AT/15.6.1/001

CSIRO DIVISION OF RADIOPHYSICS

AT COMPACT ARRAY LINE SURVEY

28/6/84

INTRODUCTION

The AT compact array line station locations are fundamentally determined by two (2) constraints. These apply without compromise.

The stations are

i) to lie in a plane normal to the rotational axis of the Earth;

ii) to lie in an absolute straight line in space and in this plane.

Weaker constraints also apply due to practical considerations ,specifically the following:

iii) Minimisation of soil transport associated with cuts and fills:

iv) Provision for adequate discharge of flood waters from the site without side effects to neighbouring land.

Below is an external view of the Earth with the compact array line shown, conforming to the constraints indicated. Note that the eastern end ,of the array, is closer to the equator in latitude than the western end, for the situation shown.

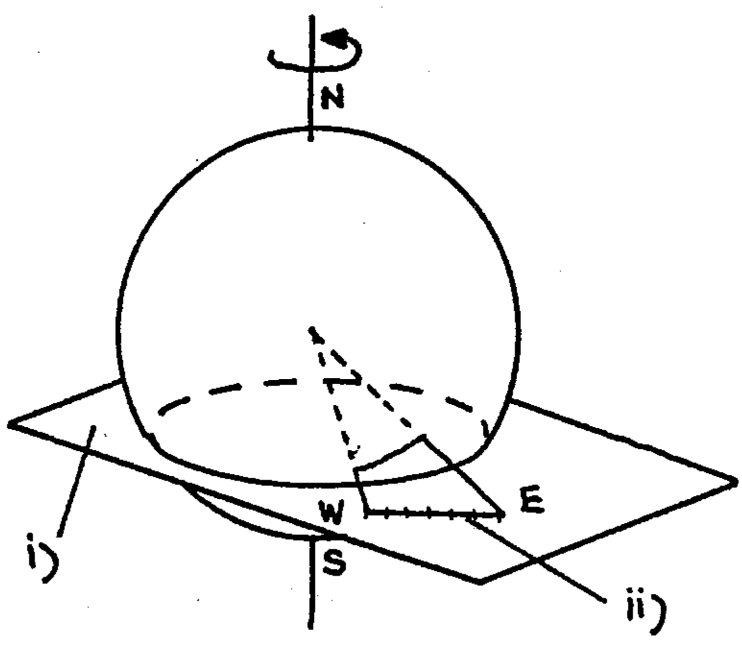


FIGURE I.

PROBLEM DISCUSSION

We now develop in a systematic way the subtle problems involved given that the ultimate aim is to position individual antenna elements to an accuracy of 6 cm (ref AT/20.1/006). We can visualise this as a requirement to locate the focus of each antenna to within the size of a billiard ball. This requirement is to exploit redundancy and simplify subsequent calculation (ie the computer/correlator group has dumped their problem on the antenna group).

Below is a series of sketches displaying the situation. Briefly column 1 displays a rudimentary view of the situation ie a flat earth. Column 2 is closer to the true situation in that it recognises that over 6 km the earth's curvature is not insignificant however assumes the Earth is smooth (constant level). Column 3 incorporates both curvature and local topology (levels). Row 1 is an uncluttered view. Row 2 shows the plane of constraint i). Row 3 shows the line of constraint ii) superimposed on the plane. Note that any line in this plane is adequate however constraints iii) and iv) imply something like the column 3/row 3 picture is required.

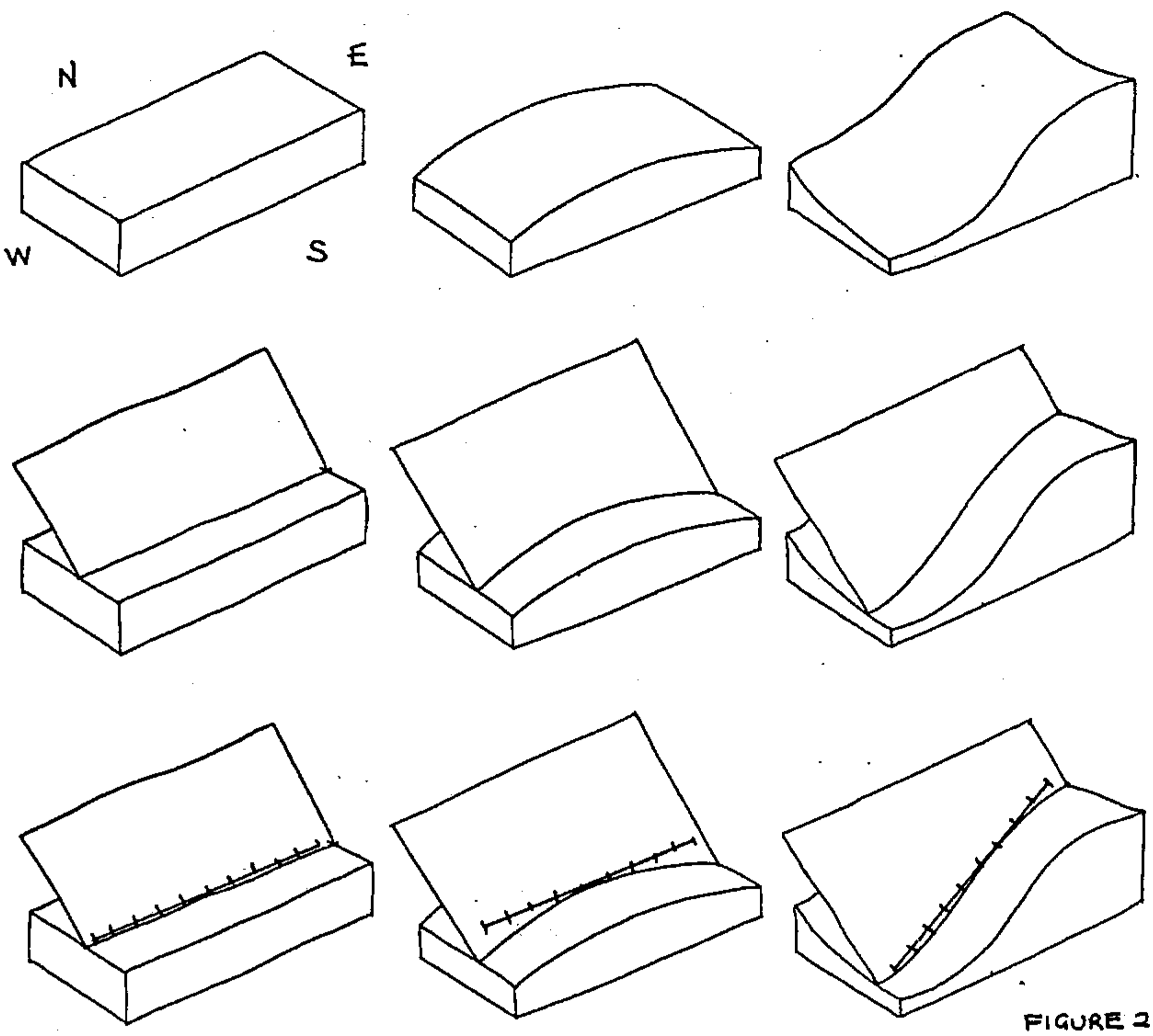


FIGURE 2.

It is also obvious that a tilt from true East-West is required since as pictured the eastern end of the Culgoora site is higher than the western end. Having said that, it is however slightly naive assuming the only compensation required is an azimuthal shift since this is just a first order description and the true picture is that the AT line ,on the plane, projects as a curve on the 'ground' even given the ideal situation of column 2. Indeed even the concept of an azimuthal shift is not well defined. Do we mean an azimuthal shift in a tangent plane? If so, tangent at which point? Theoretically surveyors like working with a standard spheroid which is just a revolution of an ellipse. Hence the fairly tangible concept of moving along a great circle of a sphere presumably blurs to the concept of moving along a geodesic of the spheroid. Indeed the surveyor needs to specify whether he/she is working in the normal section of point of measurement or from the point he/she is viewing. For the field surveyors (which there are reported sightings) ,of course, just work in a piece-wise linear fashion and given appropiate short increments reduce the resulting compounding error. We are not implying that overlooking these pedantic points can cause significant errors in approximation ,it is just that we aren't absolutely sure what assumptions can be made.

TUTORIAL ON EASTINGS AND NORTHINGS

The nature of the surveys done at Culgoora has highlighted the fact that surveyors present the information in terms of a grid of Eastings and Northings and Australian Height Data (AHD). The security ,presumably to the surveyor, in using this system, is that only they know what these terms represent. Basically Eastings and Northings are obtained from the spheroid (what we call an ellipsoid) by a transverse Mercator projection which has no simple analytic definition. Specifically the projection is conformal in that it preserves angles in the small. However since we are mapping from an ellipsoid to a cylinder (the domain of the Eastings and Northings), of zero curvature, this implies distances are corrupted. So true ellipsoid (arc) distances can only be strictly obtained from the E/N grid distances by integrating up the grid distances compensated by a continuously varying 'scale factor'. The projection results in greater distortion further away from an arbitrary origin; so to compensate somewhat for this, the Earth is split up into a number of zones with false origins. Culgoora turns out to be close to the overlap of two zones (54 and 55) hence is in a more highly distorted region of the grid. Indeed grid north is out from true north by approximately 1.3 degrees in zone 55 at Culgoora. This angular deviation is termed convergence.

A more familiar domain to work in is Latitude and Longitude. However here too one is not free from ambiguity. This is due to the fact we are referencing everything to a standard spheroid not a sphere. Hence whilst Longitude is concretely defined there are about four different definitions of Latitude. The simplest definition is the geocentric latitude which corresponds to the astronomical definition or to the elevation coordinate in spherical polars. However the standard definition corresponds to the geodetic latitude and this is what should be understood by the term 'latitude'. The appendix gives the relevant equations. There exist formulae for transforming between grid coordinates and geodetic latitude and longitude called Redfearn's formulae. These too are given in the appendix.

As if things weren't confusing enough, the height information AHD is not referenced to the spheroid but rather to a standard geoid. A geoid is a gravitational equi-potential surface and is neither a sphere or spheroid. However the geoid and spheroid are fairly close, eg at Narrabri the height of the geoid above the spheroid is *0.74 metres. ? The separation is a complicated function of location in Australia. The nature (and hence justification) of the geoid is that a level lies naturally at a tangent to the geoid whilst say a plumb bob is perpendicular to it.

*11.4 m according to table 1 in Part 2.

*16.9 m, according to R. Coleman, Mar 92.

Wrong! See intro to Part 3. and Table I

LINE DETERMINATION

We can formulate a solution following the diagrams given on page 2. We set up a tangent reference plane since neither E/N nor latitude/longitude provide a distortion-free cartesian system. We can take the levels (AHD) and say the geoid approximates a sphere and apply a curvature correction and hence reference our heights to the tangent plane. However the location of a particular point in question is not known precisely since the distances supplied by the surveyors are grid distances which differ from true metres. With this tangent plane it is simple to define an absolute straight line in space. However locating it to satisfy constraint i) is not straight forward and prone to error. Even successfully completing this we need to refer our line back to the surveyors units of E/N's and AHD. The reference plane lacks any astronomical significance, it has an arbitrary definition and it is not clear if it is tangent to the spheroid, sphere or geoid. It also helps little to solve the problem of constraint iv) where we need the AT line referenced to the geoid levels which define the 'sea levels'. This is all possible but...

The folly is working with a view such as depicted in the diagrams on page 2 rather than the view shown on page 1. With the above approach one is continually compensating for curvature and grid distortions. After all the problem is simple. A straight line in space. Anyway what has the Earth really got to do with it and why should it drive our coordinate system choice. The alternative is to use an 'exact' technique and let the mathematics handle the curvature corrections etc. As we will see we generate the station data in the format the surveyors want anyway. This approach also has the benefit of being self-checking and there is no possibility of not compensating for some subtle distortion. Any criticism of this approach as being a case of overkill can be dismissed on the grounds that the essential computer solution of the determination of the AT line is half a page long. It is simpler.

So we set up an Earth-centred cartesian coordinate system with z along the axis of rotation as depicted in the diagram below.

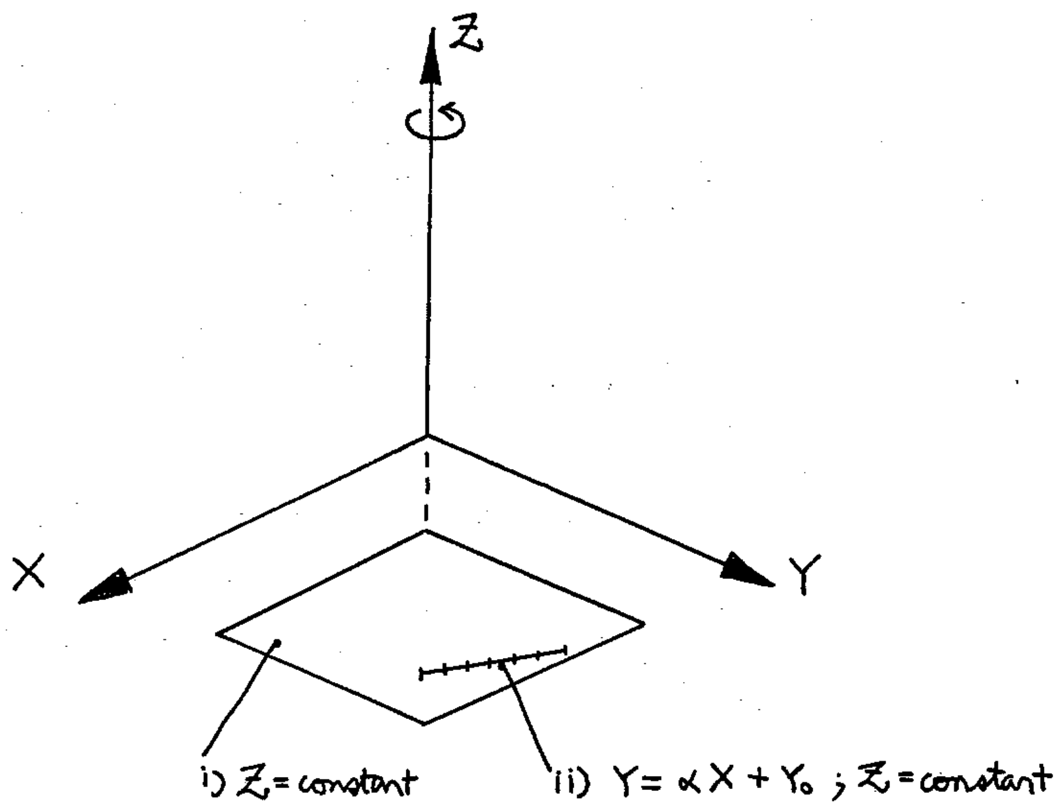


FIGURE 3.

