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S/N DEGRADATION DUE TO NON-UNIFORM BASELINE DENSITY

Let N readings be made of visibility V (u,v) with standard deviation σ_o. If the readings are averaged we have

$$\sigma_u = \sigma_o / N^{\frac{1}{2}}$$

where σ_u is the standard deviation of the (unweighted) mean.

Because baselines have a non-uniform density over the (u,v)-plane we may however wish to take *weighted* means. Let n(w) be the number of readings assigned weight w. Then $\sum_w n(w) = N$.

With uniform weighting the sum of the N readings would be NV and the variance of the sum would be $N\sigma_o{}^2$. The standard deviation of the sum is $(N\sigma_o{}^2)^{\frac{1}{2}}$ and the standard deviation of the mean is $(N\sigma_o{}^2)^{\frac{1}{2}} / N = \sigma_o / N^{\frac{1}{2}}$.

With nonuniform weighting the sum of the weighted readings is

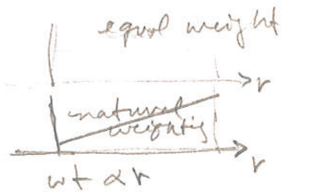
$$\sum n(w)\, w\, V$$

and the variance of the sum is

$$\sum n(w)\ w^2\ \sigma_o{}^2 .$$

The s.d of the sum is $\left[\sum n(w)\ w^2\right]^{\frac{1}{2}} \sigma_o$

and the s.d. of the mean is



$$\sigma = \frac{\left[\sum n(w)\, w^2\right]^{\frac{1}{2}} \sigma_o}{\sum n(w)\, w} .$$

The degradation factor is $\boxed{N^{\frac{1}{2}} \left[\sum n(w)\, w^2\right]^{\frac{1}{2}} / \sum n(w)\, w}$

For many baselines the S/N is reduced by ~15%.

Example 1

Readings are taken every minute for 12 hours on M concentric equi-spaced circular loci in the (u', v')-plane with standard deviation σ_o. The total number of readings is

$$N = 720\ M.$$

If all the readings are averaged without weighting, the standard deviation is

$$\sigma_u = \sigma_o / (720\ M)^{\frac{1}{2}}.$$

If we adopt weights 1, 2, ... M in proportion to the radii of the rings, which compensates for the area density* (which is inversely proportional to radius) then

$$n(w) = 720, \qquad w = 1, 2, \ldots M.$$

The new σ will be

$$\sigma = \left[\sum n(w)\, w^2\right]^{\frac{1}{2}} \sigma_o \Big/ \sum n(w)\, w \;=\; \left[720 \sum_1^M w^2\right]^{\frac{1}{2}} \sigma_o \Big/ 720 \sum_1^M w$$

$$= \left[720M(M+1)(2M+1)/6\right]^{\frac{1}{2}} \sigma_o \Big/ \left[720\, M(M+1)/2\right]$$

$$= \left[2(2M+1)/3 \times 720\, M(M+1)\right]^{\frac{1}{2}} \sigma_o$$

The degradation factor is

$$\sigma/\sigma_u = \left[2(2M+1)/3(M+1)\right]^{\frac{1}{2}}$$

M	10	20	30	40	∞
σ/σ_u	1.128	1.141	1.145	1.148	1.155 (0.6 dB)

Example 2

Readings are taken every minute for 12 hours on M concentric circular rings that are not equispaced. The radii are q_1, q_2, ... q_M where

$$q_m = q_1 (1 - G^m) / (1 - G).$$

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* For more on the concept of area density and its compensation, see A.R. Thompson and R.N. Bracewell, "Interpolation and Fourier Transformation of Fringe Visibilities", A.J. 79, 11-24, 1974 and R.N. Bracewell, "Computer Image Processing", Ann.Rev.Astron.Astrophys., 17, 113-134, 1979.

An alternative requirement might be if we define the value of the smallest spacing to be some minimum acceptable value. If we also have a largest spacing which we wish to reach, then we may tabulate the required value of G as the number of available antennas increase.

With these constraints we may tabulate the degradation as a function of the number of antennas (e.g. with N varying from 5 to 10). As a particular example we might choose a minimum spacing of 20 metres and a maximum of 6000 metres, e.g. eight antennas require G $\sim$ 1.13 .

Conclusion

The zoom array idea could lead to degradation which is significant. Other baseline arrangements that are nonuniformly distributed can be evaluated from the expression

$$\frac{\left[N \sum n(w)\, w^2\right]^{\frac{1}{2}}}{\sum n(w)\, w} \, .$$

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