Field rotation due to an antenna's pointing model

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1 introduction

In order to ensure that an antenna points towards a given direction in the sky one generally has to apply a number of corrections to allow for the antenna's pointing imperfections. These corrections are often expressed in the form of a pointing algorithm, whose parameters can be related to the construction of the antenna. The AT antennas, for example, have a 11 parameter model, defined in section 2 Only recently has it become apparent from polarisation work that a correction in the polarisation leakage terms is associated with the pointing parameters, and that an on-line correction is required. In effect, the position angle of the polarisation probes differs slightly from one antenna to the next. In general the effect is small and can be neglected; sources at high elevation can be affected. This note defines the correction procedure.

2 The Pointing Model

The AT antennas have an alt-azimuth mount; in the ideal case the azimuth axis is vertical, the elevation axis is horizontal, and the feed axis is normal to the elevation axis. The pointing model allows for small departures from this ideal:

ea	azimuth encoder zero-point
ee	elevation encoder zero-point
fz	azimuth squint (feed rotation normal to the elevation axis)
fx	elevation squint (feed rotation about the elevation axis)
ax	azimuth axis tilt towards the south
ay	azimuth axis tilt towards the west
ey	non-orthogonality of the elevation axis to the azimuth axis
se	elevation encoder eccentricity (sin)
ce	elevation encoder eccentricity (cos)
	(we set se = 0; ce is used to describe the gravitational sag)
sa	azimuth encoder eccentricity (sin) - not currently used
ca	azimuth encoder eccentricity (cos) - not currently used

The algorithm used by the ACC is :

$$\begin{aligned} azcorr &= ((-ay * \cos(A) + ax * \sin(A) + ey) * \sin(E) + fz \\ &\quad +(ca * \cos(A) + sa * \sin(A) + ea) * \cos(E))/\cos(E) \end{aligned}$$

$$\begin{aligned} elcorr &= ay * \sin(A) + ax * \cos(A) + cedz * \cos(E) + se * \sin(E) \\ &\quad +fx + ee \end{aligned}$$

applied in the sense :

$$\begin{aligned} A(true) &= A(encoder) + azcorr \\ E(true) &= E(encoder) + elcorr \end{aligned}$$

3 Rigorous Theory and the Field Rotation

To derive expressions for the pointing and field rotation, we start the a defect-free antenna and apply a series of rotations in sequence: the feed squints, the rotation in elevation; the elevation axis tilt; the rotation in azimuth; the azimuth axis tilt.

The fundamental coordinate frame has x pointing east; y pointing north and z pointing up. An ideal antenna at azimuth 0 has its elevation axis parallel to the x-axis. Azimuth increases counter-clockwise about the z-axis.

notation : let $R_{axis}(\theta)$ be a rotation matrix corresponding to a rotation about the specified axis by an angle θ. The conventions are illustrated in figure refconventions.

If (A, E) are the azimuth and elevation encoder readings, then the full rotation matrix is given by :

$$\mathbf{R} = R_x(ax)R_y(-ay)R_z(-A)R_y(ey)R_x(E - \frac{\pi}{2})R_y(fz) \tag{1}$$

Elaborating each rotation matrix - in each case we describe the old frame, (such as the feed) in the new frame (the vertex):

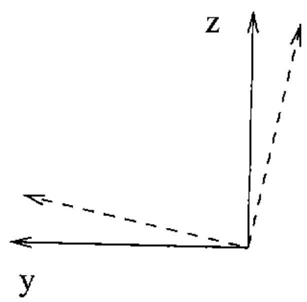
$$R_y(fz) = \begin{vmatrix} \cos(fz) & 0 & \sin(fz) \\ 0 & 1 & 0 \\ -\sin(fz) & 0 & \cos(fz) \end{vmatrix} \tag{2}$$

$$R_x(E - \frac{\pi}{2}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \sin(E) & \cos(E) \\ 0 & -\cos(E) & \sin(E) \end{vmatrix} \tag{3}$$

$$R_y(ey) = \begin{vmatrix} \cos(ey) & 0 & \sin(ey) \\ 0 & 1 & 0 \\ -\sin(ey) & 0 & \cos(ey) \end{vmatrix} \tag{4}$$

