POLARIZATION SYNTHESIS WITH THE AST

1. Introduction

Dick Manchester asked me to think about this problem presumably because I had been involved with attempts to measure polarization to high accuracy. However my experience with interferometric measurements of polarization is very limited and these notes should be read in that light. They represent my first attempt to acquaint myself with the possibilities. A detailed consideration of the effects of errors is needed before possible systems can be realistically compared.

Because of my lack of experience of interferometric polarization measurements I wrote to Kurt Weiler for advice. His initial reply to my questions has influenced these notes but he should not be blamed for anything here.

2.1 Restrictions, and a General Statement

Assume that each aerial is equipped with two (phase stable) receiving channels and restrict consideration to the case where the polarizations accepted by these two channels are opposite polarizations. Then for any pair of aerials a,b accepting polarizations p,#p and q,#q the four (complex) correlations between the accepted signals, namely

$$<S_p^a \; S_q^b> \quad (1a)$$

$$<S_p^a \; S_{\#q}^b> \quad (1b)$$

$$<S_{\#p}^a \; S_q^b> \quad (1c)$$

$$<S_{\#p}^a \; S_{\#q}^b> \quad (1d)$$

contain all the information needed to map all four Stokes parameters. This statement is evidently true* whether or

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$$\langle S^a_p \, S^b_{\#q} \rangle \qquad (1b)$$

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not the pairs of polarizations accepted by the two aerials are the same, i.e. whether or not p = q.

While this sounds a powerful general statement it is necessary to notice that the different Stokes parameters are derived from these four correlations in different ways so that imperfections can affect the accuracy of some Stokes parameters more than others. Since in addition the magnitudes of the different Stokes parameters are usually quite different (e.g. for continuum sources $I \gg Q$, $U \gg V$) special considerations are needed if it is desired to measure all the Stokes parameters with reasonable accuracy.

2.2 Both Aerials Accepting the Same Pair of Opposite Polarizations p,#p

(a) General

For this case (q = p) the four correlations become

$$\langle S^a_p \, S^b_p \rangle \qquad (2a)$$

$$\langle S^a_p \, S^b_{\#p} \rangle \qquad (2b)$$

$$\langle S^a_{\#p} \, S^b_p \rangle \qquad (2c)$$

$$\langle S^a_{\#p} \, S^b_{\#p} \rangle \; . \qquad (2d)$$

If you are familiar with the Poincare sphere representation of polarization it is easy to consider this case quite generally. Otherwise you might skip to the particular cases discussed in (b) and (c). On the Poincare sphere (Fig. 1) LH and RH circular polarization are represented by the 'north' and 'south' poles while the various linear polarizations appear around the 'equator' with the position angle of the linear polarization being equal to half the azimuthal angle ('longitude'). The pair of opposite polarizations p,#p accepted by the aerials lie at either end of a diameter of the sphere.

The correlations between like polarizations (2a and 2d) give directly the Fourier components on the sky for the polarizations p and #p. The sum of (2a) and (2d) thus gives the Fourier component of the total intensity, I. The difference (2a) - (2d) gives the Fourier component of a Stokes-like parameter corresponding to p - #p. If p and #p are two of the cardinal polarizations (LH and RH circular, linear at position angles 0°, 45°, 90°, 135°) then this difference will be one of the Stokes parameters Q, U, V - otherwise it is some linear combination of these Stokes parameters. Notice that this value is derived as the difference of two quantities. Therefore if it is a small quantity (relative to I) it will not be determined with great accuracy.

The correlations between opposite polarizations (2b and 2c) are affected only by polarized radiation whose representation on the Poincare sphere lies on the great circle perpendicular to the diameter joining p and #p. All polarizations around this great circle have an equal effect on the amplitude of the correlation, their position around the circle merely affects the phase. Both cross-polarized correlations (2b) and (2c) are needed to separate out the effects on the phase produced by the type of polarization and by the direction of arrival of the radiation. (For a point source in a known direction (2c) can be derived from (2a).) Because these cross-polarized correlations contain no contribution from the unpolarized radiation they provide an accurate measure of the Fourier components of polarizations around this great circle even when such polarization is small compared to I.

