~~ASTDOC21~~-29FEB79-RDE

What Size Antennas for the AST ?

The antenna diameter is one of the critical parameters in the array configuration and cost budgetting but it is one for which there is not yet a consensus of opinion. I have tried to summarise here some of the arguments which affect this decision and to present plots to elucidate the relations between size and other parameters.

The main factors which influence the design are; the cost, the sensitivity and the speed of the array. Total cost includes; (i) the cost of the antennas, which depends on the number and diameter; (ii) the cost of front-ends, which depends on the number of elements, and (iii) the cost of the backend, which is proportional to the number of interferometer correlations. The sensitivity depends on diameter and the number of correlations. The speed depends on the number of instantaneous correlations. The optimum design consequently depends on a balance between these main factors and I will discuss them first. There are additional factors which must also be considered and these are covered later. A one kilometer long E-W array is assumed for all the following discussion.

1. Cost

There is a large component of the total cost which depends on the diameter and number of antennas. This component can be expressed as
Cost = n.A(d) + n.B + m.C
where: n is the number of antennas,
m the number of correlations,
A(d) the cost / antenna as a function of diameter,
B the cost / frontend and
C the backend cost / correlator.

From the revised AST proposal we have

$A(d) = 0.25 \left(\frac{d}{15}\right)^{2.5}$ M \$ (antenna cost)

B = 0.13 M \$ (frontend cost)

C = 0.05 M \$ (backend cost)

However, the recent budgetary estimates from MAN and Mitsubishi give higher dish costs with weaker diameter dependance. The multiple order Alt/Az estimate for 15 and 20 m antennas from Mitsubishi can be modelled by

$$\text{Cost} = 0.26 + 0.37 \left(\frac{d}{15}\right)^{1.3} \quad \text{M \$}$$

(The transporter cost is taken from the MAN quote since this is more in line with the kind of transporter envisaged). The constant part of this estimate is caused by the drive and control system and the transporter and presumably the high contribution of the drive and control system to the total cost comes from the high precision requested. This formula may break down for diameters much greater than those used for the quotes (15 - 20m).

Figure 1 shows the dependance of the cost on both diameter and number of elements for the Mitsubishi dish cost model with the AST proposal values for B and C. In this plot, $m = \frac{1}{4}n^2$. Arrays with different numbers of correlations are discussed later. In addition to this component of cost which depends on the number and size of the dishes there is a constant component due to site development and construction for which the AST proposal estimate is \$4.4 M.

2. Sensitivity

The point source sensitivity of the array in a given time is $\sigma_s \propto \sqrt{m}\, d^2$. If the number of elements is large and all the correlations are taken $m = \frac{1}{2}n(n-1)$ so that $\sigma_s \sim nd^2$ ie the sensitivity is proportional to the total collecting area regardless of how the antennas are distributed.

Figure 2 shows the dependance of sensitivity on n and D for an array with $n^2/4$ correlations. The sensitivity curves are normalised to the sensitivity of the 14 x 15m proposal (assuming it had used $n^2/4$ correlations).

2.1 Cost minimisation for given sensitivity

By combining Figures 1 and 2 we can look for the minimum-cost array for a given sensitivity, as was done in ASTDOC2 (17/9/75). A reanalysis using the revised AST cost estimate is shown by the full line in Figure 3 and indicates a broad minimum centred on 25m (the actual AST proposal of 14 x 15m dishes violates the assumption, $m = n^2/4$, of this analysis since it did not attempt to maximise the number of correlations). However, if we use the Misubishi dish cost model the dependance of diameter is weaker than d^2 so no minimum can occur (dashed line) and the larger the dish the better.

3. Time for full synthesis

More elements give more instantaneous interferometers and consequently a shorter time for full synthesis. On this argument alone the optimum array would give full synthesis in one day but this requires at least 20 elements so a compromise is needed to get out of the unacceptably expensive part of Figure 1. Figure 4 shows the time for full synthesis as a function of n and d. Full synthesis is assumed to be 12 hours tracking in HA with a spacing increment of 0.75d up to a maximum baseline of 1 km.

3.1 How often will full synthesis be used?

Arguments depending on the time required for full synthesis only apply to observations where full synthesis is required ie when the full field of view must be mapped. This situation may arise either because the astronomical objectives require full mapping or because of confusion by grating lobes from other sources in the field. There are also two ways in which less than full synthesis can be done: (i) full 12 hour observations on less days or (ii) less than 12 hour observations at various hour angles. Both (i) and (ii) provide valid sampling for sources smaller than the full field. Case (i) usually applies when the object to be mapped is complex but completely contained within an area less than the primary beam. In this case it can be mapped completely when it is all inside its first grating lobe and no further correction has to be applied. Thus an object of diameter 1/x of the primary beam can be mapped in a time 1/x of that given in Figure 4. Case (ii) also removes the oversampling in hour angle which occurs for objects of size less than the primary beam and can be used to make a very large increase in speed for smaller sources. eg for objects with diameter 1/x of the primary beam, x different objects can be done in each 12 hour observation. In this case the UV coverage contains sharp radial discontinuities which have to be corrected eg by a CLEAN procedure. Case (ii) is also applicable if the problem is to detect or measure the position of point sources and in this case there is considerable freedom in the choice of hour angles or spacings.

The relative importance of full synthesis and the partial synthesis options (i) and (ii) depends on the astronomical objectives. At 21 cm the effects of confusion will force most high sensitivity observations into the full synthesis mode but at higher frequencies, and for line observations, this will often not be required. Using statistics from Westerbork for the distribution of observing time in 1977 we have :

Full synthesis	21%
(i) < 4 day obs.	15%
(ii) <12 hour obs.	44%
Calibration	20%

This is probably fairly representative of the expected distribution for the AST.

3.2 Field of View

The time required for full synthesis is intimately related to the field of view. In Figure 4 the effect of the decreasing field of view with increasing dish diameter is included and this weakens the dependance on n. This aspect is discussed further in Section 5.

4. Configuration

An EW array of 1 km maximum baseline is assumed throughout and the only question I will discuss is how to distribute the elements since this has some effect on all the previous discussion. Two aspects have to be considered :

(i) the number of correlations m, which influences the sensitivity (~ m) and the backend cost (~ m);

(ii) the number of non redundant spacings which determines the time for full synthesis.