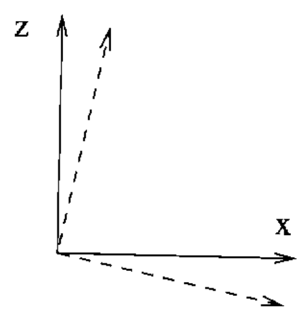
$$R_z(-A) = \begin{vmatrix} \cos(A) & \sin(A) & 0 \\ -\sin(A) & \cos(A) & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{5}$$

Rx(t)



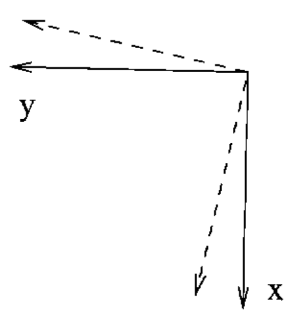
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(t) & -\sin(t) \\ 0 & \sin(t) & \cos(t) \end{bmatrix}$$

Ry(t)



$$\begin{bmatrix} \cos(t) & 0 & \sin(t) \\ 0 & 1 & 0 \\ -\sin(t) & 0 & \cos(t) \end{bmatrix}$$

Rz(t)



$$\begin{bmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 1: The definition of the rotation matrices.

$$R_y(-ay) = \begin{vmatrix} \cos(ay) & 0 & -\sin(ay) \\ 0 & 1 & 0 \\ \sin(ay) & 0 & \cos(ay) \end{vmatrix} \tag{6}$$

$$R_x(ax) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos(ax) & -\sin(ax) \\ 0 & \sin(ax) & \cos(ax) \end{vmatrix} \tag{7}$$

Some elaborations:

(1) Pointing axis, ideal antenna. Apply the rotation to the unit vector aligned along the z-axis :

The rotation matrix is :

$$\mathbf{R} = \begin{vmatrix} \cos(A) & \sin(A)\sin(E) & \sin(A)\cos(E) \\ -\sin(A) & \cos(A)\sin(E) & \cos(A)\cos(E) \\ 0 & -\cos(E) & \sin(E) \end{vmatrix} \tag{8}$$

and the unit vector of the feed axis is given by :

$$\hat{z} = \mathbf{R} \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} \sin(A)\cos(E) \\ \cos(A)\cos(E) \\ \sin(E) \end{vmatrix} \tag{9}$$

(2) The x-probe, ideal antenna:

$$\hat{x} = \mathbf{R} \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} \cos(A) \\ -\sin(A) \\ 0 \end{vmatrix} \tag{10}$$

(3) Real antenna, pointing axis.

The rotation matrix can be written in the small angle approximation :

$$\mathbf{R} = \mathbf{R_{ideal}} + \mathbf{R_{err1}} + \mathbf{R_{err2}} \tag{11}$$

$$\mathbf{R_{err1}} = \begin{vmatrix} -fz.\sin(A)\cos(E) & -ey.\cos(A)cos(E) & (fz+ey.\sin(E))\cos(A) \\ -fz.\cos(A)\cos(E) & ey.\sin(A)\sin(A) & -(fz+ey.\sin(E))\sin(A) \\ -(fz.\sin(E)+ey) & 0 & 0 \end{vmatrix} \quad (12)$$

$\mathbf{R_{err2}} =$

$$\begin{vmatrix} 0 & ay.\cos(E) & -ay.\sin(E) \\ 0 & ax.\cos(E) & -ax.\sin(E) \\ -ax.\sin(A)+ay.\cos(A) & (ax.\cos(A)+ay.\sin(A))\sin(E) & (ax.\cos(A)+ay.\sin(A))\cos(E \end{vmatrix} \quad (13)$$

$$\hat{z} = \mathbf{R}\begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} \sin(A)\cos(E)+(fz+ey.\sin(E))\cos(A)-ay.\sin(E) \\ \cos(A)\cos(E)-(fz+ey.\sin(E))\sin(A)-ax.\sin(E) \\ \sin(E)+(ax.\cos(A)+ay.\sin(A))\cos(E) \end{vmatrix} \quad (14)$$

Equating this to the unit vector of an ideal antenna pointing in the direction $A+\Delta A$, $E+\Delta E$ we find:

$$\begin{aligned} \Delta A\cos(A)\cos(E)-\Delta E\sin(A)\sin(E) &= (fz+ey.\sin(E))\cos(A)-ay.\sin(E) \\ -\Delta A\sin(A)\cos(E)-\Delta E\cos(A)\sin(E) &= -(fz+ey.\sin(E))\sin(A)-ax.\sin(E) \\ \Delta E\cos(E) &= (ax.\cos(A)+ay.\sin(A))\cos(E) \end{aligned}$$

The pointing corrections follow:

$$\begin{aligned} \Delta E &= ax.\cos(A)+ay.\sin(A) \\ \Delta A &= (fz+(ey.+ax.\sin(A)-ay.\cos(A))\sin(E))/\cos(E) \end{aligned}$$

(4) Field rotation.