(b) Opposite Circular Polarizations

If we specialise to the case where both aerials accept LH and RH circular polarization the correlations (2) may be written

$$<S^a_{LH}\, S^b_{LH}> = I^\dagger + V^\dagger \quad (3a)$$

$$<S^a_{LH}\, S^b_{RH}> = Q^\dagger - jU^\dagger \quad (3b)$$

$$<S^a_{RH}\, S^b_{LH}> = Q^\dagger + jU^\dagger \quad (3c)$$

$$<S^a_{RH}\, S^b_{RH}> = I^\dagger - V^\dagger \quad (3d)$$

where

$$\left.\begin{aligned} I^\dagger &= a + jb\,, \\ Q^\dagger &= c + jd\,, \\ U^\dagger &= e + jf\,, \\ V^\dagger &= g + jh\,, \end{aligned}\right\} \quad (4)$$

(a,b,c,d,e,f,g,h all real),

are the (complex) Fourier components corresponding to the Stokes' parameters I, Q, U, V. (In equation (3) and subsequent such equations the constant multipliers are omitted to simplify the appearance.) On the Poincare sphere (Fig. 1) the points p,#p now coincide with the poles LH, RH, and the truth of the above relations follows readily from the discussion in (a) above.

The Fourier component of the total intensity ($I^\dagger$) is given directly by the sum of (3a) and (3d). The Fourier components of the linear polarization ($Q^\dagger$ and $U^\dagger$) are given by (3b) and (3c): since these correlations contain no response to the unpolarized components $Q^\dagger$ and $U^\dagger$ are well determined even when they are small compared to $I^\dagger$. However $V^\dagger$, which is given by the difference of (3a) and (3d), will obviously be poorly determined when it is small compared to $I^\dagger$.

The system of opposite circular polarizations has been advocated by Conway and Kronberg (1969). It is used at Jodrell Bank, Green Bank and on the VLA. If one is interested in measuring total intensity and linear polarization <u>only</u>

then it seems the obvious theoretical favourite of the class being discussed in Section 2.2. In fact for synchrotron sources V/I is so small that only one of (3a),(3d) need be measured, thus saving a correlator. The system is obviously useless for measuring small degrees of circular polarization such as are found in synchrotron sources: it would be suitable for measuring the very high degrees of circular polarization found in some line sources. It has the disadvantage that circularly polarized feeds are normally made by combining two orthogonal linear polarizations. Whether this is done at r.f. or i.f. there is liable to be less isolation between the two circular polarizations than between the constituent linear polarizations; if the combining is done at r.f. there might also be increased loss.

(c) Opposite Linear Polarizations

If the accepted polarizations are in position angles 0° and 90° the four correlations become

$$\langle S_0^a\ S_0^b \rangle = I^\dagger + Q^\dagger \qquad (5a)$$

$$\langle S_0^a\ S_{90}^b \rangle = U^\dagger + jV^\dagger \qquad (5b)$$

$$\langle S_{90}^a\ S_0^b \rangle = U^\dagger - jV^\dagger \qquad (5c)$$

$$\langle S_{90}^a\ S_{90}^b \rangle = I^\dagger - Q^\dagger\ . \qquad (5d)$$

The two parameters specifying linear polarization, $Q^\dagger$ and $U^\dagger$, are now derived from different types of equations: $Q^\dagger$ is derived from (5a) minus (5d) and will be poorly determined if it is small compared to $I^\dagger$, while $U^\dagger$ is derived from (5b) and (5c) and will be well determined if (as is usual) $U^\dagger \gg V^\dagger$. To use a system employing opposite linearly polarized feeds to measure both $Q^\dagger$ and $U^\dagger$ with high accuracy (assuming $I^\dagger \gg Q^\dagger$, $U^\dagger \gg V^\dagger$) it is necessary to make an additional set of measurements in which the accepted polarizations are at position angles 45° and 135°. The correlations then become

$$\langle S_{45}^a\ S_{45}^b \rangle = I^\dagger + U^\dagger \qquad (6a)$$

$$\langle S_{45}^a\ S_{135}^b \rangle = Q^\dagger + jV^\dagger \qquad (6b)$$

$$\langle S_{135}^a\ S_{45}^b \rangle = Q^\dagger - jV^\dagger \qquad (6c)$$

$$\langle S_{135}^a\ S_{135}^b \rangle = I^\dagger - U^\dagger \qquad (6d)$$

and $Q^\dagger$ can be derived with high accuracy from (6b) and (6c). This is an obvious disadvantage of the 'opposite linear polarizations' system relative to the 'opposite circular polarizations' system if the aim is to measure total intensity and linear polarization only to high accuracy.