In all the previous plots I have used the least redundant regular array in which n/2 dishes with separation ℓ are correlated with n/2 dishes with separation $n\ell$, giving $n^2/4$ correlations.

eg   x   x   x   xxxxx

This is the scheme used in the Cambridge 5 km array. For this configuration multiple day synthesis can be done with half the antennas moveable and no spacings need be duplicated. At one extreme from this we have the all fixed element grating array giving n-1 different spacings and at the other extreme a minimum redundancy array (Moffet IEEE AP-16, 172) which has an irregular distribution of spacings to get as close as possible to the theoretical limit of $\frac{1}{2}n(n-1)$ correlations. Fig. 5 shows the effects of some different configurations on the (n,D) plot for constant sensitivity.

The fixed grating array has no moveable elements so n must be large enough to do full synthesis in 1 day whereas the minimum redundancy array has to have n-1 moveable elements to do multiple day synthesis. The choice between these is mainly a question of how many moveable elements we want.

4.1 How many moveables:

Moveable telescopes cost more but their use can result in a larger number of correlations so that the total number of elements is reduced. In order to look for the best compromise between these two extremes, I have calculated the cost as a function of the ratio of the number of fixed to the number of moveable telescopes, assuming the total number of elements required is varied to keep the same sensitivity. For this case we ha

$$\text{Cost} = n\,A(d) + n\,B + m\,C + n\,x\,D$$
$$m = n^2\,x\,(1-x)$$

and the sensitivity, $\sigma_s = d^2\, n \sqrt{x(1-x)}$ = constant

where x is the fraction of elements which are moveable and D is the extra cost for a moveable antenna.

Using the values of A, B and C discussed in 1) and taking the excess cost of a moveable, D, as $0.2 M, I obtain the results given in Figure 6. From this plot it is clearly an advantage to have > 30% of the elements moveable even though this increases the cost of the individual elements. It is an even greater advantage to go to the full minimum redundancy array although this advantage has decreasing importance as the number of elements increases. The main disadvantage of the moveable telescopes is the increased operational load (more to move and to calibrate). Another possible disadvantage would be the decreased baseline stability of a moveable telescope but this is offset by the fact that any telescope or receiver errors are distributed over more telescopes if there are more moveables. Furthermore, no particular difficulties have been found in using moveable telescopes at either Cambridge or Westerbork.

4.2 The case for minimum redundancy

From Figure 5 it would appear that a minimum redundancy array has a clear advantage, especially for small numbers of elements, but there are some extra complications which have to be considered.

(i) The multiple day minimum redundancy solutions have only been investigated for a few cases (Blum, Ribes and Biraud A & A 41, 409). Their results indicate that multiple day solutions are at least as good as the one day solutions and this assumption is used in Figure 5. Clearly this would have to be checked by finding the actual solutions for any proposed array.

(ii) Multiple day minimum redundancy solutions also have bad operational consequences because the optimum solution for an n-day map may not include any configuration of a less than n day map (eg an optimum 1 day synthesis map could not be done as part of a set of configurations needed for a 4 day map).

5. Field of View

The larger field of view obtained with smaller dishes is potentially the greatest advantage of an array of a larger number of smaller elements so it is worth looking carefully at this point. If we assume that the smaller element array has enough elements to achieve the same sensitivity (Figure 2) as the large element array then the time to map a given area goes as D^{-2} giving an enormous advantage to the small antenna array. This is of course just the kind of mapping advantage that arrays have over single dishes and it cannot be offset unless the larger elements are equipped with multiple feeds. If we make a comparison for speed instead of sensitivity (ie we only want complete synthesis and do not care about the final sensitivity) then the time taken goes as D^{-3} ! With such an enormous advantage going for the small dish array one might well ask why even consider larger dishes. There are three major reasons :

(i) Figure 3 shows that a very heavy price paid for going to dishes that are too small;

(ii) the astronomical objectives only rarely require mapping of very large areas and

(iii) the required dynamic range is larger for smaller disnes (see 7).

5.1 Mapping speed for constant cost

If we operate with a constant cost constraint the situation is changed completely. From Figure 1 we could compare a 6 x 20m array with a 7 x 15m array. Using Figure 4 we see that the time for full synthesis is about the same so that the time advantage to map a given area is $15^2 / 20^2 = 1.8$. However, Figure 2 shows that we are down in sensitivity by a factor of 1.5 so we will need to spend 1.5^2 times as long to map the area to the same sensitivity so in this example the larger element array is actually faster by a factor $1.5^2/1.8 = 1.25$.

5.2 Astronomical objectives of large field of view

This is harder to answer since it depends on the kind of astronomy to be done and the actual fields of view in question. Very large area mapping is usually required for the survey type observation and if these are sensitivity limited the consideration in 5.1 will apply. The only case where the small elements have a real advantage is when the survey is not sensitivity limited. It is also useful to note that in the sensitivity limited survey mode the 64m dish is just about as good as any of the proposed AST configurations as long as confusion is not a problem.

The other case for large fields of view is when the object to be mapped is large. In the case of the WSRT, mapping of objects larger than the primary beam of the 25m antennas occurs about 10% of the time. Perhaps the greatest but completely unpredictable advantage of the larger field is the increased chance of accidental discoveries.

6. Cassegrain Optics

Use of the Cassegrain focus has the advantage of minimising system noise and allowing heavier receivers but small diameter Cassegrain systems cannot have as high an efficiency as larger diameter systems at the lower frequencies.

7. Dynamic Range and Antenna Pointing Specifications

A very substantial increase in antenna cost may have been incurred by the very tight stability specifications on the antennas. Furthermore, it has not been demonstrated that the specification on the electronic stability can be met. These specifications have been set by the requirement of reaching the sensitivity limit of the telescope in the presence of field sources. The required dynamic range is related to the expected flux density, S, of field sources. From the source count relations we have $N(S) \propto S^{-\alpha}$ so the most probable flux density of a field source is $\langle S \rangle \propto D^{-2/\alpha}$. For the relevant flux density range for the AST $0.7 < \alpha < 1.0$ so we have a quite strong dependance of dynamic range on diameter. Any attempt to decrease the cost of the elements by reducing these specifications would have to be associated with an increase in antenna diameter if we are to achieve the theoretical continuum sensitivity. This argument will not apply in general to line work nor to continuum work at frequencies higher than about 5 GHz.

