

Spatial Filtering and Convolution

Spatial Filtering

- apply a filter (also sometimes called a kernel or mask) to an image

Spatial Filtering

- apply a filter (also sometimes called a kernel or mask) to an image
- a new pixel value is calculated, one pixel at a time

Spatial Filtering

- apply a filter (also sometimes called a kernel or mask) to an image
- a new pixel value is calculated, one pixel at a time
- the neighbouring pixels influence the result

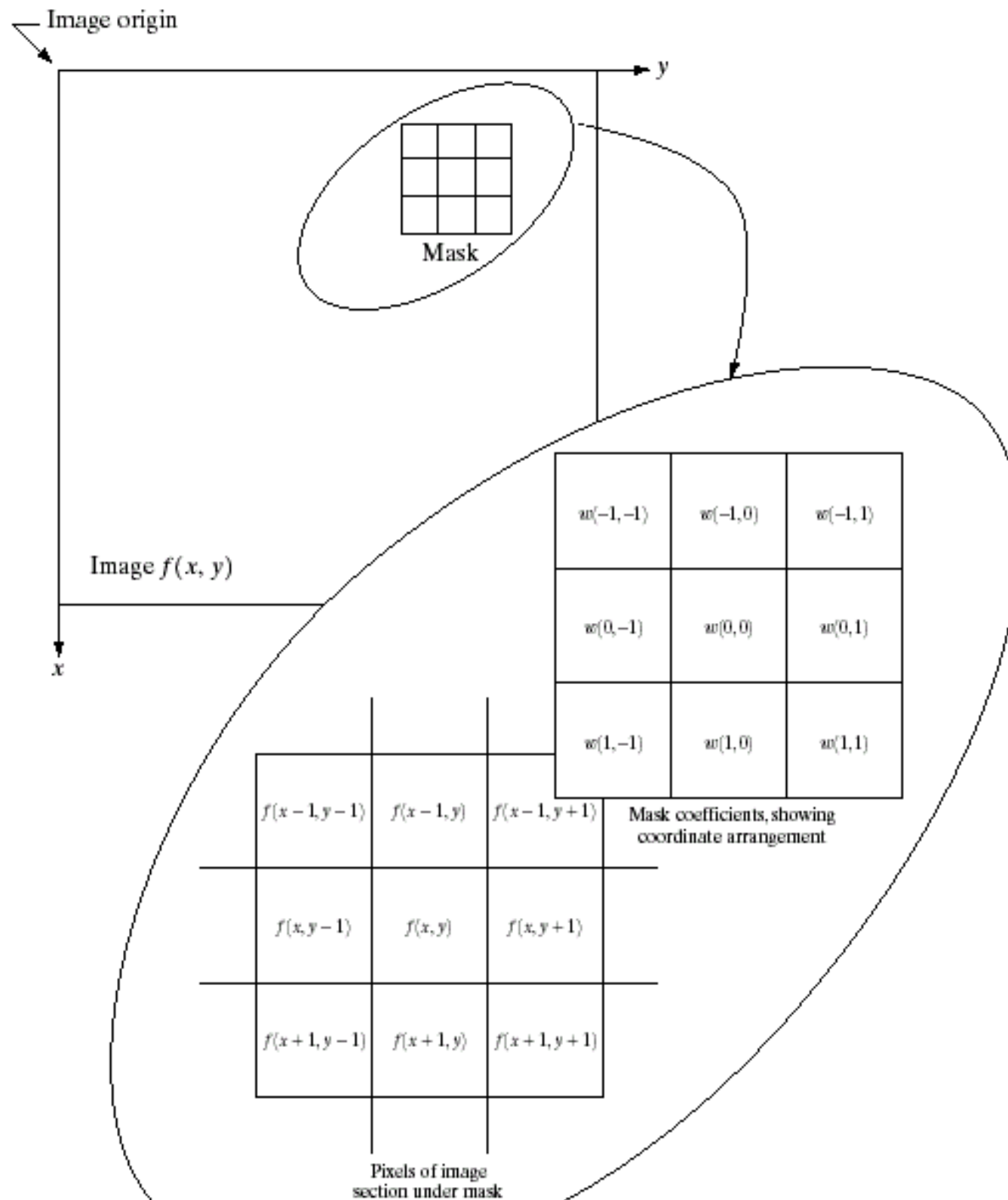


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3×3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

