IMPACT OF NEW FREQUENCY STANDARDS ON THE INTERNATIONAL TIME SCALES

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Elements of a time scale

Minimum requisite: 1 clock

Reliable
Stable and accurate in frequency
Accessible

Algorithm
Atomic standards performances

• Secondary standards

Cs standard (st. tube)  
Stability: $5 \times 10^{-14}$  
Accuracy: $1 \times 10^{-12}$  
@ 5 days

Cs standard (high perf.)  
Stability: $1 \times 10^{-14}$  
Accuracy: $5 \times 10^{-13}$  
@ 5 days

H- maser (active)  
Stability: < $2 \times 10^{-16}$  
Accuracy:  
@ 1 day

• Primary standards

Stability: few $10^{-16}$
Principles of calculation of UTC/TAI with the algorithm ALGOS

Time and frequency differences

Clock data → data → Time transfer

PFS data → Frequency corrections

EAL

- Optimized frequency stability
- No constrained to be accurate in frequency

TAI + (-n x s) → UTC

- Optimized frequency stability
- Accurate in frequency
EAL

Weighting algorithm

\[ EAL(t) = \sum_{i=1}^{N} w_i \left[ h_i(t) + h'_i(t) \right] \]

Prediction algorithm

- \( N \) is the number of atomic clocks
- \( w_i \) the relative weight of the clock \( H_i \).
- \( h_i(t) \) is the reading of clock \( H_i \) at time \( t \)
- \( h_i'(t) \) is the prediction of the reading of clock \( H_i \)

The weights of the clocks obey the relation:

\[ \sum_{i=1}^{N} w_i = 1 \]

\( w_{\text{max}} = 2.5/N \)
Prediction Algorithm

In two different intervals the clock ensemble can change.

The consequences:

**Time step**

![Graph showing time step]

**Frequency step**

![Graph showing frequency step]
The prediction term $h'_i(t)$ for clock $H_i$ is the sum of two terms:

\[ h'_i(t) = a_{i,I_i} + B_{ip,I_i}(t - t_i) \]

- $a_{i,I_i}$ is the time correction relative to EAL of clock $H_i$ at date $t_i$
- $B_{ip,I_i}$ is the frequency of clock $H_i$, relative to EAL, predicted for the period $[t_i, t]$

Linear model: the frequency is considered constant during the month
Prediction Analysis - Cesium Clock

3-year test period (2006-2008)

The difference (prediction-reality) of the EAL-CS Clock with standard deviation (red lines).

Considering 100 Cesium Clocks

The mean value of the difference (prediction-reality) for 100 EAL-CS Clock at 30 days is about 0.2 ns and the standard deviation is about 21 ns
Prediction Analysis - H-maser

3-year test period (2006-2008)

The difference (prediction-reality) of the EAL- H Maser

Considering 20 H-masers

The mean value of the difference (prediction-reality) after 30 days is about -30 ns and the standard deviation at 30 days is about 40 ns

As expected, linear model does not take care of the H-maser frequency drift
EAL compared to TT(BIPM)

EAL shows a frequency drift w.r.t. TT

$\frac{f(\text{EAL})-f(\text{TT(BIPM08)})}{10^4}$

4 x $10^{-16}$ / month
To show the influence of H-masers on EAL drift we consider TT as an independent reference:

Frequency drift on \( f(EAL) - f(TT) \) is: \( 4 \times 10^{-16} / \text{month} \)

Frequency drift on \( f(EAL_{\text{without Hmaser}}) - f(TT) \) is: \( 2.4 \times 10^{-16} / \text{month} \)

About 40% of EAL frequency drift is due to the H-masers.
New prediction algorithm for the H-masers

The obtained relation for the prediction algorithm:

$$h_i(t) = a_{i,I_i} + B_{ip,I_i} (t-t_i) + \frac{1}{2} C_{ip,I_{i+1}} (t_i-t_{i-1}) (t-t_i) + \frac{1}{2} C_{ip,I_i} (t-t_i)^2$$

Now the frequency is not constant on the interval!

- $a_{i,I_i}$ is the time correction relative to EAL of clock $H_i$ at date $t_i$
- $B_{ip,I_i}$ is the frequency of clock $H_i$, relative to EAL, predicted for the period $[t_i, t]$
- $C_{ip,I_i}$ is the frequency drift of the clock $H_i$, relative to EAL, predicted for the period $[t_i, t]$
- $C_{ip,I_{i+1}}$ is the frequency drift of the clock $H_i$, relative to EAL, predicted for the period $[t_{i-1}, t_i]$
Effect of the new prediction algorithm

The difference (prediction-reality) of the EAL-H maser using two different prediction techniques

Considering 20 (EAL-H-masers)

The mean value of the difference (prediction-reality) after 30 days is about 2 ns and the standard deviation at 30 days is about 45 ns
To show the influence of new prediction algorithm on EAL drift we use TT as independent frequency reference:

\[ \text{Frequency drift on } f(\text{EAL}) - f(\text{TT}) \text{ is: } 4 \times 10^{-16}/\text{month} \]

\[ \text{Frequency drift on } f(\text{EAL}_{\text{new prediction algorithm}}) - f(\text{TT}) \text{ is: } 3.2 \times 10^{-16}/\text{month} \]

About 20% of EAL frequency drift is due to the linear prediction used in ALGOS.
Difference of total weight of H-masers when a linear and a quadratic prediction is used. Weights increase. The drift of EAL increases when the prediction model is quadratic. Four months used for estimating the drift.