Another implication of the dynamic range requirement is the ability to measure and calibrate the pointing of the antennas and here we are caught twice since the required pointing precision goes as $D^{2/\alpha}$ while our capability to measure the pointing goes as D^{-1}.

8. Compound array using the 64m antenna

From Figure 5 it can be seen that the extra collecting area of the 64m antenna can have a big influence on sensitivity and hence the array optimisation. However because of the limited zenith angle coverage of the 64m antenna results in incomplete coverage for $\delta > -60^{o}$, the slower speed and lack of short spacings it will have only limited application to the full synthesis observations. But for sensitivity limited incomplete synthesis (small objects, point source detections) it can play a very important role. The ratio of the sensitivity of the compound to the stand-alone array is

$$\frac{\text{S/N compound}}{\text{S/N stand-alone}} = \frac{n\,d}{m\,d}\,\frac{64}{d} = \frac{2}{n}\,\frac{64}{d} \qquad (\text{for } m = n^2/4)$$

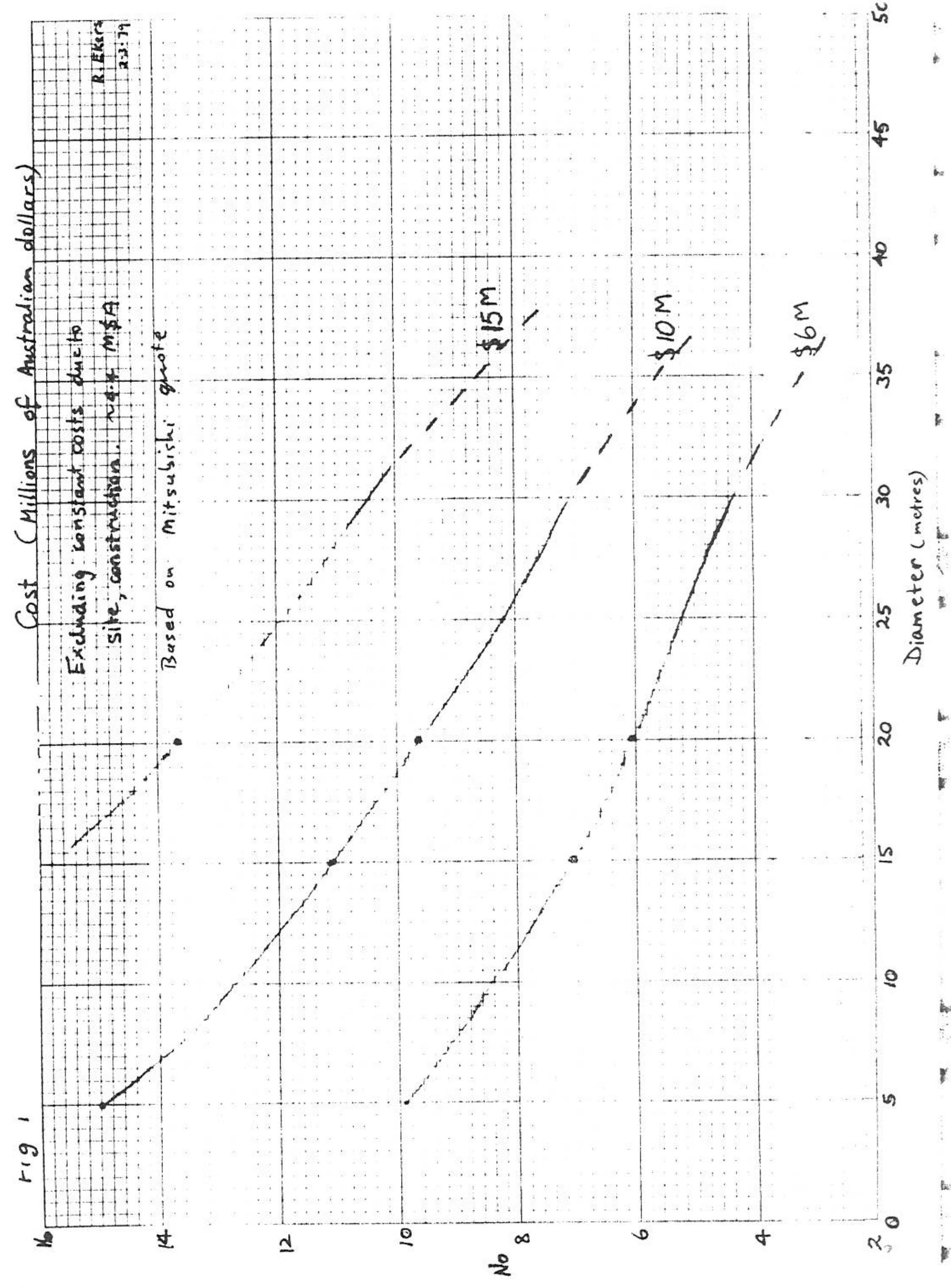
eg for a 6 x 20m $\frac{1}{4}n^2$ array the sensitivity of the compound system is better by a factor 2.7, giving a factor 7 advantage in observing time. This sensitivity advantage is proportional to $\sqrt{n}.d$, which contains the same relation between n and d as the expression for the sensitivity of the stand alone array. Consequently, the choice of element diameter is not influenced by the compound mode as long as the constant sensitivity relation in Figure 2 is satisfied. If this relation has to be violated to follow a constant cost relation in Figure 1 then the larger diameter elements are also best in the compound mode.

9. Operational Considerations

So far the analysis has been based exclusively on the construction costs, however there are some very important implications for the operating budget which has almost no dependance on antenna diameter but has a very strong dependance on the number of elements (front end maintenance and future developments) and on the number of correlators (backend maintenance and future development). These factors mitigate very strongly against having too many elements - the difference between operating a 14 element array of 15m antennas and a 4 element array of 25m antennas could easily be a factor of 2.

