Some Simulations of Self-Calibration for the AT

R.J. Sault

April 16, 1991

This memo attempts to characterise how well we can expect self calibration to work with AT data. The results are purely based on computer simulations, rather than experience with real data. This also addresses whether east-west arrays are fundamentally worse, for self calibration purposes, than other array configurations. The results are that self calibration works with the AT, though more selfcal iterations may be required than for, say, VLA data. Using several configurations of data helps to improve the robustness of selfcal. Finally it does appear that an east-west array is less robust than other array configurations, when using standard selfcal.

Computer Simulations

A number of simulations of both one and four 12 hour syntheses, using standard 3km AT configurations were performed. Various different integration times, signal-to-noise ratios, self-cal solution intervals and phase error time scales, self-cal initial model, rms phase errors, etc, were used. Parameter space is quite large, and only some more "typical" parameters were simulated. Amplitude gain errors were never simulated. All self-cal solutions were phase only. The self-cal solution interval was always set equal to the time-scale of the modeled phase. The results are for a system temperature of 50 K and a bandwidth of 128 MHz. The observation frequency was set at 1.4 GHz. A 'system gain' of 9 Jy/K was assumed (which accounts for dish size, aperture efficiency and sensitivity loss by the correlator). No bandwidth or primary beam effects were simulated. Each simulation run always performed 50 selfcal iterations – to be confident that the final state had been reached. Only in one run did it appear that more than 50 iterations were needed. All simulations were of a source representing a spiral galaxy (Figure 1). This needs a dynamic range of approximately 100 to detect the weaker features. The declination of the source was −50°. The source was intended to be very similar to that used in earlier AT studies. It does, however, differ from the earlier source, as the earlier source was not readily available when these investigations were started.

The Miriad package (a reduction package developed by the BIMA consortium) was used for all simulation and analysis work. The solution algorithm used was a least-squares (L2) algorithm, as suggested by Schwab ("Adaptive calibration of radio interferometer data", International Optical Computing Conference, SPIE vol. 231, pp 18–24). This is very similar to the algorithm used in AIPS tasks CALIB and ASCAL.

The initial simulations used the CLEAN components of the dirty maps as the first and subsequent selfcal model. CLEAN was stopped when either the theoretical 4 sigma level (based on receiver noise) was reached, or a negative component was encountered, or 10000 components were cleaned off. Generally the 'negative component' criteria was the one first met, but sometimes, for the higher dynamic range maps, the 4 sigma level came into play (even when the map was still of quite poor quality). Images were 128 × 128, with each pixel being 4 arcsec square. Uniform weighting and Schwab's spheroidal gridding functions were used in mapping (the same as the AIPS defaults). Note that the receiver noise calculation correctly includes the effect of uniform weighting.

Table 1 summarises the results of simulations for a single 12 hour synthesis (using the "Config No. 3" of the 3 km Minimum Redundancy antenna configurations – which gives a modestly uniform single day uv coverage). Table 2 gives the results for simulations of 4 12 hour syntheses (using configurations 1, 2, 3 and 4 of the 3km minimum redundancy configurations). Unfortunately these configurations have now been superceded. However I feel confident that the results here will be quite applicable to the new configurations as well.

The parameters given in Table 1 and 2 are:

SNR This gives the rms signal to noise ratio of a visibility, for a 20 second integration time. Because all structures on the test source are fully resolved, there is essentially no signal on the longer baselines, whereas there is significant signal at the shorter baselines. See Figure 2 for an amplitude vs uv distance plot of noiseless data.

DRange The theoretical dynamic range of the resultant image. This is the peak in the CLEAN image divided by the theoretical rms noise.

Interval The time-scale of the phase fluctuations. As noted above, the selfcal solution interval was always set equal to this.

PError The rms antenna gain phase error. The units are degrees. Though the phase errors were gaussian, because of 2π wrap around of phase, an rms phase error of 80° on each antenna, leads to phase errors which are more akin to uniformly distributed. That is to say that you cannot get phase errors much worse than 80°.

Initial The initial error ratio. That is the ratio of the error in the initial CLEANed image, before any selfcal, to the receiver noise.

Final The final error ratio. That is the ratio of the error in the CLEANed selfcal'ed image, after 50 iterations, to the receiver noise.

Iters The number of selfcal iterations required to bring the error ratio within 10% of its final value.

C? A "bullet" here indicates that the simulation 'failed' (see below).

The error in a selfcal image was determined from the difference between it and an image that was mapped and cleaned from data without noise or phase errors. The error was measured in a rectangle of 130 × 160 arcsec centered on the source. Because selfcal loses absolute position, the selfcal'ed image was aligned with the noiseless image, using a maximum likelihood algorithm. For small phase errors, the misalignment was small (fractions of a pixel). For large phase errors (80 degrees) the misalignment could be large (many pixels), which is a result of the very poor initial selfcal model.

