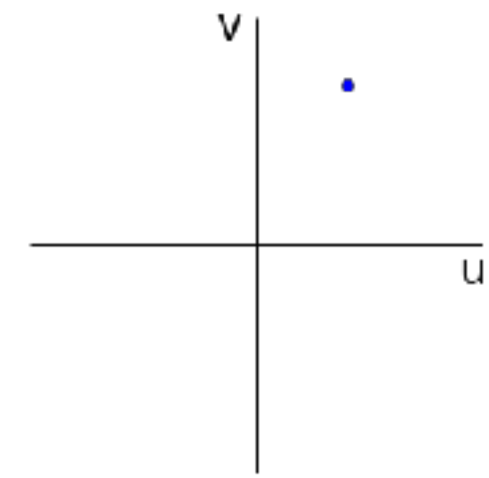




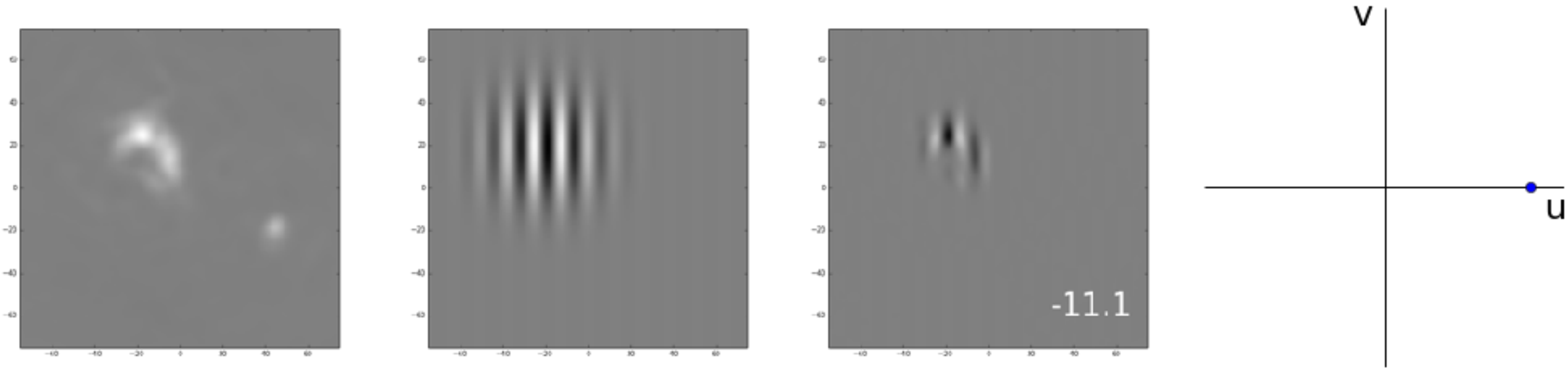


V(u,v) = 9.1 - 4.7 i

Amplitude = 10.3
Phase = -27.5 degrees

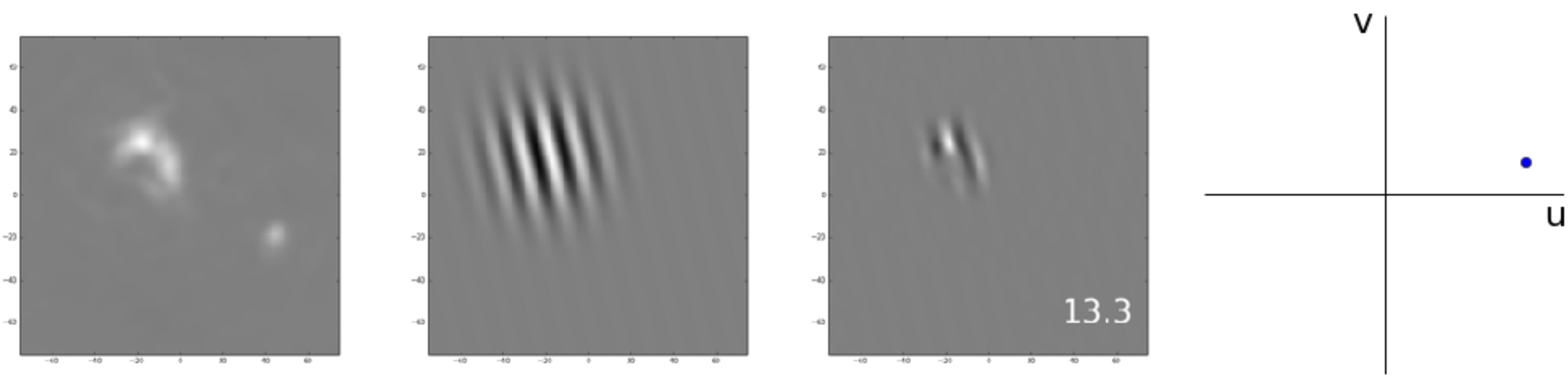
The visibility

$$V(u,v) = \int\int I_\nu(l,m)e^{-i2\pi(ul+vm)}dldm$$



The visibility

$$V(u,v) = \int\int I_\nu(l,m) e^{-i2\pi(ul+vm)} dl dm$$



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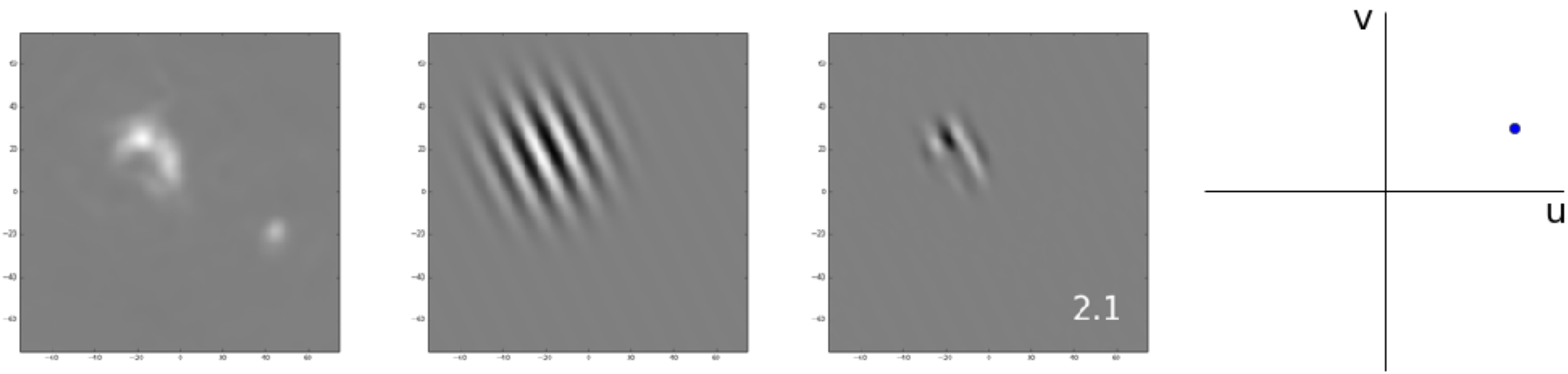

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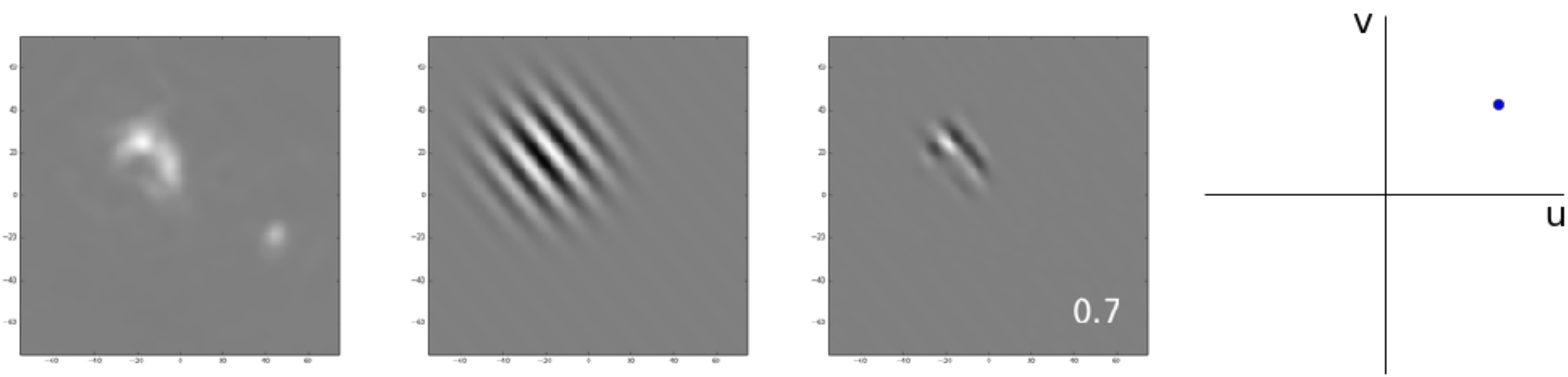