Constraint i) is disposed thus: Z = constant.

Constraint ii) degenerates to a linear relationship between X and Y (with Z constant).

We can quickly generate the station locations in X,Y and Z for an feasible compact array line, and we can set upon optimising constraints iii) and iv). The first step is to recognise that five (5) parameters specify our line. Determining stations 1 and 37 say, each of three (3) coordinates gives six (6) parameters less one (1) since their Z coordinates are necessarily identical. We reduce the optimisation to two (2) parameters for the following reasons. Station 14 which is the central (1500m) station of the 3km array will have its position fixed in E/N or equivalently in latitude and longitude. This station has been located and pegged at Culgoora. The (first order) azimuthal shift which compensates for the fall in the land shall pivot about this point. The height (AHD) at station 14 will be a flexible parameter. The remaining parameters are accounted for by the height of station 37 (flexible) and by the fact that station 37 is exactly 4500m from station 14. The reason for incorporating station 37 in this fashion is that the topology survey does not cover the region of interest at the 6km site. So we essentially fix the height of station 37 at a reasonable level and as we select different trial heights for station 14, station 37 will swing on an arc (radius 4500m) at a constant height above the 6km site. The 'vertical' rise above station 14 and the arc traced out by station 37 are not independent (ie are parametrized in one variable) due to the fact that both stations must lie in the plane of constraint i). This is depicted diagramatically below.

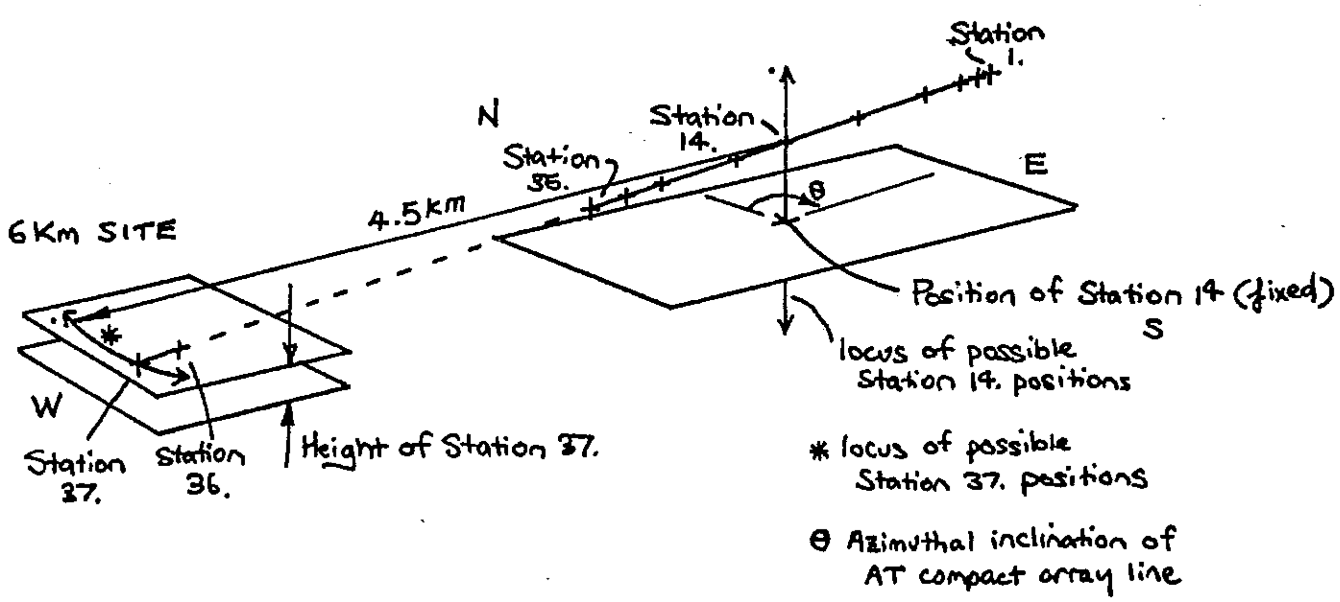


FIGURE 4.

The picture is an aid only and the computer program written handles the situation in the Earth-centred XYZ system. One enters only two parameters, the desired heights of stations 14 and 37.

It should be clear that we need to be able to convert between coordinate systems. This is simple in principal and appropiate FORTRAN subroutines have been written based on equations given in the appendix accurate to 1mm over the scale of the Earth (effectively exact). Hence once we generate the cartesian XYZ of the stations the subroutines return the equivalent Grid coordinates and/or the Geographic coordinates, complete with curvature correction automatically. The diagram below depicts the general relationship.

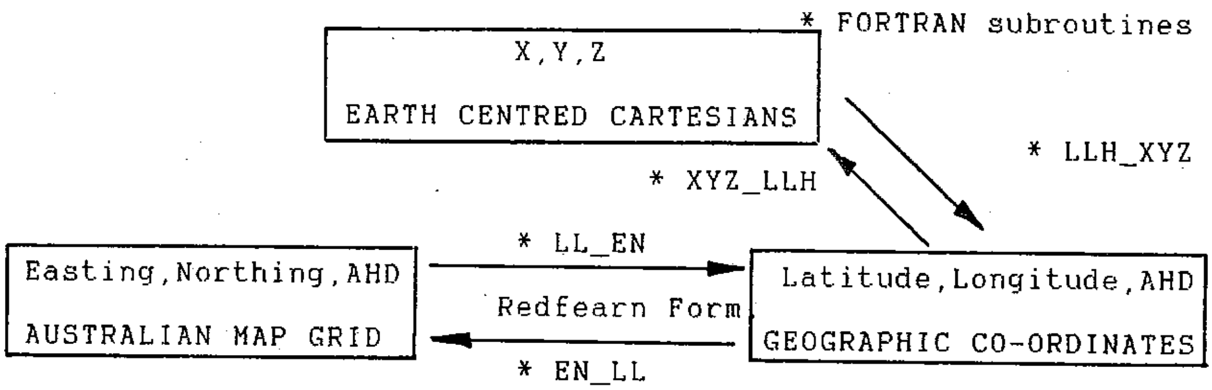


FIGURE 5.

SOIL MOVEMENT

For each station generated, we take the Easting and Northing and interpolate off the surveyors data a (AHD) height. We then compare this with our required station height and adjust station 14 height appropiately to minimise the difference for stations 1 to 35. This will be done in the next report.

WATER COURSING

For each station we have the AHD data which is in a one to one correspondence with the 'contours' of lying water ('sea-levels'). We graphically plot the height vs distance along the array and determine the regions of ground most likely to give us trouble, in terms of flooding, and re-adjust the line, through station 14 and 37 heights, appropiately to provide sufficient clearance. This too will be done in the next report.

RESULTS

The table in the following pages gives the results of a preliminary line based on a rough best fit based on a graph given in the appendix. The critical data in determining this line is given below:

Location of station 14 (which is fixed and based on a calculation given in the appendix. This figure may be subject to a slight re-evaluation).

EASTING E746605.3600
NORTHING N6643571.6000

Input Heights

Station 14 AHD 210.0 metres
Station 37 AHD 206.0 metres

16.9m? (Mar 92)
x 11.4m ??

The analysis assumes the geoid is a constant 0.740 metres above the spheroid at all station locations. Also given is the north-south distance between the indicated stations as measured along the Australian standard spheroid via the Meridian distance formula found in the appendix. This formula is a function of latitude only. From this we estimate the required first order azimuthal shift required to compensate for the fall in the land.

Azimuth shift = arctan(6000.0/3.530)
= 89 deg 58.0 min East of North

(In reality we require an azimuthal shift to induce the required 3.530 metre meridial shift which differs from the value given in the above calculation). This value differs marginally from an estimate of 89 deg 57.9 min calculated based on preliminary information and which has been the basis of the locations of soil tests done at the proposed station sites. The error at station 37 thus introduced is 13cm which is insignificant for a soil sample. The value of the azimuthal shift will be refined given that the next report will optimise with respect to constraints iii) to v). It must be reemphasised however that this figure isn't useful for obtaining true accurate results for it denies the fact that the stations project as a subtle curve and definitely not a line of constant azimuth from station 14.

CONCLUSIONS

A set of numbers which relate the exact station locations has been given. We in no way imply these numbers should form the basis of determining the final locations of the antennas for two reasons (i) they aren't optimised and (ii) it is clear the locations of the stations will need to be interactively surveyed and determined at a future date (ie a straight line in space can be verified by a laser beam etc). However the formulation of the results provides a means through which constraints iii) and iv) can be tackled and any resulting error is due solely to the field survey errors not approximations in analysis.

R.A.KENNEDY

NOTES

a) The principal reference point is a national mapping mark called TRIG NM/C/59 on top of the Culgoora building.

b) The relative heights of the indicated stations is the critical notion not the absolute value indicated which may be in error of measurement (for TRIG NM/C/59).

c) The accuracy, shown in the table, is for purpose of verifying the coordinate system transformation formulae rather than absolute figures ie we don't really believe the plane is 3200631541 mm below the equator.

d) The derivation, of the required azimuthal shift, may not give the 'best possible value' given that, for a surveyor moving from a point along a line of constant azimuth, presumably traverses a geodesic of an ellipsoid and measures distance along the arc, not a straight line in space.

TABLE OF COMPUTED STATION POSITIONS

STATION #	DIST metres	EARTH CENTRED CARTESIAN COORDINATES X metres	Y metres	Z metres	GEODETIC LATITUDE DD.MMSSsss	LONGITUDE DDD.MMSSsss	AHD metres	AUSTRALIAN MAP GRID EASTING metres	NORTHING metres	ZONE
					-30.3144400	149°.5801700				
1	0.000	-4752303.310	2790371.166	-3200631.541	-30.1851984	149.3448612	212.038	E748105.4821	N6643538.7747	55
2	30.612	-4752287.765	2790397.537	-3200631.541	-30.1851985	149.3447466	211.993	E748074.8675	N6643539.4446	55
3	61.224	-4752272.220	2790423.908	-3200631.541	-30.1851986	149.3446320	211.948	E748044.2529	N6643540.1145	55
4	91.837	-4752256.674	2790450.281	-3200631.541	-30.1851987	149.3445174	211.903	E748013.6372	N6643540.7845	55
5	122.449	-4752241.129	2790476.652	-3200631.541	-30.1851988	149.3444029	211.859	E747983.0226	N6643541.4544	55
6	153.061	-4752225.584	2790503.023	-3200631.541	-30.1851989	149.3442883	211.814	E747952.4080	N6643542.1243	55
7	183.673	-4752210.039	2790529.395	-3200631.541	-30.1851989	149.3441737	211.770	E747921.7935	N6643542.7942	55
8	214.286	-4752194.494	2790555.767	-3200631.541	-30.1851990	149.3440591	211.726	E747891.1779	N6643543.4641	55
9	244.898	-4752178.949	2790582.138	-3200631.541	-30.1851991	149.3439446	211.681	E747860.5633	N6643544.1340	55
10	489.796	-4752054.588	2790793.111	-3200631.541	-30.1851998	149.3430280	211.334	E747615.6448	N6643549.4933	55
11	688.776	-4751953.545	2790964.526	-3200631.541	-30.1852003	149.3422832	211.059	E747416.6484	N6643553.8477	55
12	979.592	-4751805.867	2791215.056	-3200631.541	-30.1852010	149.3411947	210.667	E747125.8089	N6643560.2117	55
13	1285.714	-4751650.416	2791478.771	-3200631.541	-30.1852018	149.3400490	210.270	E746819.6627	N6643566.9107	55
14	1500.000	-4751541.600	2791663.373	-3200631.541	-30.1852023	149.3352469	210.000	E746605.3600	N6643571.6000	55
15	1530.612	-4751526.055	2791689.744	-3200631.541	-30.1852024	149.3351324	209.962	E746574.7456	N6643572.2699	55
16	1561.224	-4751510.510	2791716.115	-3200631.541	-30.1852024	149.3350178	209.924	E746544.1313	N6643572.9398	55
17	1668.367	-4751456.103	2791808.416	-3200631.541	-30.1852027	149.3346168	209.793	E746436.9801	N6643575.2845	55
18	1683.673	-4751448.330	2791821.602	-3200631.541	-30.1852027	149.3345595	209.775	E746421.6729	N6643575.6194	55
19	1698.980	-4751440.557	2791834.788	-3200631.541	-30.1852028	149.3345022	209.756	E746406.3648	N6643575.9544	55
20	1714.286	-4751432.785	2791847.974	-3200631.541	-30.1852028	149.3344449	209.738	E746391.0576	N6643576.2893	55
21	1729.592	-4751425.012	2791861.160	-3200631.541	-30.1852028	149.3343876	209.719	E746375.7504	N6643576.6243	55
22	1959.184	-4751308.424	2792058.947	-3200631.541	-30.1852034	149.3335283	209.446	E746146.1412	N6643581.6487	55
23	1974.490	-4751300.651	2792072.132	-3200631.541	-30.1852034	149.3334710	209.429	E746130.8340	N6643581.9837	55
24	2142.857	-4751215.154	2792217.176	-3200631.541	-30.1852038	149.3328409	209.234	E745962.4546	N6643585.6681	55
25	2250.000	-4751160.746	2792309.476	-3200631.541	-30.1852040	149.3324398	209.113	E745855.3038	N6643588.0127	55
26	2265.306	-4751152.973	2792322.662	-3200631.541	-30.1852040	149.3323825	209.096	E745839.9966	N6643588.3477	55
27	2494.898	-4751036.385	2792520.449	-3200631.541	-30.1852045	149.3315232	208.842	E745610.3881	N6643593.3718	55
28	2571.429	-4750997.522	2792586.378	-3200631.541	-30.1852047	149.3312368	208.760	E745533.8516	N6643595.0466	55
29	2632.653	-4750966.432	2792639.121	-3200631.541	-30.1852048	149.3310076	208.694	E745472.6233	N6643596.3863	55
30	2647.959	-4750958.660	2792652.306	-3200631.541	-30.1852048	149.3309503	208.678	E745457.3162	N6643596.7213	55
31	2785.714	-4750888.707	2792770.978	-3200631.541	-30.1852051	149.3304348	208.533	E745319.5516	N6643599.7358	55
32	2892.857	-4750834.299	2792863.279	-3200631.541	-30.1852053	149.3300337	208.423	E745212.4011	N6643602.0804	55
33	2908.163	-4750826.526	2792876.465	-3200631.541	-30.1852053	149.3259765	208.407	E745197.0940	N6643602.4153	55
34	2984.694	-4750787.664	2792942.394	-3200631.541	-30.1852055	149.3256900	208.330	E745120.5578	N6643604.0901	55
35	3000.000	-4750779.891	2792955.580	-3200631.541	-30.1852055	149.3256327	208.314	E745105.2507	N6643604.4250	55
36	5938.776	-4749287.562	2795487.251	-3200631.541	-30.1852098	149.3106334	206.033	E742166.2981	N6643668.7330	55
37	6000.000	-4749256.472	2795539.994	-3200631.541	-30.1852099	149.3104043	206.000	E742105.0709	N6643670.0727	55

HEIGHT OF GEOID ABOVE SPHEROID 0.740 metres

16.9m (mar 92)

11.4m

NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS 1 AND 37 3.530 metres
NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS 1 AND 14 1.192 metres
NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS 14 AND 37 2.338 metres

John,

Ignore these figures.