Analysis of the Simulations

Although Tables 1 and 2 give 'number of iterations' for the selfcal process to get within 10% of the final error ratio, these iteration counts must be treated with some caution – they are only a rough guide. The error ratio from one iteration to the next tended to have an appreciable jitter. The error ratio can also go through a 'minimum' well before the 50 selfcal iterations always used, and then start to increase slightly (note that the tables give the error ratios after 50 iterations, not at the minimum). Also the number of iterations is heavily dependent on the stopping criteria for CLEANing. In the presence of large phase errors, the "first negative component" criteria probably still allows too many extraneous components into the model – at least in the early iterations of the high dynamic range maps. For these cases, a more conservative early strategy would have led to faster convergence.

Figure 3 gives a plot of the error ratio varying with iteration, for both a 1 and 4 day synthesis (and a 'random' array – see next section). These are for SNR=50, Interval=10 minutes, PError=40°.

For low to moderate phase errors, the initial dirty maps were quite reasonable, and recognisable. For phase errors of 80 degrees rms, the initial dirty map was essentially rubbish, with nothing of the true source being recognisable, and with as much negative as positive flux.

Figure 4 shows an example of initial and final images, for a 1 day synthesis, SNR=10, Interval=10, PError=40°. This is a case where selfcal succeeded. Figure 5 is an example where it was less successful.

This shows the initial and final images, for the same parameters, except PError=80°. Though in this latter example, selfcal has clearly improved the image substantially, it is still substantially poorer than it could be (this run counted as a 'failure').

Indeed 'failures' of the simulations can be divided into three cases:

- It occasionally happened that, for the 1 day syntheses, the maximum in the original dirty map was negative. In this case the simulation failed (this is not a failing of selfcal, but rather of the automated way in which all the simulations were performed).
- The initial model was so far from the true source position that the CLEAN box cut the model. This again is a failing of the automated procedure, and could be avoided by a proficient user.
- Though the iterations continued normally, the procedure "converged" to an image which was far from the true image. In this case, Tables 1 and 2 continue to show the selfcal statistics, but note that the final error is substantially worse than for other simulations. This failing is clearly a more insidious failure – as it conceivably could lead to the astronomer believing a image of poor quality. Indeed the images of Figure 5 are considered 'failures'.

With one exception, all selfcal failures were when the phase error was 80°, and for the 1 day syntheses. Four day syntheses always converged.

Note that for cases where there was not phase errors (selfcal was used all the same) that the error ratio is degraded by using selfcal. Even in the case of 20° phase errors, and signal-to-noise ratio of 1, the error ratio degrades with selfcal. The fact that we can make an image worse, by using selfcal, should not be a surprise. We can never expect the deconvolution process to be perfect, so the selfcal model is alway imperfect. So selfcal'ing data without phase errors on an imperfect model is bound to degrade the image quality.

Varying the phase error time scale (and the solution interval time) had surprisingly little effect on the results. Increasing the time scale from 1 minute to 100 minutes reduced the error ratio by typically only 10 to 20 percent. Here a detail of the simulation should be mentioned. In the selfcal solution, the phase is assumed to be constant over a solution interval. In fact the phase errors were continuously varying. The phases varied linearly from one random phase to another, over a time given by the time scale. So the selfcal algorithm was incorrectly modelling the phase fluctuations. However, when applying the selfcal gains, the phases were varied linearly from one solution time to the next.

It is gratifying that 4 day syntheses appear better to selfcal. Selfcal runs of 4 day syntheses converge much faster, and appear to be more robust (no selfcal 'failures'). Though the selfcal algorithm separately solves for the gains for the different days (indeed the different solution intervals), the selfcal algorithm benefits from the better model that combining multiple days of data achieves. This is clearly an important rule to remember in the reduction process.

Note that for the SNR 50 and 100, the final error was substantially poorer than theoretical. This applies even when the data had no phase errors (the initial error ratio of the 0 phase error runs). So, at least partially, the error is due to deconvolution errors, rather than an inability of selfcal. Also, however, machine rounding effects start to make their presence felt – especially for the 4 day SNR 100 case. So, unfortunately, the final error ratios must be treated with some caution. This is particularly unfortunate as it makes it less straight forward to compare the differences between the 1 and 4 day syntheses.

