AUSTRALIAN TELESCOPE PROJECT

# PRELIMINARY ERROR BUDGET FOR SERVO SYSTEM*



by

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## 1. INTRODUCTION

This brief report develops a preliminary servo pointing error budget for the radiotelescope structures proposed for the Australia Telescope Project. The results so far show that the encoder errcrs constitute the most significant part of the error budget, and servo errors wind and friction are considerably smaller. Transient wind errors, assuming step-type gusts, produce peak errors of the same order of magnitude as the position encoder errors.

The results have been produced as follows. A mathematical model of the telescope drive system and structure. The model is presented in Section 2. Based upon this model a position control loop is derived using a CAD package to study closed loop stability, robustness and bandwidth characteristics. The controller is described briefly in Section 3. Finally the controller is tested on a detailed computer simulation of the telescope. The errors are based upon the observed performance of this controller on the simulated telescope. The structure of the simulation is presented in Section 4 along with the error budget.

## 2. MATHEMATICAL MODEL

The complete mathematical model of the telescope drive system and structure is extremely complex and it is difficult to design controllers based upon this model. The reduced model presented below allows for considerable design insight in the controller design phase while still achieving very good performance when tested on the complete system model.

### Motor Model

$$E_g = IR + V_1$$

$$V_1 = K_E\dot{\theta}_M$$

$$T_M = K_J I$$

$$T_M - [\frac{B}{N}(\frac{\dot{\theta}_M}{N} - \dot{\theta}_0) + \frac{C}{N}(\frac{\theta_M}{N} - \theta_0)] = J_M\ddot{\theta}_M$$

### Load Model

$$B(\frac{\dot{\theta}M}{N} - \dot{\theta}_0) + C(\frac{\theta_M}{N} - \theta_0) + T_D = J_L\ddot{\theta}_0$$

where

$E_g$ = motor input voltage

$I$ = motor armature current

$V_1$ = motor back-EMf voltage

$R$ = motor armature resistance

$T_M$ = motor shaft torque

$\dot{\theta}_M$ = motor angular velocity

$K_T$ = motor torque constant

$K_E$ = motor back-emf constant

N = gear ratio

B = friction at load

$J_M$ = motor inertia

C = drive compliance

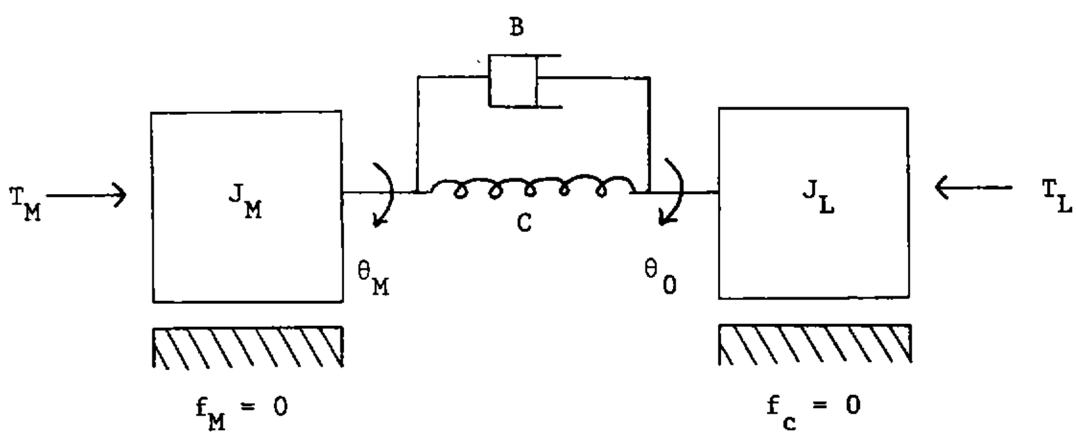
$J_L$ = load inertia

$T_L$ = load torque

$T_D$ = disturbance torque

$\dot{\theta}_0$ = load angular velocity

$f_M$ = friction at motor.