FIGURE 3.33

Another
representation of
a general 3×3
spatial filter mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

New Pixel Value

- the pixel at location (x, y) in the new image will have a value:

New Pixel Value

- the pixel at location (x, y) in the new image will have a value:

$$\begin{aligned} s(x, y) = & w_1 f(x-1, y-1) + w_2 f(x-1, y) + w_3 f(x-1, y+1) \\ & + w_4 f(x, y-1) + w_5 f(x, y) + w_6 f(x, y+1) \\ & + w_7 f(x+1, y-1) + w_8 f(x+1, y) + w_9 f(x+1, y+1) \end{aligned}$$

New Image

- slide the mask over by one pixel and repeat
- NOTE: original image requires a 1-pixel border be added or else the new image will be smaller than the original

Example

Multiply the values in the mask with the corresponding values in the image and add them together.

- Find the new pixel value...

The original image

The mask

$1/9$

1	1	1
1	1	1
1	1	1

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

Normalization factor.
Equal to 1 over the
sum of the elements in
the mask.

$1/9$

1	1	1
1	1	1

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

The mask

1	1	1
1	1	1
1	1	1

$1/9$

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

The mask

1	1	1
1	1	1
1	1	1

$1/9$

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

The mask

1	1	1
1	1	1
1	1	1

$1/9$

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

The mask

1	1	1
1	1	1
1	1	1

$1/9$

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

The mask

1	1	1
1	1	1
1	1	1

$1/9$

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

The mask

1	1	1
1	1	1
1	1	1

$1/9$

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

The mask

1	1	1
1	1	1
1	1	1

$1/9$

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

The mask

1	1	1
1	1	1
1	1	1

$1/9$

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

- Find the new pixel value...

Multiply the values in the mask with the corresponding values in the image and add them together.

The mask

1	1	1
1	1	1
1	1	1

$1/9$

The original image

135	146	133	134	141
140	159	148	156	160
137	147	133	123	120
137	138	129	122	120
139	130	125	119	113

Example

The result

142	142	139
141	139	135
135	130	123

What did you just do?

$1/9$

1	1	1
1	1	1
1	1	1

What did you just do?

$1/9$



1	1	1
1	1	1
1	1	1

- this is the averaging mask!

What did you just do?

