### Better redundant configurations for the AT.

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Because of other pressures the original redundant configurations for the AT were chosen without a full investigation of error propagation. It now turns out that after a few days observations with these configurations, error propagation becomes a serious problem. This is illustrated by Table 1. On the left side of this table are listed the aerial positions for each of 10 days, for the original 3km redundant array. Below each aerial position is the variance of the estimate of the phase error for that aerial. On the right side of the table are listed the lengths of the spacings observed, and below each the variance of the estimate of the spacing phase. A \* beside a spacing indicates that the same spacing was measured twice on this day. A ^ beside a spacing indicates that that spacing was also measured on a previous day.

The variances are estimated by assuming that all measured phases have equal, unit variance. The mathematical details of this calculation are given in the Appendix. A variance of 9 means an error 3 times the measurement error, and a variance of 43 means that the error estimate is nearly 7 times the measurement error. Table 1 shows that after 5 days the errors are quite large and the situation continues to deteriorate as more days are added.

The solution is to allow a little more redundancy. Table 2 illustrates what can be achieved with the existing station locations, for a 3-km 6-day sequence. After a least-squares fit to the observational data (see Appendix), the variance of all deduced quantities is now less than 4, i.e. the errors are less than double the measurement errors. This significant improvement has been achieved at a very modest cost; the new 6-day scheme measures 33 different spacings, whereas the original scheme measured 37 different spacings in 6 days. ( A non-redundant sequence measures 60 uncalibrated spacings in the same time).

This particular solution is not optimum in any sense; it has been chosen to give acceptable error levels and reasonably uniform coverage. It seems probable that longer sequences can be found which retain these two essential properties, and that similar 1.5-km and 6-km sequences can be found, using the existing station locations. However significant computing effort will be required to do this. In the mean time the solution already found can be used to

compare the performance of the redundant scheme with a non-redundant scheme, calibrated by using self-cal.

These results do not modify the requirements for the station locations. The station locations were selected to include the positions needed for the first three days of a redundant sequence, but the requirement of a redundant observing sequence did not otherwise affect the selection of stations. The first two days of the original redundant sequence are retained in the present solution, and indeed are essential to any redundant configuration. Therefore the reasons for selecting the present set of stations are still valid.

# Appendix Evaluation of phases using over-determined, redundant arrays.

#### Notation:

From observations on day d with a pair of aerials i, and j at east-west positions  $x_{d,i}$ , and  $x_{d,j}$ , we determine the measured phase  $\phi_{d,i,j}$ , which is related to the aerial phase errors  $e_{d,i,j}$ , and the true phase due to the sky  $\psi(s_{d,i,j})$  where the spacing  $s_{d,i,j} = x_{d,i} - x_{d,j}$ 

$$\phi_{dij} = \psi(s_{dij}) + e_{di} - e_{dj}$$
 . . (1)

#### <u>Constraints</u>

From a number of such observations we obtain a set of linear equations relating the observations,  $\{\phi_{d,i}\}$  to the quantities that we need to know,  $\{\psi(s_{d,i})\}$  and the errors,  $\{e_{d,i}\}$ .

This set of equations will be singular unles we add constraints which serve to define the zero phase for each day, and fix the slope of the errors for a series of days. For example, for each d, for the zero of phase:

$$\sum e_{di} = 0 \qquad . \qquad . \qquad . \qquad (2)$$

and for all d's taken to-gether, for the slope:

$$\sum \sum x_{di} e_{di} = 0 \qquad (3)$$

## Solution - minimal redundancy

Writing equations (1), (2), and (3) to-gether in matrix notation, we have

$$A x = b$$
 . . . (4)

where the vector x is defined as:

$$x^{T} = (\psi(s_{1 12}), \psi(s_{1 13}), \dots, e_{1 1}, e_{1 2}, \dots)$$

and b by:

$$b^{T} = (\phi_{112}, \phi_{113}, ...)$$

and  $\lambda$  is a matrix which contains only +1, -1, or 0, except in the row which represents equation (3).

For the original redundant configuration the redundancy was chosen carefully to ensure that A was square and non-singular, i.e.  $A^{-1}$  exists, and

$$x = A^{-1} b$$
 (5)

## <u> Solution - extra redundancy</u>

When extra redundancy is introduced, equations (4) are overdetermined, provided the configurations are chosen sensibly. We then seek a least squares solution for x:

$$(A \times -b)^{T} (A \times -b) = minimum$$
 $A^{T} A \times = A^{T} b$ 
 $X = (A^{T} A)^{-1} A^{T} b$  . . . (6)

Here  $({\tt A}^{\intercal} - {\tt A})$  is a square matrix, and should be non-singular.

 $A^{i\,n\,v}=(A^T\ A)^{-\,i}\ A^T$  is known as the generalised inverse. There is a NAG subroutine, F01BLF, which calculates this generalised inverse, and detects any problems, such as singularity.

## Variance of solution values.

The variance of an element  $x_i$  of x can be calculated from (6):

$$\sigma_i^2 = \sum (A_{ij}^{inv})^2 v_i$$

where  $\sigma_i^2$  is the variance of  $x_i$ , and  $v_i$  is the variance of  $b_i$ .