We proceed in three steps: we determine the orientation of the real antenna's "x-probe" for a given encoder azimuth and elevation; we then locate the orientation of the x-probe for the ideal antenna which is pointing in the same direction as the real antenna; finally we determine the angle between the two x-probes. Since the two antennas have a common z-axis, the computed angle is indeed the field rotation.

$$\hat{x}_{real} = \mathbf{R} \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} \cos(A) - fz.\sin(A)\cos(E) \\ -\sin(A) - fz.\cos(A)\cos(E) \\ -(fz.\sin(E) + ey) - ax.\sin(A) + ay.\cos(A) \end{vmatrix} \tag{15}$$

while the ideal antenna pointing in the same direction will have

$$\hat{x}_{ideal} = \begin{vmatrix} \cos(A + \Delta A) \\ -\sin(A + \Delta A) \\ 0 \end{vmatrix} \tag{16}$$

The cosine of the angle betwen these two vectors can be obtained from' the dot product :

$$\begin{aligned} \cos(\chi) &= (\cos(A) - fz.\sin(A)\cos(E))\cos(A + \Delta A) + (\sin(A) + fz.\cos(A)\cos(E))\sin(A + \\ &= \cos(A + fz.\cos(E))\cos(A + \Delta A) + \sin(A + fz.\cos(E))\sin(A + \Delta A) \\ &= \cos(fz.\cos(E) - \Delta A) \end{aligned}$$

and, finally,

$$|\chi| = (fz.\sin(E) + ey + (ax.\sin(A) - ay.\cos(A)))\tan(E) \tag{17}$$

Since $\hat{x}_{ideal}$ is always horizontal we can determine the sign of χ from the sign of the z-component of $\hat{x}_{real}$ - the rotation is positive (in the sense of real to ideal) is the z-component is negative.

On the AT antennas the pointing parameters are all small, typically less than one arcminute, so that the field rotation is in almost all cases insignificant. It has been observed to affect the polarisation leakage and lead to "significant" stokes V for sources transitting close to the zenith. Figure 2 illustrates these algorithms applied to a particularly testing observation, with a source passing particularly close to the zenith.