Based upon these equations the system transfer function becomes

$$\begin{bmatrix} J_M S^2+(f_M+B)S+C & -N(BS+C) \\ -1/N(BS+C) & JLS^2+BS+C \end{bmatrix} \begin{bmatrix} \theta_M \\ \theta_0 \end{bmatrix} = \begin{bmatrix} N^2K_T/R & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} E_g \\ T_D \end{bmatrix}$$

Define

$$G_1 \triangleq \left.\frac{\theta_M}{E_g}\right|_{T_D=0} = \frac{N^2K_T}{R} \frac{J_LS^2+BS+C}{\Delta}$$

$$G_2 \triangleq \left.\frac{\theta_M}{T_L}\right|_{E_g=0} = N \frac{BS+C}{\Delta}$$

$$G_3 \triangleq \left.\frac{\theta_0}{E_g}\right|_{T_L=0} = \frac{NK_T}{R} \frac{BS+C}{\Delta}$$

$$G_4 \triangleq \left.\frac{\theta_0}{T_L}\right|_{E_g=0} = \frac{J_MS^2+(f_M+B)S+C}{\Delta}$$

where

$$\Delta = J_MJ_LS^4 + ((J_M+J_L)B+J_Lf_M)S^3 + ((J_M+J_L)C+f_MB)S^2 + Cf_MS$$

Substituting the current best estimates of system parameters we obtain the following system transfer functions.

(a) Azimuth

$$G_1(s) = \frac{2.85(S+0.214\pm j21.38)}{\Delta}$$

$$G_2(s) = \frac{3.91\times10^{-4}(S+1069.14)}{\Delta}$$

$$G_3(s) = \frac{9.78\times10^{-5}(S+1069.14)}{\Delta}$$

$$G_4(s) = \frac{4.81\times10^{-7}(S+2.883\pm j7.84)}{\Delta}$$

where

$$\Delta = S^4 + 6.194\ S^3 + 529.422\ S^2 + 2606.4\ S$$

$$= S(S+4.98)(S+0.607\pm j22.87)$$

(b) Elevation

$$G_1(s) = \frac{2.85(S+0.244\pm j24.36)}{\Delta}$$

$$G_2(s) = \frac{2.11\times10^{-4}(S+1218.42)}{\Delta}$$

$$G_3(s) = \frac{5.28\times10^{-5}(S+1218.42)}{\Delta}$$

$$G_4(s) = \frac{6.41\times10^{-7}(S+2.86\pm j2.66)}{\Delta}$$

where

$$\Delta = S^7 + 6.202\ S^3 + 511\ 626\ S^2 + 3385.1\ S$$

$$= S(S+5.567)(S+0.3176\pm j24.66)$$

Notes:

(i) locked rotor frequency for azimuth

$W_n$ = 21.38 rad/sec

$\xi$ = 0.01

(ii) free rotor frequency for azimuth

$W_n$ = 22.878

$\xi$ = 0.0265

(iii) locked rotor frequency for elevation

$W_n$ = 24.36 rad/sec

$\xi$ = 0.01

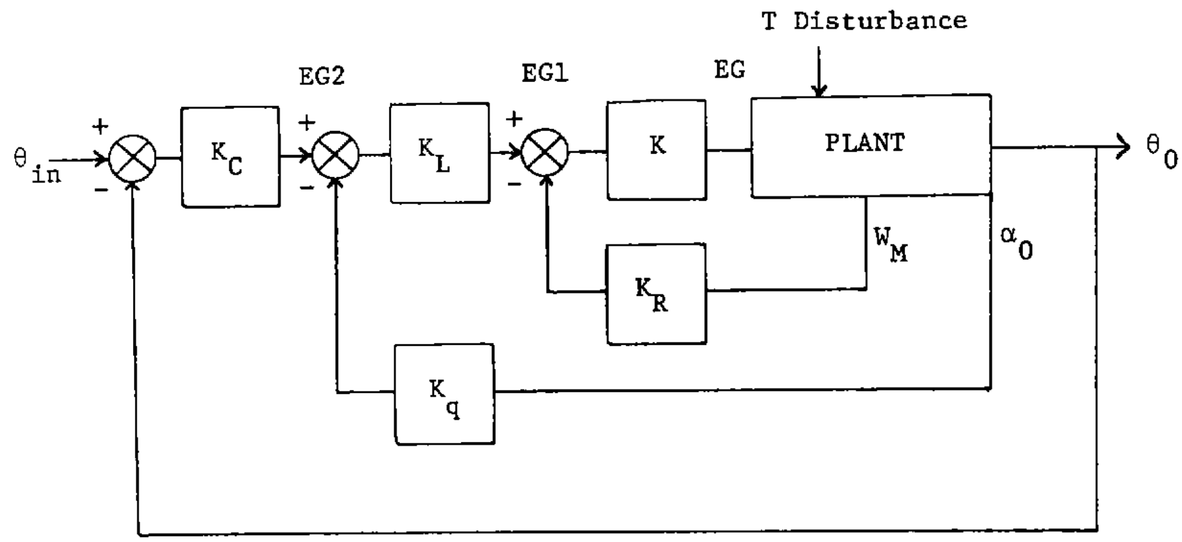
(iv) free rotor frequency for elevation

$W_n$ = 24.662

$\xi$ = 0.0129

## 3. CONTROLLER DESIGN

The form of controller to be used is given in the figure below. This structure was obtained by experimentation with the model of Section 2.




If we make the gain $K$ very large the disturbance is cancelled at low frequencies.

So let

$K_R = 0.1$

$K = 5000$

(for the same overall gain, we could make $K_R$ smaller, and $K$ larger giving same C.L.P., but better disturbance rejection.)

(a) Azimuth

$$\frac{\theta_0}{EG} = \frac{9.78\times10^{-5}(S+1069.14)}{S(S+4.98)(S+0.607\pm j22.87)}$$

$$\frac{\theta_0}{EG1} = \frac{9.78\times10^{-5}\times5000(S+1069.14)}{S[(S+4.98)(S+0.607\pm j22.87)+2.85\times5000\times6.1(S+0.214\pm j21.38)]}$$

$$= \frac{0.4989(S+1069.14)}{S(S+1431.0)(S+0.2387\pm j21.38)}$$

Now use accelleration feedback to move the resonant poles to higher frequency with improved damping closed loop poles minimum damping = 0.7.

Let

$$\left.\begin{aligned} K_q &= 0.1 \\ K_L &= \frac{GAIN}{0.0489} \end{aligned}\right\}$$

GAIN = 80; (S+14.75±j15.37)(S+1440.9)

GAIN = 120; (S+16.59)(S+27.28)(S+1446.1)

GAIN = 160; (S+9.22)(S+48.89)(S+1451.5)

GAIN = 200; (S+6.87)(S+65.38)(S+1456.9)

$$\frac{\theta_0}{EG2} = \frac{0.489K_L(S+1069.14)}{S[(S+0.2387\pm j21.38)(S+1431.0)+K_L\times0.1\times0.489S(S+1069.14)]}$$

$$\frac{\theta_0}{EG2} = \frac{800(S+1069.14)}{S(S+14.75\pm j15.37)(S+1490.9)} \rightarrow \frac{2000(S+1069.14)}{(S(S+6.87)(S+65.38)(S+1456.9)}$$

The position loop must be closed around the whole system, and the gain used that will produce a crossover frequency (3.5 Hz) or a closed loop bandwidth of the same. This is because under the simulation or the "real thing" situation some parts with the low resonant frequency will not be included in the feedback loops and therefore are not controllable.

Case (a)

$$\frac{800(S+1069.14)}{S(S+14.75\pm j15.37)(S+1440.9)}$$

Phase margin = 70° at $W_C$ = 3.5 rad/sec.

This is quite satisfactory so just adjust the gain for the required crossover.

$$K_C = 2.75$$

$$\text{CLTF} = \frac{2200S+2352108}{S^4+1470.4S^3+42960.35S^2+656079.56S+2382108}$$

$$= \frac{2200(S+1069.14)}{(S+4.889)(S+12.31\pm j13.51)(S+1441.0)}$$

Case (b)

$$\frac{2000(S+1069.14)}{S(S+6.87)(S+65.38)(S+1456.9)}$$

Phase margin = 60° at $W_C$ = 3.5 rad/sec.