APPENDIX

Relevant mathematical equations are given in the following pages. (The reference is "The Australian Map Grid Technical Manual" by the National Mapping Council of Australia, Canberra 1972). These form the basis of the FORTRAN subroutines which do the coordinate transformations.

The final page gives a graph which drove the initial estimates for the position of the AT compact array line. There are a number of points which must be discussed concerning this diagram in order to avoid misinterpretation. The diagram displays the plane of constraint i) looking along positive Z (ie ~north). The solid wriggly line is the intersection of the local Culgoora topology with the plane. The dotted smooth curve is merely a convenient reference at a totally arbitrary fixed height above sea level. (This can be interpreted as the level of an ideal flood, ignoring the porosity of the soil etc.). The origin indicated is also arbitrary though the (small) x axis zero corresponds to the building location. Depending on one's choice of this zero, the picture looks different due to the unequal scales on the axes (specifically distortion is introduced due to the fact matrices don't commute). Elaborating, the zero in the x axis defines the tangent position to the dotted curve. Moving the zero 4km west produces the picture, as in the indent, seemingly bearing little resemblance to the principal plot. However they are equivalent. If the axes scales were justified (unexploded) the two views would be related by a simple rotation. As it is, the 'stretch' distorts the real situation. Of course linearity is preserved and a straight line in the 'sectional plane' is necessarily a straight line in space (independent of the stretch). It is not a simple matter to deduce the required azimuthal shift from this diagram.

The analytic technique to produce these plots is not based on the XYZ system as outlined before, rather the Earth was assumed spherical and standard spherical polars were used for latitude and longitude. Levels were interpolated by eye at specific points from the surveys done by the Australian Survey Office and Radiophysics.

Also included is a calculation done at the Australian Survey Office (ASO) determining the location of station 14 in terms of Eastings and Northings, based on the following information.

Station 14 is exactly 117.0 metres north (true) and 350.0 metres east (true) of a point known as the centre of the heliograph. We have determined this latter point to be 2.89 metres south and 3.72 metres west of NM/C/59. This may be marginally in error. Further data is given for stations 1 (E1500), 14 (AERIAL CENTRE), 35 (W1500) and 37 (W4500). The small discrepancy between these calculated figures and the tabulated results we present can be accounted for by a) ASO used the older figure for the azimuth shift b) the ASO shifts of say 4500 metres west are along the spheroid ie at an AHD = 0.0 metres (along the arc and not compensating for the height change which approximately matches the gradient).

GLENN BUSH (ASO) CALCULATIONS

BASCAL Version 2.10 - HARDCOPY Output 12.10PM THU, JUN 21 1984

From	To	Fwd Grid Brg	Sph Dist	Rev Grid Brg	Easting	Northing	Pt
	50				746256.471	6643465.298	50
50-	51	1 17 36.00	114.110	181 17 35.86	746259.047	6643579.419	51
51-	52	91 17 36.00	346.280	271 17 36.01	746605.360	6643571.600	52
52-	53	91 15 30.00	1500.000	271 15 30.04	748105.531	6643538.648	53

Traverse Heading:CALCULATING CO-ORDINATES OF AERIAL CENTRE

Filename: AMG Grid: AMG Zone: 55

From	To	Fwd Grid Brg	Sph Dist	Rev Grid Brg	Easting	Northing	Pt
	50				746256.471	6643465.298	50
50-	51	1 17 36.00	114.110	181 17 35.86	746259.047	6643579.419	51
51-	52	91 17 36.00	346.280	271 17 36.01	746605.360	6643571.600	52
52-	53	91 15 30.00	1500.000	271 15 30.04	748105.531	6643538.648	53

Input data DELETED

Points in Traverse stored in file AMG

AMG - TRAVERSE WITH BEARINGS
****

Traverse Heading:CALCULATION OF POINTS ALONG AERIAL LINE

Filename: AMG Grid: AMG Zone: 55

From	To	Fwd Grid Brg	Sph Dist	Rev Grid Brg	Easting	Northing	Pt
	52				746605.360	6643571.600	52
52-	54	271 15 30.00	1500.000	91 15 29.96	745105.203	6643604.552	54
54-	55	271 15 30.00	3000.000	91 15 29.92	742104.931	6643670.454	55

Traverse Heading:CALCULATION OF POINTS ALONG AERIAL LINE

Filename: AMG Grid: AMG Zone: 55

From	To	Fwd Grid Brg	Sph Dist	Rev Grid Brg	Easting	Northing	Pt
	52				746605.360	6643571.600	52
52-	54	271 15 30.00	1500.000	91 15 29.96	745105.203	6643604.552	54
54-	55	271 15 30.00	3000.000	91 15 29.92	742104.931	6643670.454	55

Input data DELETED

Points in Traverse stored in file AMG

GRID TO SPHEROID
****

A GROUP OF POINTS FROM FILE AMG

GLENN BUSH (ASO) CALCULATIONS (con't)

RANGE OF POINTS : 50 TO 60

Point Number	Filename: AMG Description	Notes 1 3 5	Grid: AMG Easting	Zone: 55 Northing	Datum: AHD Height
50	TRIG NM/C/59		746256.471	6643465.298	
LATITUDE	30 18 55.72883		CONVERGENCE	1 17 35.99	
LONGITUDE	149 33 39.50842				
51			746259.047	6643579.419	
LATITUDE	30 18 52.02323		CONVERGENCE	1 17 35.85	
LONGITUDE	149 33 39.50843				
52	AERIAL CENTRE		746605.360	6643571.600	
LATITUDE	30 18 52.02306		CONVERGENCE	1 17 42.40	
LONGITUDE	149 33 52.46949				
53	E1500		748105.531	6643538.648	
LATITUDE	30 18 51.98844		CONVERGENCE	1 18 10.78	
LONGITUDE	149 34 48.61362				
54	W1500		745105.203	6643604.552	
LATITUDE	30 18 52.05098		CONVERGENCE	1 17 14.02	
LONGITUDE	149 32 56.32535				
55	W4500		742104.931	6643670.454	
LATITUDE	30 18 52.08672		CONVERGENCE	1 16 17.26	
LONGITUDE	149 31 4.03704				

We attempt now to briefly correlate our calculations with those of ASO as follows. The resulting spheroidal North-South deviation between W4500 and E1500, from the ASO calculations, is 3.03 metres using the meridian distance formula elsewhere in this appendix. We simulate this same meridial distance change by adjusting the height of station 37 to 206.64 metres (leaving station 14 height at 210.0 metres), in our calculations. The resulting discrepancy corresponds to 0.15 metres at station 37 and 0.05 metres at station 1 as can be seen from the following results:

Station 1	E748105.480	N6643538.650	(AHD 211.83)
Station 37	E742105.079	N6643670.447	(AHD 206.64)

The difference corresponds to the errors introduced for not accounting for the height change ,between stations (ie not all AHD 0.00), and a curvature correction term since distances were measured along the arc not a straight line in space, plus more subtle, higher order terms.

2 Symbols, Definitions and Sign Conventions

2.1 SYMBOLS

The sybmols used with the Australian Map Grid are listed below. Many terms are more fully defined in paragraph *2.2* below, and illustrated in Annex D. Some symbols used with the Australian National Spheroid are listed in paragraph *3.2.*

ϕ	= Geodetic latitude, negative south of the equator.	
ϕ_1, ϕ_2	= Latitude at points 1 and 2 respectively	and similarly for other terms.
ϕ_m	= $(\phi_1 + \phi_2) / 2$	and similarly for other terms.
$\Delta\phi$	= $\phi_2 - \phi_1$	and similarly for other terms.
$\Delta\phi''$	= $\Delta\phi$ expressed in seconds of arc	and similarly for other terms.
λ	= Geodetic longitude measured from Greenwich, positive eastwards.	
λ_0	= Geodetic longitude of a central meridian.	
ω	= Geodetic longitude measured from a central meridian, positive eastwards: $\omega = \lambda - \lambda_0$	
E'	= Easting measured from a central meridian, positive eastwards.	
N'	= Northing measured from the equator, negative southwards.	
E	= E' + 500 000 metres.	
N	= N' in the northern hemisphere.	
	= N' + 10 000 000 metres in the southern hemisphere.	
ρ, ν	= Radii of curvature of the spheroid in meridian and prime vertical respectively.	
α	= Azimuth, clockwise through 360° from true north.	
β	= Grid bearing, clockwise through 360° from grid north.	
θ	= Plane bearing, clockwise through 360° from grid north.	
γ	= Grid convergence, positive when grid north is west of true north, negative when grid north is east of true north. $\beta = \alpha + \gamma$	
δ	= Arc-to-chord correction, with sign defined by the equations: $\theta = \beta + \delta = \alpha + \gamma + \delta$	
$\Delta\alpha$	= Meridian convergence.	
$\Delta\beta$	= Line curvature.	
s	= Spheroidal distance.	
S	= Grid distance.	
L	= Plane distance.	
m	= Meridian distance, true distance from the equator, negative southwards.	
a, b	= Major and minor semi-axes of the spheroid.	
e^2	= $(a^2 - b^2) / a^2 = (\text{eccentricity})^2$	
e'^2	= $(a^2 - b^2) / b^2 = (\text{second eccentricity})^2$	
k_0	= Central scale factor = 0·999 6	
k	= Point scale factor.	
K	= Line scale factor.	
t	= $\tan\phi$	
ψ	= ν/ρ	
ϕ'	= Latitude for which $m = N'/k_0$	

t', ψ', ρ', ν' are functions of the latitude ϕ'

R^2	= $\rho\nu$
r^2	= $R^2 k_0^2 = \rho\nu k_0^2$

Note: E', N', E, N, S, L, r, k and K include the central scale factor, k_0.
s, ρ, ν, R and m are true distances, which must be specifically multiplied by k_0 when necessary.

3 Rigorous Formulae on the Spheroid

3.1 AIMS

3.1.1 This chapter gives formulae and numerical examples for highly accurate computations on the spheroid:

3.1.1.1 *The Direct Problem:* latitude and longitude from spheroidal distance and azimuth.

3.1.1.2 *The Reverse Problem:* spheroidal distance and azimuth from latitude and longitude.

The formulae are those given by Dr A.R. Robbins in *Empire Survey Review* No. 125, 1962. They are accurate to better than 20mm over distances of 1 500 kilometres. The errors can reach 16 metres at 4 500 kilometres, and more than 2 000 metres at 9 000 kilometres.

3.1.2 It is not foreseen that these formulae will ever be used for hand computation: they require ten-figure trigonometrical functions, and it is easier and better to use the programs available for electronic computers. In the numerical examples, all trigonometrical functions and intermediate results are given, which should be adequate for checking similar programs; but they have not been laid out in a form suitable for hand computation, and they may give a false idea of ease and brevity.

3.1.3 Robbins's formulae are for the normal section. For conversion to the geodesic, see paragraph *3.7*.

3.2 FUNCTIONS FOR THE AUSTRALIAN NATIONAL SPHEROID

3.2.1 By definition (see paragraph *1.2.1*):

Major semi-axis, $a = 6\,378\,160$ metres
Flattening, $f = 1/298{\cdot}25$ exactly

3.2.2 From these figures can be derived:

Flattening, f	$= 0{\cdot}003\,352\,891\,869$
Minor semi-axis, $b = a\,(1-f)$	$= 6\,356\,774{\cdot}719$ metres
$e^2 = 2f - f^2 = (a^2 - b^2)/a^2$	$= 0{\cdot}006\,694\,541\,855$
e	$= 0\,081\,820\,180\,0$
$e'^2 = e^2/(1-e^2) = (a^2 - b^2)/b^2$	$= 0{\cdot}006\,739\,660\,796$
e'	$= 0{\cdot}082\,095\,437\,1$

3.2.3 For the computation of radii of curvature:

$c = a/(1-e^2)^{1/2} = 6\,399\,617{\cdot}225$ metres
$V^2 = 1 + e'^2 \cos^2\phi = \nu/\rho = \psi$ (in Chapter 4)
$\rho = c/V^3$;
$\nu = c/V$;
$R = (\rho\nu)^{1/2} = c/V^2$

3.2.4 For computing meridian distances on the Australian National Spheroid, the formula given in paragraph *4.2* reduces to:

$$m = 111\,133{\cdot}348\,785\,\phi^0 - 16\,038{\cdot}954\,6 \sin 2\phi + 16{\cdot}833\,1 \sin 4\phi - 0{\cdot}021\,8 \sin 6\phi \text{ metres}$$

where ϕ^0 = latitude in degrees

The values of A′, B′, C′, D′ quoted on page i of TM5-241-33 Latitude Functions include higher order terms. The difference in meridian distance is less than 0·5mm in latitude 45°.

3.2.5 The following 11-figure values are often useful:

$\sin 1'' = 0{\cdot}000\,004\,848\,136\,811\,1$
$\pi = 3{\cdot}141\,592\,653\,6$

4 Rigorous Formulae Between Spheroid and Grid

4.1 AIMS

4.1.1 The aims of this chapter are to provide:

4.1.1.1 The adopted formula for meridian distance;

4.1.1.2 Redfearn's formulae for obtaining easting, northing, grid convergence and point scale factor, from latitude and longitude;

4.1.1.3 Redfearn's formulae for the reverse computation, from grid to spheroid;

4.1.1.4 Numerical examples.

4.1.2 Redfearn's formulae were published in *Empire Survey Review* No. 69, 1948. They are accurate to better than 1 mm in any zone of the Australian Map Grid. For purposes of definition, they are to be regarded as exact, not merely as the opening terms of infinite series. All angles are in radians. For the definition of symbols, see paragraph *2.1*.

4.1.3 It is not foreseen that Redfearn's formulae will ever be used for hand computation. For routine work, computations using the tables published by the US Army give results which should be adequate for all practical purposes – see paragraphs *5.8* and *5.10*. If extreme precision is required for an occasional job, it is easier and better to use an electronic computer – see paragraph *1.8.3*.