$1/9$

1	1	1
1	1	1
1	1	1

- this is the averaging mask!
- the result is to smooth out the image

Original image



Averaged image



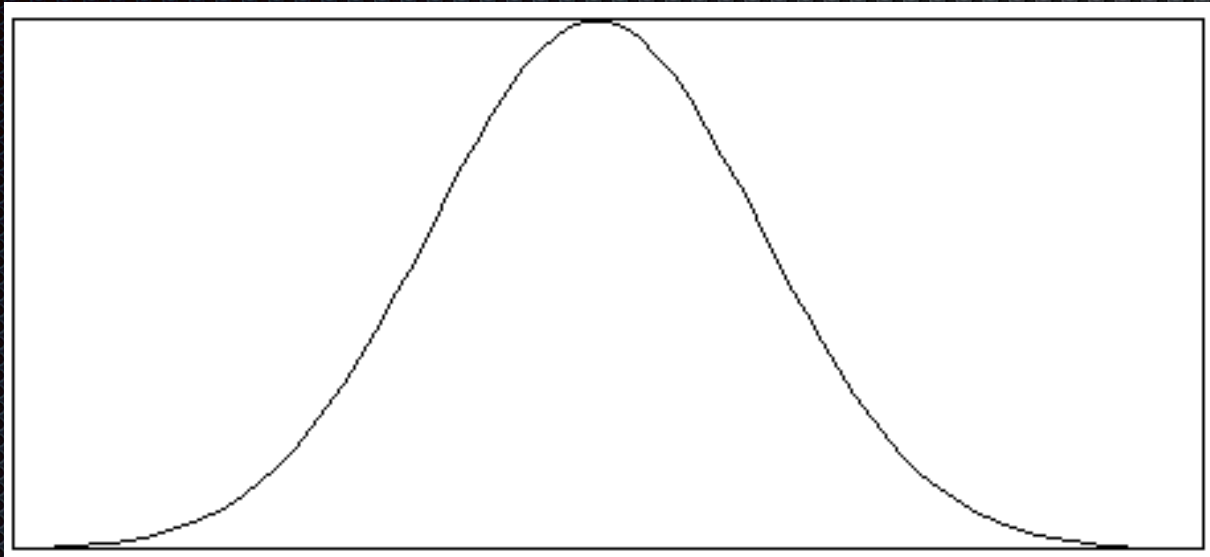