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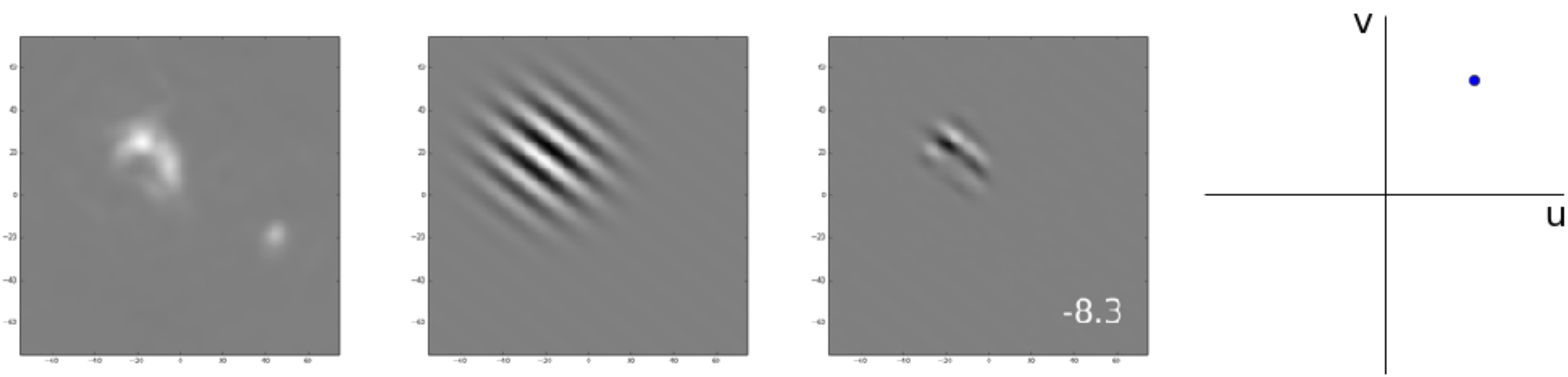

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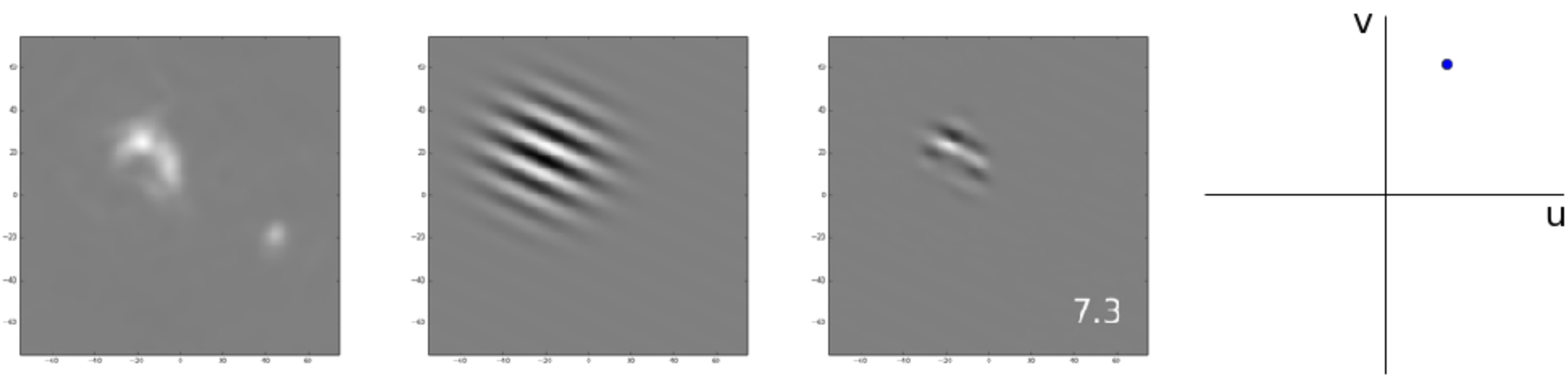

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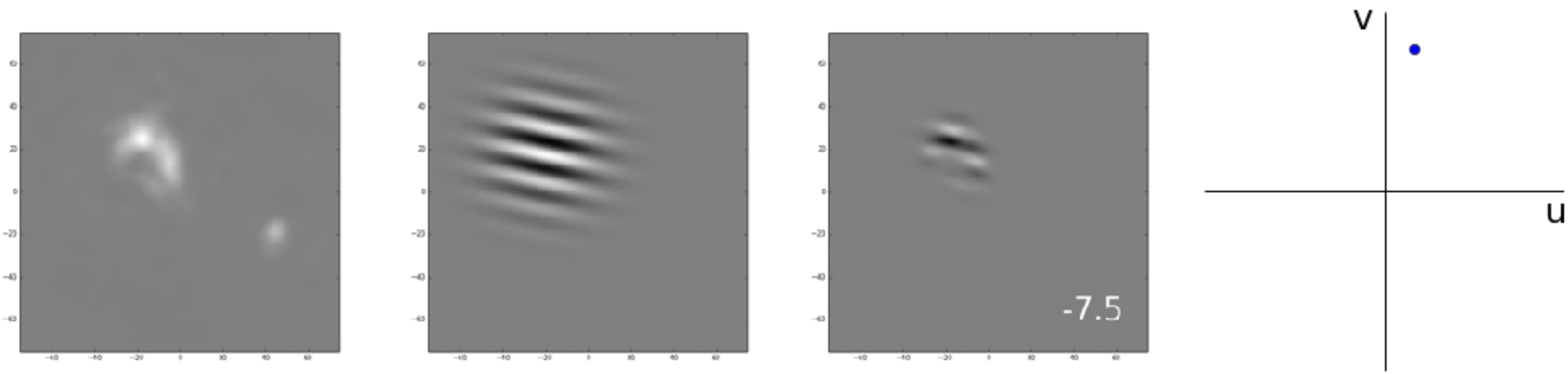

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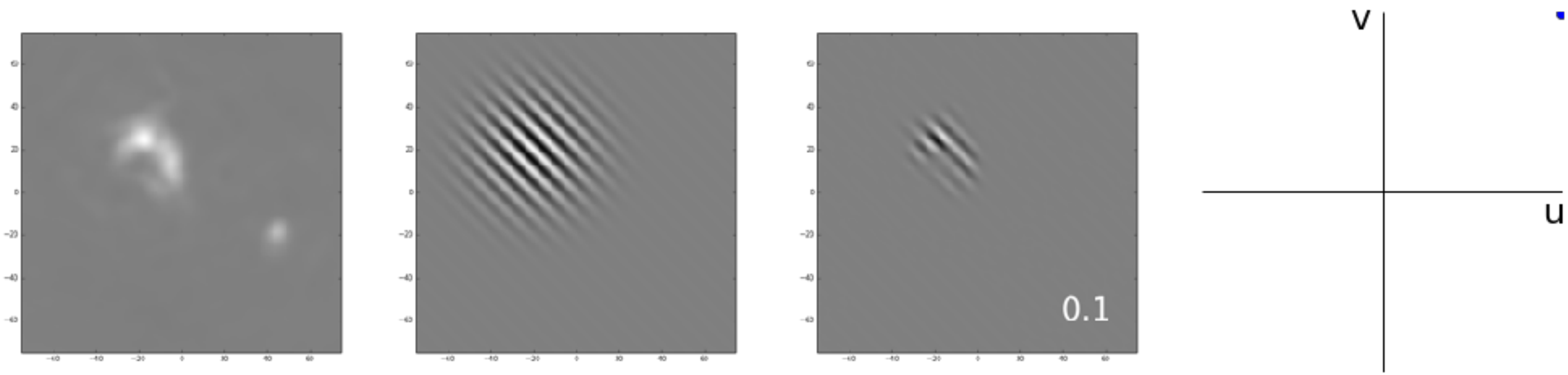
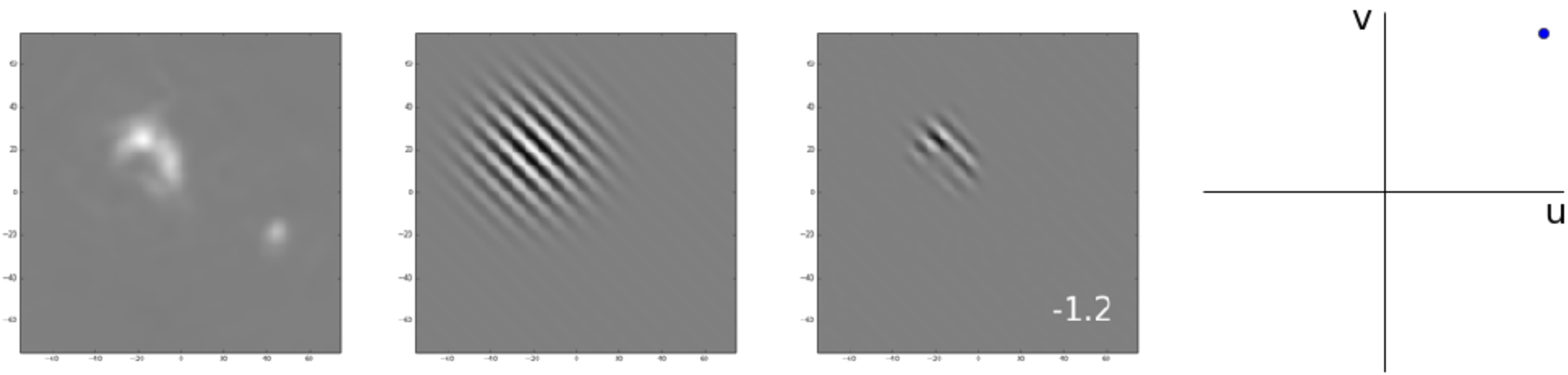