4.1.4 In the numerical examples, all trigonometrical functions and intermediate results are given. They should be adequate for checking other computer programs. They have not been laid out in a form specifically designed for hand computation, and they may give a false idea of ease and brevity.

4.2 MERIDIAN DISTANCE

In Australia, meridian distance is defined by the following terms only of an infinite series:

$$m = a(A_0\phi - A_2 \sin 2\phi + A_4 \sin 4\phi - A_6 \sin 6\phi)$$

where

$$A_0 = 1 - e^2/4 - 3e^4/64 - 5e^6/256$$
$$A_2 = (3/8)(e^2 + e^4/4 + 15e^6/128)$$
$$A_4 = (15/256)(e^4 + 3e^6/4)$$
$$A_6 = 35e^6/3072$$

This limited formula is correct to less than 0·5 mm in latitude 45°. See paragraph *3.2.4*.

4.3 SPHEROID TO GRID: REDFEARN'S FORMULAE

4.3.1 **Easting**

$$E' = k_0 \Big\{ \nu\omega\cos\phi + \nu\frac{\omega^3}{6}\cos^3\phi(\psi - t^2) + \nu\frac{\omega^5}{120}\cos^5\phi\,[4\psi^3(1-6t^2) + \psi^2(1+8t^2) - \psi(2t^2) + t^4] + \nu\frac{\omega^7}{5040}\cos^7\phi(61 - 479t^2 + 179t^4 - t^6)\Big\}$$

4.3.2 **Northing**

$$N' = k_0 \Big\{ m + \nu\sin\phi\frac{\omega^2}{2}\cos\phi + \nu\sin\phi\frac{\omega^4}{24}\cos^3\phi(4\psi^2 + \psi - t^2)$$

$$+\nu \sin\phi \frac{\omega^6}{720} \cos^5\phi \, [8\psi^4(11 - 24t^2) - 28\psi^3(1 - 6t^2) + \psi^2(1 - 32t^2) - \psi(2t^2) + t^4]$$

$$+\nu \sin\phi \frac{\omega^8}{40320} \cos^7\phi \, (1385 - 3111t^2 + 543t^4 - t^6)\}$$

4.3.3 **Grid Convergence** (radians)

$$\gamma = -\sin\phi \; \omega$$
$$- \sin\phi \frac{\omega^3}{3} \cos^2\phi \, (2\psi^2 - \psi)$$
$$- \sin\phi \frac{\omega^5}{15} \cos^4\phi \, [\psi^4(11 - 24t^2) - \psi^3 (11 - 36t^2) + 2\psi^2 (1 - 7t^2) + \psi t^2]$$
$$- \sin\phi \frac{\omega^7}{315} \cos^6\phi \, (17 - 26t^2 + 2t^4)$$

4.3.4 **Point Scale Factor**

$$k = k_0 \{1 + \frac{\omega^2}{2} \cos^2\phi \, \psi$$
$$+ \frac{\omega^4}{24} \cos^4\phi \, [4\psi^3(1 - 6t^2) + \psi^2(1 + 24t^2) - 4\psi t^2]$$
$$+ \frac{\omega^6}{720} \cos^6\phi \, (61 - 148t^2 + 16t^4)\}$$

4.4 FROM AUSTRALIAN GEODETIC DATUM TO AUSTRALIAN MAP GRID: NUMERICAL EXAMPLE

Station: BUNINYONG -37.6543214167

Latitude ϕ = − 37° 39′ 15″557 1 | Longitude λ = + 143° 55′ 30″633 0 143.925 1758333

Zone: 54 | λ_0 = 141° | ω = + 2° 55′ 30″633 0

ω in radians = + 0·051 053 949 6

Functions:

ϕ radians	= − 0·657 191 886 3		
sin ϕ	= − 0·610 896 049 5		A_0 = 0·998 324 257 9
sin 2ϕ	= − 0·967 306 020 1	e^2 = 0·006 694 541 9	A_2 = 0·002 514 668 0
sin 4ϕ	= − 0·490 640 893 3	e^4 = 0·000 044 816 9	A_4 = 0·000 002 639 2
sin 6ϕ	= + 0·718 441 150 9	e^6 = 0·000 000 300 0	A_6 = 0·000 000 003 4

Meridian Distance:

1st term	= − 4 184 650·835
2nd term	= + 15 514·577
3rd term	= − 8·259
4th term	= − 0·016
Sum = m	= − 4 169 144·533

Radii of Curvature:

ρ = 6 359 277·924

ν = 6 386 142·439

Powers:

Power	cos ϕ	ω	t= tan ϕ	$\psi = \nu/\rho$
1	0·791 710 816 3	0·051 053 949 6	− 0·771 615 136 4	1·004 224 459 9
2	0·626 806 016 6	0·002 606 505 8	0·595 389 918 7	1·008 466 765 9
3	0·496 249 103 1	0·000 133 072 4		1·012 726 993 3
4	0·392 885 782 5	0·000 006 793 9	0·354 489 155 3	1·017 005 217 8
5	0·311 051 923 6	0·000 000 346 9		
6	0·246 263 172 3	0·000 000 017 7	0·211 059 269 4	
7	0·194 969 217 2	0·000 000 000 9		
8		(+4·6.10^{-11})		

* Cartesian XYZ – Geodetic Latitude & Longitude Transformation

$$X = (\nu + h)\cos\phi\cos\lambda$$
$$Y = (\nu + h)\cos\phi\sin\lambda$$
$$Z = [\nu(1-e^2) + h]\sin\phi$$

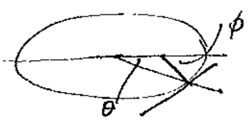
ν radius of curv. in prime vertical

$$\nu = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

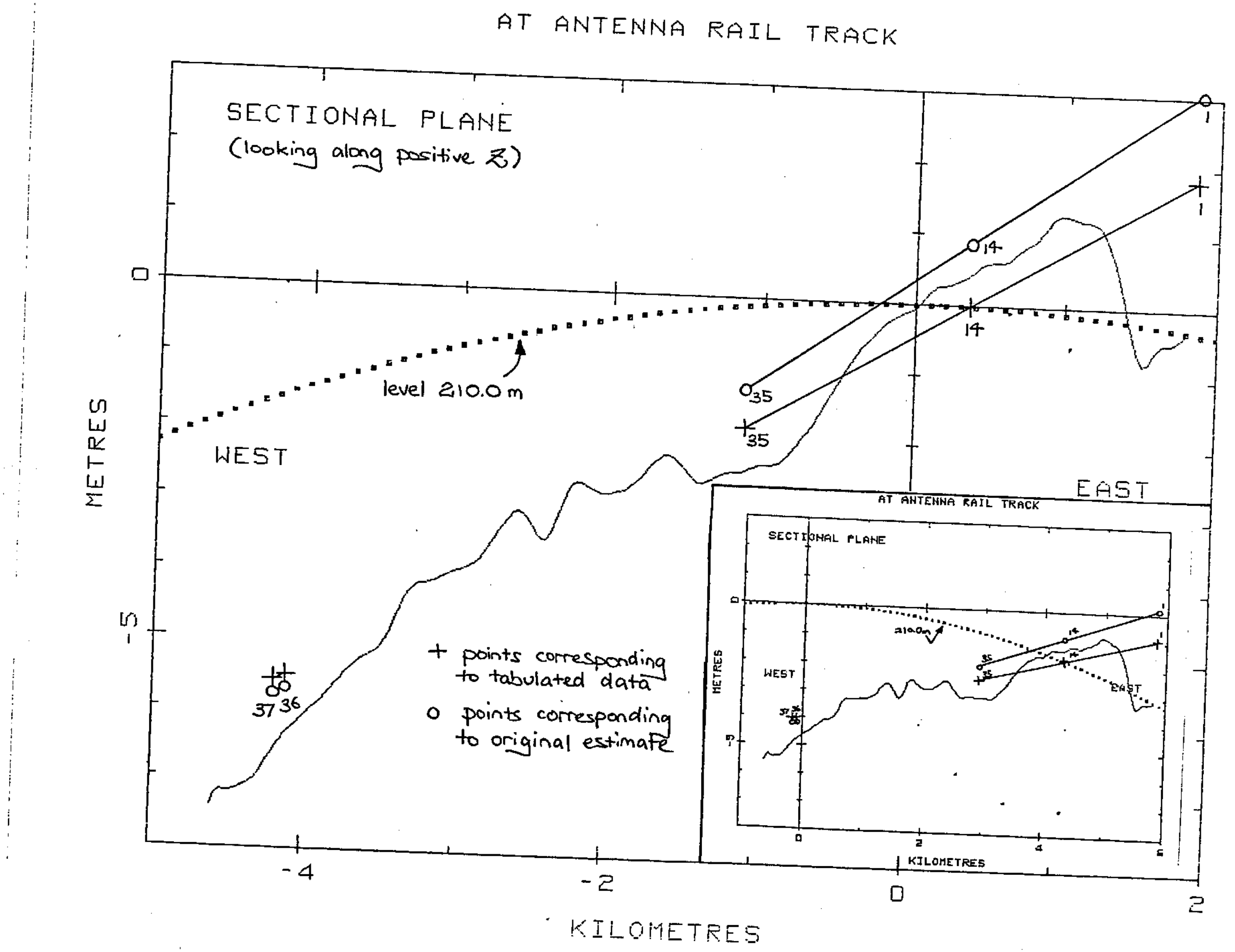
where h = spheroidal height ($\equiv$ geoidal height + geoid/spheroid sep.)

$$\phi_1 = \operatorname{atn}\left(\frac{Z}{\sqrt{x^2+y^2}}\right) + \operatorname{asn}\left(\frac{\nu e^2 \sin\phi_0 \cos\phi_0}{\sqrt{x^2+y^2+z^2}}\right)$$

and iterate for ϕ.



$\tan\theta = (1-e^2)\tan\phi$



AT ANTENNA RAIL TRACK
SECTIONAL PLANE
(looking along positive Z)
METRES
0
-5
level 210.0 m
WEST
EAST
1
14
35
36
37
+ points corresponding to tabulated data
o points corresponding to original estimate
-4
-2
0
2
KILOMETRES

FIGURE 6.

AT/15.6.1/002

CSIRO DIVISION OF RADIOPHYSICS

AT COMPACT ARRAY LINE SURVEY 2

16/8/84

INTRODUCTION

This brief paper is an adjunct to the first report (AT/15.6/001) and gives station locations subject to water coursing and soil escavation constraints. The accompanying figures and tables give the relevant data, along with the program code for future reference.

RESULTS

Figure 1 gives the generated profile of the AT compact array line, for an optimised design. The actual station locations are indicated, these points representing the level of the top of the supporting piers (note that stations 36 and 37 have been included and have been displaced 5 km east). It is apparent that these stations, as plotted, lie on a gentle curve, a manifestation of representing an absolute straight line in space in the surveyor's coordinate system (AHD). The solid wriggly curve represents the local topology ground height directly under the line of the antenna elements.

The driving constraint of the configuration of figure 1 is that the pier tops should clear by 0.6 metres the potential bog at -2500 W. This is to provide clearance of rain runoff except during major floods which occur about once every five years (as outlined in AT/15.6/100). These figures are based on civil engineering considerations. Table 2 gives information concerning the (AHD) heights of the pier and the ground level (and their difference), complementing the information already given in Table 1, (which corresponds to the table given in the first report). The ground level figures were interpolated, by the program, from the numerical data supplied by the survey office.

Included on figure 1 is a dotted line representing the level of the anticipated escavation required. This curve is roughly 0.85 metres below the pier top or 0.15 metres below the ground level whichever was the lesser. The former figure relates to the anticipated level of foundation escavation required for the pier (to reach firm substrata) and rail structure and the latter figure relates to the removal of structurally unsound topsoil before filling.

R.A.KENNEDY
G.VAN DER MEULEN

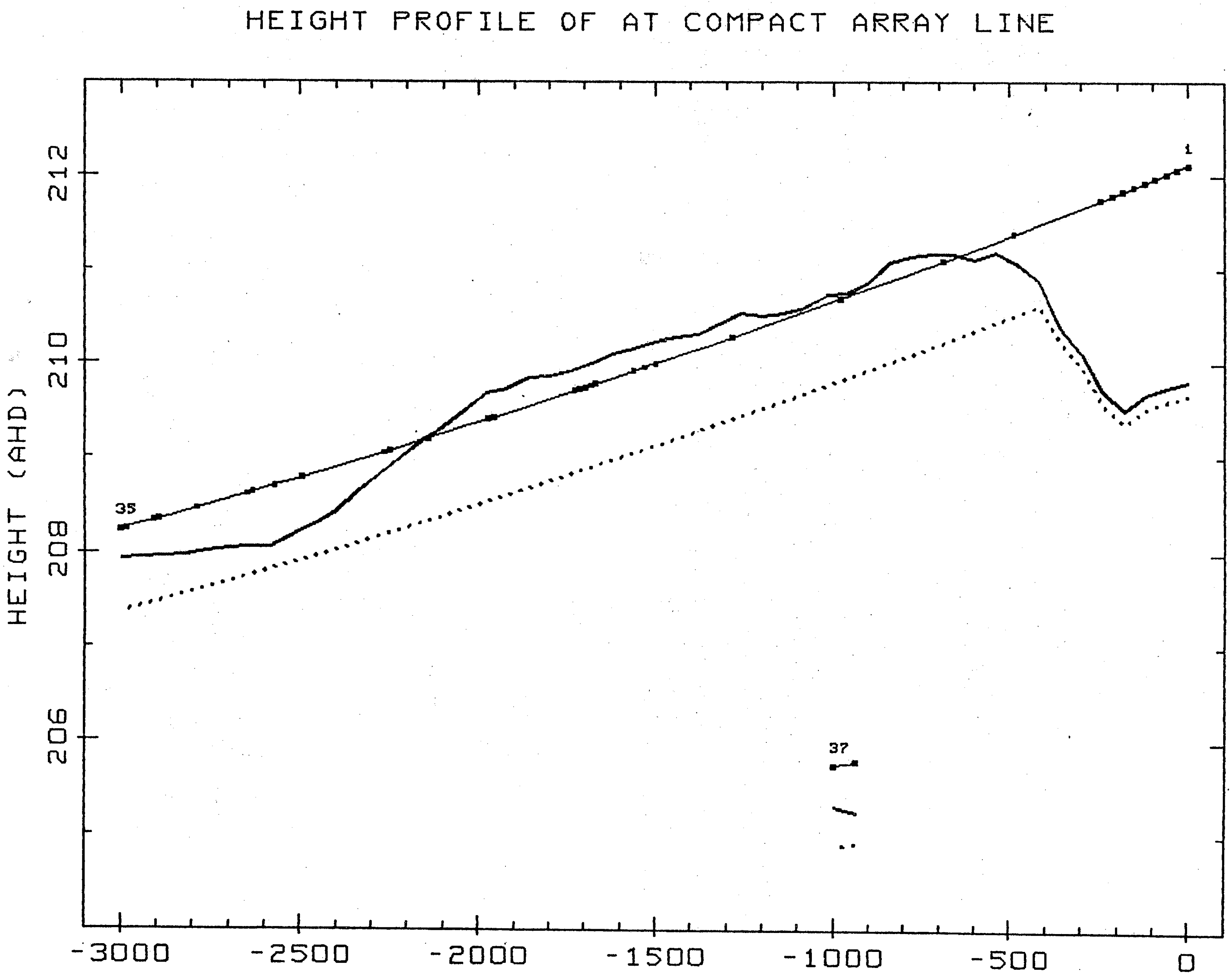

HEIGHT PROFILE OF AT COMPACT ARRAY LINE
HEIGHT (AHD)
212
210
208
206
-3000
-2500
-2000
-1500
-1000
-500
0
W
DISTANCE ALONG ARRAY
E
1
35
37

FIGURE 1.