Comparison with non-East-West Arrays

Some simulations were devoted towards answering the question "Is an east-west array worse for selfcal than other configurations". This was motivated by earlier studies by Norris and Kesteven (Selfcal and the AT, memo number AT/25.1.1/011). Comparisons between runs for an east-west array and another array are made very difficult, because the uv coverage of the two arrays will not be identical. This means that, all other parameters being the same, signal-to-noise ratios and sidelobe levels will be different. To avoid this problem, non-east-west observations were formed by taking the uv points from an east-west observation, and giving them a 'random' observation time (within the bounds of the total observation). This results in an array which, overall, has identical coverage to the AT. But at a given time, the uv points are scattered over the uv plane (rather than lying along a line). Of course, in the simulations, the phase error applied to the uv point was the phase error for the new observation time. This "random" array clearly gives better instantaneous uv coverage.

The results of the simulations for 1 day syntheses are given in Table 3. In all respects, other than array configuration, the runs were identical to those given in Table 1. Several features are worth noting. The random array appears to converge substantially faster, and in cases of large phase errors, the random array appears more robust. In only one instance did the random array fail to converge. However, when both east-west and random arrays converged, they both converged to similar error ratios. The random array seems to show somewhat lower (10-30%) error ratios, for both the initial and final error. However this was not universally so.

To further compare an east-west and a random array, some simulations were performed using bad initial models. Rather than using CLEAN components derived from the initial dirty map, either a point source or a 'reverse spiral' (the true source rotated by 180°) as the initial selfcal model. Some results for these simulations are presented, in Figures 6 through 9. These figures clearly show that the random array is significantly more robust to a bad initial model than the east-west array. In these simulations, the phase error on the data is not important – once we have a model, and use it for self calibration, then the original phase errors become irrelevant. The phase error originally in the data is only important when it effects the initial model.

It has been said that an east-west array is more prone to 'symmetrise' the source. It would be more accurate to say that the east-west array is less robust at rejecting an initial bad model. If the initial model was symmetric (i.e. zero phase, such as a point source), then the initial selfcal iterates will also tend to be symmetric (tend to have zero phase). If the original model was a 180° rotation of the true source, then the initial selfcal iterates will also tend to be 180° rotations of the true source. Putting it another way – an east-west observation is more malleable, in that the phases can be more easily adjusted to make the data into an incorrect image.

Conclusion

If selfcal were given a perfect model to start with, it would determine (as best it ever could) the correct antenna gains on the first iteration, regardless of the array configuration or the length of observation. The need for iterating selfcal comes about only because the initial model is far from perfect, and because it is generated from the data that is being corrected. The imperfections in the model are difficult to quantify – they are highly image dependent and are also affected by the deconvolution step. Several days of data should produce a model, with proportionally fewer degrees of freedom, than a 1 day synthesis. The ratio of unknowns (the model pixel values and the antenna gains) to knowns (the measured data) decreases as more configurations are observed.

It also appears that a 'random' array produces a model with fewer degrees of freedom than does an east-west array. In these case, the model is more constrained to be nearer the correct solution.

At a more practical level, the results given here should stress the importance of using all days configurations, when making a model for selfcal.

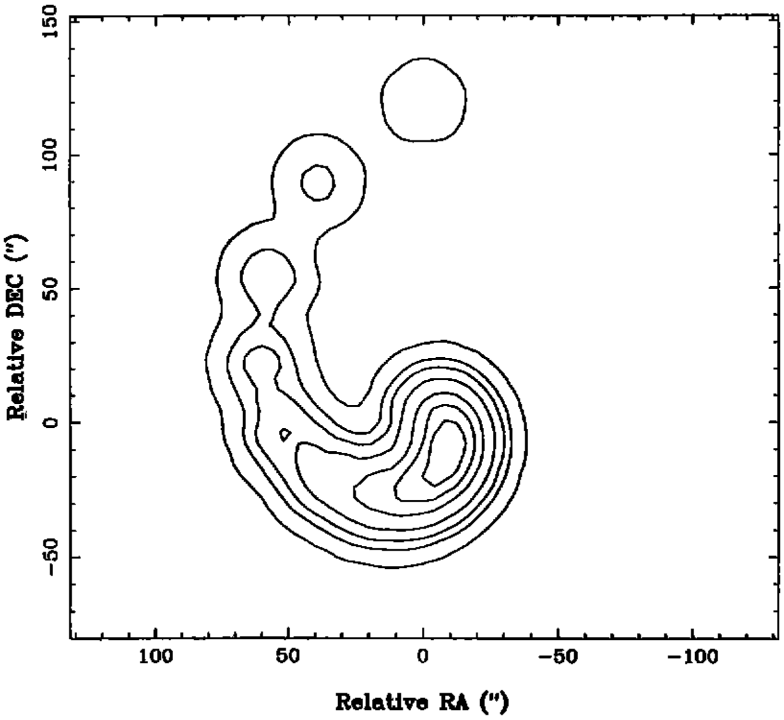

Figure 1: The spiral source.

