ZOOM ARRAYS

Introduction

Rotation synthesis antenna arrays may be arranged east-west to give equi-spaced concentric circles (or ellipses) in the (u,v)-plane. However, it is thought that there may be some advantage in (u,v)-plane loci that are more tightly packed near the origin than in the outer regions. A zoom array is one where the gaps between adjacent (u-v)-plane circles increase outward in geometric progression.

Terminology

Let N antennas be distributed along a straight line of length 100 units. The first antenna is at x=0 and the Nth is at x=100. The nth antenna is at $x=x_n$. Thus $x_1=0$ and $x_N=100$.

A "baseline" is the line segment defined by an antenna pair. We are not interested in the absolute location of this line segment or its direction but only in its absolute length. Consequently the phrase "absolute length of a baseline" may be abbreviated to "baseline". With N antennas there will be $M = N(N-1)/2$ baseline lengths, some of which may be equal or "redundant".

The closest pair of antennas define the shortest baseline B_1 and the extreme antennas define the longest baseline B_M which is being taken as 100. The baselines B_m are arranged in increasing order of length.

A zoom array, by definition, is one in which the gap between adjacent baselines $B_{m+1} - B_m$ increased geometrically. Thus, with an expansion factor G,

$$B_{m+2} - B_{m+1} = G(B_{m+1} - B_m) \qquad (1)$$

The Zoom Array is an Ideal

The number of baselines M corresponding to various short arrays with number of elements N ranging from 3 to 8 is as follows:

N	3	4	5	6	7	8
M	3	6	10	15	21	28

No array exists which meets precisely the zoom array definition, for these reasons. With N=3 there are only three baselines and thus too few gaps to exhibit geometrical progression. With N=4 there are five baseline gaps and therefore three conditions of the form

$$(B_{m+3} - B_{m+2})\ (B_{m+1} - B_m) = (B_{m+2} - B_{m+1})^2 \qquad (2)$$

With these three equations one might hope to solve for x_2, x_3 and x_4 (x_1 being zero). However, the baseline gaps must be all different, if $G > 1$ as required, and this can never happen. The reason is as follows. Suppose that the most closely spaced pair of antennas are a distance B_1 apart, and that B_1 is a small value such as 5. Then two baselines p and q differing by 5 exist between the given pair and one of the other antennas. But there are two other baselines r and s also differing by 5 which exist between the given pair and the remaining antenna. Therefore there will be two equal baseline gaps of size 5, *unless* p and q straddle r or s. In that case the equal gaps are not equal to $|p-q| = 5$, but are equal to $|p-r|$.

Approximate Zoom Arrays

Achievable baselines $\hat{B}_m$ cannot all coincide with the ideal values B_m but the degree of dispersion away from the ideal may be characterized by

$$D = \left[\sum_1^M \ (\hat{B}_m - B_m)^2 / M \right]^{1/2} \qquad (3)$$

This is the rms departure of the baselines from the corresponding ideal values.

As was shown in memoranda of 6 and 20 September 1982, respectively entitled "Logarithmic and Zoom Arrays" and "A Six-Element Zoom Array", rather low dispersions of the order of two or three percent of the maximum baseline could readily be found.

The following cases illustrate the quality of fit that can be obtained. Figure 1 is for five elements arranged as shown by the small circles according to the listed values of x_n, namely 0, 33.95, 77.2, 84.78, 100. This arrangement produces the baselines indicated by small crosses. As can be seen the progression of baselines is quite regular. When the baseline length B_m is plotted against m the agreement with the full curve representing exact geometric progression is seen to be excellent. The dispersion D is only 1.6 units in a total baseline range of 100. The expansion factor of the ideal curve is $G = 1.06$.

In Figures 2 to 4 similar examples are given for N=6 and N=7.

Method of Calculation

There is a basic relationship between the first baseline B_1 and the expansion factor G desirable as follows. From the definition,

$$\left.\begin{aligned} B_2 &= B_1 + GB_1 \\ B_3 &= B_1 + GB_1 + G^2B_1 \\ &\dots \\ B_M &= B_1\,(1 + G + G^2 + \dots + G^{M-1}) \end{aligned}\right\} \tag{4}$$

Since $B_M = 100$, we have

$$100 = B_1(1 - G^M) / (1 - G) \tag{5}$$

Plausible array arrangements were tested by first determining the shortest baseline $\hat{B}_1$, then solving (5) for G iteratively and using this value to calculate ideal baselines B_M from (4). A value of D was obtained from (3) and the array was then perturbed progressively until D was reduced to a small value. Following a talk entitled "Logarithmic and Zoom Arrays" of 17 September 1982 reporting that good approximate arrangements could be found, I received further arrangements from Geoff Poulton (Figs 1 and 2) and Jim Maguire (Fig. 3).