STATION #	DIST metres	EARTH CENTRED CARTESIAN COORDINATES X metres	Y metres	Z metres	GEODETIC LATITUDE DD.MMSSsss	LONGITUDE DDD.MMSSsss	AHD metres	AUSTRALIAN MAP GRID EASTING metres	NORTHING metres	ZONE
1	0.000	-4752303.393	2790371.215	-3200631.541	-30.1851983	149.3448612	212.122	E748105.4830	N6643538.8235	55
2	30.612	-4752287.846	2790397.585	-3200631.541	-30.1851984	149.3447466	212.075	E748074.8684	N6643539.4924	55
3	61.224	-4752272.300	2790423.955	-3200631.541	-30.1851985	149.3446320	212.028	E748044.2538	N6643540.1613	55
4	91.837	-4752256.752	2790450.327	-3200631.541	-30.1851985	149.3445174	211.982	E748013.6381	N6643540.8302	55
5	122.449	-4752241.206	2790476.697	-3200631.541	-30.1851986	149.3444029	211.935	E747983.0235	N6643541.4991	55
6	153.061	-4752225.659	2790503.067	-3200631.541	-30.1851987	149.3442883	211.889	E747952.4089	N6643542.1680	55
7	183.673	-4752210.112	2790529.438	-3200631.541	-30.1851988	149.3441737	211.843	E747921.7943	N6643542.8369	55
8	214.286	-4752194.565	2790555.809	-3200631.541	-30.1851989	149.3440591	211.797	E747891.1787	N6643543.5059	55
9	244.898	-4752179.019	2790582.179	-3200631.541	-30.1851990	149.3439446	211.751	E747860.5641	N6643544.1748	55
10	489.796	-4752054.644	2790793.144	-3200631.541	-30.1851997	149.3430280	211.390	E747615.6455	N6643549.5261	55
11	688.776	-4751953.590	2790964.553	-3200631.541	-30.1852002	149.3422832	211.104	E747416.6490	N6643553.8740	55
12	979.592	-4751805.896	2791215.073	-3200631.541	-30.1852010	149.3411947	210.696	E747125.8092	N6643560.2287	55
13	1285.714	-4751650.428	2791478.778	-3200631.541	-30.1852018	149.3400490	210.282	E746819.6628	N6643566.9177	55
14	1500.000	-4751541.600	2791663.373	-3200631.541	-30.1852023	149.3352469	210.000	E746605.3600	N6643571.6000	55
15	1530.612	-4751526.054	2791689.743	-3200631.541	-30.1852024	149.3351324	209.960	E746574.7456	N6643572.2689	55
16	1561.224	-4751510.507	2791716.113	-3200631.541	-30.1852025	149.3350178	209.921	E746544.1312	N6643572.9378	55
17	1668.367	-4751456.093	2791808.411	-3200631.541	-30.1852027	149.3346168	209.784	E746436.9800	N6643575.2790	55
18	1683.673	-4751448.320	2791821.596	-3200631.541	-30.1852028	149.3345595	209.764	E746421.6728	N6643575.6135	55
19	1698.980	-4751440.546	2791834.782	-3200631.541	-30.1852028	149.3345022	209.745	E746406.3646	N6643575.9479	55
20	1714.286	-4751432.773	2791847.967	-3200631.541	-30.1852028	149.3344449	209.726	E746391.0575	N6643576.2824	55
21	1729.592	-4751424.999	2791861.152	-3200631.541	-30.1852029	149.3343876	209.706	E746375.7503	N6643576.6168	55
22	1959.184	-4751308.398	2792058.932	-3200631.541	-30.1852034	149.3335283	209.421	E746146.1409	N6643581.6338	55
23	1974.490	-4751300.625	2792072.117	-3200631.541	-30.1852034	149.3334710	209.402	E746130.8337	N6643581.9683	55
24	2142.857	-4751215.118	2792217.155	-3200631.541	-30.1852038	149.3328409	209.199	E745962.4542	N6643585.6472	55
25	2250.000	-4751160.704	2792309.452	-3200631.541	-30.1852041	149.3324398	209.071	E745855.3033	N6643587.9884	55
26	2265.306	-4751152.931	2792322.637	-3200631.541	-30.1852041	149.3323825	209.053	E745839.9962	N6643588.3228	55
27	2494.898	-4751036.330	2792520.416	-3200631.541	-30.1852046	149.3315232	208.787	E745610.3874	N6643593.3395	55
28	2571.429	-4750997.463	2792586.343	-3200631.541	-30.1852048	149.3312368	208.700	E745533.8509	N6643595.0118	55
29	2632.653	-4750966.369	2792639.084	-3200631.541	-30.1852049	149.3310076	208.631	E745472.6226	N6643596.3496	55
30	2647.959	-4750958.596	2792652.269	-3200631.541	-30.1852049	149.3309503	208.614	E745457.3155	N6643596.6840	55
31	2785.714	-4750888.636	2792770.936	-3200631.541	-30.1852052	149.3304348	208.462	E745319.5507	N6643599.6940	55
32	2892.857	-4750834.222	2792863.234	-3200631.541	-30.1852054	149.3300337	208.345	E745212.4002	N6643602.0352	55
33	2908.163	-4750826.448	2792876.419	-3200631.541	-30.1852055	149.3259765	208.329	E745197.0931	N6643602.3696	55
34	2984.694	-4750787.581	2792942.345	-3200631.541	-30.1852056	149.3256900	208.247	E745120.5568	N6643604.0418	55
35	3000.000	-4750779.808	2792955.531	-3200631.541	-30.1852057	149.3256327	208.231	E745105.2497	N6643604.3763	55
36	5938.776	-4749287.316	2795487.106	-3200631.541	-30.1852103	149.3106334	205.787	E742166.2951	N6643668.5888	55
37	6000.000	-4749256.223	2795539.847	-3200631.541	-30.1852104	149.3104043	205.750	E742105.0679	N6643669.9265	55

HEIGHT OF GEOID ABOVE SPHEROID 0.740 metres

NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS 1 AND 37 3.725 metres
NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS 1 AND 14 1.240 metres
NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS 14 AND 37 2.485 metres

John,
Ignore these figures.

TABLE 1.

STATION #	DIST metres	EASTING metres	NORTHING metres	PIER TOP metres	GND LEVELS metres	DIFF metres
1	0.000	E748105.4830	N6643538.8235	212.122	209.817	2.304
2	30.612	E748074.8684	N6643539.4924	212.075	209.816	2.259
3	61.224	E748044.2538	N6643540.1613	212.028	209.762	2.266
4	91.837	E748013.6381	N6643540.8302	211.982	209.721	2.260
5	122.449	E747983.0235	N6643541.4991	211.935	209.659	2.276
6	153.061	E747952.4089	N6643542.1680	211.889	209.563	2.326
7	183.673	E747921.7943	N6643542.8369	211.843	209.512	2.331
8	214.286	E747891.1787	N6643543.5059	211.797	209.544	2.253
9	244.898	E747860.5641	N6643544.1748	211.751	209.760	1.991
10	489.796	E747615.6455	N6643549.5261	211.390	211.102	0.289
11	688.776	E747416.6490	N6643553.8740	211.104	211.247	-0.144
12	979.592	E747125.8092	N6643560.2287	210.696	210.734	-0.038
13	1285.714	E746819.6628	N6643566.9177	210.282	210.462	-0.180
14	1500.000	E746605.3600	N6643571.6000	210.000	210.240	-0.240
15	1530.612	E746574.7456	N6643572.2689	209.960	210.207	-0.247
16	1561.224	E746544.1312	N6643572.9378	209.921	210.166	-0.245
17	1668.367	E746436.9800	N6643575.2790	209.784	210.042	-0.258
18	1683.673	E746421.6728	N6643575.6135	209.764	209.994	-0.229
19	1698.980	E746406.3646	N6643575.9479	209.745	209.957	-0.212
20	1714.286	E746391.0575	N6643576.2824	209.726	209.937	-0.212
21	1729.592	E746375.7503	N6643576.6168	209.706	209.915	-0.209
22	1959.184	E746146.1409	N6643581.6338	209.421	209.689	-0.268
23	1974.490	E746130.8337	N6643581.9683	209.402	209.673	-0.271
24	2142.857	E745962.4542	N6643585.6472	209.199	209.225	-0.027
25	2250.000	E745855.3033	N6643587.9884	209.071	208.913	0.159
26	2265.306	E745839.9962	N6643588.3228	209.053	208.897	0.156
27	2494.898	E745610.3874	N6643593.3395	208.787	208.207	0.580
28	2571.429	E745533.8509	N6643595.0118	208.700	208.095	0.606
29	2632.653	E745472.6226	N6643596.3496	208.631	208.050	0.581
30	2647.959	E745457.3155	N6643596.6840	208.614	208.058	0.556
31	2785.714	E745319.5507	N6643599.6940	208.462	207.993	0.469
32	2892.857	E745212.4002	N6643602.0352	208.345	207.971	0.375
33	2908.163	E745197.0931	N6643602.3696	208.329	207.966	0.363
34	2984.694	E745120.5568	N6643604.0418	208.247	207.939	0.308
35	3000.000	E745105.2497	N6643604.3763	208.231	207.929	0.301
36	5938.776	E742166.2951	N6643668.5888	205.787	205.250	0.537
37	6000.000	E742105.0679	N6643669.9265	205.750	205.300	0.450