The visibility

$$V(u,v) = \int\int I_\nu(l,m)e^{-i2\pi(ul+vm)}dldm$$



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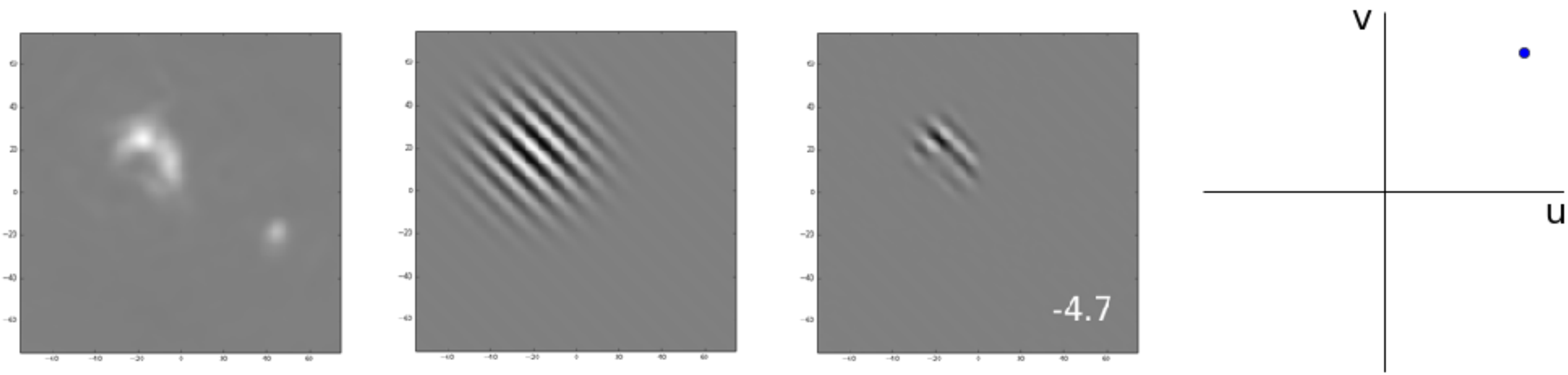

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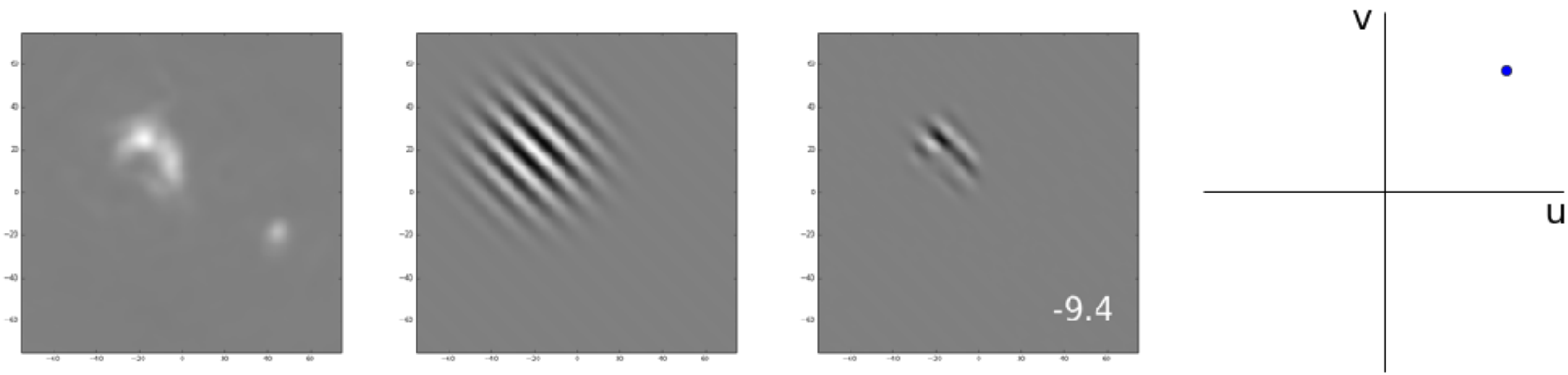

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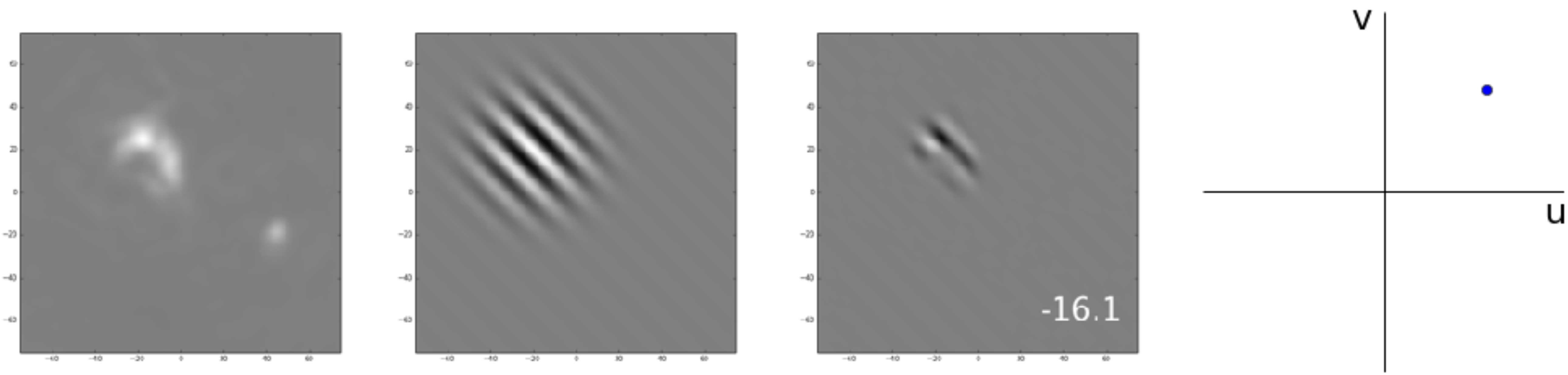

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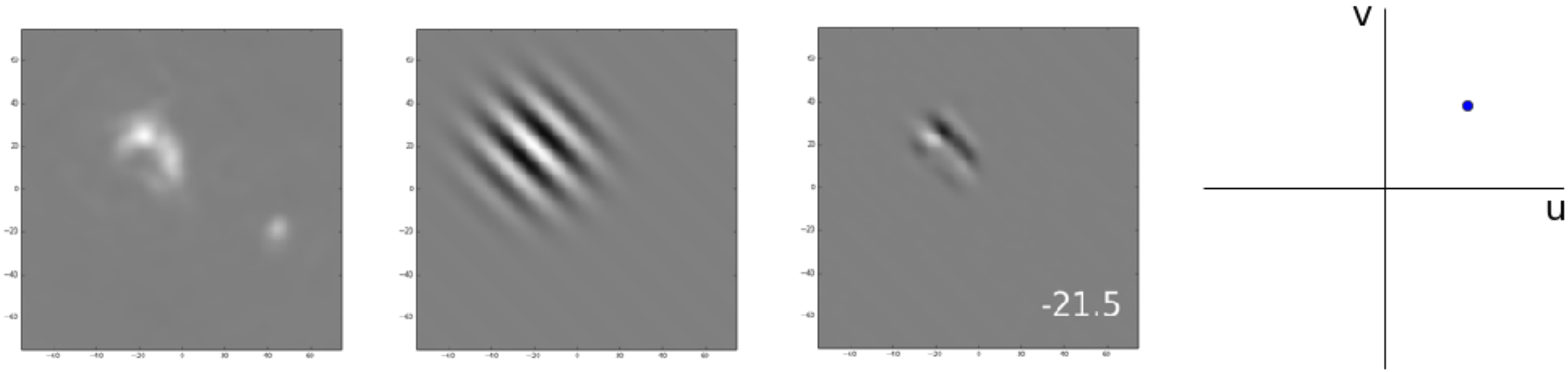