Conclusion

Some rather interesting array configurations have been arrived at for sets of 5, 6 and 7 antennas. They deliver baselines that approximate quite well to a regular geometric progression. The expansion factors are in the range 1.04 to 1.06.

The rationale for zoom arrays is not discussed here. The main conclusion is that such arrays do indeed appear to be available if needed.

13 October 82 R.N. Bracewell

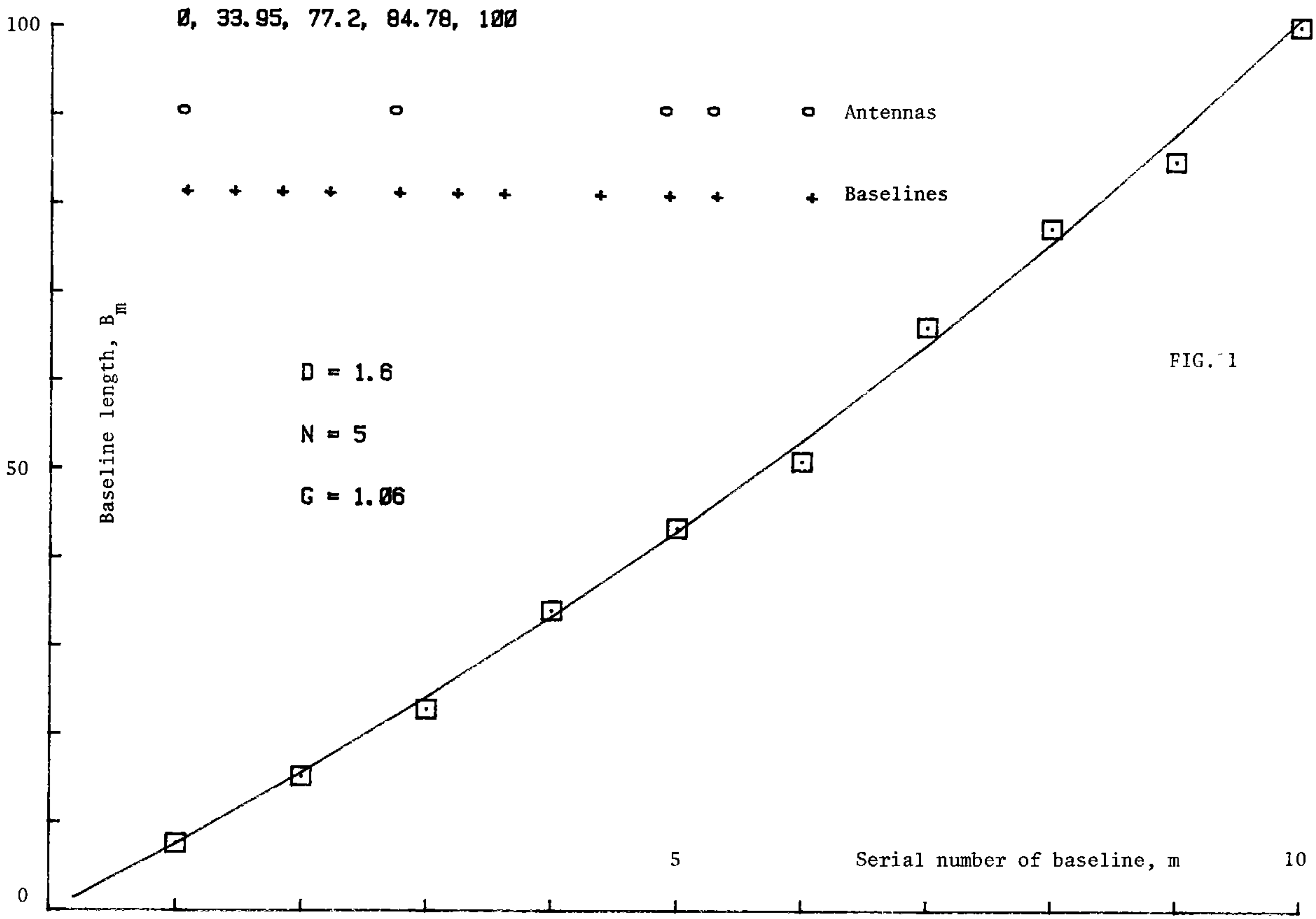
Ø, 33.95, 77.2, 84.78, 100
Antennas
Baselines
D = 1.6
N = 5
G = 1.06
Baseline length, B_m
Serial number of baseline, m
100
50
0
5
10