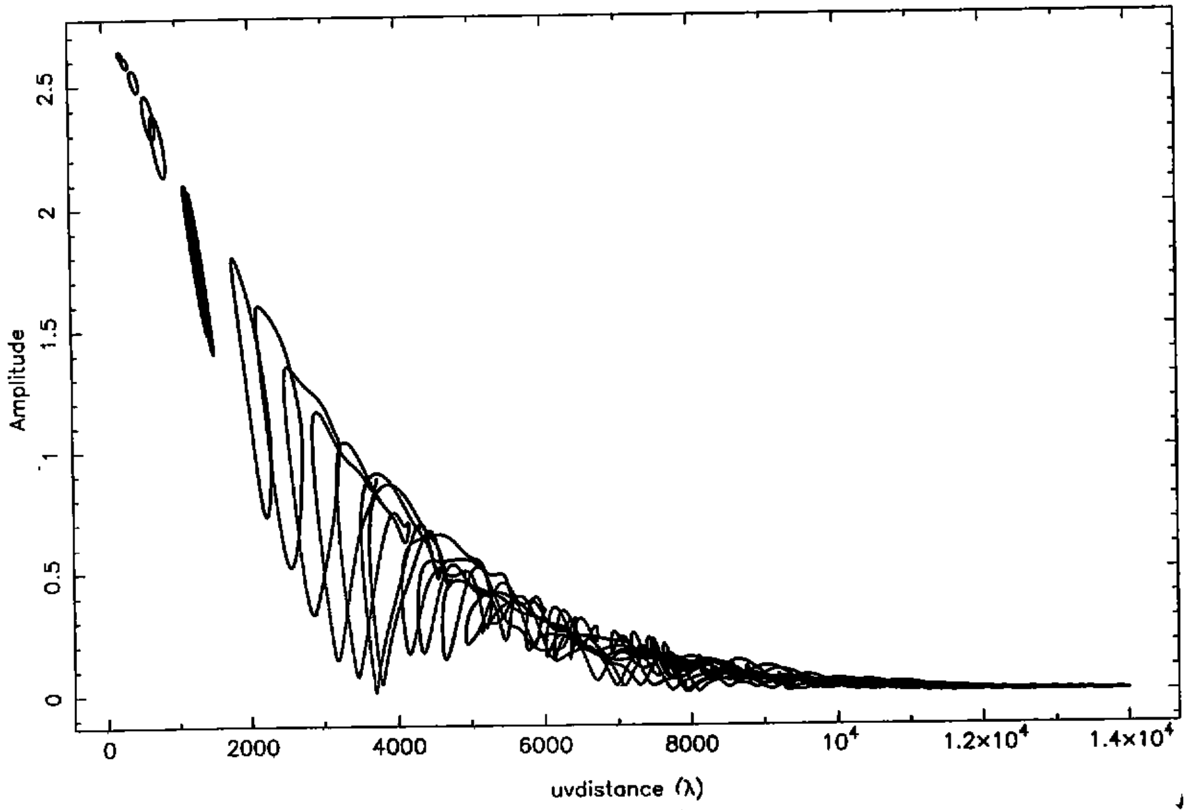

Figure 2: Amplitude vs uv distance of the spiral source, for a 1 day synthesis. The data is noiseless.