TABLE 2.

```
PROGRAM STATIONS

THIS PROGRAM GIVES THE COORDINATES OF THE AT ARRAY STATIONS IN A
VARIETY OF FORMATS AND FORMS SPECIFICALLY
        ABSOLUTE SPACE DISTANCE
        GEODETIC LATITUDE AND LONGITUDE
        EASTING, NORTHING AND ZONE
        HEIGHT ABOVE GEOID (AUSTRALIAN HEIGHT DATA
        HEIGHT ABOVE LOCAL TOPOLOGY

IMPLICIT DOUBLE PRECISION (A-G,O-R,T-Z)
DOUBLE PRECISION LAT,LONG,NORTHING,DL(100),NN,NNN,NS
DOUBLE PRECISION HEIGHT,HO,HH,HHH,HT
DIMENSION HARRAY(100),HLINE(100),HTOP(100),SX(2),SY(2),SZ(2),SU(2)
DIMENSION HESC(100),HD(100)
INTEGER ZONE
LOGICAL STILL_ERROR,ENOUGH

COMMON /DATA/ AE(2000),AN(2000),AH(2000),NPTS

OPEN(UNIT=1,FILE='STATIONS.DAT',STATUS='OLD',FORM='FORMATTED')
OPEN(UNIT=2,FILE='STATIONS.OUT',STATUS='NEW',FORM='FORMATTED')
OPEN(UNIT=3,FILE='MOSS.ROD',STATUS='OLD',FORM='FORMATTED')
OPEN(UNIT=4,FILE='HEIGHTS.OUT',STATUS='NEW',FORM='FORMATTED')

PI=3.141592653589793
DTR=PI/180.0

READ(1,1000) CENT_E,CENT_N,CENT_H,GS_SEP,ZONE
TYPE 4000,CENT_H
ACCEPT 4010,CT
IF(CT.NE.0) CENT_H=CT
CALL LL_EN(CENT_E,CENT_N,ZONE,LAT,LONG)
CALL XYZ_LLH(LAT,LONG,CENT_H,GS_SEP,XO,YO,ZO)

Z=ZO DEFINES THE PLANE OF THE ANTENNNAS (METRES SOUTH OF EQUATOR)

READ(1,1010) HO  ! DESIRED HEIGHT OF STATION 37 ABOVE GEOID
TYPE 4020,HO
ACCEPT 4010,CT
IF(CT.NE.0) HO=CT

TYPE 4030
ACCEPT 4010,F1
TYPE 4040
ACCEPT 4010,F2

READ IN MOSS DATA

ES=741000.0
NS=6643000.0
I=0
DO WHILE(.NOT.ENOUGH)
    READ(3,2000) EE,NN,HH,EEE,NNN,HHH
    I=I+1
    AE(I)=EE+ES
    AN(I)=NN+NS
    AH(I)=HH
    IF(EEE.NE.-1) THEN
        I=I+1
        AE(I)=EEE+ES
        AN(I)=NNN+NS
        AH(I)=HHH

    END IF
    ENOUGH=EE.EQ.999
END DO
NPTS=I-2

AL=90.0+ATAN2(YO,XO)/DTR-1.0
AU=AL+2.0
STILL_ERROR=.TRUE.

DO WHILE (STILL_ERROR)
    ALPHA=(AL+AU)/2.0
    X=XO-4500.0*DCOS(ALPHA*DTR)
    Y=YO-4500.0*DSIN(ALPHA*DTR)
    CALL LLH_XYZ(X,Y,ZO,LAT,LONG,HEIGHT)
    HEIGHT=HEIGHT-GS_SEP
    IF(HEIGHT.LT.HO) THEN
        AL=ALPHA
    ELSE
        AU=ALPHA
    END IF
    STILL_ERROR=DABS(HEIGHT-HO).GT.0.0001
END DO

READ(1,1020) N
WRITE(2,1070)
WRITE(2,1050)
WRITE(2,1060)
DO I=1,N
    READ(1,1030) D
    X=XO+(1500.0-D)*DCOS(ALPHA*DTR)
    Y=YO+(1500.0-D)*DSIN(ALPHA*DTR)
    CALL LLH_XYZ(X,Y,ZO,LAT,LONG,HEIGHT)
    CALL EN_LL(LAT,LONG,0,EASTING,NORTHING,ZONE)
    DL(I)=LAT
    LONG=DMS(LONG/DTR)
    LAT=DMS(LAT/DTR)
    HEIGHT=HEIGHT-GS_SEP
    IF(I.LE.35) THEN
        CALL TOP_HEIGHT(EASTING,NORTHING,HT)
    ELSE
        IF(I.EQ.36) THEN
            HT=205.25
        ELSE
            HT=205.30
        END IF
    END IF
    HARRAY(I)=-D
    HLINE(I)=HEIGHT
    IF(I.EQ.1) WRITE(4,1110)
    WRITE(4,1100) I,D,EASTING,NORTHING,HEIGHT,HT,HEIGHT-HT
    WRITE(2,1040) I,D,X,Y,ZO,LAT,LONG,HEIGHT,EASTING,NORTHING,ZONE
END DO
WRITE(2,1080) GS_SEP

E_SQRD=    0.006694541855 D00
A=               6378160.00 D00
CALL MERDIST(A,E_SQRD,DL(1),D1)
CALL MERDIST(A,E_SQRD,DL(14),D14)
CALL MERDIST(A,E_SQRD,DL(37),D37)
WRITE(2,1090) 1,37,DABS(D37-D1)
WRITE(2,1090) 1,14,DABS(D14-D1)
WRITE(2,1090) 14,37,DABS(D37-D14)
        DO I=0,50
            D=60.0*I
            X=XO+(1500.0-D)*DCOS(ALPHA*DTR)
            Y=YO+(1500.0-D)*DSIN(ALPHA*DTR)
            CALL LLH_XYZ(X,Y,ZO,LAT,LONG,HEIGHT)
            CALL EN_LL(LAT,LONG,0,EASTING,NORTHING,ZONE)
            HEIGHT=HEIGHT-GS_SEP
            CALL TOP_HEIGHT(EASTING,NORTHING,HT)
            HTOP(I+1)=HT
            HD(I+1)=-D
            HESC(I+1)=MIN(SNGL(HT-F1),SNGL(HEIGHT-F2))
        END DO

C       NOW A GRAPHIC PLOT

        CALL PGBEGIN(5,'STATIONS.PLT',1,1)
        CALL PGENV(-3100.0,100.0,204.0,213.0,0,0)
        CALL PGLABEL('W        DISTANCE ALONG ARRAY      E','HEIGHT (AHD)',
     & 'HEIGHT PROFILE OF AT COMPACT ARRAY LINE')
        CALL PGLINE(35,HARRAY,HLINE)
        CALL PGPOINT(35,HARRAY,HLINE,15)
        SZ(1)=205.250
        SZ(2)=205.300
        DO I=1,2
            SX(I)=HARRAY(35+I)+5000.0
            SY(I)=HLINE(35+I)
            SU(I)=MIN(SZ(I)-F1,SY(I)-F2)
        END DO
        CALL PGLINE(2,SX,SY)
        CALL PGPOINT(2,SX,SY,15)
        CALL PGSETC(0.5)
        CALL PGTEXT(HARRAY(1),HLINE(1)+0.2,'1')
        CALL PGTEXT(HARRAY(35),HLINE(35)+0.2,'35')
        CALL PGTEXT(SX(2),SY(2)+0.2,'37')
        CALL GRSETLW(2)
        CALL PGLINE(51,HD,HTOP)
        CALL PGLINE(2,SX,SZ)
        CALL GRSETLS(4)
        CALL PGLINE(51,HD,HESC)
        CALL PGLINE(2,SX,SU)
        CALL PGEND

1000    FORMAT(4F15.0,I5)
1010    FORMAT(F10.0)
1020    FORMAT(I10)
1030    FORMAT(F10.0)
1040    FORMAT(X,I5,F10.3,3F14.3,2F13.7,F9.3,'  E',F11.4,'  N',F12.4,I5)
1050    FORMAT(X,'STATION',4X,'DIST',7X,'X',13X,'Y',13X,'Z',10X,
     &  'LATITUDE',4X,'LONGITUDE',5X,'AHD',7X,'EASTING',7X,'NORTHING',
     &  3X,'ZONE')
1060    FORMAT(4X,'#',2(5X,'metres'),2(8X,'metres'),6X,'DD.MMSSsss',
     &  2X,'DDD.MMSSsss',3X,'metres',6X,'metres',9X,'metres',/)
1070    FORMAT(20X,'EARTH CENTRED CARTESIAN COORDINATES',13X,'GEODETIC',
     &  23X,'AUSTRALIAN MAP GRID')
1080    FORMAT(/,84X,'HEIGHT OF GEOID ABOVE SPHEROID ',F6.3,' metres',//)
1090    FORMAT(58X,'NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS ',
     &  I2,' AND ',I2,'  ',F6.3,' metres')
1100    FORMAT(X,I5,F10.3,'  E',F11.4,'  N',F12.4,3F10.3)
1110    FORMAT(X,'STATION',4X,'DIST',4X,'EASTING',7X,'NORTHING',5X,
     &  'PIER TOP',X,'GND LEVELS',5X,'DIFF',/,4X,'#',5X,'metres',5X,
     &  'metres',9X,'metres',7x,'metres',4x,'metres',4x,'metres',/)
2000    FORMAT(6F10.0)
4000    FORMAT(' ENTER GEOIDAL HEIGHT OF STATION 14 (',F7.3,') -')
4010    FORMAT(F20.0)
4020    FORMAT(' ENTER GEOIDAL HEIGHT OF STATION 37 (',F7.3,') -')
4030    FORMAT(' ENTER AMOUNT OF TOP SOIL TO BE REMOVED        -')
4040    FORMAT(' ENTER DEPTH OF ESCAVATION BELOW PIER TOP      -')

        END


        SUBROUTINE EN_LL(LAT,LONG,I,EASTING,NORTHING,ZONE)
C
C       USES REDFEARN'S FORMULAE TO DETERMINE EASTINGS AND NORTHINGS
C
C       LAT     IS THE GEODETIC LATITUDE
C       LONG    IS THE GEODETIC LONGITUDE
C       I     , SELECTS THE DESIRED ZONE OFFSET FROM STANDARD (I=0) ZONE
C
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DOUBLE PRECISION LAT,LONG,NORTHING,N1,N2,N3,N4,M,MU,KO,L1,L2,L3
        INTEGER ZONE

        PI=3.141592653589793
        DTR=PI/180.0

        KO=              0.9996 D00
        ED_SQRD=  0.006739660796 D00
        E_SQRD=   0.006694541855 D00
        A=              6378160.00 D00

        C=A/DSQRT(1-E_SQRD)

        ZONE=DNINT((LONG/DTR-3.0)/6.0)+31+I
        SHIFT=(6.0*ZONE-183.0)*DTR
        OMEGA=LONG-SHIFT

        PHI=1+ED_SQRD*DCOS(LAT)**2
        V=DSQRT(PHI)
        MU=C/V
        RHO=C/V**3
        T=DTAN(LAT)

        CALL MERDIST(A,E_SQRD,LAT,M)

        E1=MU*OMEGA*DCOS(LAT)+MU*OMEGA**3*DCOS(LAT)**3*(PHI-T**2)/6.0
        E2=MU*OMEGA**5*DCOS(LAT)**5*(4.0*PHI**3*(1-6.0*T**2)+PHI**2*
     &  (1.0+8.0*T**2)-PHI*(2.0*T**2)+T**4)/120.0
        E3=MU*OMEGA**7*DCOS(LAT)**7*(61.0-479.0*T**2+179*T**4-T**6)/5040.0

        EASTING=KO*(E1+E2+E3)+500000.0

        N1=M+MU*DSIN(LAT)*OMEGA**2*DCOS(LAT)/2.0
        N2=MU*DSIN(LAT)*OMEGA**4*DCOS(LAT)**3*(4.0*PHI**2+PHI-T**2)/24.0
        N3=MU*DSIN(LAT)*OMEGA**6*DCOS(LAT)**5/720.0*
     &  (8.0*PHI**4*(11-24*T**2)-28.0*PHI**3*(1-6.0*T**2)+
     &  PHI**2*(1.0-32.0*T**2)-PHI*2.0*T**2+T**4)
        N4=MU*SIN(LAT)*OMEGA**8*DCOS(LAT)**7/40320.0*
     &  (1385.0-3111.0*T**2+543.0*T**4-T**6)

        NORTHING=KO*(N1+N2+N3+N4)+10000000.00

        RETURN
        END
```

PROGRAM LISTING 1.

```
      SUBROUTINE MERDIST(A,E_SQRD,LAT,M)

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION LAT,M

      A0=-5.0*E_SQRD**3/256.0-3.0*E_SQRD**2/64.0
      A0=A0-E_SQRD/4.0
      A0=A*(A0+1.0)*LAT
      A2=15.0*E_SQRD**3/128.0+E_SQRD**2/4.0
      A2=-0.375*(E_SQRD+A2)*A*DSIN(2.0*LAT)
      A4=(15.0/256.0)*(E_SQRD**2+0.75*E_SQRD**3)*A*DSIN(4.0*LAT)
      A6=-(35.0/3072.0)*E_SQRD**3*A*DSIN(6.0*LAT)
      M=A0+A2+A4+A6

      RETURN
      END


      SUBROUTINE LL_EN(EASTING,NORTHING,ZONE,LAT,LONG)
C
C     GET GEODETIC LATITUDE AND LONGITUDE FROM REDFEARN'S FORMULA
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION LAT,LATU,LATL,LONG,NORTHING,M,MD,MU,KO,K
      INTEGER ZONE

      PI=3.141592653589793
      DTR=PI/180.0

      KO=             0.9996 D00
      ED_SQRD=  0.006739660796 D00
      E_SQRD=   0.006694541855 D00
      A=           6378160.00 D00
C
C     FIND LAT CORRESPONDING TO MERIDIAN DISTANCE MD
C
      E=EASTING-500000.0
      MD=(NORTHING-10000000.0)/KO
      LATU=MD/(0.998*A)+0.1
      LATL=LATU-0.2

      DO WHILE(ABS(M-MD).GE.0.0005)
          LAT=(LATU+LATL)/2.0
          CALL MERDIST(A,E_SQRD,LAT,M)
          IF(M.GT.MD) THEN
              LATU=LAT
          ELSE
              LATL=LAT
          END IF
      END DO

      C=A/DSQRT(1-E_SQRD)
      P=1+ED_SQRD*DCOS(LAT)**2
      V=DSQRT(P)
      MU=C/V
      R=C/V**3
      T=DTAN(LAT)
      S=1.0/DCOS(LAT)

      K=T/(KO*R)
      U=KO*MU

      LAT=LAT-K*E**2/(2.0*U)

      T2=K*((E/U)**3*E*(-4.0*P**2+9.0*P*(1.0-T**2)+12.0*T**2))/24.0
      T3=-K*(E/U)**5*E*(8.0*P**4*(11.0-24.0*T**2)-
     & 12.0*P**3*(21.0-71.0*T**2)+15.0*(15.0-98.0*T**2+15*T**4)+
     & 180.0*P*(5.0*T**2-3.0*T**4)+360.0*T**4)/720.0
      T4=K*(E/U)**7*E*(1385.0+3633*T**2+4095*T**4+1575*T**6)/40320.0
      LAT=LAT+T2+T3+T4

      U=E/(KO*MU)

      LONG=S*(U-U**3*(P+2.0*T**2)/6.0)
      T3=S*(U**5*(-4.0*P**3*(1.0-6.0*T**2)+P**2*(9.0-68.0*T**2)+
     & 72.0*P*T**2+24.0*T**4))/120.0
      T4=-S*(U**7*(61.0+662.0*T**2+1320.0*T**4+720.0*T**6))/5040.0

      SHIFT=(6.0*ZONE-183.0)*DTR

      LONG=LONG+T3+T4+SHIFT

      RETURN
      END


      DOUBLE PRECISION FUNCTION DMS(ARG)

      PUTS INTO DEGREE MINUTE AND SECONDS DDD.MMSSssss

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)

      A1=DINT(ARG)
      A2=DINT(60.0*(ARG-A1))/100.0
      A3=((ARG-A1)*0.6-A2)*0.6
      DMS=A1+A2+A3

      END

      DOUBLE PRECISION FUNCTION DEC(ARG)

      PUTS INTO DECIMAL DEGREES

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)

      A1=DINT(ARG)
      A2=DINT((ARG-A1)*100.1)/60.0
      A3=(ARG*100.0-INT(ARG*100.0))/36.0
      DEC=A1+A2+A3

      END


      SUBROUTINE XYZ_LLH(LAT,LONG,HEIGHT,GS_SEP,X,Y,Z)

              LAT IS THE GEODETIC LATITUDE
              LONG IS THE GEODETIC (AND GEOCENTRIC) LONGITUDE
              HEIGHT IS THE HEIGHT ABOVE THE GEOID (A.H.D.)
              GS_SEP IS THE GEOID-SPHEROID SEPARATION

      DERIVED QUANTITIES

              H IS THE SPHEROIDAL HEIGHT , WHICH EQUALS THE GEOIDAL HEIGH
      (HEIGHT ABOVE THE STANDARD GEOID A.H.D. (AUSTRALIAN HEIGHT DATA))
      PLUS THE GEOID-SPHEROID (AUSTRALIAN STANDARD ELLIPSOID) SEPARATION.
              MU IS THE RADIUS OF CURVATURE IN THE PRIME VERTICAL

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION MU,LONG,LAT

      C=6399617.225
      E_SQR =0.006694541855
      ED_SQR=0.006739660796

      V=DSQRT(1+ED_SQR*DCOS(LAT)*DCOS(LAT))
      MU=C/V
      H=HEIGHT+GS_SEP

      X=(MU+H)*DCOS(LAT)*DCOS(LONG)
      Y=(MU+H)*DCOS(LAT)*DSIN(LONG)
      Z=(MU*(1-E_SQR)+H)*DSIN(LAT)

      RETURN
      END

      SUBROUTINE LLH_XYZ(X,Y,Z,LAT,LONG,SPH_HEIGHT)
C
C     GIVES THE GEODETIC LATITUDE AND LONGITUDE FORM X Y Z
C     VALID FOR ABS(LATITUDE)<40
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION LAT,LONG,LATO,LONGO,MU

      PI=3.141592653589793
      DTR=PI/180.0
      C=6399617.225
      E_SQR =0.006694541855
      ED_SQR=0.006739660796
```