FIG. 1

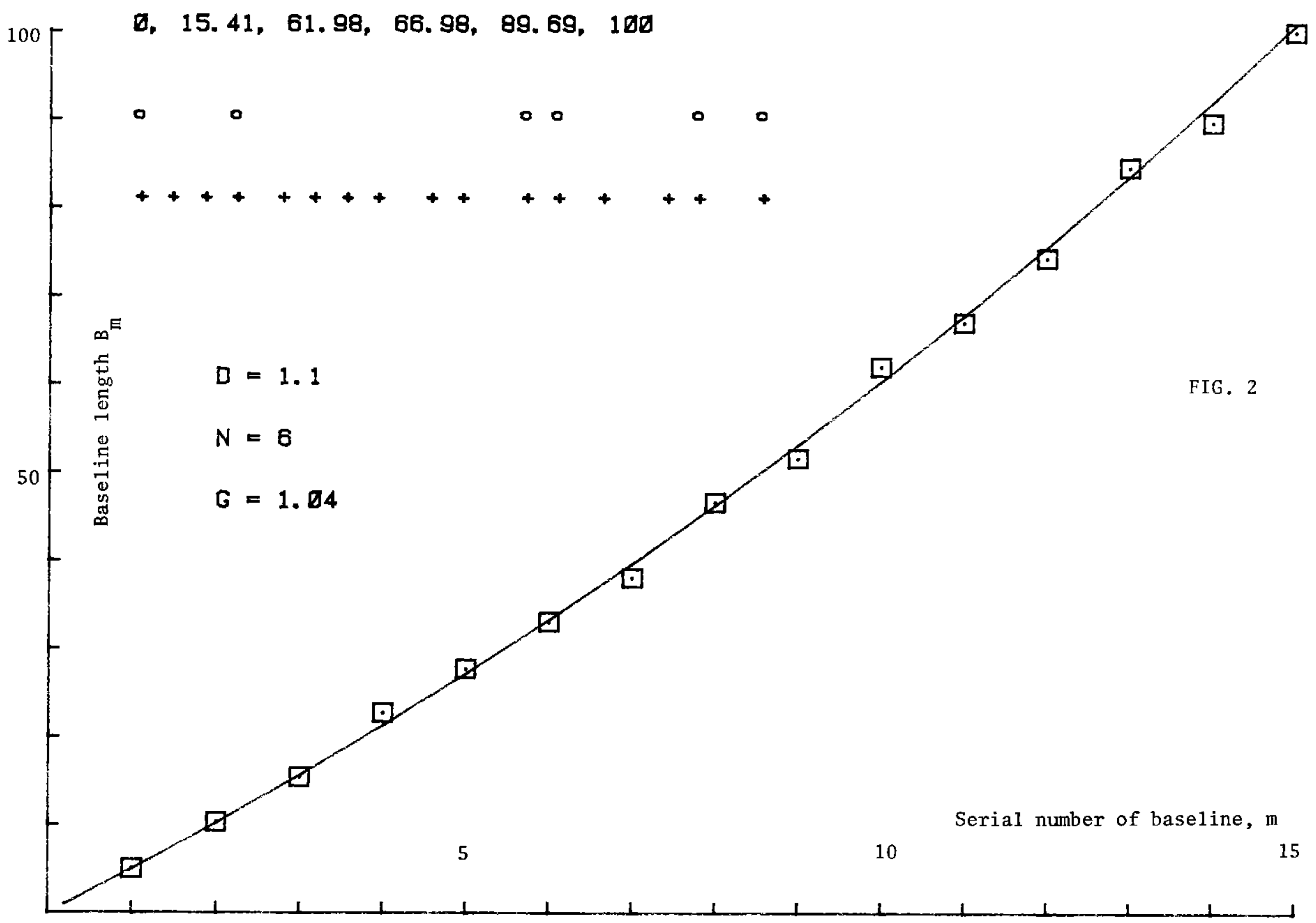
Ø, 15.41, 61.98, 66.98, 89.69, 100
D = 1.1
N = 6
G = 1.Ø4
Baseline length B_m
Serial number of baseline, m
100
50
5
10
15