10. Future extensions

To increase the resolution of the array additional elements can be added at larger spacings. The motivation for such extensions is strong since each additional element adds n additional spacings. For any extensions it is most likely that all possible extra correlations are taken and this case is the same as for the 64m compound array (Sect. 8) so there are no additional consequences for the choice of d. However, unlike the 64m compound array any extension is very likely to be used in a synthesis mode so that there are implications for the number of elements and their configuration. Minimum redundancy extensions are probably excluded (except for possibly the first step) since the length of track needed for the moveables is increased enormously. The best that can be done would be to correlate the new elements with the main array in a uniform spacing configuration. This means that the speed would be proportional to n, favouring the small dishes although not so strongly as in Figure 4. It also has implications for the configuration since the largest possible number of uniformly spaced elements is required. This can be best satisfied with the two extremes discussed in Sect. 4. The fixed grating array is obviously good but so is the minimum redundancy array since the elements can be moved to an equal spacing configuration.



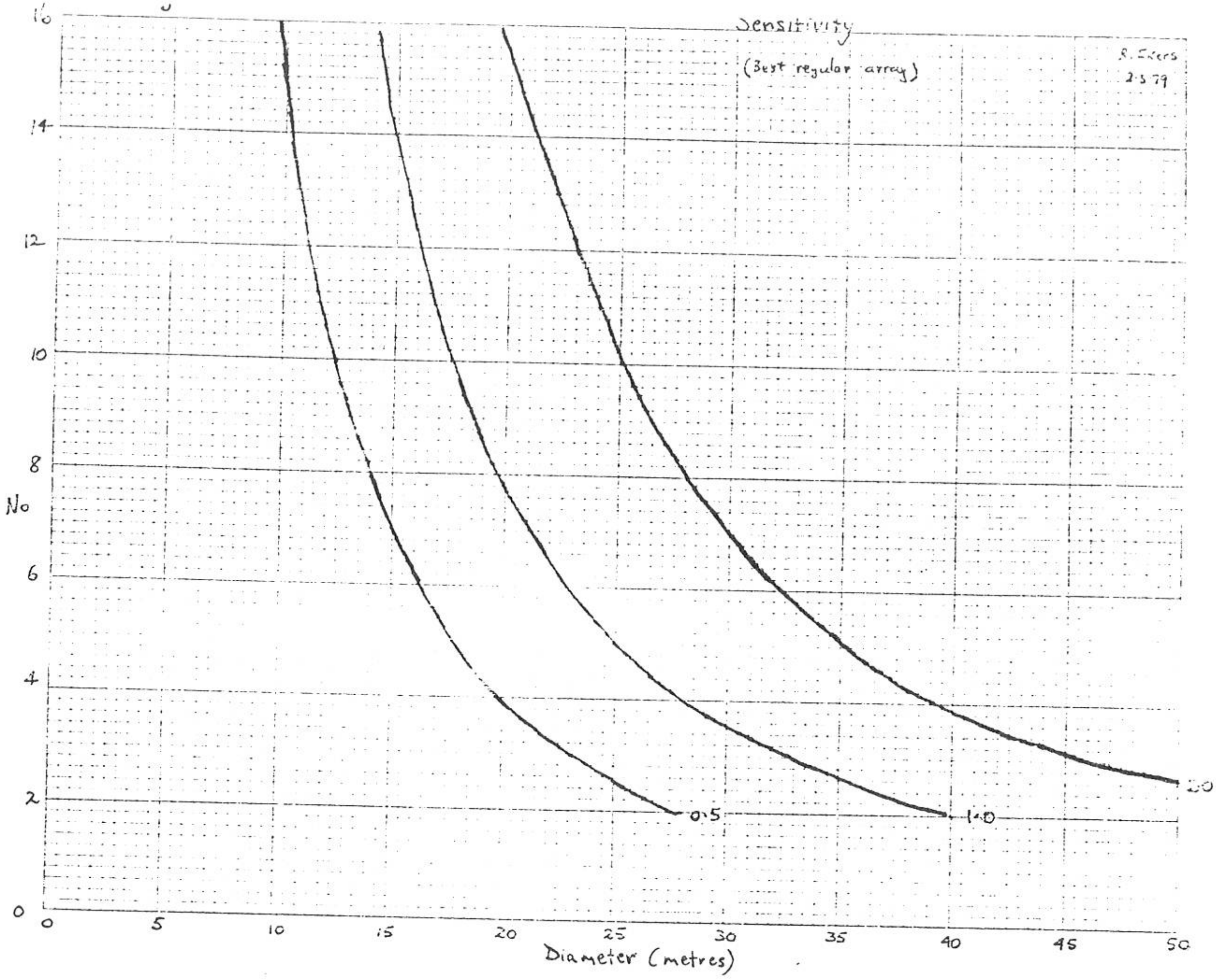

Sensitivity
(Best regular array)
R.
2.3.79
No
Diameter (metres)
0.5
1.0
2.0
0
5
10
15
20
25
30
35
40
45
50
2
4
6
8
10
12
14
16