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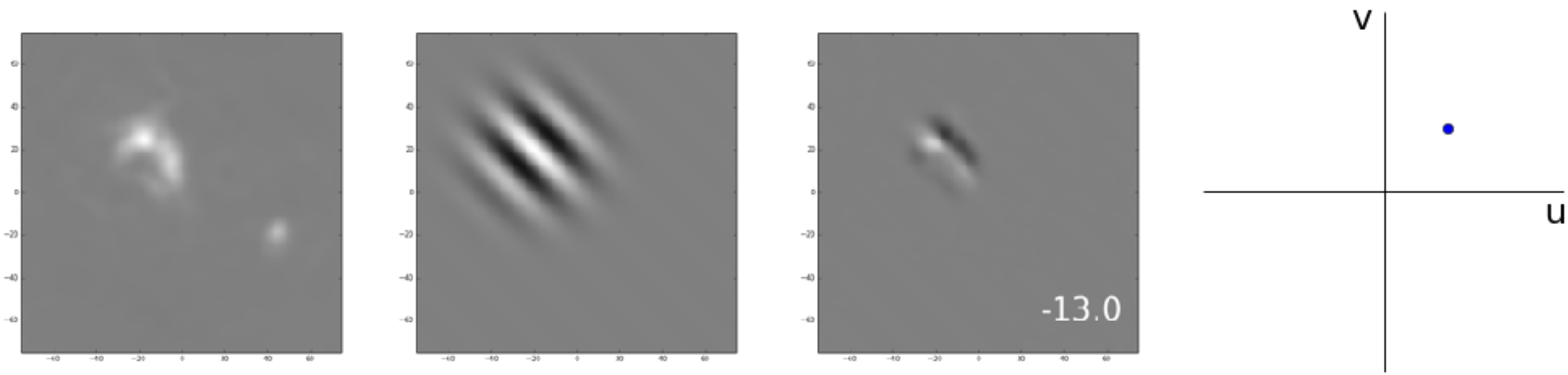

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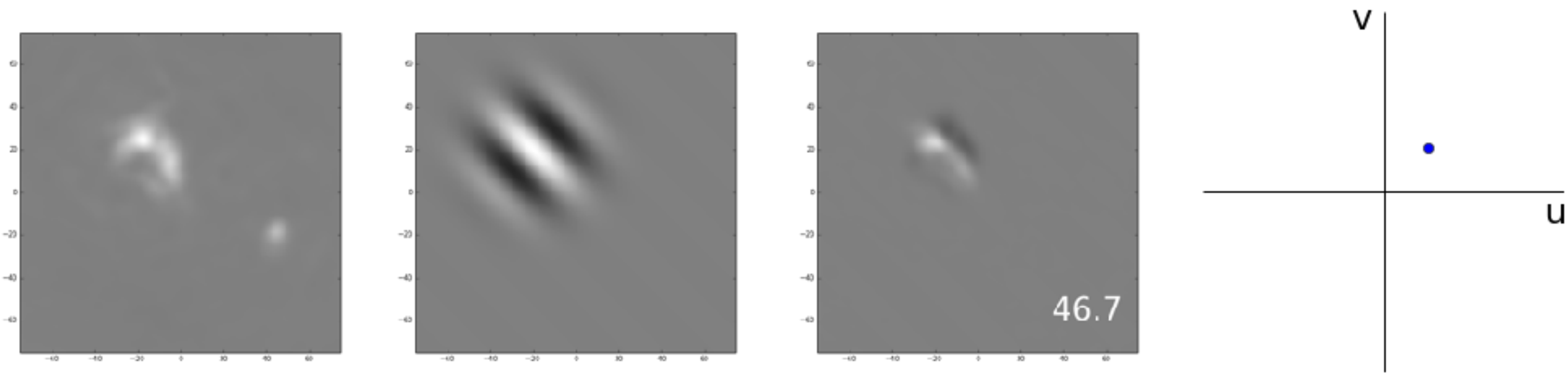

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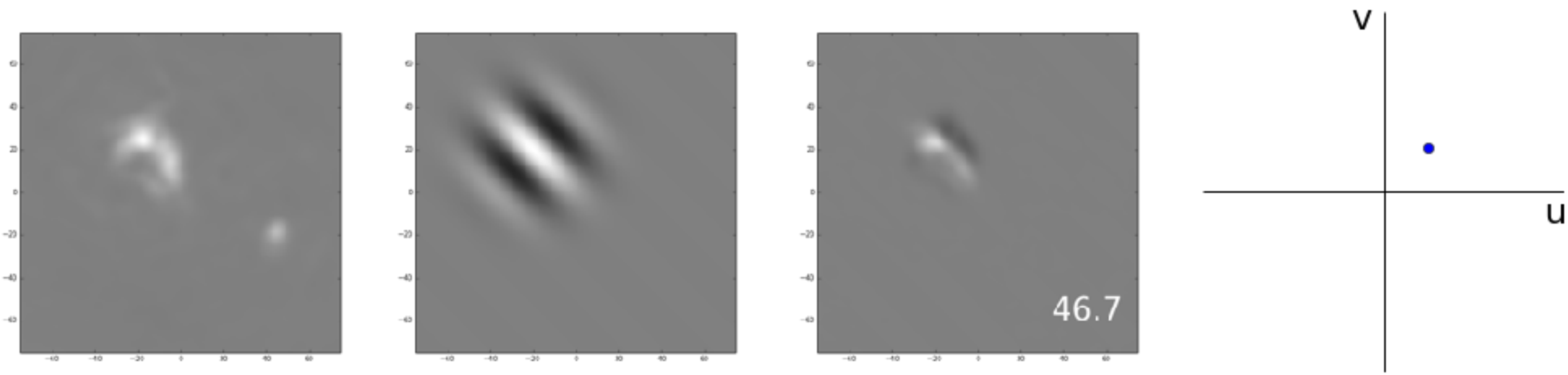

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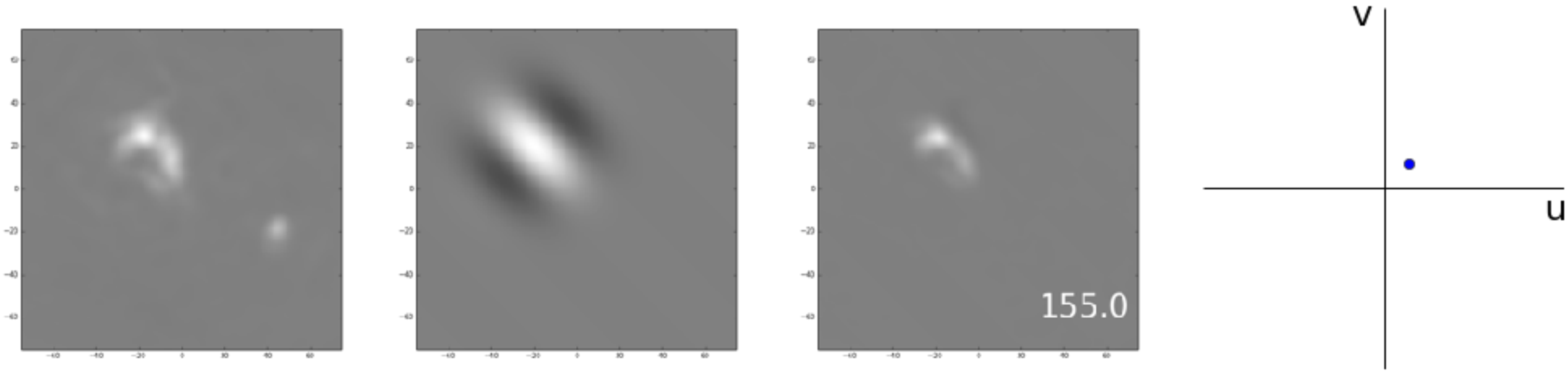