FIG. 2

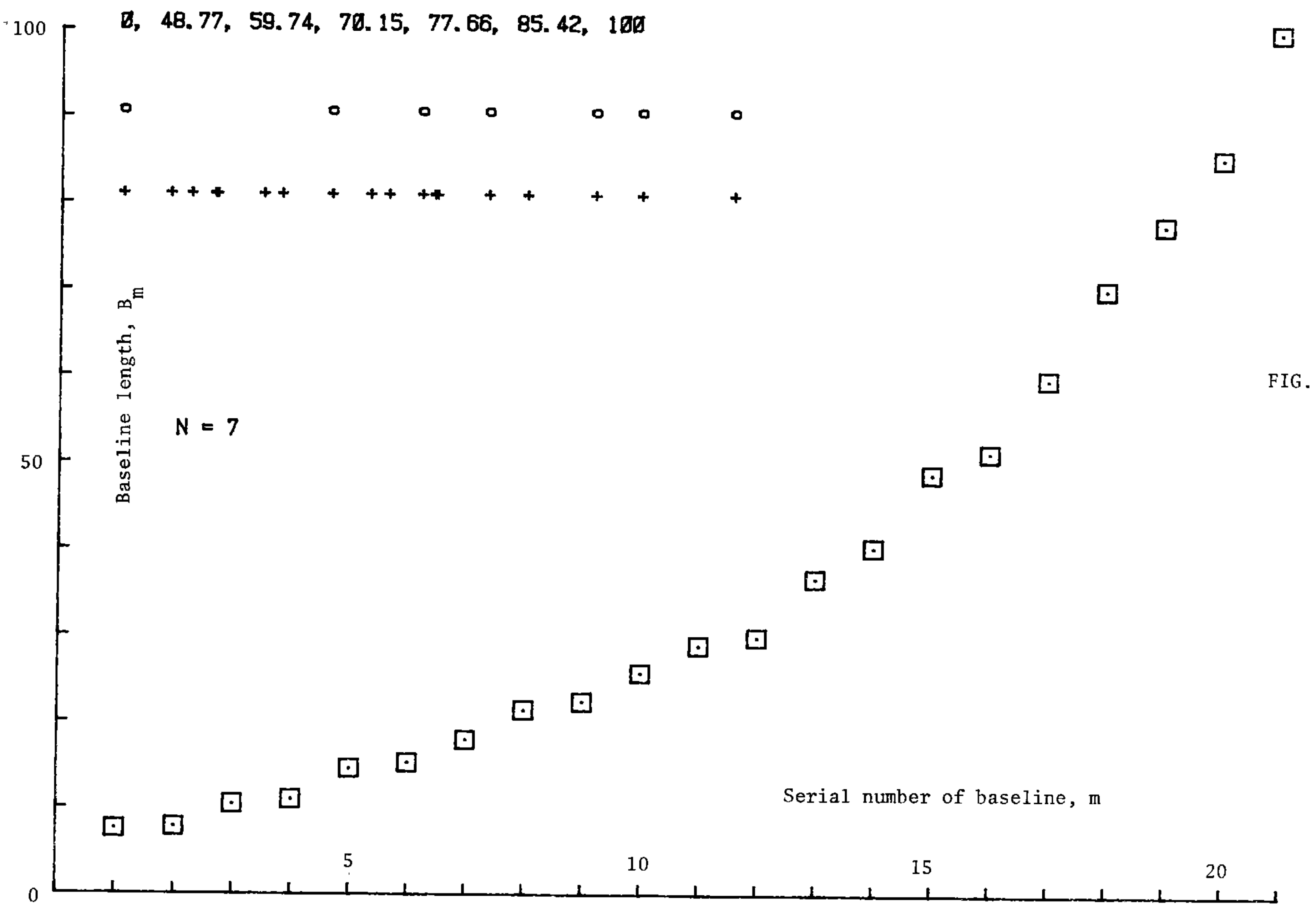
Ø, 48.77, 59.74, 7Ø.15, 77.66, 85.42, 1ØØ
100
50
0
Baseline length, B_m
N = 7
Serial number of baseline, m
5
10
15
20