Uses 6378160 & 1/298.25 for figure.

```
      R=DSQRT(X*X+Y*Y+Z*Z)
      LONG=DATAN2(Y,X)
      LAT=DATAN2(Z,DSQRT(X*X+Y*Y))

      CALL XYZ_LLH(LAT,LONG,0.0,0.0,XO,YO,ZO)

      ERROR=1111.0

      DO WHILE (ERROR.GT.0.0001)
          DELLAT=DACOS(ZO/R)-DACOS(Z/R)
          LAT=LAT+DELLAT
          V=DSQRT(1+ED_SQR*DCOS(LAT)*DCOS(LAT))
          MU=C/V
          HEIGHT=DSQRT(X*X+Y*Y)/DCOS(LAT)-MU
          CALL XYZ_LLH(LAT,LONG,HEIGHT,0.0,XO,YO,ZO)
          ERROR=DABS(DSQRT(X*X+Y*Y+Z*Z)-DSQRT(XO*XO+YO*YO+ZO*ZO))
      END DO

      SPH_HEIGHT=HEIGHT

      RETURN
      END

      SUBROUTINE TOP_HEIGHT(EE,NN,VAL)
C
C     FINDS THE HEIGHT OF THE SPECIFIED POINT ABOVE THE LOCAL TOPOLOGY
C     BY INTERPLOATION
C
      IMPLICIT DOUBLE PRECISION(A-H,N,O-Z)
      INTEGER NPTS,A,B,C,D

      DOUBLE PRECISION X(4),Y(4),Z(4)

      COMMON /DATA/ E(2000),N(2000),H(2000),NPTS
C
C     FIRST FIND THE FOUR CLOSEST POINTS TO GIVEN POINT
C
      A=NPTS+1
      B=A
      C=A
      D=A
      E(A)=0.0
      N(A)=0.0
      DO I=1,NPTS
          DIST=DSQRT((EE-E(I))**2+(NN-N(I))**2)
          DISTA=DSQRT((EE-E(A))**2+(NN-N(A))**2)
          DISTB=DSQRT((EE-E(B))**2+(NN-N(B))**2)
          DISTC=DSQRT((EE-E(C))**2+(NN-N(C))**2)
          DISTD=DSQRT((EE-E(D))**2+(NN-N(D))**2)
          IF(DIST.LT.DISTA) THEN
              D=C
              C=B
              B=A
              A=I
          ELSE IF(DIST.LT.DISTB) THEN
              D=C
              C=B
              B=I
          ELSE IF(DIST.LT.DISTC) THEN
              D=C
              C=I
          ELSE IF(DIST.LT.DISTD) THEN
              D=I
          END IF
      END DO

      AA=DISTB*DISTC*DISTD
      AB=DISTA*DISTC*DISTD
      AC=DISTA*DISTB*DISTD
      AD=DISTA*DISTB*DISTC

      VAL=(AA*H(A)+AB*H(B)+AC*H(C)+AD*H(D))/(AA+AB+AC+AD)

      RETURN
      END
```

PROGRAM LISTING 2.

A.T file

AT/15.6.1/003
AT/15.6.2/012

CSIRO DIVISION OF RADIOPHYSICS

AT COMPACT ARRAY LINE SURVEY 3

R.A.KENNEDY
29/3/85

INTRODUCTION

This brief paper is third in the series and gives an up to date (definitive) set of coordinates for the AT station locations. This line has been optimised for civil works by MWP through trial and error runs of a modified version of the program given in the first report (AT/15.6.1/001). We note that the geoid spheroid separation has been corrected.* This has minimal effect on the Easting/Northing (or Latitude/Longitude) and the heights (AHD). The following constraints were used to define the AT line station locations (specifically the mid-point of the four pier-tops).

i) Station 1 AHD 212.230

ii) Station 37 AHD 207.780

iii) Station 14 E746605.360 N6643571.600

We note that the remaining coordinates of stations 1,14 and 37 come out in the wash and can be found in the attached table. Implicit constraints are, of course, that these three stations are in an absolute straight line (in space) and all lie in a common plane perpendicular to the rotational axis of the earth.

HISTORICAL NOTE

Hopefully the obscurity of the choice of constraint iii) has been established. (Constraints i) and ii) follow from money considerations.) The origins of this selection will now be expounded. Basically the 'East-West' offset was chosen as a compromise between having station 35 sitting in a bog (wet-season) and protecting an antenna sitting on station 1 from pot-shots from the local intelligensia with rifles on the eastern road, (dimples don't help antennas like they help golf balls). So selecting station 14, 350 metres east of the heliograph centre gave sufficient range to make target practice difficult (a westerly displacement) and to get station 35 to higher ground (an easterly displacement). The 'North-South' offset was selected in order to minimise destruction of the existing heliograph telegraph pole network and kept a cosy distance from the main control building. So the number 117 metres north of the heliograph centre was chosen. These north-south and east-west offsets were then compensated (by -3.72 and -2.89 metres respectively) by the estimated discrepancy between the heliograph centre and the National Mapping Mark TRIG NM/C/59 nearby. So traversing the spheroid at AHD=0.0 metres 114.11 metres true north then 346.28 metres true east from TRIG NM/C/59 yields the coordinates given in constraint iii). The appendix of report 1 in this series has this calculation by the Australian Survey Office. This original product of overkill has stuck through all the calculations to date.

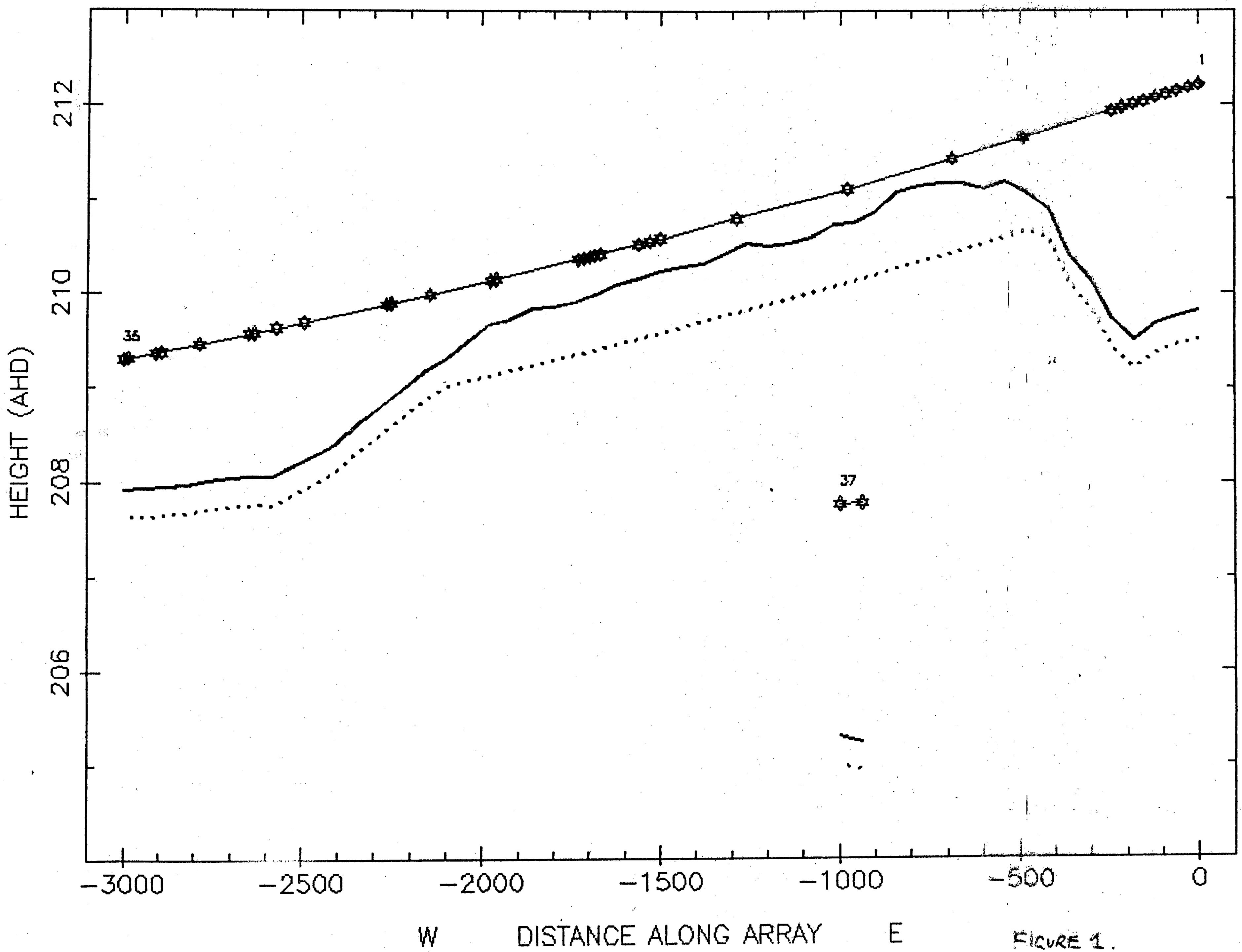
ACCURACY

The calculations have been confirmed to the 1mm level by an independent computer calculation by National Mapping in Canberra. The required positioning accuracy needs not be this precise and the anticipated final position accuracy should be within a few centimetres.

LEANING TOWERS OF CULGOORA

Unfortunately the telescope mounts will be built on the earths surface. All the mounts will of course have parallel axes (in space) but due to the earth's curvature it is impossible to have more than one telescopes azimuth axis 'vertical'. If we set station 35's azimuth axis parallel to its local geodetic normal then all other stations will be skew to their local normal by amounts roughly proportional to their linear rail distance from station 35. Qualitatively an effect is there. The question is of what magnitude? A second table gives the AHD of the four piers around the station centre. Note station 35 has the privelege of being 'normal' (pun) ie the AHD of the four piers is the same. The greatest discrepancy is for stations 1 and 37 (each 3km from station 35) where we require an east-west tilt corresponding to 5mm vertical over appoximately 10 metres horizontal (the distance between the piers). Fortunately the north-south tilt required is negligable as the table shows.

--+--



HEIGHT (AHD)
212
210
208
206
35
37
1
−3000
−2500
−2000
−1500
−1000
−500
0
W
DISTANCE ALONG ARRAY
E

FIGURE 1.

STATION #	DIST metres	EARTH CENTRED CARTESIAN COORDINATES X metres	Y metres	Z metres	GEODETIC LATITUDE DD.MMSSsss	GEODETIC LONGITUDE DDD.MMSSsss	AHD metres	AUSTRALIAN MAP GRID EASTING metres	AUSTRALIAN MAP GRID NORTHING metres	ZONE
			375!		-30°31'44222	149°58'17000	246.4872			
1	0.000	-4752311.286	2790375.851	-3200637.219	-30.1851992	149.3448612	212.230	E 748105.475	N 6643538.543	55
2	30.612	-4752295.749	2790402.227	-3200637.219	-30.1851993	149.3447466	212.193	E 748074.860	N 6643539.217	55
3	61.224	-4752280.212	2790428.604	-3200637.219	-30.1851993	149.3446320	212.156	E 748044.246	N 6643539.892	55
4	91.837	-4752264.675	2790454.981	-3200637.219	-30.1851994	149.3445174	212.119	E 748013.630	N 6643540.567	55
5	122.449	-4752249.138	2790481.357	-3200637.219	-30.1851995	149.3444029	212.083	E 747983.016	N 6643541.241	55
6	153.061	-4752233.601	2790507.733	-3200637.219	-30.1851995	149.3442883	212.046	E 747952.402	N 6643541.916	55
7	183.673	-4752218.064	2790534.109	-3200637.219	-30.1851996	149.3441737	212.010	E 747921.787	N 6643542.590	55
8	214.286	-4752202.527	2790560.486	-3200637.219	-30.1851997	149.3440591	211.974	E 747891.172	N 6643543.265	55
9	244.898	-4752186.990	2790586.862	-3200637.219	-30.1851997	149.3439446	211.938	E 747860.557	N 6643543.940	55
10	489.796	-4752062.694	2790797.873	-3200637.219	-30.1852003	149.3430280	211.655	E 747615.640	N 6643549.337	55
11	688.776	-4751961.703	2790969.349	-3200637.219	-30.1852007	149.3422832	211.433	E 747416.645	N 6643553.722	55
12	979.592	-4751814.102	2791219.894	-3200637.219	-30.1852013	149.3411947	211.118	E 747125.806	N 6643560.131	55
13	1285.714	-4751658.732	2791483.657	-3200637.219	-30.1852019	149.3400490	210.802	E 746819.662	N 6643566.878	55
14	1500.000	-4751549.973	2791668.292	-3200637.219	-30.1852023	149.3352469	210.589	E 746605.360	N 6643571.600	55
15	1530.612	-4751534.436	2791694.668	-3200637.219	-30.1852024	149.3351324	210.559	E 746574.746	N 6643572.275	55
16	1561.224	-4751518.899	2791721.044	-3200637.219	-30.1852024	149.3350178	210.529	E 746544.132	N 6643572.949	55
17	1668.367	-4751464.520	2791813.361	-3200637.219	-30.1852026	149.3346168	210.427	E 746436.981	N 6643575.311	55
18	1683.673	-4751456.751	2791826.549	-3200637.219	-30.1852026	149.3345595	210.412	E 746421.674	N 6643575.648	55
19	1698.980	-4751448.982	2791839.738	-3200637.219	-30.1852027	149.3345022	210.398	E 746406.366	N 6643575.985	55
20	1714.286	-4751441.214	2791852.926	-3200637.219	-30.1852027	149.3344449	210.383	E 746391.059	N 6643576.322	55
21	1729.592	-4751433.445	2791866.114	-3200637.219	-30.1852027	149.3343876	210.369	E 746375.752	N 6643576.660	55
22	1959.184	-4751316.918	2792063.937	-3200637.219	-30.1852031	149.3335283	210.157	E 746146.143	N 6643581.720	55
23	1974.490	-4751309.149	2792077.125	-3200637.219	-30.1852032	149.3334710	210.143	E 746130.836	N 6643582.057	55
24	2142.857	-4751223.696	2792222.195	-3200637.219	-30.1852034	149.3328409	209.993	E 745962.458	N 6643585.767	55
25	2250.000	-4751169.316	2792314.512	-3200637.219	-30.1852036	149.3324398	209.900	E 745855.307	N 6643588.129	55
26	2265.306	-4751161.548	2792327.700	-3200637.219	-30.1852036	149.3323826	209.887	E 745840.000	N 6643588.466	55
27	2494.898	-4751045.020	2792525.523	-3200637.219	-30.1852040	149.3315232	209.695	E 745610.393	N 6643593.526	55
28	2571.429	-4751006.178	2792591.464	-3200637.219	-30.1852041	149.3312368	209.632	E 745533.857	N 6643595.212	55
29	2632.653	-4750975.104	2792644.216	-3200637.219	-30.1852042	149.3310076	209.583	E 745472.629	N 6643596.561	55
30	2647.959	-4750967.335	2792657.404	-3200637.219	-30.1852042	149.3309504	209.571	E 745457.322	N 6643596.899	55
31	2785.714	-4750897.419	2792776.078	-3200637.219	-30.1852044	149.3304348	209.463	E 745319.558	N 6643599.935	55
32	2892.857	-4750843.039	2792868.415	-3200637.219	-30.1852046	149.3300337	209.380	E 745212.408	N 6643602.296	55
33	2908.163	-4750835.271	2792881.603	-3200637.219	-30.1852046	149.3259765	209.369	E 745197.101	N 6643602.633	55
34	2984.694	-4750796.428	2792947.544	-3200637.219	-30.1852047	149.3256900	209.311	E 745120.565	N 6643604.320	55
35	3000.000	-4750788.660	2792960.732	-3200637.219	-30.1852048	149.3256327	209.300	E 745105.258	N 6643604.657	55
36	5938.776	-4749297.107	2795492.861	-3200637.219	-30.1852076	149.3106334	207.797	E 742166.320	N 6643669.420	55
37	6000.000	-4749266.034	2795545.613	-3200637.219	-30.1852076	149.3104043	207.780	E 742105.094	N 6643670.769	55
13 revised		-4751786.592	(-)2791430.107	-3200484.469						
1 revised		-4752439.1	(-)2790322.3							
		-127.86	-53.55	+152.75						

HEIGHT OF GEOID ABOVE SPHEROID 11.400 metres ? 16.9m

N(ANS) = 16.870m from R Coleman

NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS 1 AND 37 2.602 metres
NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS 1 AND 14 0.960 metres
NORTH-SOUTH SPHEROID DISTANCE BETWEEN STATIONS 14 AND 37 1.642 metres

N(WGS84) = 29.724m OSU86E model

John,

These are the positions actually used.