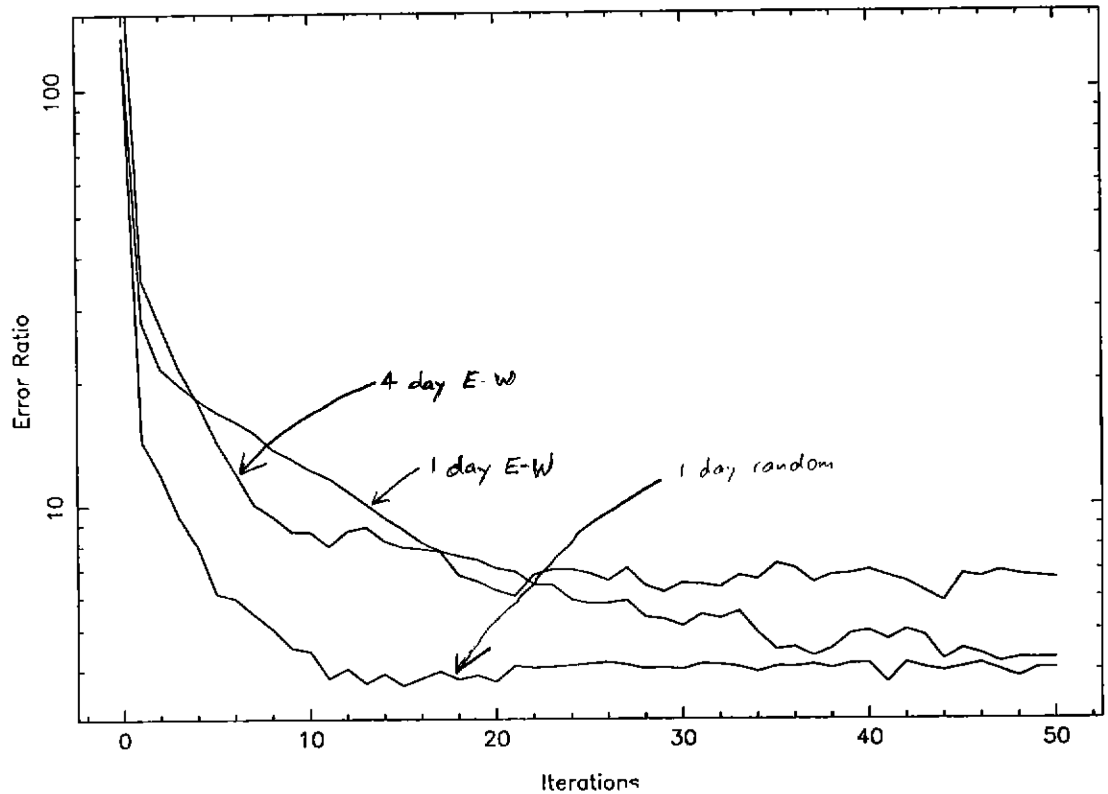

Figure 3: Error ratio vs selfcal iterations: The error ratio indicates how much worse than theoretical the error in the image is. The three plots correspond to a 1 day synthesis, a 4 day synthesis, and a 1 day synthesis for the 'random' array. Note that the 1 day synthesis for the east-west array converges more slowly than the other two. Simulation parameters are SNR=50, Interval=10 minutes, PError=40°.