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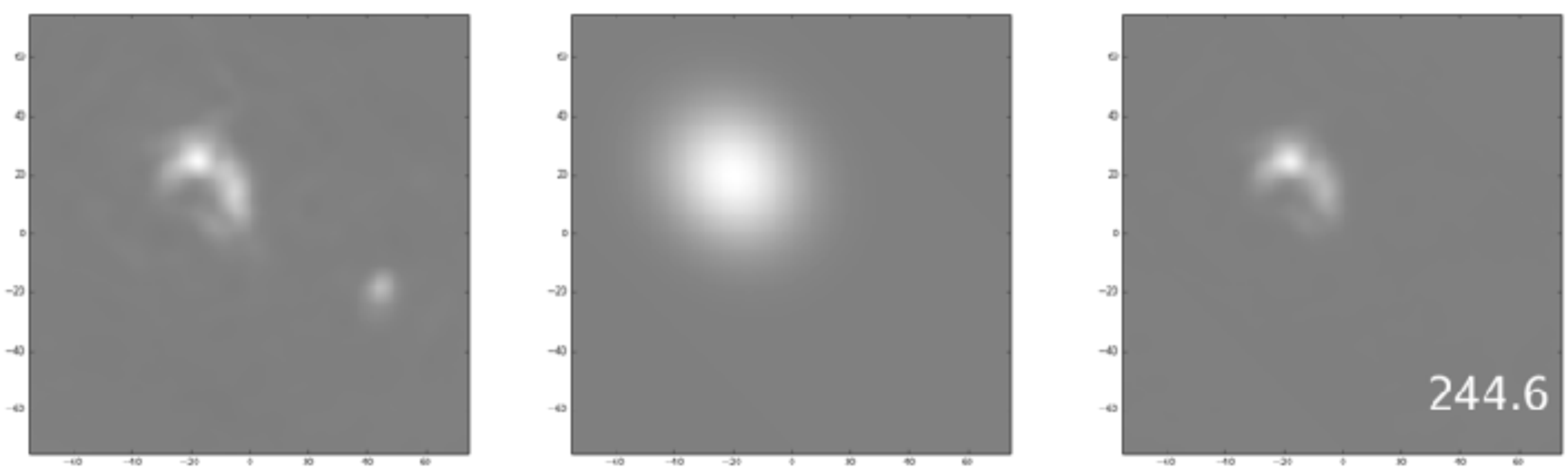
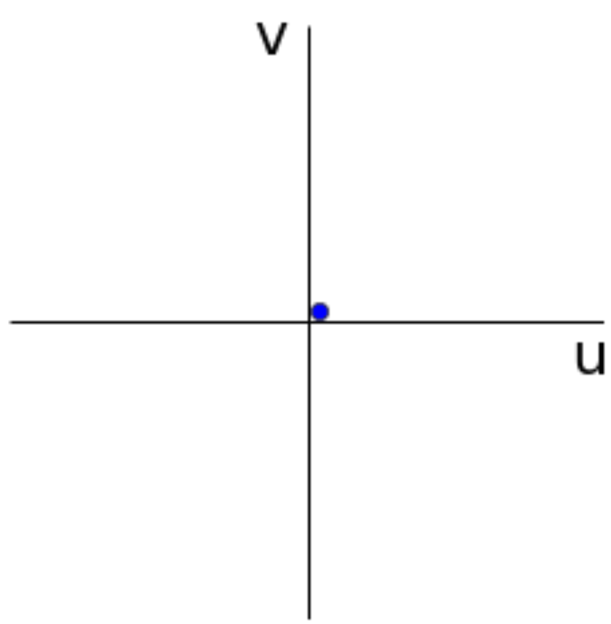




A Fourier relation

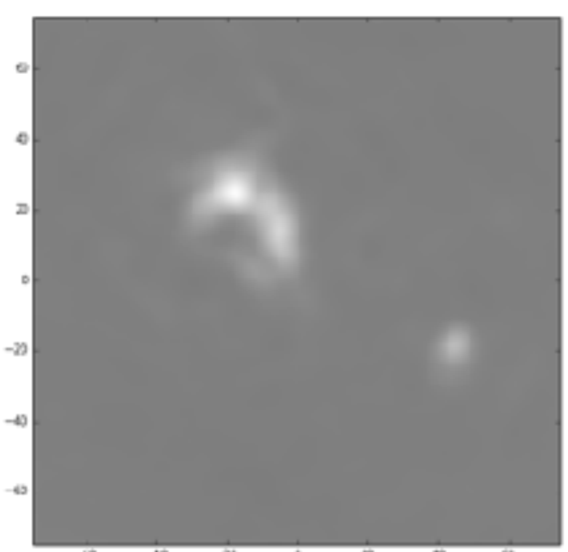
$$V(u,v) = \int\int I_\nu(l,m) e^{-i2\pi(ul+vm)} dl dm$$

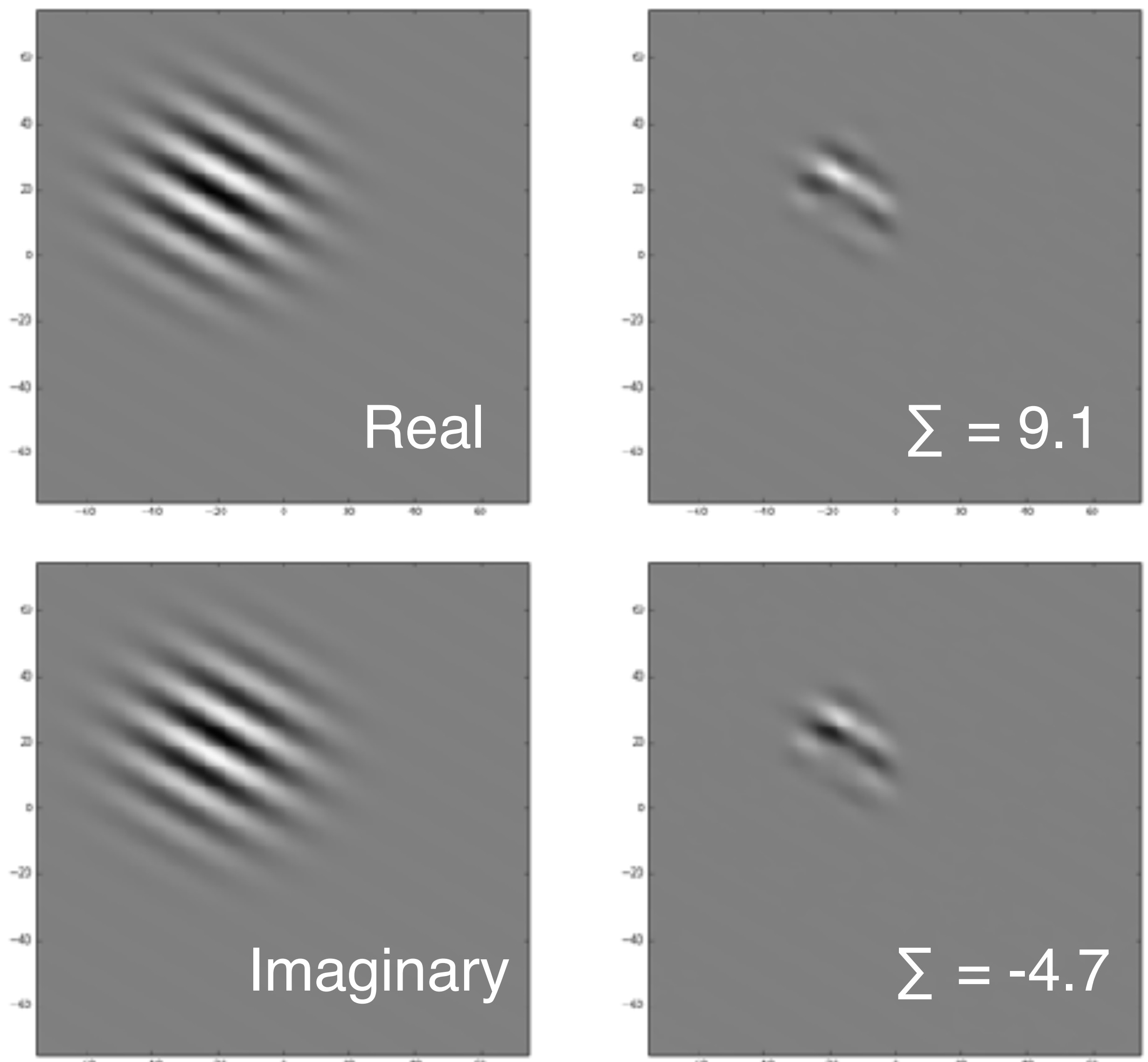
$$I_\nu(l,m) = \int\int V(u,v) e^{i2\pi(ul+vm)} du dv$$

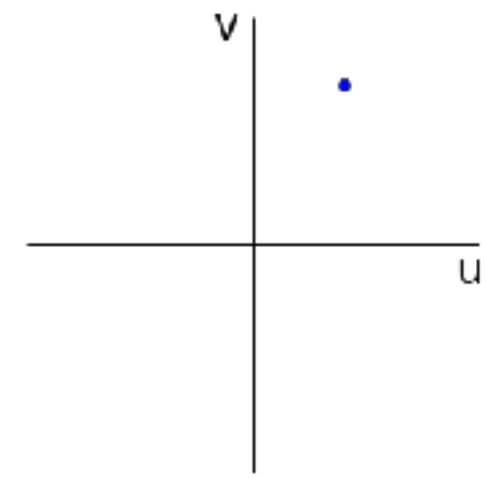
To deduce the sky brightness distribution we need to measure the visibilities for many values of (u,v)