Fig. 3

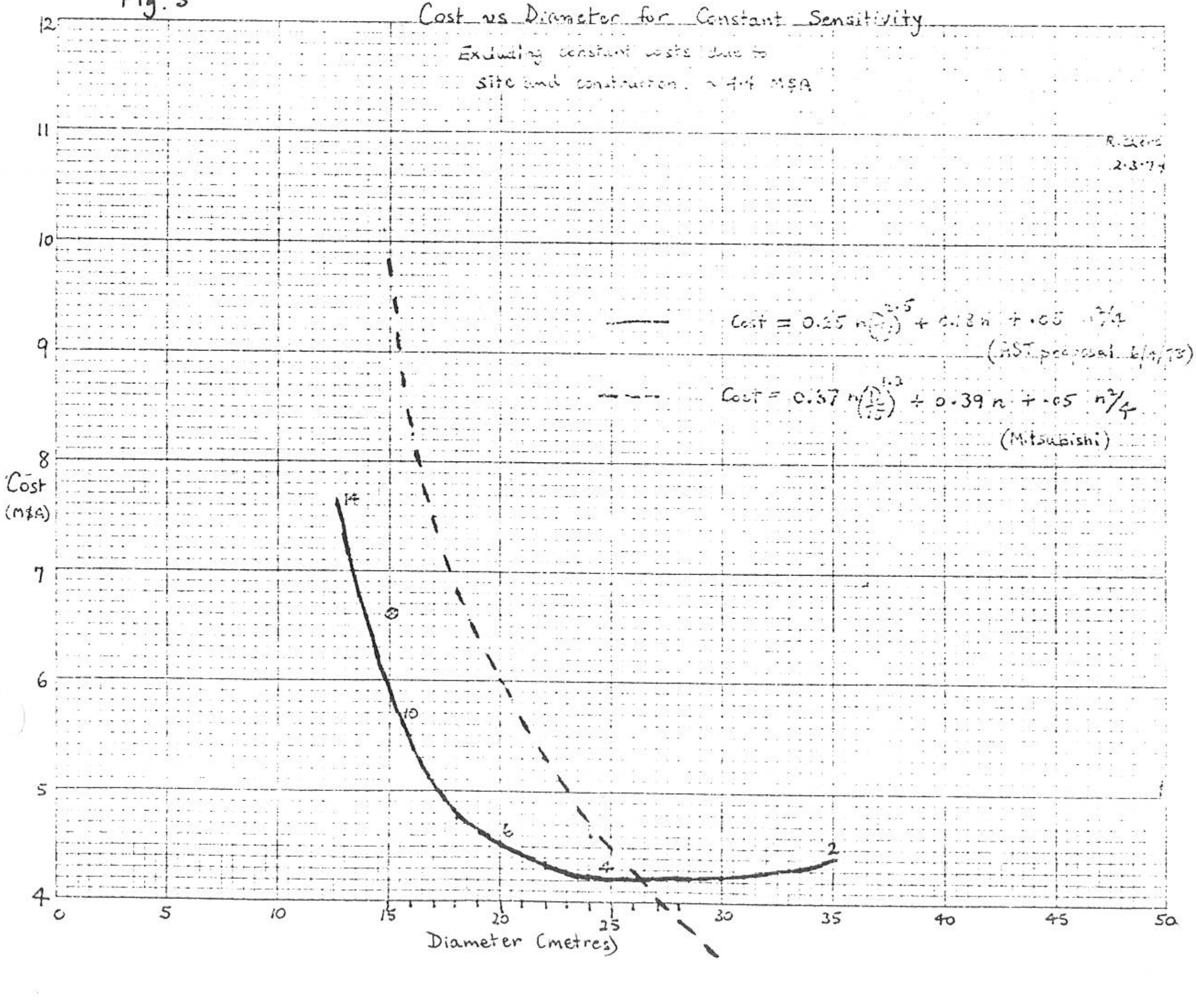

Cost vs Diameter for Constant Sensitivity
Excluding constant costs due to
site and construction
2.3.79
Cost = 0.25 n
(AST proposal
Cost = 0.57
+ 0.39 n + .05 n²/4
(Mitsubishi)
Cost
(M$A)
Diameter (metres)
14
10
6
4
2
0
5
10
15
20
25
30
35
40
45
50
4
5
6
7
8
9
10
11
12