3, 11, 16, 30, 34, 37

TABLE 1.

STATION #	DIST metres	EASTING metres	NORTHING metres	STAT CTR metres	SW PIER metres	NW PIER metres	SE PIER metres	NE PIER metres	GND LEVELS metres	DIFF metres
1	0.000	E 748105.475	N 6643538.543	212.23000	212.22765	212.22766	212.23235	212.23236	209.81778	2.41222
2	30.612	E 748074.860	N 6643539.217	212.19298	212.19066	212.19066	212.19531	212.19532	209.82455	2.36843
3	61.224	E 748044.246	N 6643539.892	212.15612	212.15382	212.15382	212.15842	212.15842	209.76232	2.39380
4	91.837	E 748013.630	N 6643540.567	212.11939	212.11712	212.11712	212.12167	212.12167	209.72134	2.39805
5	122.449	E 747983.016	N 6643541.241	212.08282	212.08056	212.08057	212.08507	212.08507	209.65945	2.42337
6	153.061	E 747952.402	N 6643541.916	212.04639	212.04416	212.04416	212.04862	212.04862	209.56424	2.48215
7	183.673	E 747921.787	N 6643542.590	212.01010	212.00790	212.00790	212.01231	212.01232	209.51273	2.49738
8	214.286	E 747891.172	N 6643543.265	211.97397	211.97179	211.97179	211.97615	211.97615	209.54422	2.42974
9	244.898	E 747860.557	N 6643543.940	211.93798	211.93582	211.93583	211.94014	211.94014	209.76077	2.17721
10	489.796	E 747615.640	N 6643549.337	211.65535	211.65339	211.65339	211.65732	211.65732	211.10188	0.55347
11	688.776	E 747416.645	N 6643553.722	211.43263	211.43083	211.43083	211.43445	211.43445	211.24691	0.18573
12	979.592	E 747125.806	N 6643560.131	211.11828	211.11670	211.11670	211.11987	211.11987	210.73394	0.38434
13	1285.714	E 746819.662	N 6643566.878	210.80170	210.80036	210.80036	210.80304	210.80304	210.46199	0.33971
14	1500.000	E 746605.360	N 6643571.600	210.58882	210.58765	210.58765	210.59000	210.59000	210.23962	0.34920
15	1530.612	E 746574.746	N 6643572.275	210.55900	210.55785	210.55785	210.56015	210.56015	210.20709	0.35191
16	1561.224	E 746544.132	N 6643572.949	210.52932	210.52820	210.52820	210.53045	210.53045	210.16560	0.36372
17	1668.367	E 746436.981	N 6643575.311	210.42661	210.42557	210.42557	210.42765	210.42766	210.04178	0.38483
18	1683.673	E 746421.674	N 6643575.648	210.41208	210.41105	210.41106	210.41312	210.41312	209.99356	0.41852
19	1698.980	E 746406.366	N 6643575.985	210.39759	210.39658	210.39658	210.39861	210.39861	209.95696	0.44063
20	1714.286	E 746391.059	N 6643576.322	210.38314	210.38213	210.38214	210.38415	210.38415	209.93726	0.44588
21	1729.592	E 746375.752	N 6643576.660	210.36872	210.36773	210.36773	210.36972	210.36972	209.91501	0.45371
22	1959.184	E 746146.143	N 6643581.720	210.15688	210.15607	210.15607	210.15770	210.15770	209.68897	0.46791
23	1974.490	E 746130.836	N 6643582.057	210.14305	210.14225	210.14225	210.14386	210.14386	209.67293	0.47012
24	2142.857	E 745962.458	N 6643585.767	209.99335	209.99268	209.99269	209.99403	209.99403	209.22528	0.76807
25	2250.000	E 745855.307	N 6643588.129	209.90040	209.89982	209.89982	209.90099	209.90099	208.91273	0.98768
26	2265.306	E 745840.000	N 6643588.466	209.88727	209.88670	209.88670	209.88785	209.88785	208.89736	0.98991
27	2494.898	E 745610.393	N 6643593.526	209.69469	209.69430	209.69430	209.69509	209.69509	208.20755	1.48715
28	2571.429	E 745533.857	N 6643595.212	209.63234	209.63200	209.63201	209.63268	209.63268	208.09458	1.53775
29	2632.653	E 745472.629	N 6643596.561	209.58311	209.58283	209.58283	209.58340	209.58340	208.05029	1.53282
30	2647.959	E 745457.322	N 6643596.899	209.57090	209.57063	209.57063	209.57118	209.57118	208.05839	1.51250
31	2785.714	E 745319.558	N 6643599.935	209.46262	209.46246	209.46246	209.46279	209.46279	207.99300	1.46962
32	2892.857	E 745212.408	N 6643602.296	209.38046	209.38038	209.38038	209.38055	209.38055	207.97076	1.40970
33	2908.163	E 745197.101	N 6643602.633	209.36887	209.36880	209.36880	209.36894	209.36894	207.96559	1.40328
34	2984.694	E 745120.565	N 6643604.320	209.31146	209.31146	209.31146	209.31148	209.31148	207.93902	1.37244
35	3000.000	E 745105.258	N 6643604.657	209.30009	209.30010	209.30010	209.30010	209.30010	207.92883	1.37126
36	5938.776	E 742166.320	N 6643669.420	207.79693	207.79924	207.79923	207.79463	207.79463	205.25000	2.54693
37	6000.000	E 742105.094	N 6643670.769	207.78000	207.78235	207.78235	207.77765	207.77765	205.30000	2.48000

CA1 : λ = 149°.581 322 8
φ = -30°.312 874 62
H = 218.291 m

CA1S 149°.567 956 09
φ = -30.31288209
H = 216.875m

#1 149.5813228 -30.3128746 218.3

TABLE 2.

AT/15.6.1/004

CSIRO DIVISION OF RADIOPHYSICS

AT COMPACT ARRAY LINE SURVEY 4
POINT AND LINE SCALE FACTORS

R.A.KENNEDY
2/5/85

DEFINITIONS

The scale factors give the conversion between Easting Northing Grid metres and *true* Spheroid metres. So a map, with Australian Map Grid data, isn't in metres at all, despite the fact the word 'metre' appears frequently on it. Care must also be taken with the difference between grid and true north.

SCALE FACTORS

POINT SCALE FACTOR $k = \frac{dL}{ds} = \frac{dS}{ds}$

This is a function of a *single* point.

LINE SCALE FACTOR $K = \frac{L}{s} \simeq \frac{S}{s}$

This is a function of *two* points say 1 and 3 in the figure below.

where L is the plane (straight line) distance on the Easting/Northing grid.
s is the arc length of the geodesic between points 1 and 3 (through 2) on the spheroid (which defines latitude and longitude).
S is the arc length of the projection of the geodesic arc onto the Easting/Northing grid plane. Note 1', 2' and 3' are on the (flat) plane.

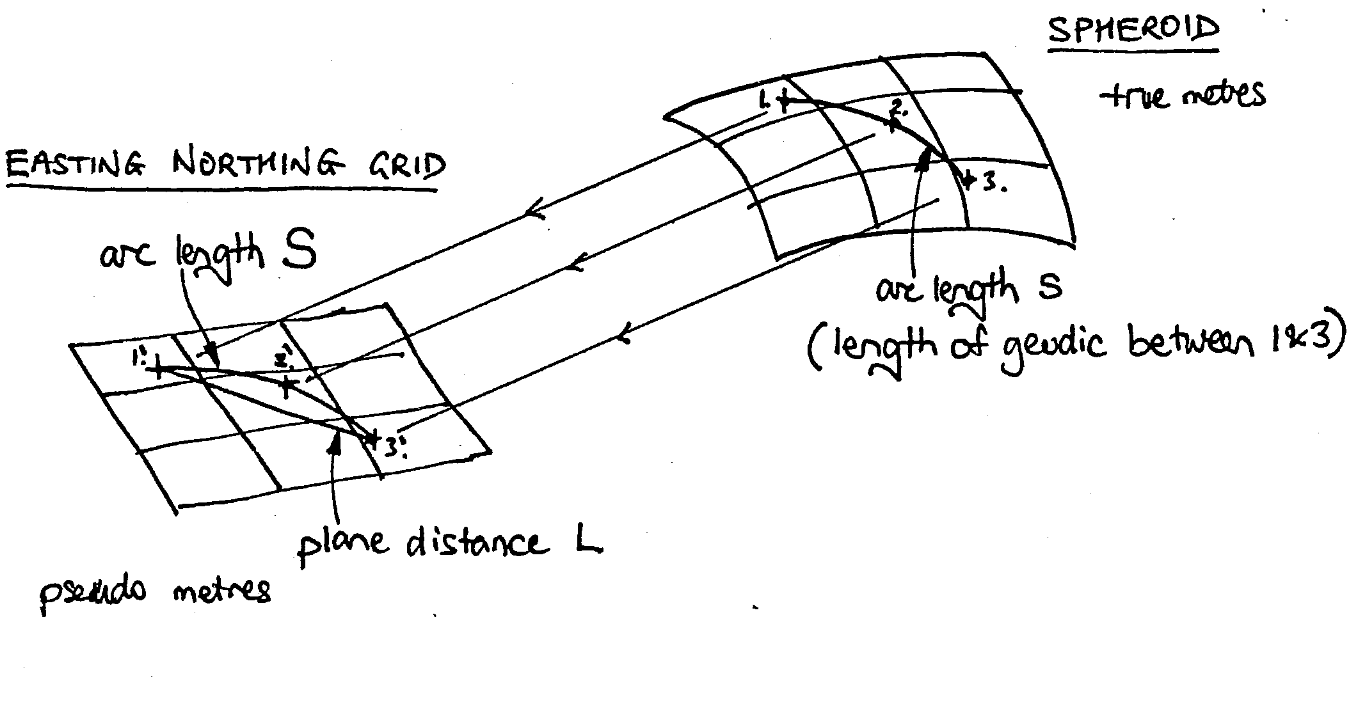


FIGURE 1.

1985 MAY

Next we give the point scale factors for stations 1 and 37. The point scale factor has a rigorous definition see page 13 Australian Map Grid Reference Manual (if your mad). However the line scale factor corresponds to gross movements so is effectively some integral over the point scale factor.

NOTE

These scale factors as defined relate to moving over the spheroid surface or over the Easting/Northing grid and have no reference to height ie these figures do not correct for heights or height differences. So caution must be taken with these numbers. Without correction these numbers cannot be used in the field.

DATA

PSF for the point ASO 4, (E746607.105 N6643646.939) is
1.00035033

AT STATION PSF DATA:

STATION # 1	POINT SCALE FACTOR	1.00035948
STATION # 2	POINT SCALE FACTOR	1.00035929
STATION # 3	POINT SCALE FACTOR	1.00035910
STATION # 4	POINT SCALE FACTOR	1.00035891
STATION # 5	POINT SCALE FACTOR	1.00035873
STATION # 6	POINT SCALE FACTOR	1.00035854
STATION # 7	POINT SCALE FACTOR	1.00035835
STATION # 8	POINT SCALE FACTOR	1.00035816
STATION # 9	POINT SCALE FACTOR	1.00035798
STATION #10	POINT SCALE FACTOR	1.00035648
STATION #11	POINT SCALE FACTOR	1.00035526
STATION #12	POINT SCALE FACTOR	1.00035349
STATION #13	POINT SCALE FACTOR	1.00035162
STATION #14	POINT SCALE FACTOR	1.00035032
STATION #15	POINT SCALE FACTOR	1.00035013
STATION #16	POINT SCALE FACTOR	1.00034995
STATION #17	POINT SCALE FACTOR	1.00034929
STATION #18	POINT SCALE FACTOR	1.00034920
STATION #19	POINT SCALE FACTOR	1.00034911
STATION #20	POINT SCALE FACTOR	1.00034901
STATION #21	POINT SCALE FACTOR	1.00034892
STATION #22	POINT SCALE FACTOR	1.00034753
STATION #23	POINT SCALE FACTOR	1.00034743
STATION #24	POINT SCALE FACTOR	1.00034641
STATION #25	POINT SCALE FACTOR	1.00034576
STATION #26	POINT SCALE FACTOR	1.00034567
STATION #27	POINT SCALE FACTOR	1.00034428
STATION #28	POINT SCALE FACTOR	1.00034381
STATION #29	POINT SCALE FACTOR	1.00034344
STATION #30	POINT SCALE FACTOR	1.00034335
STATION #31	POINT SCALE FACTOR	1.00034251
STATION #32	POINT SCALE FACTOR	1.00034187
STATION #33	POINT SCALE FACTOR	1.00034177
STATION #34	POINT SCALE FACTOR	1.00034131
STATION #35	POINT SCALE FACTOR	1.00034122
STATION #36	POINT SCALE FACTOR	1.00032355
STATION #37	POINT SCALE FACTOR	1.00032318