A Fourier relation

$$V(u,v) = \int\int I_\nu(l,m)e^{-i2\pi(ul+vm)}dldm$$





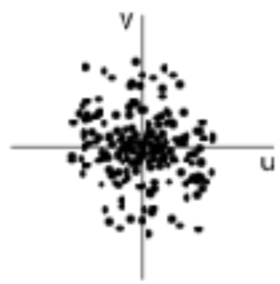


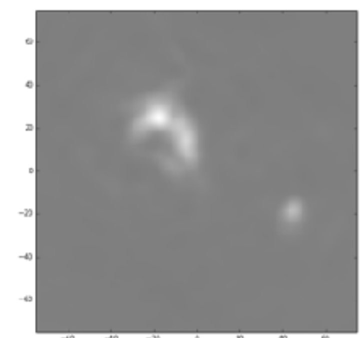
V(u,v) = 9.1 - 4.7 i

Amplitude = 10.3
Phase = -27.5 degrees

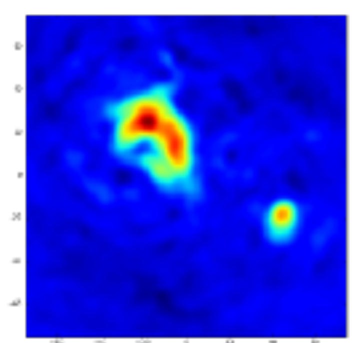
A Fourier relation

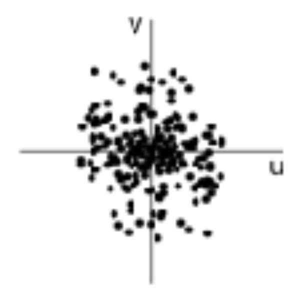
$$V(u,v) = \int\int I_\nu(l,m)e^{-i2\pi(ul+vm)}dldm$$