This is also quite satisfactory, thus only a gain adjustment is needed, once again.

$$K_C = 1.25$$

$$\text{CLTF} = \frac{2500S+2672850}{S^4+1529.06S^3+105573.18S^2+648309.375S+2672850}$$

$$= \frac{2500(S+1069.14)}{(S+3.158\pm j4.23)(S+65.84)(S+1457.0)}$$

(b) Elevation

$$\frac{\theta_0}{EG} = \frac{5.28\times10^{-5}(S+1218.42)}{S(S+5.567)(S+0.3176\pm j24.66)}$$

$K_R = 0.1$

$K = 5000$

$$\frac{\theta_0}{EG1} = \frac{5.28\ 10^{-5}\times5000(S+1218.42)}{S[(S+5.576)(S+0.3176\pm j24.66)+2.85\times5000\times0.1(S+0.249\pm j24.361)]}$$

$$= \frac{0.264(S+1218.42)}{S(S+1431)(S+0.2494\pm j24.36)}$$

Now use acceleration feedback to move resonant poles.

Minimum damping = 0.7.

Let

$K_q = 0.1$

$$K_L = \frac{GAIN}{0.0264}$$

GAIN = 40; $(S+17.21\pm j17.17)(S+1436.8)$

GAIN = 50; $(S+21.43\pm j11.46)(S+1438.4)$

GAIN = 60; $(S+17.44)(S+33.89)(S+1439.9)$

GAIN = 70; $(S+12.5)(S+47.18)(S+1441.5)$

Finally we obtain the elevation controller

$$\frac{\theta_0}{EG2} = \frac{400(S+1218.42)}{S(S+1436.8)(S+17.21\pm j17.17)}$$

Gain for $W_C$ = 3.5 rad/sec = $K_C$ = 6.5

## 4. ERROR BUDGET

The controllers presented in Section 3 were tested on a detailed antenna model (see Figure at end of Section). Several assumptions and conditions are presented then the elevation and azimuth error budgets are presented. In summary it can be seen that in azimuth the error in low wind conditions is totally deminated by the position encoder error and is about 5 arc sec. With wind disturbance the worst error is about 6.5 arc sec. Similarly in elevation the low wind error is about 5 arc sec while the error in windy conditions is 8.5 arc sec.

### (a) Static Friction

An error is developed, at start up or at stall gearbox static friction = 3Nm. It is assumed that a torque step function is applied to the gearbox.

Stiction effects at "creep speeds" will be investigated at a later stage.

### (b) Gearing/Transmission Errors

Transmission errors were explicitly modelled into the simulation.

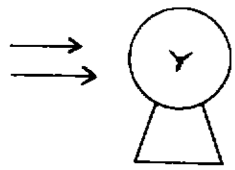
The results show that the errors vary less than 5% from their values, when transmission erorrs are neglected.

Gearing errors in transducers must also be investigated.

### (c) Required Motion

The acceleration of the antenna is the only contributer to steady state errors.

(d) Wind and Wind Gusts

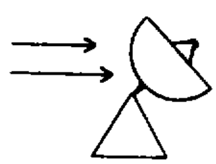


Azimuth: max wind torque occurs when the antenna is in the horizon position, at 90° to wind.

$$\text{Maximum Wind Torque at Wind Speed V m/s} = 36.61\times10^{3} \left(\frac{V}{8}\right)^{2} ; \; V \geq 8$$

$$= 36.61\times10^{3}\times k\left(\frac{V}{8}\right)^{2} ; \; V \leq 8$$

where k ≥ 1, aerodynamic correction factor



Elevation: max wind torque occurs when antenna is midway between horizon and zenith positions and at 180° to the wind.

$$\text{Maximum Wind Torque at Wind Speed V m/s} = -37.89\times10^{3} \left(\frac{V}{8}\right)^{2} ; \; V \geq 8$$

$$= -37.89\times10^{3} \left(\frac{V}{8}\right)^{2} ; \; V \leq 8$$

where k ≥ 1, aerodynamic correction factor

Peak Error Budget For Azimuth

(i) Zero Wind Condition

(a) Static friction (0.04"/Nm)