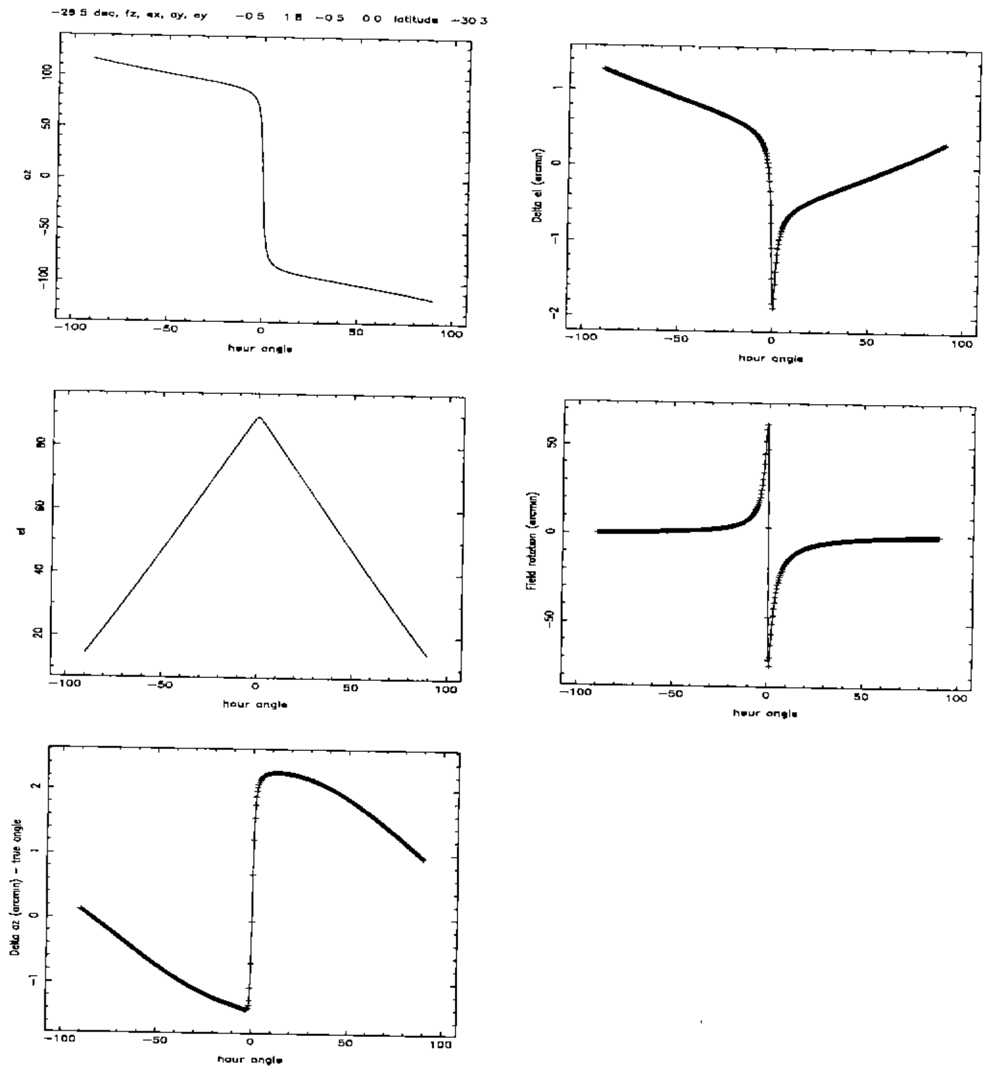


Figure 2: The pointing and rotation characteristics for a source transitting close to the zenith. The continuous curves are based on the full theory; the crosses shows the small angle approximations, which clearly are good enough.

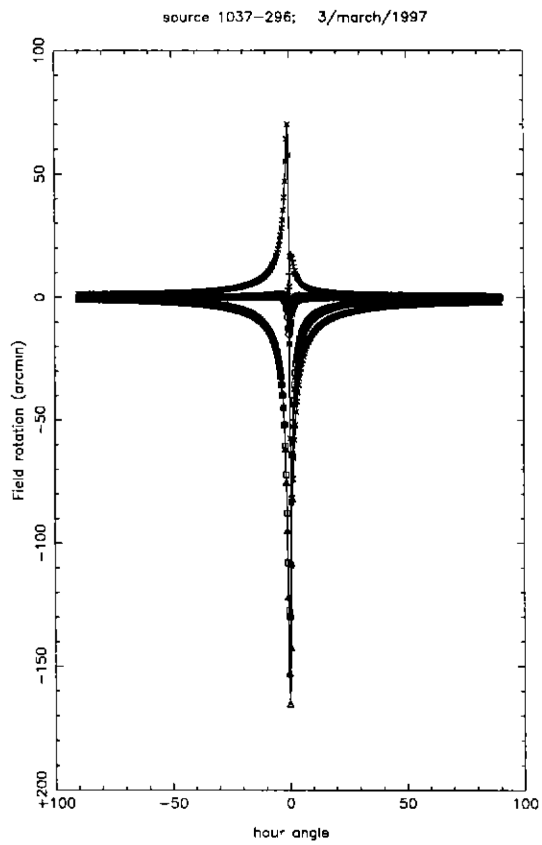


Figure 3: The field rotation predicted for the observation of 1037-296 of 3 March 1997.

4 The motivating observation

The field rotation problem arose in connection with observations made by Dave Rayner in march, 1997. Figure 3 shows the field rotation computed for all the antennas in the array, using the pointing parameters in place at the time. The curves fall into three groups : antennas 1,2,3 all have small rotations. Antenna 4 has an asymetric curve, positive before transit, negative after. Antennas 5 and 6 have strong, symmetric, negative curves.

Bob Sault has provided a task in Miriad (transfix) which will correct data for the rotation, using these algorithms. CAOBS will compute these corrections on-line and have them written to the RPFITS file.