Observing



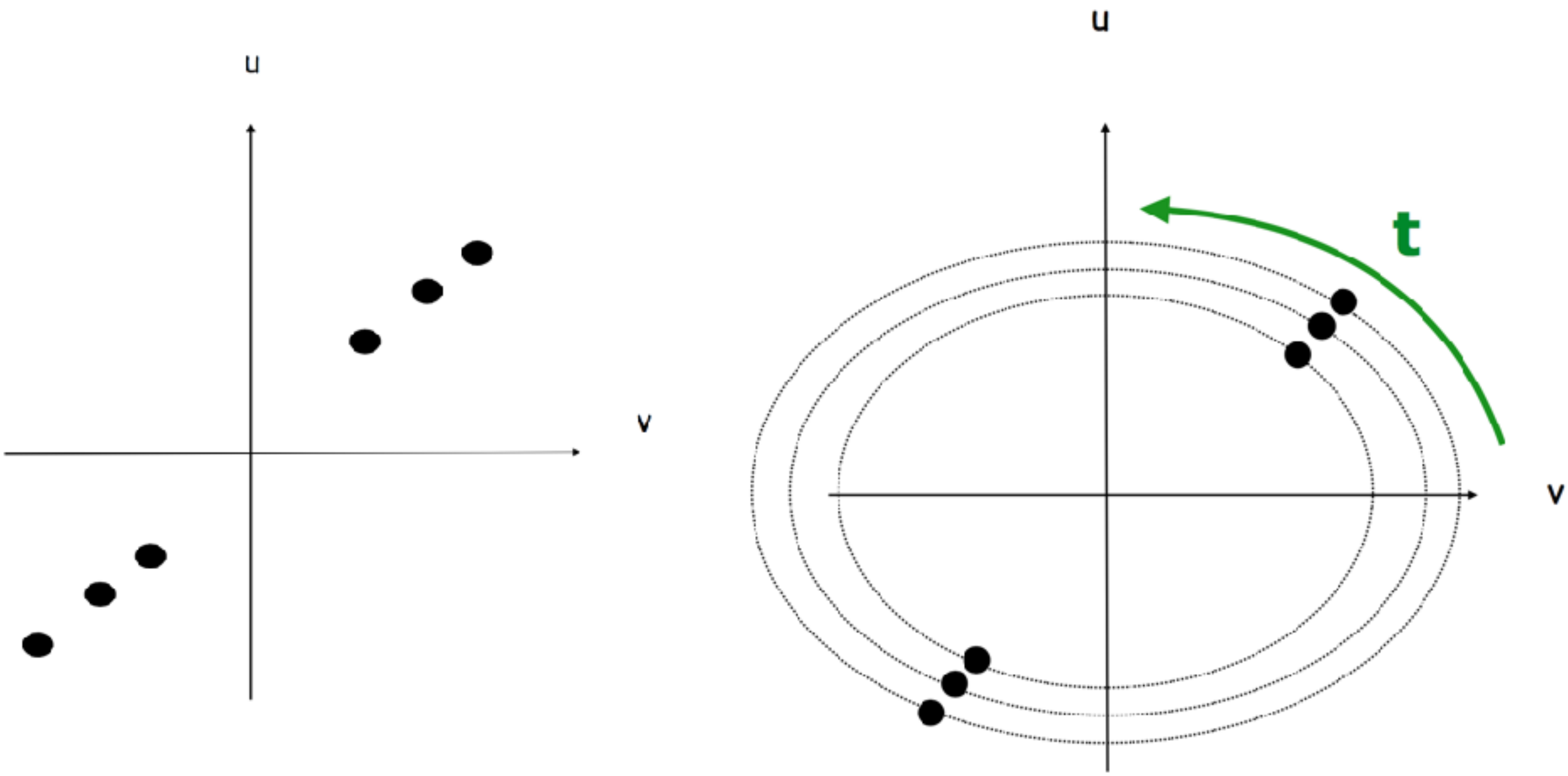


Imaging

$$I_\nu(l,m) = \int\int V(u,v)e^{i2\pi(ul+vm)}dudv$$

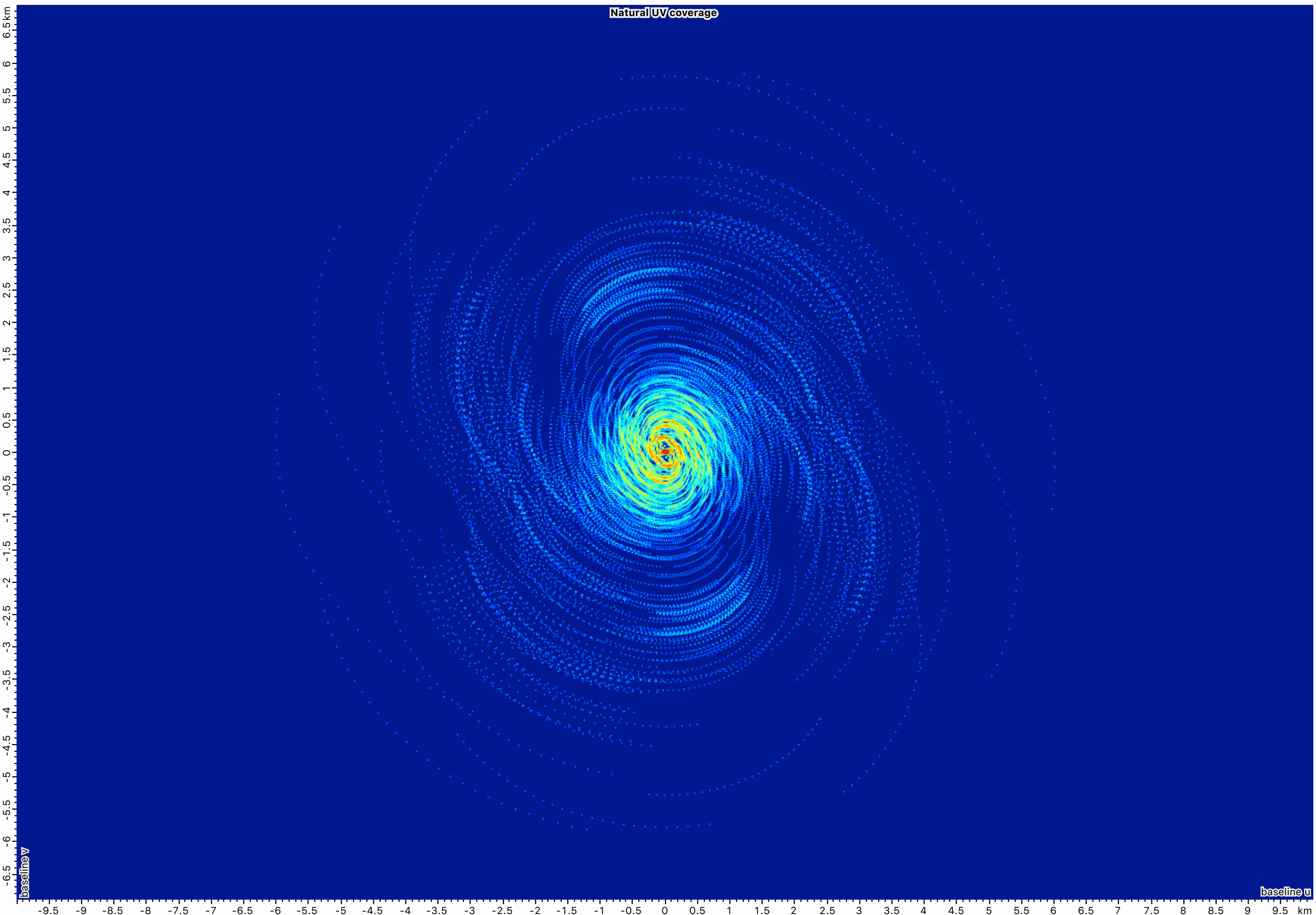
Earth-rotation synthesis

At any instant a E-W interferometer probes a line

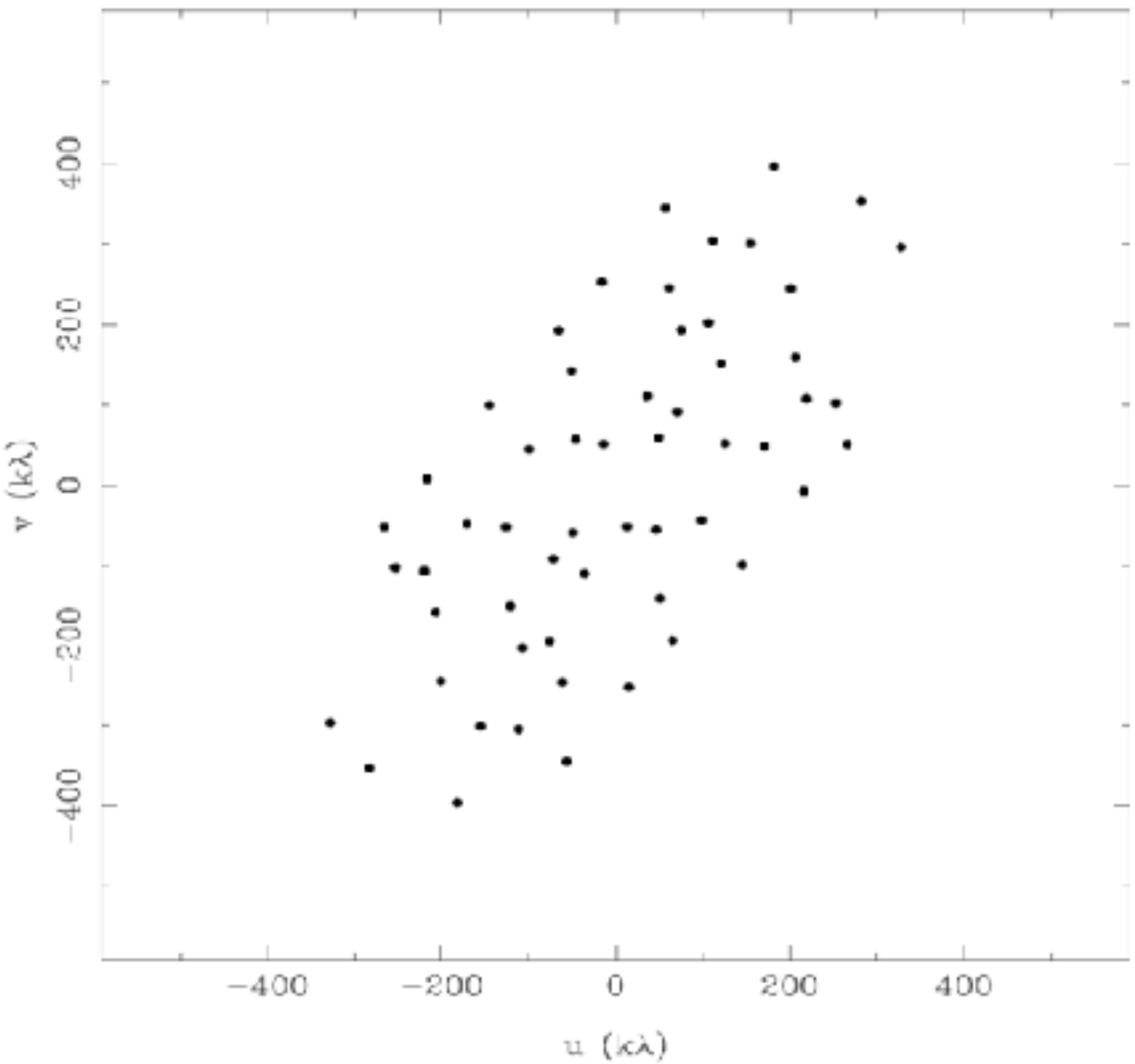


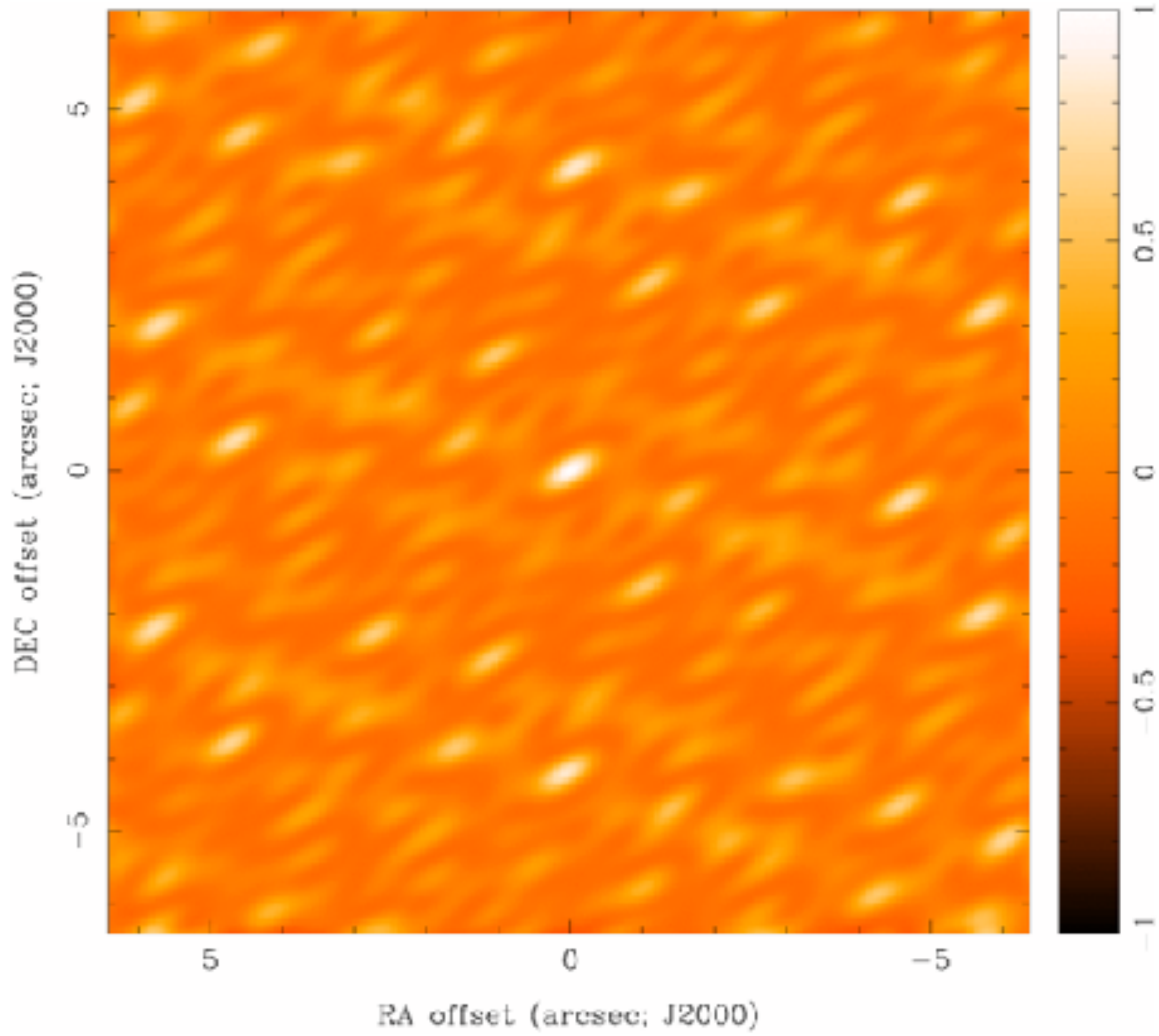
Letting the earth rotate sweeps out an ellipse in u-v space