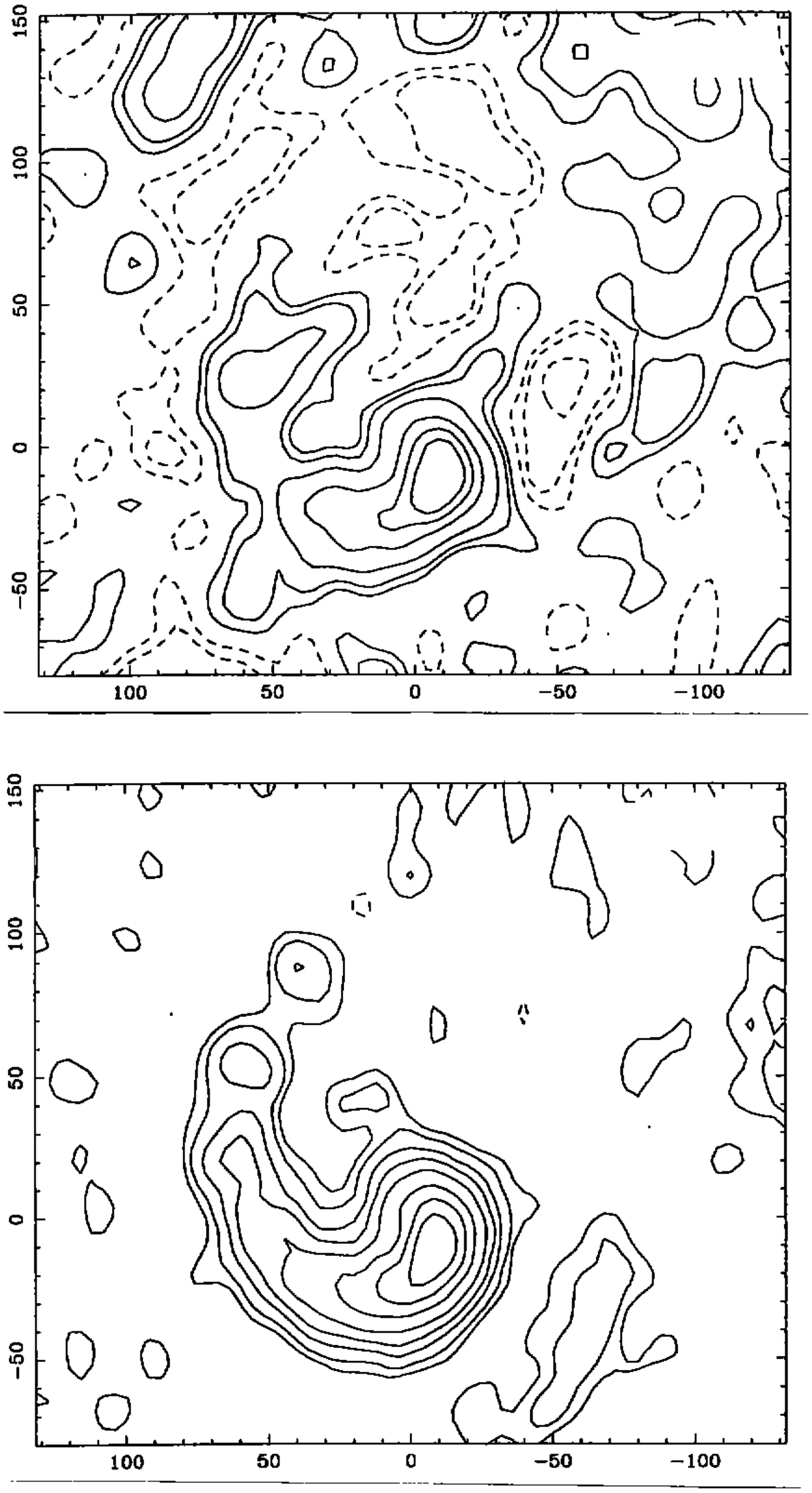

Figure 4: The initial and final CLEANed images, for a 1 day synthesis, for SNR=10, Interval=10 minutes, PError=40°. The final image is a substantial improvement over the initial image.