Fig. 4

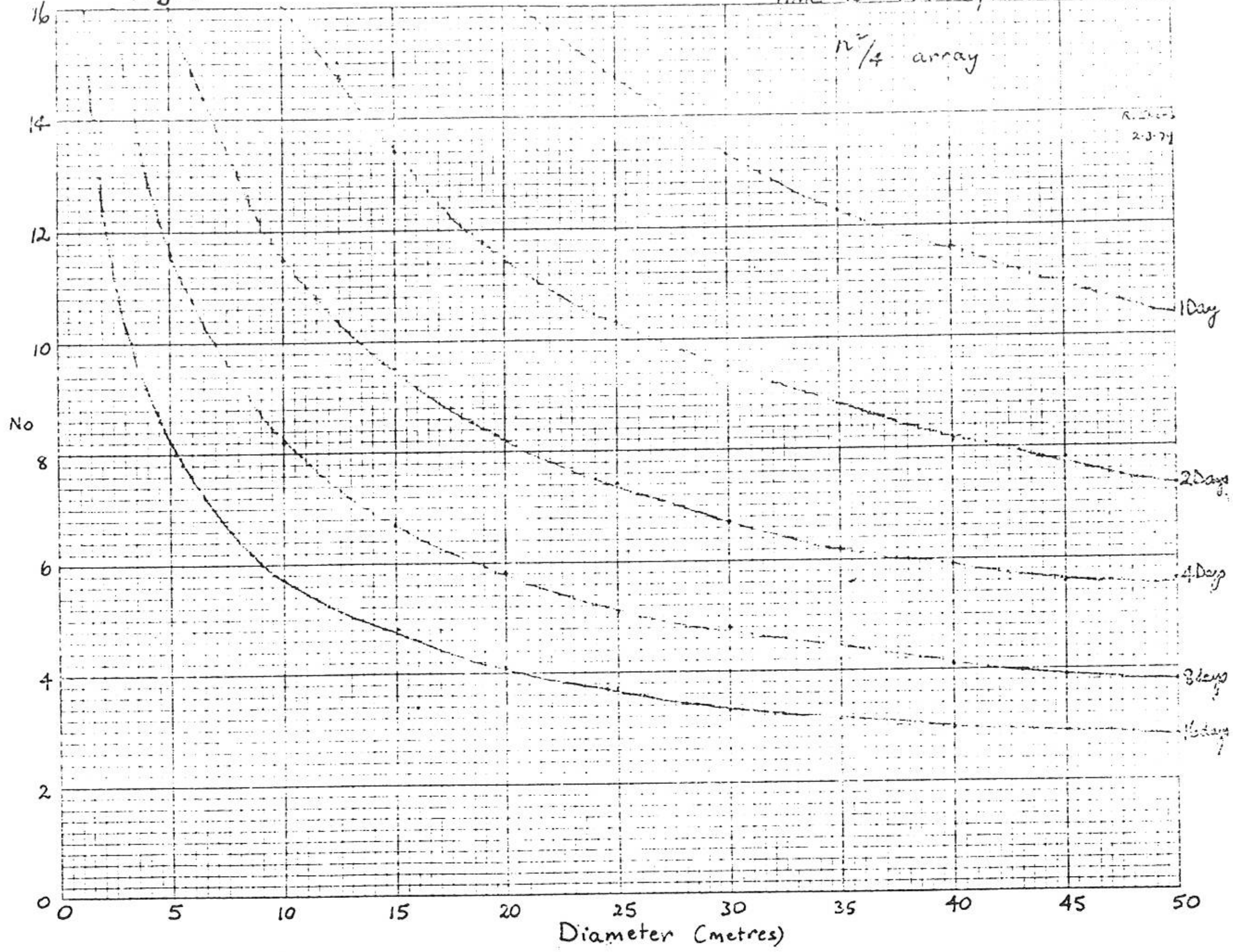
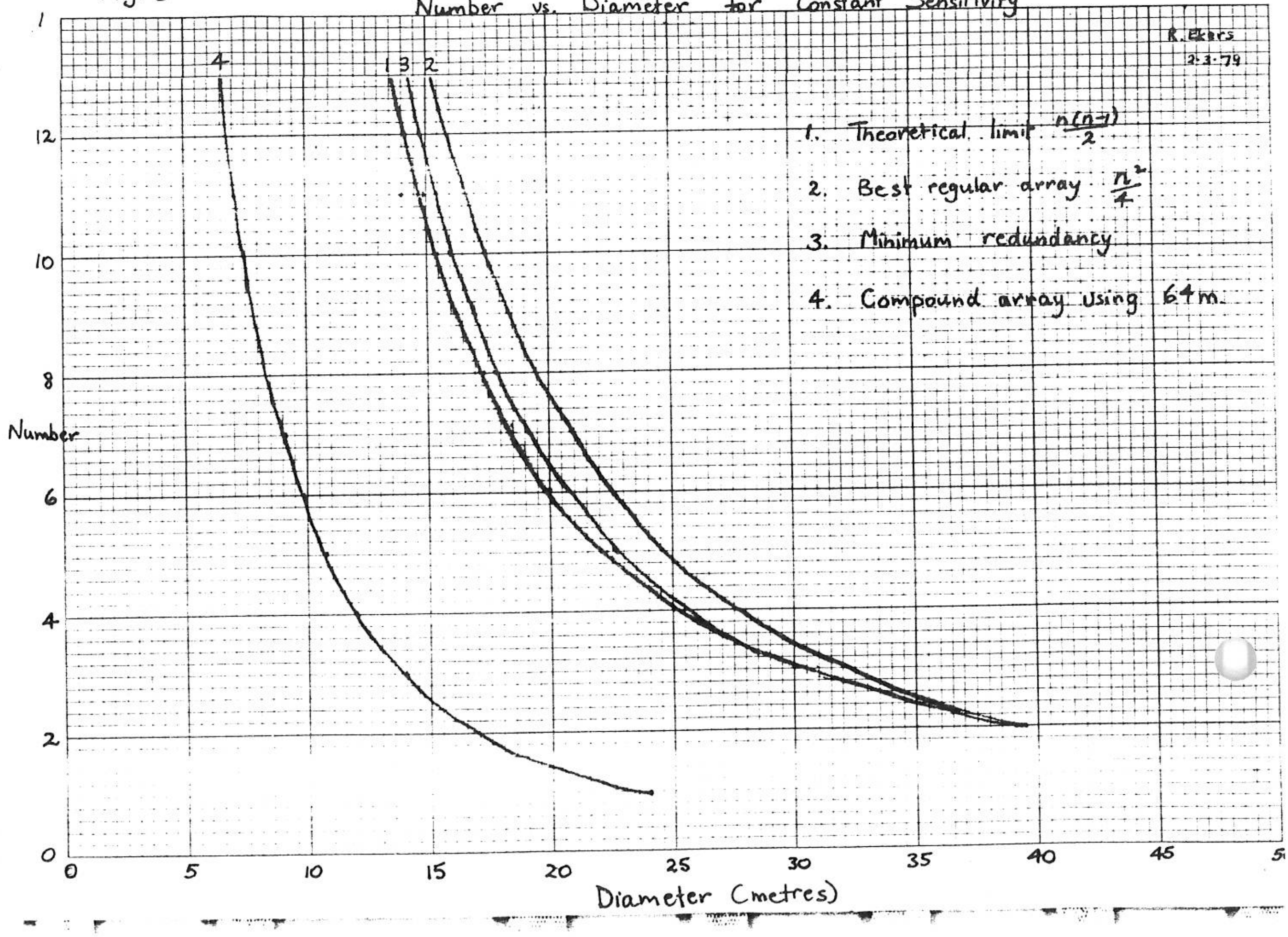
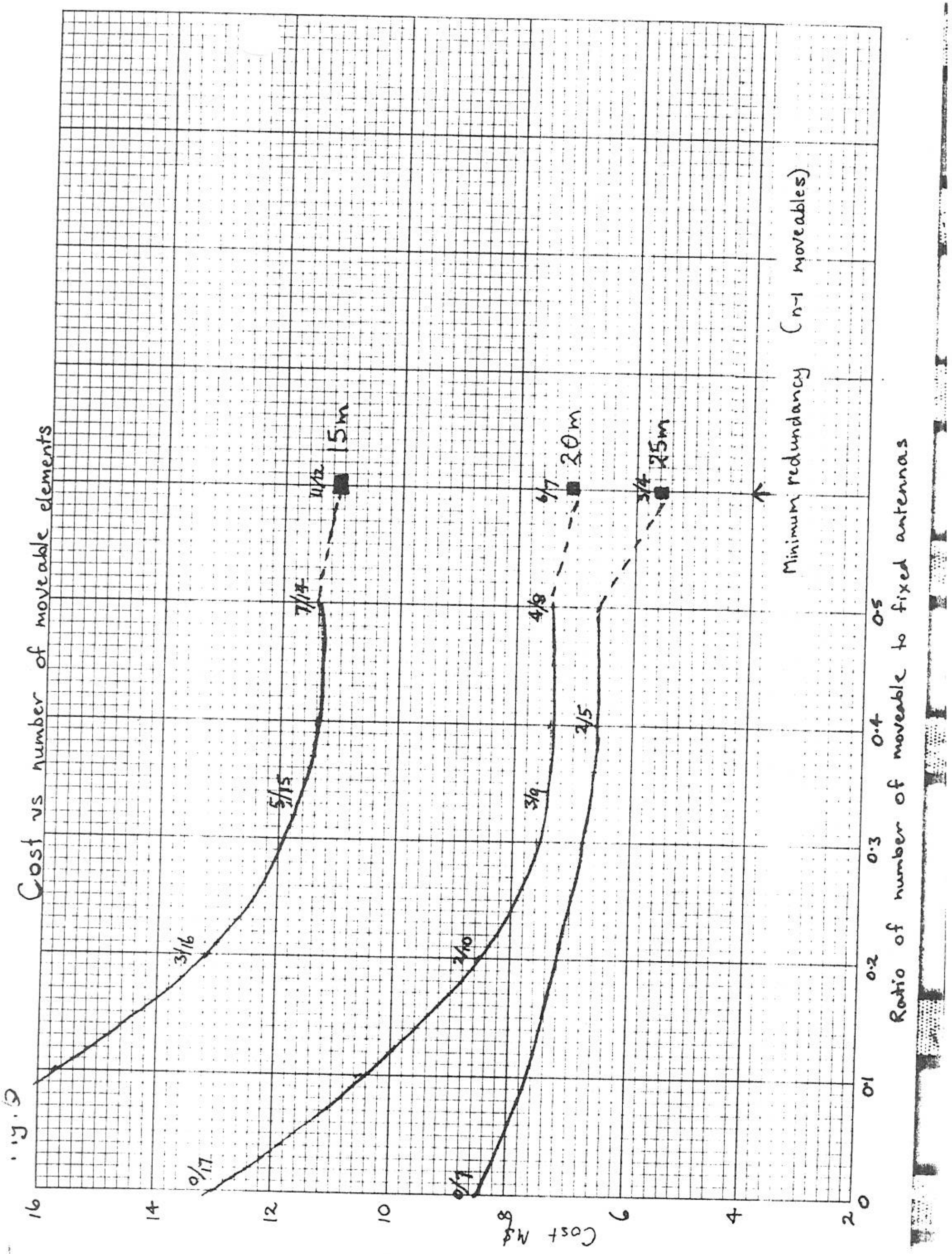