The 'opposite linear polarizations' arrangement seems to be the best available for measuring small degrees of circular polarization (Weiler and Raimond 1976, 1977; Ryle, Odell and Waggett 1975). For a point source in a known direction one can allow for the effect on phase caused by the direction of arrival and force $U^\dagger$ and $V^\dagger$ in (5b), (5c) to be real. Then $V^\dagger$ is given by the quadrature component of (5b) or (5c) and can be determined to high accuracy even when $U^\dagger \gg V^\dagger$. However for an extended source the situation is more difficult. Using (4) to expand (5b) and (5c) and collecting terms

$$\langle S_0^a\ S_{90}^b \rangle = (e - h) + j(f + g) \qquad (7b)$$

$$\langle S_{90}^a\ S_0^b \rangle = (e + h) + j(f - g) \qquad (7c)$$

and if $e \gg h$ and $f \gg g$ then g and h, which must be determined from differences, will be poorly determined. Weiler and Raimond do not appear to discuss this problem. A possible, but messy, way around it is to choose an orientation for the crossed linear polarizations (possibly different for every projected baseline!) such that there is no correlation contributed by the linear polarization (i.e. the linear component lies in the direction of one of the accepted polarizations).

2.3 Each of the Two Aerials Accepting a Different Pair of Opposite Polarizations ($q \neq p$)

Because of imperfections systems intended to have $q = p$ actually fall into this category, but at this stage such imperfections are not being considered in detail. However at Westerbork, for all standard observations, a system with $q \neq p$ is deliberately chosen (Weiler 1973). In each pair of aerials being correlated the accepted polarizations are in position angles 0° and 90° in one aerial and 45° and 135° in the other aerial. The four correlations then become

$$<S_0^a \, S_{45}^b> \; = I^\dagger + Q^\dagger + U^\dagger + iV^\dagger \qquad (8a)$$

$$<S_0^a \, S_{135}^b> \; = I^\dagger + Q^\dagger - U^\dagger - iV^\dagger \qquad (8b)$$

$$<S_{90}^a \, S_{45}^b> \; = I^\dagger - Q^\dagger + U^\dagger - iV^\dagger \qquad (8c)$$

$$<S_{90}^a \, S_{135}^b> = I^\dagger - Q^\dagger - U^\dagger + iV^\dagger \, . \qquad (8d)$$

As is obvious from the geometry, or from the equations, all four correlations are likely to be of comparable magnitude - each responds to the unpolarized component. This is one of the basic reasons* for choosing the system since it allows the gain and phase errors of each channel to be readily calibrated by observations of (unpolarized) sources. This contrasts with the systems discussed in Section 2.2 where the correlations between opposite polarizations will be small, making it difficult to calibrate the gain and phase of those channels.

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*Additional advantages claimed:

(i) Feeds do not have to be rotated.
(ii) Conventional gain and phase calibration procedures for total intensity observations are also suitable for polarization observations.
(iii) Reduction procedures are simple and straightforward.

It is clear from the equations (8) that $I^\dagger$, $Q^\dagger$, $U^\dagger$ and $V^\dagger$ all have comparable influences on all correlations. If $I^\dagger$, $Q^\dagger$, $U^\dagger$ and $V^\dagger$ were all of comparable magnitude this would seem an ideal system. However when, as in most cases, $I^\dagger \gg Q^\dagger$, $U^\dagger \gg V^\dagger$ the system yields $Q^\dagger$ and $U^\dagger$ with only moderate accuracy and is quite useless for measuring $V^\dagger$. It seems to have proved a satisfactory system at Westerbork - the automatic calibration feature is clearly a great advantage. However for accurate polarization work Westerbork converts to the system discussed in Section 2.2(c). Hence any other instrument adopting the Westerbork system presumably should be capable of conversion in some fashion for occasions when accurate measurements of polarization are needed.

3. Considerations for the AST

Let's have a preliminary look at these generalities in relation to some of the specific design ideas for the AST.

3.1 The Altaz Telescope

If the altaz telescope output is to be correlated with outputs from equatorially mounted telescopes then for all the systems using linear polarization the feed on the altaz telescope must be rotated continuously during the observations to compensate for the changing parallactic angle. For the system using circular polarizations the altaz feed need not rotate - a phase compensation could be added in the reduction program.