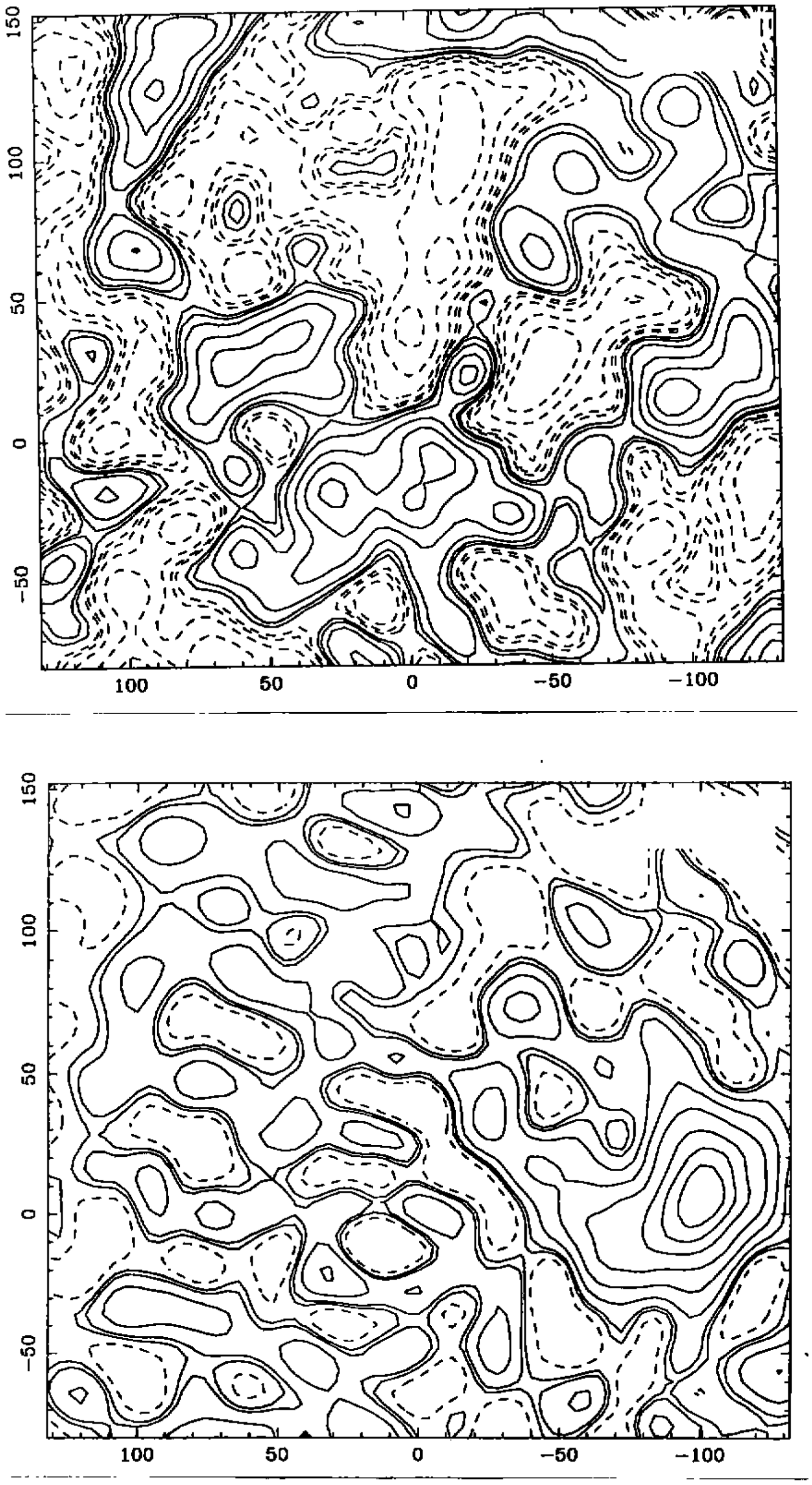

Figure 5: The initial and final CLEANed images, for a 1 day synthesis, for SNR=10, Interval=10 minutes, PError=80°. The initial image is complete rubbish. While the final image is clearly a substantial improvement, it still was deemed not to have converged. Compare with Figure 4.

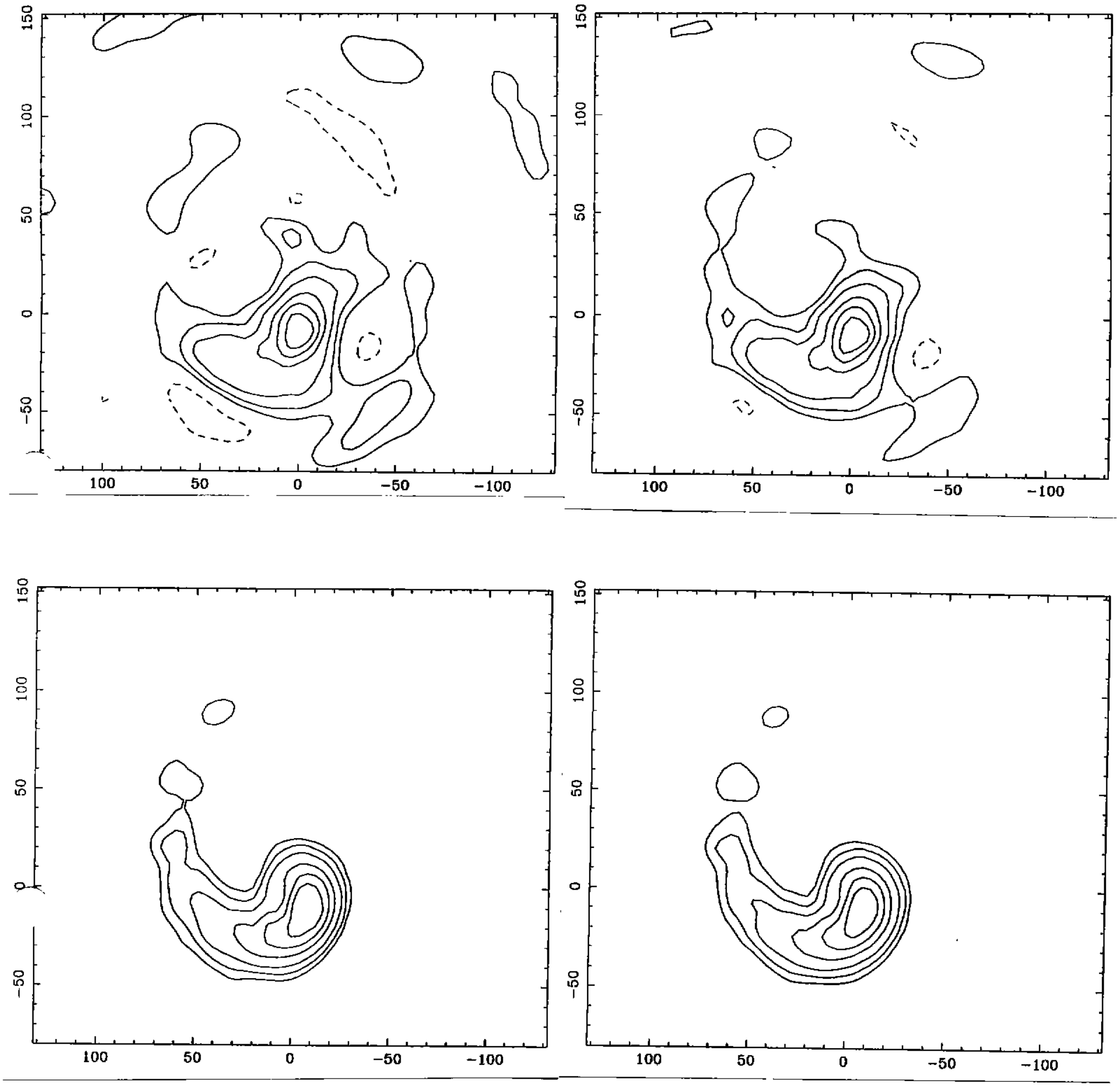

Figure 6: Selfcal iterates for the random array, using a point source as the initial model. SNR=100, Interval=5 minutes, PError=0°. Iterations 0, 1, 10 and 20 are shown.