- this operator is known as convolution

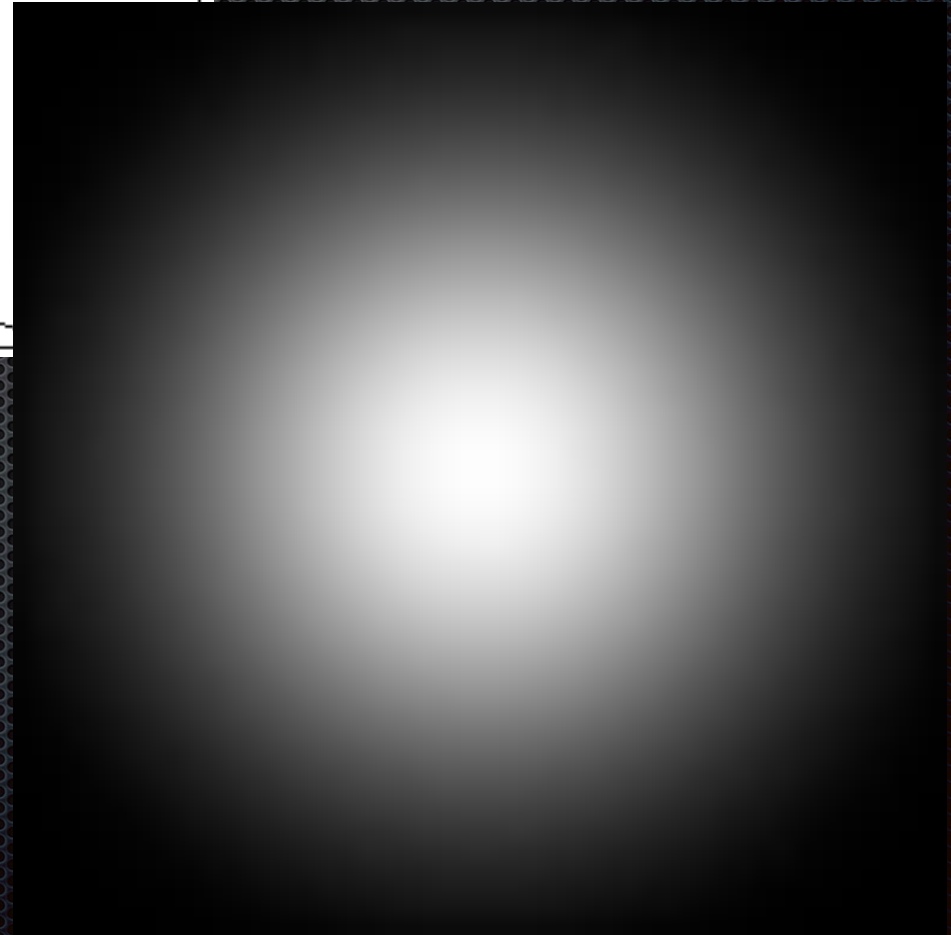
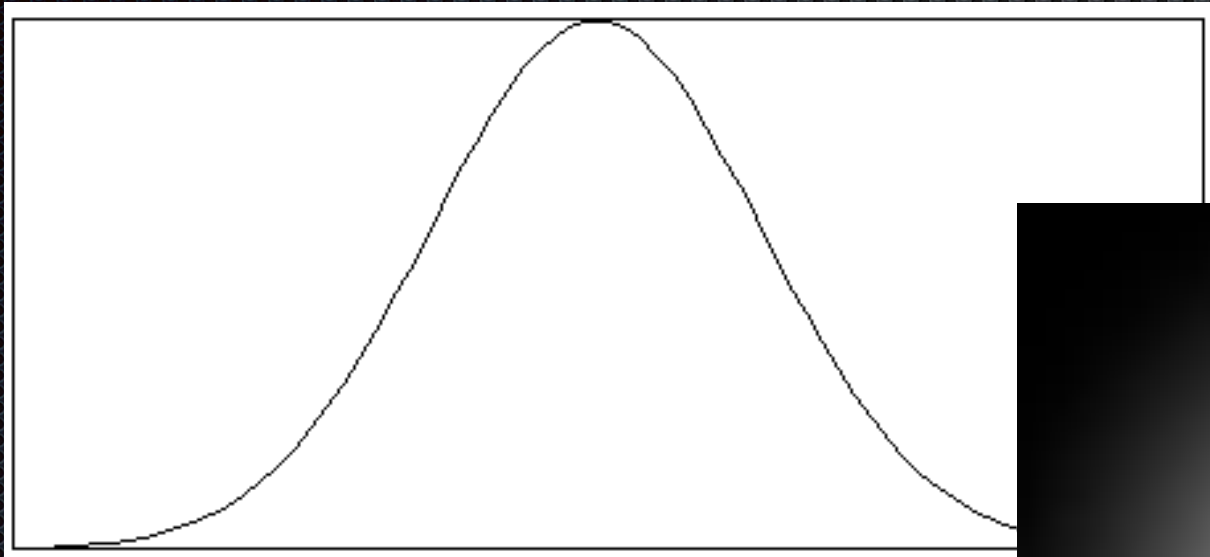
- ✦ this operator is known as convolution
- ✦ one convolves an image with a filter

