AT/10.1/035

Further thoughts on choosing redundant arrays.

Don McLean

The following is a sequel to AT/10.1/033. It consists of two sections: the first is a revision of the original Appendix A on the selection of two-day sequences; the second section is concerned with three day sequences. The reasons for these changes and additions are:-

The original two-day sequences required the use of the six-km aerial even during three-km observations, the new discussion selects configurations which can be used either as 6-km or 3-km arrays.

A three-day sequence is required, because after 2 days all the known spacings are multiples of some factor (e.g. 4 for configuration A) and using day-to-day redundancy for subsequent days we can only ever calibrate spacings which are multiples of that factor, because all new spacings will be sums or differences of known spacings. The third day is used to intoduce spacings which are multiples of 7, giving a starting set from which all spacings can ultimately be built from sums and differences.

Method of selecting 2-day redundant sequences.

To build up sequences such as (A), (B), and (C) of AT/10.1/033 we are limited by the following constraints.

The 3km array should be close to 3km long, because the spacing 1-5 must equal spacing 5-6; this is the only way that the aerial 6 can be calibrated.

The 3km array should be one of the configurations which gives 7 different spacings, listed in section (b). The length of the first of those is 7 times the shortest spacing and so we must choose this shortest spacing about 200/7=28.6 unit spacings long. The other configurations all have lengths equal to 8 times the shortest spacing, and so must be built up of multiples of about 200/8=25 unit spacings.

Consider now the following, generalised 2-day sequence:

```
  *                            *   *   *      *           *
 -8n                           0   n   2n     4n          8n

  *                            *              *      *    *    *
 -7m                           0              3m     5m   6m   7m
```

To minimise movement at the 6km point the arrays are aligned at the 3km point.

Possible values for n are close to 25, and possible values for m are close to 29:

n=	23	24	25	26	27	28	29
8n=	184	192	200	208	216	224	232

m=	25	26	27	28	29	30	31	32
7m=	175	182	189	196	203	210	217	224

To minimise the length of track needed at the 6km point we must choose values of n and m such that $16n \sim 14m$. Also to relate the calibration from the first day to the calibration from the second day we need at least one common baseline. Since all the baselines on the first day are multiples of n and all the baselines on the second day are multiples of m, we require that the least common multiple of m and n be less than 400, or less than 200 if the arrangement is to work for the 3km array. For a given pair m & n, we set the larger of 7m or 8n equal to 3km - this fixes the unit spacing. $|8n-7m|$*unit is then the travel needed at the 6km point.

The following table lists some possible combinations:

n	m	unit spacing	common spacing	6km travel	example	comments
23	25	16.30	-	146.74		No common baseline
23	26	16.30	-	32.61		No common baseline.
23	27	15.87	-	79.35		No common baseline.
24	27	15.63	216	46.88		Common baseline>200.
24	28	15.31	168	61.23		
25	28	15.00	-	45.00		No common baseline.
25	29	14.78	-	44.30		No common baseline.
25	30	14.29	150	142.9		Large movement at 6km.
26	28	14.42	364	173.1		Common baseline >200.
26	29	14.42	-	72.1		No common baseline
26	30	14.28	390	28.56	B	Common baseline >200.
27	30	13.89	270	83.34	C	Common baseline >200.
27	31	13.82	-	13.82		No common baseline.
27	32	13.39	-	107.1		No common baseline.
28	32	13.39	224,448	0	A	Redundant redundancy.
29	32	12.93	-	103.4		No common baseline

The combination n=24, m=28 appears to be the most satisfactory. Other possibilities are 25, 30 and 28, 32.

Three-day redundant sequences.

The two-day sequences listed above do not form an adequate basis for building day-to-day redundant sequences because all baselines are multiples of some factor (often 4). With only two stations allowed at the 6-km point it is not possible to construct a 3-day sequence with each day independently calibratable; the following sequences achieve this aim for the 3-km array and use day-to-day redundancy for the 6-km point for the third day. They are both based on the (n=24, m=28) two-day sequence

Consider the following three-day observing sequence:

```
   *                                    *   *   *       *              *
 -192                                   0  24  48      96   .         192

 *                                      *    *    *         *             *
-196                                    0   28   56        112           196

    *                                   ***   *   *
  -192                                  0^^  28  49
                                         ||
                                        /  \
                                       7   14
```

Spacings recorded:

Day:	1(3km)	1(+6km)	2(3km)	2(+6km)	3(3km)	3(+6km)
	24*2	192	28*2	196	7*2	192
	48*2	206	56*2	224	14*2	199
	72	230	84*2	252	21*2	206
	96*2	288	112	308	28	220
	144	384	140	392	35	241
	168		168		42	
	192		196		49	

Spacings involving or not involving the 6-km point are shown separately. Spacings which are underlined are repeated in some other column. The number of stations required is 12 on the 3km track + 2 at the 6km "point".

The array is fully determined, whether the 6km point is included or omitted. If day-to-day redundancy were used after the first two days, only spacings which are a multiple of 4 (the HCF of 24 and 28) could be formed. After the third day day-to-day redundancy can be used to form all spacings.

An alternative sequence which achieves the same aims (the first two days are the same) is:

```
   *                                     *   *   *        *              *
 -192                                    0  24  48       96             192

  *                                      *    *    *          *              *
-196                                     0   28   56         112            196

  *                                           *   *   *       *          *
-196                                         28  49  70      112        173
```

Spacings recorded:

Day:	1(3km)	1(+6km)	2(3km)	2(+6km)	3(3km)	3(+6km)
	24*2	192	28*2	196	21*2	224
	48*2	206	56*2	224	42*2	245
	72	230	84*2	252	63*2	266
	96*2	288	112	308	84	308
	144	384	140	392	95	369
	168		168		116	
	192		196		137	

The disadvantage of this second arrangement is that it contains one "wasted" redundant spacing (both 224 & 308 are repeated in the last column). This could be avoided at the expense of an extra station, by using the station at -192 on the third day and moving the 3km array to start at station 0.