Thanks to Naomi McClure-Griffiths for u-v coverage slides


Natural UV coverage
baseline v
km
baseline u
km

Importance of u-v coverage





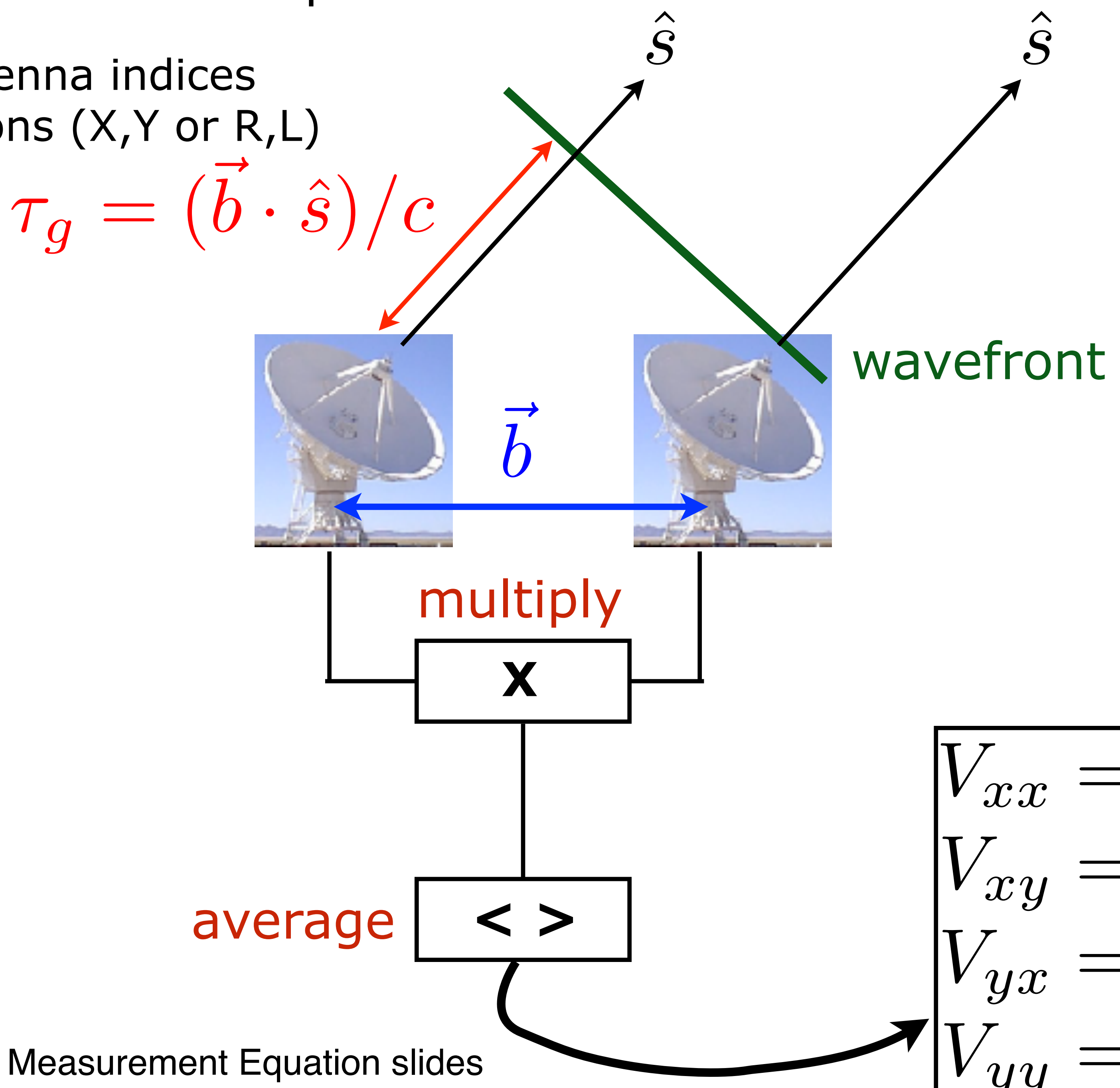
u-v coverage and imaging

- Bandwidth synthesis - include visibility samples from frequency channels across the band. This effectively broadens the sampled tracks on the u-v plane
- Sampling the u-v plane is normally incomplete, and the image is the sky brightness distribution convolved with the “point spread function”.
- There are various schemes for deconvolving this “dirty” image. Lectures on this.

The Measurement Equation

- The measurement equation (Hamaker, Bregman & Sault) is a matrix formalism for expressing the polarimetric response of an interferometer

Here: p,q are antenna indices
x,y are polarizations (X,Y or R,L)



$$V_{xx} = \langle \mathcal{V}_{px} \mathcal{V}_{qx}^* \rangle$$
$$V_{xy} = \langle \mathcal{V}_{px} \mathcal{V}_{qy}^* \rangle$$
$$V_{yx} = \langle \mathcal{V}_{py} \mathcal{V}_{qx}^* \rangle$$
$$V_{yy} = \langle \mathcal{V}_{py} \mathcal{V}_{qy}^* \rangle$$

Thanks to George Heald for Measurement Equation slides

The Measurement Equation

- The measurement equation (Hamaker, Bregman & Sault) is a matrix formalism for expressing the polarimetric response of an interferometer

$$
\begin{aligned}
V_{xx} &= \langle \mathcal{V}_{px} \mathcal{V}_{qx}^{*} \rangle \\
V_{xy} &= \langle \mathcal{V}_{px} \mathcal{V}_{qy}^{*} \rangle \\
V_{yx} &= \langle \mathcal{V}_{py} \mathcal{V}_{qx}^{*} \rangle \\
V_{yy} &= \langle \mathcal{V}_{py} \mathcal{V}_{qy}^{*} \rangle
\end{aligned}
$$

can be more easily and elegantly expressed in matrix form, as:

$$
\mathbf{V}_{pq} = \langle \vec{\mathcal{V}}_p \vec{\mathcal{V}}_q^{\dagger} \rangle = \langle \begin{pmatrix} \mathcal{V}_{px} \\ \mathcal{V}_{py} \end{pmatrix} \begin{pmatrix} \mathcal{V}_{qx}^{*} & \mathcal{V}_{qy}^{*} \end{pmatrix} \rangle = \begin{pmatrix} V_{xx} & V_{xy} \\ V_{yx} & V_{yy} \end{pmatrix}
$$

The Measurement Equation

- The measurement equation (Hamaker, Bregman & Sault) is a matrix formalism for expressing the polarimetric response of an interferometer
- Introducing the coherency matrix **C**, which describes the intensity distribution:

$$\mathbf{C}_{pq} = \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} = \langle \vec{E}\vec{E}^{\dagger} \rangle$$

- and the "Jones matrix" J which contains all of the information about what happens to the signal, from the source to the correlator,

$$\mathcal{V}_p = \mathbf{J}_p \vec{E} \qquad \mathcal{V}_q = \mathbf{J}_q \vec{E}$$

- then with a bit of math we can write down the measurement equation:

$$\mathbf{V}_{pq} = \mathbf{J}_p \mathbf{C}_{pq} \mathbf{J}_q^{\dagger} = \mathbf{J}_{pq} \mathbf{C}_{pq}$$

The Measurement Equation

- The Jones matrices contain all of the stuff that we have to calibrate

$$\mathbf{V}_{pq} = \mathbf{J}_{pq}\mathbf{C}_{pq}$$

- For example,
  - G: the (complex) antenna gain
  - B: bandpass
  - F: Faraday rotation
  - E: antenna response pattern

$$\mathbf{V}_{pq} = \mathbf{M}_{pq}\mathbf{B}_{pq}\mathbf{G}_{pq}\mathbf{D}_{pq}\mathbf{E}_{pq}\mathbf{P}_{pq}\mathbf{T}_{pq}\mathbf{F}_{pq}\mathbf{C}_{pq}$$

The Measurement Equation

- So what?

$$\mathbf{V}_{pq} = \mathbf{J}_{pq}\mathbf{C}_{pq}$$

- The measurement equation makes it more straightforward to handle polarization calibration, and direction dependent effects
- Explicit separation of dependencies (for example, G(t) and B(ν) are the antenna gain and bandpass)
- Necessary in order to understand advanced calibration!

- We'll return to this in the Calibration lecture