On the Relativistic Theory of Earth Rotation

S.A. Klioner

Lohrmann Observatory, Dresden Technical University

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Relativity and Earth rotation: why to bother?

- Earth rotation is the only astronomical phenomenon
  - which is observed with a high accuracy

and

- which has no widely-used consistent relativistic model

- Modern theories of precession/nutation (IAU2000) are based on purely Newtonian theories with geodetic precession and nutation added in an inconsistent way

- Modern theories of rigid Earth nutation are intended to attain formal accuracy of 1 μas (expected relativistic effects are much larger)
How to model?

• Early attempts ( - 1986 )

  - One single reference system **BCRS** for both translational motion of solar system and for rotational motion of all the bodies…

  - The results were clearly physically inadequate coming from bad choice of coordinates:

    E.g. spurious annual variations in LOD with an amplitude of 75 µs…

Reason: “bad” coordinates that provide no analogy of Newtonian tidal forces at the post-Newtonian level

“Better” coordinates are clearly needed: **GCRS**
How to model?

• More sophisticated way (1986 - )

  - A physically adequate local GCRS

  - Still some coordinates, but chosen in such a way that the influence of external gravitational fields is as small as possible:

    full analogy of Newtonian tidal forces at the post-Newtonian level
Main goal of the project

• Derivation of a new consistent and improved precession/nutation series for a rigidly rotating multipole model of the Earth in the post-Newtonian approximation of general relativity

• using post-Newtonian definitions of:
  - potential coefficients
  - moment of inertia tensor

• dynamical equations in the GCRS

• correct relativistic time scales

• rigorous treatment of the geodetic precession and nutation
Equations of rotational motion in the GCRS

- Post-Newtonian equations of rotational motion in the GCRS (Damour, Soffel, Xu, 1993, Klioner, Soffel et al 1996-)

\[
\frac{d}{dT_{CG}} \left( C^{ab} \omega^b \right) = \sum_{l=1}^{\infty} \frac{1}{l!} \varepsilon_{abc} M_{bL} G_{cL} + L^a(C, \omega, \Omega_{\text{iner}}) + \ldots
\]

The last terms is the Coriolis torque from the relativistic precessions:

\[
\Omega_{\text{iner}}^a = -\frac{3}{2c^2} \varepsilon_{aij} v^i_E \frac{\partial}{\partial x^j} w_{\text{ext}}(x_E) + \frac{2}{c^2} \varepsilon_{aij} \frac{\partial}{\partial x^j} w_{\text{ext}}^i(x_E) - \frac{1}{2c^2} \varepsilon_{aib} v^i_E G_b
\]

geodetic precession

Lense-Thirring precession

Thomas precession (negligible)
Rigidly rotating multipoles in the GCRS

• Klioner, Soffel, Xu, Wu, 2001 (based on many previous results):

- Post-Newtonian equations of rotational motion in the GCRS

\[
\frac{d}{dT_{CG}}\left(C^{ab}\omega^{b}\right) = \sum_{l=1}^{\infty} \frac{1}{l!} \varepsilon_{abc} M_{bL} G_{cL} + L^{a}(C, \omega, \Omega_{\text{iner}}) + \ldots
\]

- Rigidly rotating multipoles:
  several assumptions on the multipole moments and
  the tensor of inertia

\[
C^{ab} = P^{ac} P^{bd} \tilde{C}^{ab}, \quad \tilde{C}^{ab} = \text{const},
\]

\[
M_{a_{1}a_{2}...a_{l}} = P^{a_{1}b_{1}} P^{a_{2}b_{2}} ... P^{a_{l}b_{l}} \tilde{M}_{b_{1}b_{2}...b_{l}}, \quad \tilde{M}_{b_{1}b_{2}...b_{l}} = \text{const}, \quad 1 \geq 2,
\]

\[
\omega^{a} = \frac{1}{2} \varepsilon_{abc} P^{db} \frac{d}{dT_{CG}} P^{dc}
\]

\[
P^{ab}(T_{CG}) \text{ is an orthogonal matrix defining the orientation of the ITRS in GCRS}
\]
Numerical code: an overview

• Fortran 95, about 20000 lines
• careful coding to avoid excessive numerical errors
• two numerical integrators: ODEX and ABM with dense output
• automatic accuracy check: forth and back integrations
• any available arithmetic: 64 bit, 80 bit, 128 bit
• extended-precision arithmetic for precision-critical operations (switchable)
• the STF code has been automatically generated by Mathematica
• baseline:

  ODEX with 80 bits on Intel architecture gives errors <0.001 μas for 150 years

• in the Newtonian limit reproduces SMART within the errors of the latter
• performance:

  Newtonian case: 2.2 sec per yr
  all the relativity on: 8.8 sec per yr
Long-term numerical integrations

A first step to a relativistic theory of precession …

A long solar system ephemeris is needed:

DE404 is used to check the situation: 6000 yr
Noise from the downgrade: DE404 vs DE403
Effects of the post-Newtonian torque

6000 years with DE404
Effects of the post-Newtonian torque

6000 years with DE404:

\[
\Delta \varphi = -146.67 + 640.60 t - 7921.70 t^2 \\
+ 11375.50 t^3 + 1308.23 t^4 \\
\Delta \psi = -0.61 - 1560.31 t + 3.22 t^2 + 2.13 t^3 - 0.06 t^4 \\
\Delta \omega = -0.23 + 0.014 t - 7.99 t^2 + 0.64 t^3 + 0.08 t^4
\]

in μas, \( t \) is in thousand years
Effects of the post-Newtonian torque

6000 years with DE404: minus 4th-order polynomial
Effects on the LOD

6000 years with DE404:

\[ \dot{\varphi} + \dot{\psi} \cos \omega = \Omega \]
Effects on the LOD

6000 years with DE404:

\[ \dot{\phi} + \dot{\psi} \cos \omega = \Omega \]

\( \text{rad/s} \)

\[ \left| \delta \Omega_N \right| < 450 \, \mu \text{as} / \text{d} \quad \Rightarrow \quad \Delta \text{LOD}_N < 30 \, \mu \text{s} \]
Effects on the LOD

6000 years with DE404:

\[ \dot{\varphi} + \dot{\psi} \cos \omega = \Omega \]

\[ \Omega_{pN} - \Omega_{\text{Newt}} < 0.8 \mu\text{as} / d \quad \Rightarrow \quad \Delta LOD_{pN} < 0.06 \mu s \]