Time for full synthesis
$n^2/4$ array
R. Ekers
2.3.79
No
1 Day
2 Days
4 Days
8 days
16 days
Diameter (metres)

Fig. 5


Number vs. Diameter for Constant Sensitivity
R. Ekers
2.3.79
1. Theoretical limit $\frac{n(n-1)}{2}$
2. Best regular array $\frac{n^2}{4}$
3. Minimum redundancy
4. Compound array using 6.4m.
Number
Diameter (metres)



11 Conclusions

In ASTDOC21 I have tried to summarise all the main criteria which influence the dish diameter. It should be clear from this that there are many interrelated factors and that no matter what conclusion is reached, some compromises will be necessary. Consider the first three items, which are probably the most important viz cost, sensitivity and speed. In Figure 7 a number versus diameter diagram is constructed from Figs. 1, 2 and 4. If we require a cost for antennas and receivers < 6M $A, a sensitivity as good as that proposed in the original AST proposal and a speed such that full synthesis should not exceed 8 days then we can find only one solution - 5 x 25m dishes. The total cost (including erection and site development) for this array would then be ~ 12 M $A. If we used the old AST dish cost estimate and the much steeper cost $\propto D^{2.5}$ scaling law then the curves have a broad optimum region between 18 and 30m (Fig. 3) with the speed argument then favouring the small diameter end of the range. The most recent data from ASTDOC 22 has a stronger and more realistic diameter dependence for D > 20m and there is no solution unless we give up speed or sensitivity, or pay more. By giving up $\sqrt{2}$ in sensitivity we can find a solution at 6 x 20m Fig. 8. This analysis shows how critical the shapes of some of these relations are on determining the antenna size and the main conclusion to be drawn is that we must have the best possible cost estimate to optimize the design.

To give an overall impression of the other factors discussed I have made a rough assessment of whether each favours smaller dishes ($d \leq 20m$) or larger dishes (≥ 20) and this is shown in Table I.

TABLE I FACTORS WHICH INFLUENCE DISH DIAMETER

Factor	Small ($d < 20m$)	Large ($d > 20m$)	Irrelevant
Sensitivity-cost		xx	
Speed-cost	x		
Min. redundancy		x	
Field of view	xx		
Cassegrain focus		x	
Dynamic range		x	
Compound with 64m			x
Operating costs		x	
Reliability		x	
Baseline extensions			x

Excluding the three main points already discussed I would consider the dynamic range and the operating costs most important and these both favour the largest possible elements. The strongest argument for the small elements is the larger field of view but whereas larger fields can be mapped by taking more time there is no other way to decrease operating costs or to significantly improve dynamic range.

11.1 Possible Configurations

If the preceding arguments force us to a small number (≤ 6) of elements then the advantages of flexibility and minimum redundancy argue for all moveable elements. For example, in the compound mode, the high frequency point source detection observations are best done with all elements near the 64m telescope to minimise atmospheric phase effects, while accurate position measurements are better done with at least some far from the 64m. Any future baseline extensions will probably need the proposed array equally spaced. With all elements moveable there is a free choice between using the minimum redundancy configuration and a grating configuration for normal observations.

11.2 Summary

An array with the sensitivity of the original proposal can still be built within the estimated budget. This array would consist of 4-6 movable elements with diameter between 20 and 25m.