TABLE 1

4	Œ	7	0-	v	*	ω	N	•	0	Sequeno
5.65	0 27.30	15.29	2.17	1.66	4.14	0.83	147 4:10	0.55	1.98	Aerial
14 6.42	9.26		2.62	16 2.95	6 19.34	5.34	168 0.27	84 0.35	100 0.33	a) positions
32 19.06	102 15,72	12 4.38	113 5.77	100	10 2.80	1.66	182 0.48	. 140 0.28	148	tions
168 12.76	109 16.79	172 14.92	148 0.84	147 6.76	172 5.45	189 1.53	189 1.01	168 0.34	172 0.42	Ø
190   17.66	196   28.22	190 I 8.23 I	196	196	196   3.53	196   0.63	196	196	196	Spacings
14^ 0.99	2° 62.67	62.67		16 7.76	9.54	9.54	7. 0.74	28* 0.81	24* 0.82	7
187	7. 0.74	10° 11.39	35 <i>°</i> 7.57	47 13.70	60.84	0.74	14 <b>*</b> 0.99	56# 0.87	0.98	
22 58.76	87 87.29	12 30.31	48° 0.98	49^	10 <sup>-</sup> 11.39	10 11.39	21+ 2.68	94.	72 2.19	3
32 45.38	94 85.71	18 42.93	83 11.64	94.0	24° 0.82	147	28° 0.81	112 2.50	96 <b>+</b> 1.90	77
136	100^	160 28.63	105 12.27	96° 1.90	162 15.80	175 4.87	35 7.57	1.69	144	Z (1)
154 5.77	102 84.33	170 60.97	113 12.02	100 5.13	166	182 3.04	42 9.73	1687	16B 2.44	スキャックタンナ
158 10.05	107 9.03	172° 3.32	1407	131 15.84	172 3.32	185 10.59	9.39	196	192 6.92	
168	109 86.18	178 12.64	148	147	186 12.54	189				
176 43.94	194 66.77	1887	1 <b>88</b> 5.52	180 6.48	190 39.25	1927				
1 <b>9</b> 0^ 39.25	196	190° 39.25	196	1967	196	196° 0.74				

TABLE 2

<b>≥</b> 6₹	
₩ X Z	
6 DAY	
REDUNDANT	

Amerial positions Spacings 4 100 148 172 196   2 0.64 0.28 0.28 0.26 0.52   0. 0 84 140 168 196   2 0.27 0.34 0.27 0.20 0.51   0. 0 8 16 32 4   0 0.61 0.21 0.29 0.55 0.28   0. 0 98 147 168 196   2 0.29 0.26 0.42 0.61 0.31   2 0.41 0.28 0.66 0.41 1.20   3 0.51 0.28 0.66 0.41 1.20   3 0.51 0.28 0.69 0.39 0.60   1	ľ	•	떠	N	**	0	Sequence:
Spacings       8     172     196   24* 48* 72 96* 144 168 192       8     0.26     0.52   0.56 0.73 1.77 0.98 2.31 0.53 2.90       9     168 196   28* 56* 84* 112 140 168° 196       10     168 196   28* 56* 84* 112 140 0.53 0.81       17     0.20 0.51   0.62 0.85 0.60 2.38 1.80 0.53 0.81       18     0.55 0.28   0.75 0.80 1.85 0.99 0.56 0.62 2.98       19     0.55 0.28   0.75 0.80 1.85 0.99 0.56 0.62 2.98       10     168 196   21 28° 49* 70 98* 147 168° 196°       10     195   13 35* 48° 70° 83 98° 133 168°       10     14     195   13 35* 48° 70° 83 98° 133 168°       10     196   7* 14 84° 91 98* 182 189 195       10     196   7* 14 84° 0.60 1.98 0.38 2.97 2.34 0.81	0.51	112 0.64	0.29	0.61	0.27	0.64	¥1384
Spacings       8     172     196   24* 48* 72 96* 144 168 192       8     0.26     0.52   0.56 0.73 1.77 0.98 2.31 0.53 2.90       9     168 196   28* 56* 84* 112 140 168* 196       10     168 196   28* 56* 84* 112 140 168* 196       17     0.20 0.51   0.62 0.65 0.60 2.38 1.80 0.53 0.81       18     0.55 0.28   0.75 0.80 1.85 0.99 0.56 0.62 2.98       19     0.55 0.28   0.75 0.80 1.85 0.99 0.56 0.62 2.98       10     168 196   21 28* 49* 70 98* 147 168* 196*       10     10.31   2.62 0.62 0.66 1.71 0.38 2.07 0.53 0.81       10     14     195   13 35* 48* 70* 83 98* 133 168*       12     14     195   13 35* 48* 70* 83 98* 133 168*       12     189 196   7* 14 64* 91 98* 182 189 196*       13     14     15     14       14     195   13     35* 48* 70* 83 98* 182 189 196*       15     189 196   7* 14 64* 91 98* 182 189 196*       16     18     18     18	98 0.28	147 0.28	98 0.26	0.21	0.34	100 0.28	il posit
196   24* 48* 72 96* 144 168 192 0.52   0.56 0.73 1.77 0.98 2.31 0.53 2.90 196   28* 56* 84* 112 140 168° 196 0.51   0.62 0.85 0.60 2.38 1.80 0.53 0.81 4	182 0.89	182 0.66	147 0.42	16 0.29	140 0.27	148 0.28	078
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181	1967	1687	1967				
		181 3.27					