FIG. 3

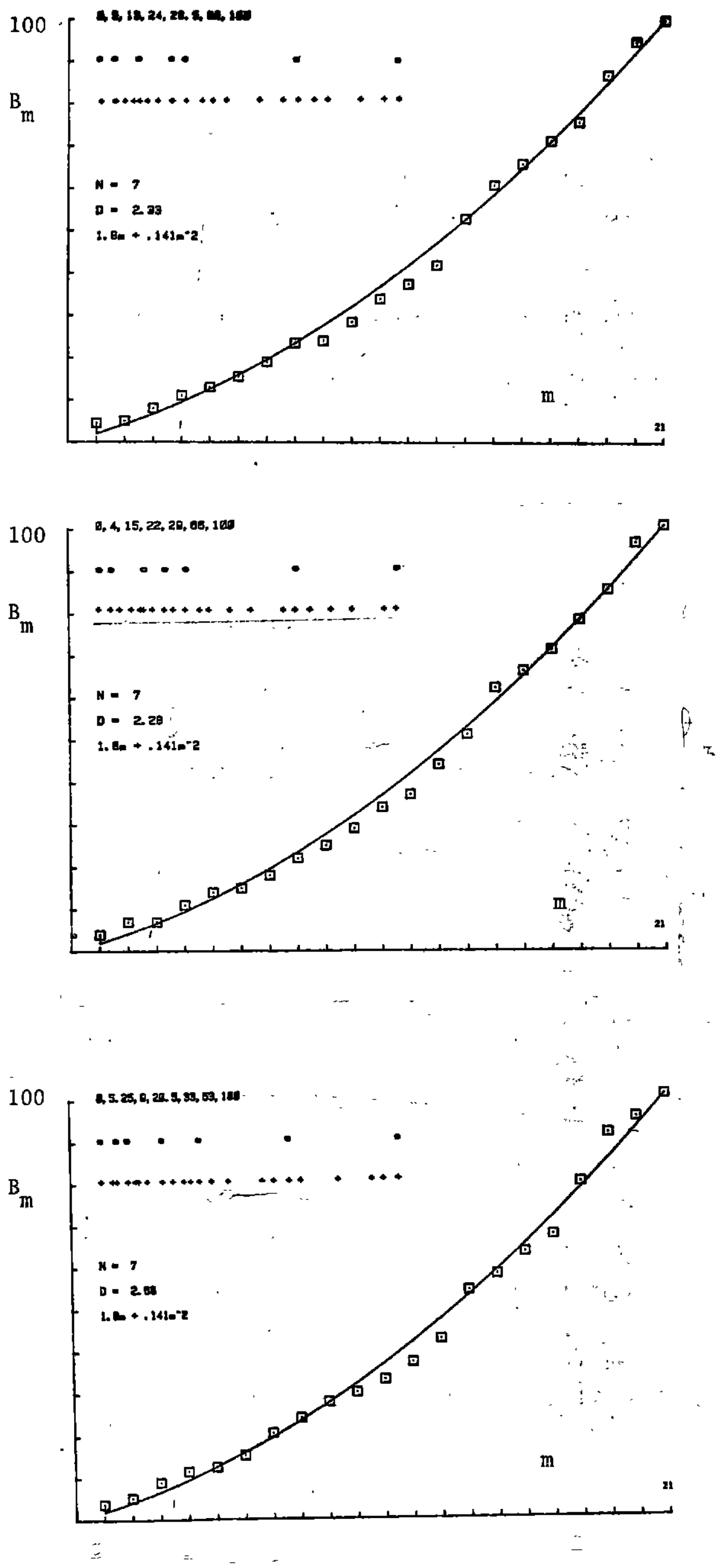
100
B m
N = 7
D = 2.33
1.8m + .141m^2
m
21
100
0, 4, 15, 22, 29, 66, 100
B m
N = 7
D = 2.28
1.8m + .141m^2
m
21
100
B m
N = 7
m
21

Fig. 4