More complicated filters

Gaussian Filter



Gaussian Filter



What would a Gaussian function look like expressed as a 3x3 mask?

3x3 Gaussian

- a Gaussian function expressed as a 3x3 mask

3x3 Gaussian

1/16

1	2	1
2	4	2
1	2	1

- a Gaussian function expressed as a 3x3 mask

Not all masks are 3x3

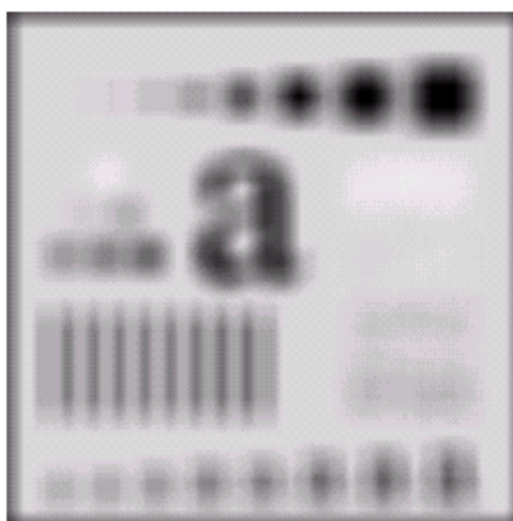
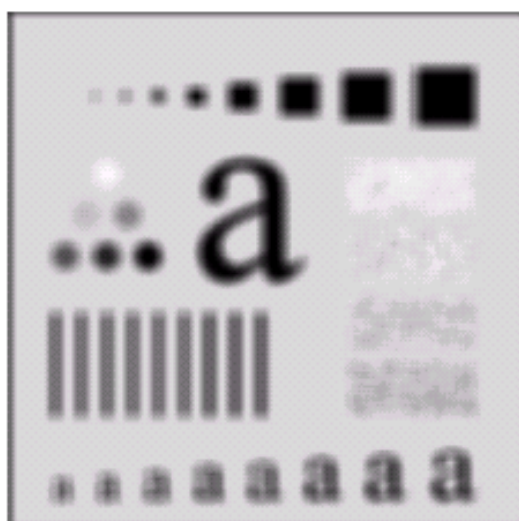
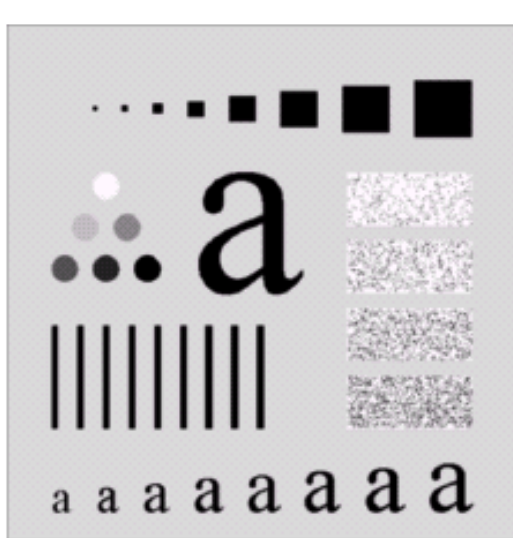
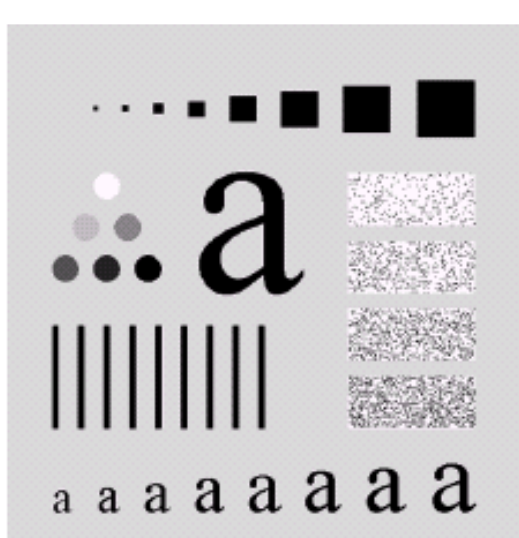
0.01	0.03	0.05	0.08	0.1	0.11	0.1	0.08	0.05	0.03	0.01
0.03	0.06	0.11	0.18	0.23	0.25	0.23	0.18	0.11	0.06	0.03
0.05	0.11	0.21	0.32	0.42	0.46	0.42	0.32	0.21	0.11	0.05
0.08	0.18	0.32	0.5	0.65	0.71	0.65	0.5	0.32	0.18	0.08
0.1	0.23	0.42	0.65	0.84	0.92	0.84	0.65	0.42	0.23	0.1
0.11	0.25	0.46	0.71	0.92	1	0.92	0.71	0.46	0.25	0.11
0.1	0.23	0.42	0.65	0.84	0.92	0.84	0.65	0.42	0.23	0.1
0.08	0.18	0.32	0.5	0.65	0.71	0.65	0.5	0.32	0.18	0.08
0.05	0.11	0.21	0.32	0.42	0.46	0.42	0.32	0.21	0.11	0.05
0.03	0.06	0.11	0.18	0.23	0.25	0.23	0.18	0.11	0.06	0.03
0.01	0.03	0.05	0.08	0.1	0.11	0.1	0.08	0.05	0.03	0.01

0.01	0.03	0.05	0.08	0.1	0.11	0.1	0.08	0.05	0.03	0.01
0.03	0.06	0.11	0.18	0.23	0.25	0.23	0.18	0.11	0.06	0.03
0.05	0.11	0.21	0.32	0.42	0.46	0.42	0.32	0.21	0.11	0.05
0.08	0.18	0.32	0.5	0.65	0.71	0.65	0.5	0.32	0.18	0.08
0.1	0.23	0.42	0.65	0.84	0.92	0.84	0.65	0.42	0.23	0.1
0.11	0.25	0.46	0.71	0.92	1	0.71	0.46	0.25	0.11	0.11
0.1	0.23	0.42	0.65	0.84	0.92	0.84	0.65	0.42	0.23	0.1
0.08	0.18	0.32	0.5	0.65	0.71	0.65	0.5	0.32	0.18	0.08
0.05	0.11	0.21	0.32	0.42	0.46	0.42	0.32	0.21	0.11	0.05
0.03	0.06	0.11	0.18	0.23	0.25	0.23	0.18	0.11	0.06	0.03
0.01	0.03	0.05	0.08	0.1	0.11	0.1	0.08	0.05	0.03	0.01

11 x 11 mask

Mask Sizes

- the larger the mask, the greater the effect



- averaging filter
- mask sizes:

no mask 3x3

5x5 9x9

15x15 35x35