- Revise the weighting strategy
- Re-considering the use of clocks in the scale formation (caesium-based scale for long-term stability + H-masers for improving short term stability)
## Primary standards in TAI

<table>
<thead>
<tr>
<th>Primary Standard</th>
<th>Type /selection</th>
<th>Type B std. Uncertainty</th>
<th>Operation</th>
<th>Comparison with</th>
<th>Number/typical duration of comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT-CSF1</td>
<td>Fountain</td>
<td>$(0.5$ to $0.7)$x$10^{-15}$</td>
<td>Discontinuous</td>
<td>H maser</td>
<td>6 / 10 to 20 d</td>
</tr>
<tr>
<td>NICT-CSF1</td>
<td>Fountain</td>
<td>$(0.8$ to $1.5)$x$10^{-15}$</td>
<td>Discontinuous</td>
<td>UTC(NICT)</td>
<td>2 / 10-15 d</td>
</tr>
<tr>
<td>NIST-F1</td>
<td>Fountain</td>
<td>$0.3x10^{-15}$</td>
<td>Discontinuous</td>
<td>H maser</td>
<td>5 / 15 to 25 d</td>
</tr>
<tr>
<td>NMIJ-F1</td>
<td>Fountain</td>
<td>$3.9x10^{-15}$</td>
<td>Discontinuous</td>
<td>H maser</td>
<td>7 / 15 to 25 d</td>
</tr>
<tr>
<td>PTB-CS1</td>
<td>Beam /Mag.</td>
<td>$8x10^{-15}$</td>
<td>Continuous</td>
<td>TAI</td>
<td>10 / 30 d</td>
</tr>
<tr>
<td>PTB-CS2</td>
<td>Beam /Mag.</td>
<td>$12x10^{-15}$</td>
<td>Continuous</td>
<td>TAI</td>
<td>12 / 30 d</td>
</tr>
<tr>
<td>PTB-CSF1</td>
<td>Fountain</td>
<td>$0.9x10^{-15}$</td>
<td>Discontinuous</td>
<td>H maser</td>
<td>2 / 25 d</td>
</tr>
<tr>
<td>SYRTE-F01</td>
<td>Fountain</td>
<td>$(0.4$ to $0.6)$x$10^{-15}$</td>
<td>Discontinuous</td>
<td>H maser</td>
<td>8 / 10 to 30 d</td>
</tr>
<tr>
<td>SYRTE-F02</td>
<td>Fountain</td>
<td>$(0.4$ to $0.6)$x$10^{-15}$</td>
<td>Discontinuous</td>
<td>H maser</td>
<td>9 / 10 to 30 d</td>
</tr>
<tr>
<td>SYRTE-FOM</td>
<td>Fountain</td>
<td>$(0.7$ to $0.9)$x$10^{-15}$</td>
<td>Discontinuous</td>
<td>H maser</td>
<td>6 / 10 to 30 d</td>
</tr>
<tr>
<td>SYRTE-JPO</td>
<td>Beam /Opt.</td>
<td>$6.3x10^{-15}$</td>
<td>Discontinuous</td>
<td>H maser</td>
<td>12 / 10 to 30 d</td>
</tr>
</tbody>
</table>
By using the PFS we evaluate the systematic variation of EAL.
Uncertainty of Optical Clocks

In the future a new definition for the second will be required.

F. Rihele, Report to CCL-CCTF WG, June 2009
### Secondary Representations (CIPM 2006)

<table>
<thead>
<tr>
<th>µ-wave clock</th>
<th>Studied at</th>
<th>Value / Hz</th>
<th>Relative uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{87}\text{Rb}$</td>
<td>LNE-SYRTE, USNO, NPL, ...</td>
<td>6 834 682 610.904 324</td>
<td>$3 \times 10^{-15}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Optical clocks</th>
<th>Studied at</th>
<th>Value / Hz</th>
<th>Relative uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{199}\text{Hg}^+$</td>
<td>NIST</td>
<td>1 064 721 609 899 145</td>
<td>$3 \times 10^{-15}$</td>
</tr>
<tr>
<td>$^{88}\text{Sr}^+$</td>
<td>NPL, NRC</td>
<td>444 779 044 095 484</td>
<td>$7 \times 10^{-15}$</td>
</tr>
<tr>
<td>$^{171}\text{Yb}^+$</td>
<td>PTB</td>
<td>688 358 979 309 308</td>
<td>$9 \times 10^{-15}$</td>
</tr>
<tr>
<td>$^{87}\text{Sr}$</td>
<td>NMJ / U. Tokyo, JILA, LNE-SYRTE, PTB, ...</td>
<td>429 228 004 229 877</td>
<td>$1.5 \times 10^{-14}$</td>
</tr>
</tbody>
</table>

These clocks can be used alternatively to the Cs clock, but can never have an uncertainty better than the Cs clock.
Conclusions

The effect of the linear prediction algorithm has been studied

• on the Cesium clock it works well
• on the H-masers it does not work well

The impact of the H-masers on EAL frequency drift has been analyzed.

A new mathematical expression for the prediction has been found. It includes the treatment of H-maser frequency drift.

More work is needed to adapt the algorithm to the proposed frequency prediction.

PFS number and accuracy has increased; the accuracy of TAI is approaching $10^{-16}$.

The challenge is to be able to compare the optical frequency standards at the level of their performances.