3Nm 0.12"

(b) Bearing Friction (0.03"/KNm)

8KNm 0.24"

(c) Position Encoder

(18 bit repeatable accuracy) 4.94"

(d) Desired motion - 6.6% overshoot

e.g. 10 arc second step 0.66"

$$\text{R.S.S.} = \sqrt{0.12^2 + 0.24^2 + 0.66^2 + 4.94^2}$$

$$= 4.99''$$

(ii) Wind Disturbance 0.25"/KNm

8m/s - max wind speed for precision 1 tracking

Max torque for worst orientation to wind 36.6 KNm error 9.15"

Torque for R.M.S. antenna orientation 16.27 KNm error 4.07"

(or R.M.S. torque for all antenna positions)

R.S.S. = 6.44"

Note:

The position encoder error is actually a peak error and the R.M.S. repeatable error would be reasonably expected to be much smaller than this.

Peak Error Budget For Elevation

(i) Zero Wind Conditions

(a) Static friction (0.018"/Nm)

3Nm 0.054"

(b) Bearing friction (0.00068"/KNm)

500Nm 0.00034"

(c) Position encoder

(18 bit repeatable accuracy) 4.94"

(d) Desired motion - 6.7% undershoot

e.g. 10 arc second step 0.67"

$$R.S.R. = \sqrt{0.054^2 + 0.00034^2 + 4.94^2 + 0.67^2}$$

$$= 4.985''$$

(ii) Wind Disturbance 0.37"/KNm

8m/s - max wind speed for precision 1 tracking

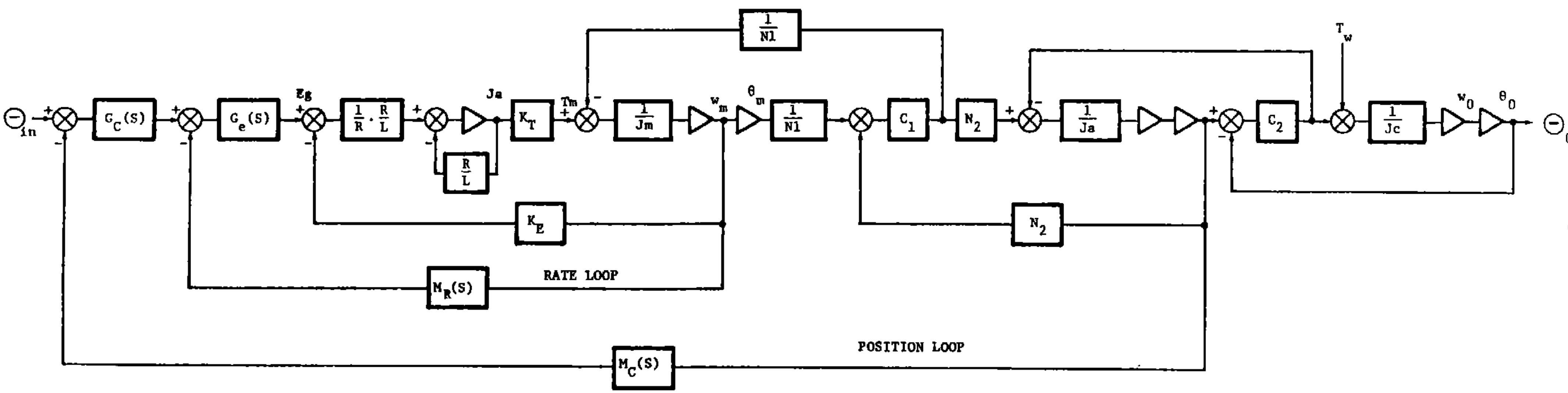
Max torque for worst orientation to the wind 37.89 KNm error 14.02"

Torque for R.M.S. antenna position 18.71 KNm error 6.92"

R.S.S. = 8.53"

Note:

The position encoder error is actually a peak error and the R.M.S. repeatable error would typically be reasonably expected to be much smaller than this.




DETAILED ANTENNA MODEL