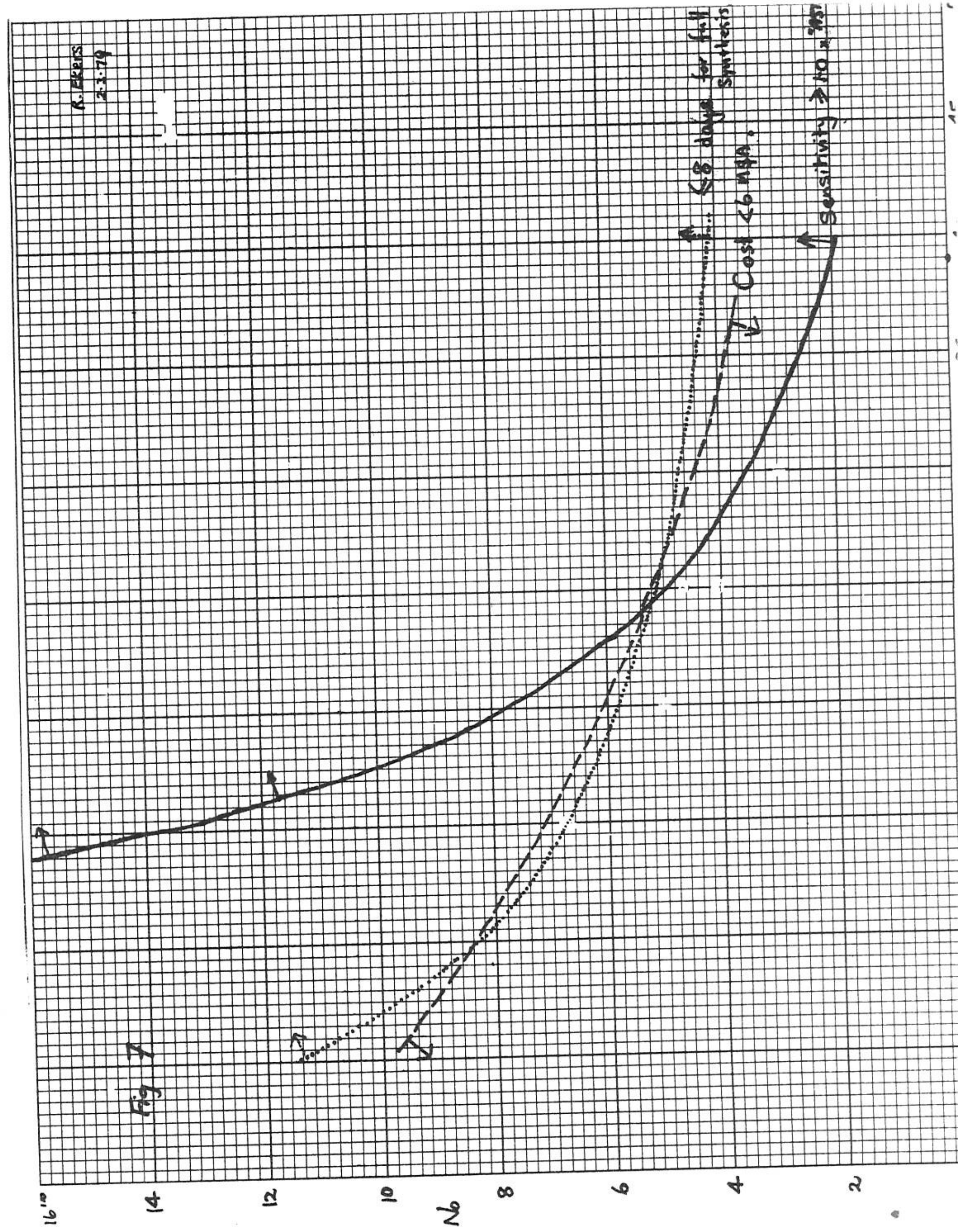


Fig 8

R Ekers
1.5.79

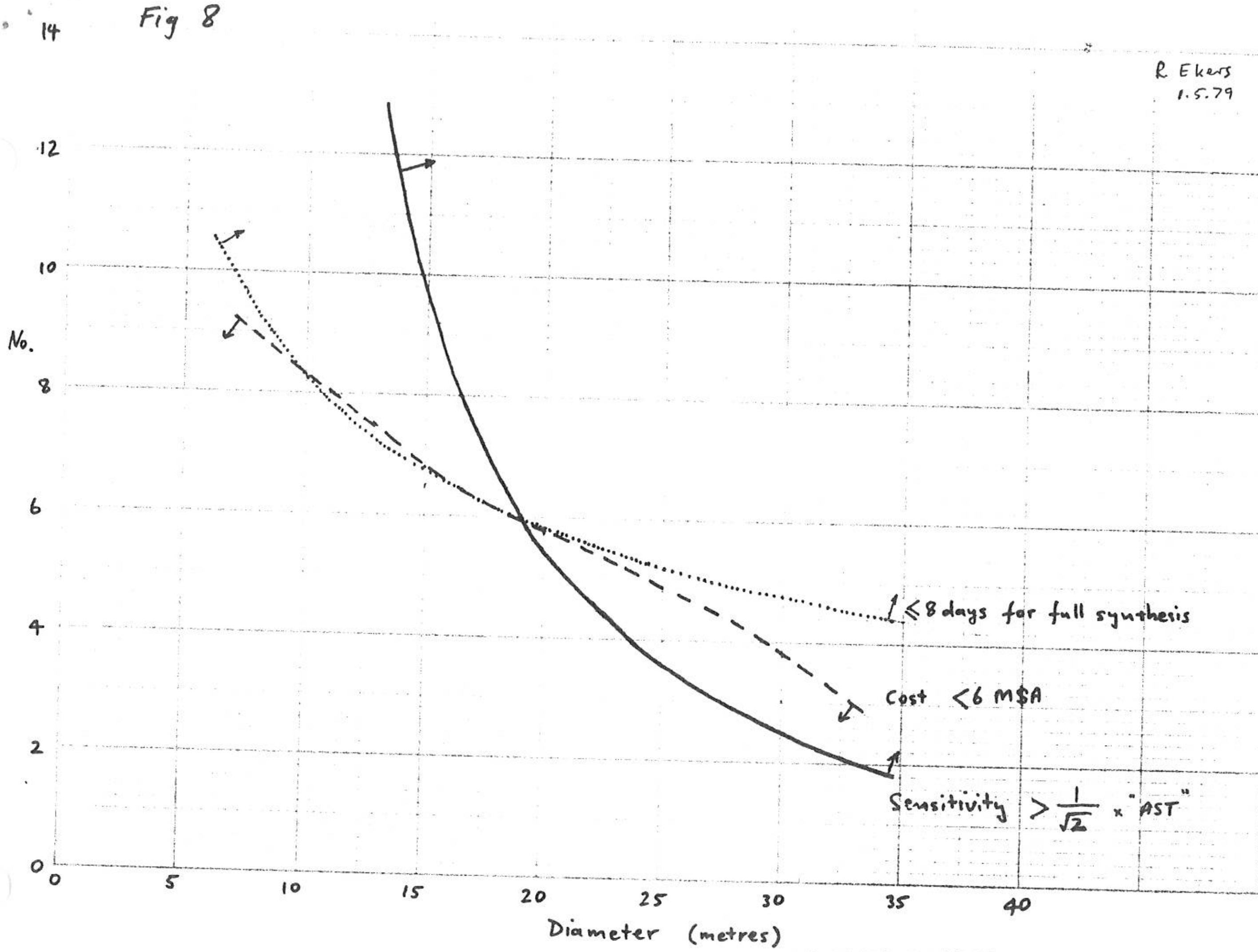


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FIG. 8