Using two such very different aerials in an interferometer might create some extraneous polarization responses. However the fact that the polarization rotates relative to the 64-m aerial should at least allow such imperfections to be discovered.

3.2 Correlations between all possible Pairs of Aerials

This is not done at Westerbork. To do this with the Westerbork polarization system would require every aerial to accept linear polarizations in P.A.s 0° and 90°, and also in P.A.s ±45°! These apparently conflicting requirements can in fact be met by equipping the aerials with feeds accepting linear polarizations in position angles 0° and 90°, and then, after some amplification, splitting off some of the power to provide an additional pair of outputs in which the signals are combined in phase and in anti-phase so as to correspond to accepting polarizations in position angles ±45°. When you realize the truth of this you are forced to question the point of the Westerbork system. But their claim is simply that with their system all the correlations are of comparable magnitude (because they contain a response to the unpolarized component) and that this facilitates the calibration of the gain and phase errors for each channel.

To adopt this system would require the provision of high quality 0°,180° hybrids (at i.f.?) for every aerial of the stand alone array. Problems arising from possible imperfections of these hybrids need examination. If this system were adopted it could readily be converted to the cross-polarized mode (Section 2.2(c)) for especially accurate measurements of polarization.

3.3 Research Programs

It is quite obvious that no system is ideal for measuring all types of polarization. The 'standard' polarization system adopted presumably should be one which will suit the majority of observations. However, unless one has to pay a very high penalty in some direction it would seem logical to adopt a system which can - albeit with some effort - be converted so as to measure any nominated types of polarization with high accuracy.

3.4 Summary of Advantages and Disadvantages

Table 1 summarizes my present understanding of the relative merits of the different systems.

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TABLE 1. Polarization Systems for the AST

System	Advantages	Disadvantages	Questions
Opposite circular polarizations	(i) No feeds need to be rotatable, not even on the altaz dish. (ii) Linear polarization is determined with high accuracy even when $I \gg Q,U$. (iii) When $V \ll I$ only three correlations need be measured.	(i) Cannot make accurate measurements of V. (ii) Calibration difficult because of low signals in cross-polarized channels. (iii) Less isolation between channels than in a linearly polarized system (?).	Feeds more lossy than linearly polarized feeds?
Opposite linear polarizations - all dishes the same	(i) Best system for measuring small values of V.	(i) Feed on altaz dish must rotate to compensate for changing parallactic angle. (ii) Accurate measurements of linear polarization require all feeds to be rotatable through 45°. (iii) Calibration difficult because of low signals in cross-polarized channels.	Can disadvantage (ii) be overcome by combining signals in position angle 0° and 90° in a hybrid to form signals in position angle 45° and 135° ?
Opposite linear polarizations. Polarizations in second aerial at 45° to those in first (Westerbork system).	(i) Easy to calibrate.	(i) Feed on altaz dish must rotate to compensate for changing parallactic angle. (ii) When every element is to be correlated with every other element a further set of hybrids is needed. (iii) Does not measure Q and U with high accuracy; hopeless for V.	How realistic is it to generate the signals in position angles 45°, 135° by combining the signals in position angles 0°, 90° in a hybrid?

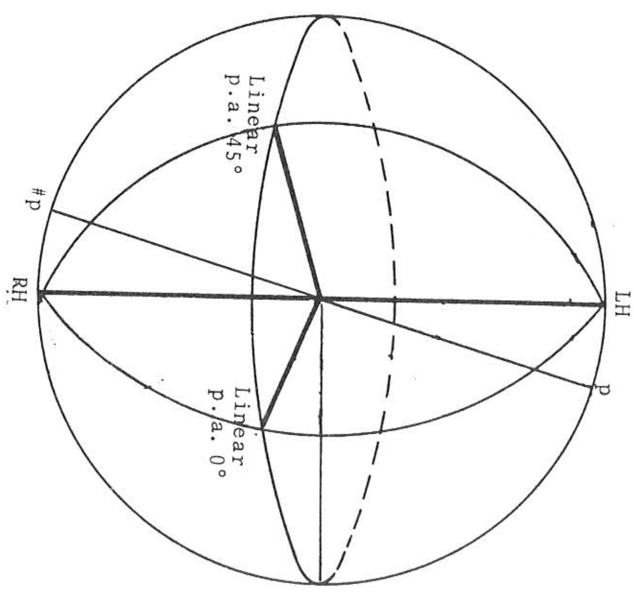

Fig. 1 - The Poincare sphere.