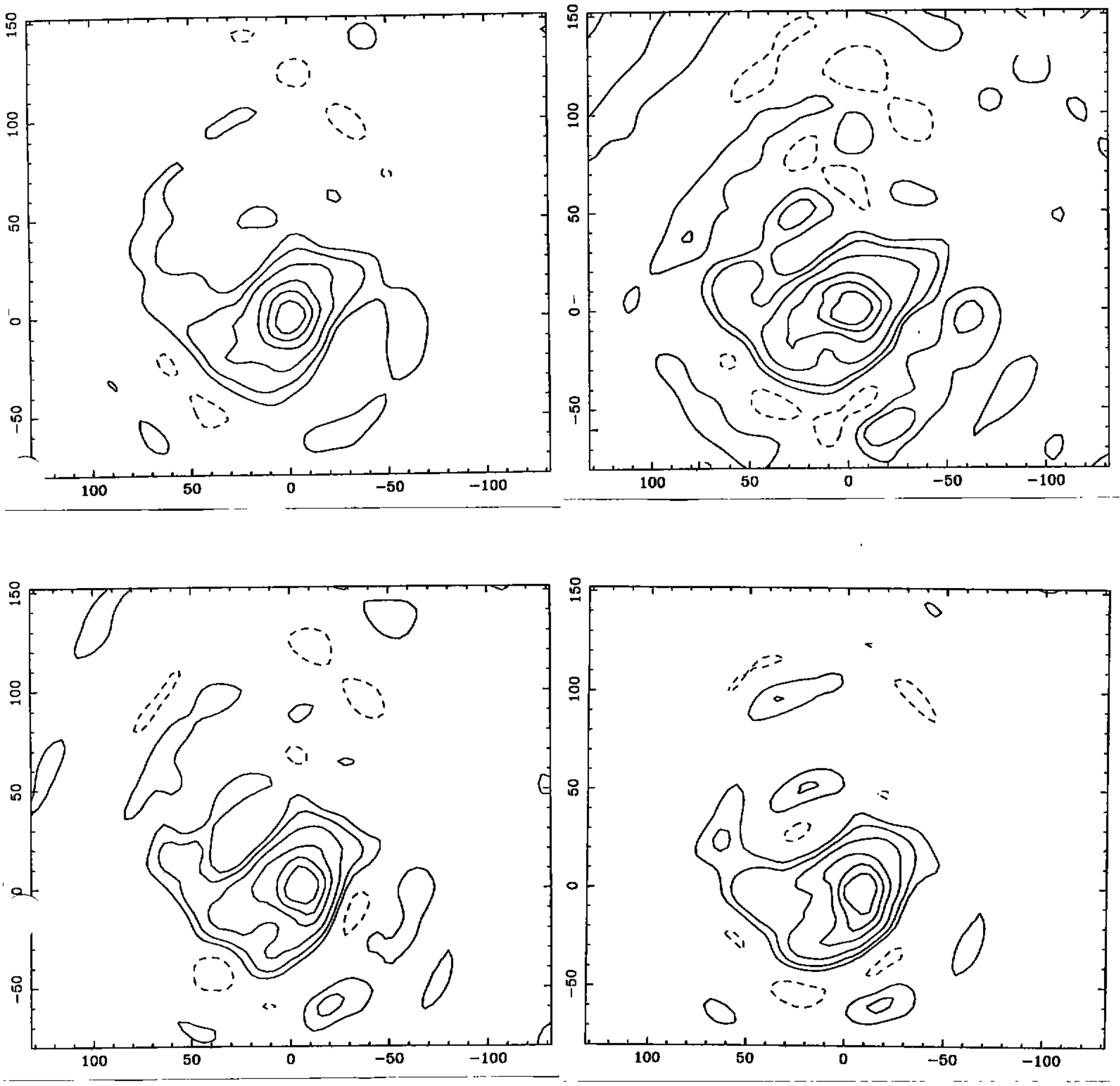

Figure 7: Selfcal iterates for an east-west array, using a point source as the initial model. SNR=100, Interval=5 minutes, PError=0°. Iterations 0, 1, 10 and 20 are shown. Compare with Figure 6, to note that the east-west array takes significantly more iterations to reject the bad initial model, and converge to the correct image. This simulation run eventually converged to the correct image, after about 30 iterations.

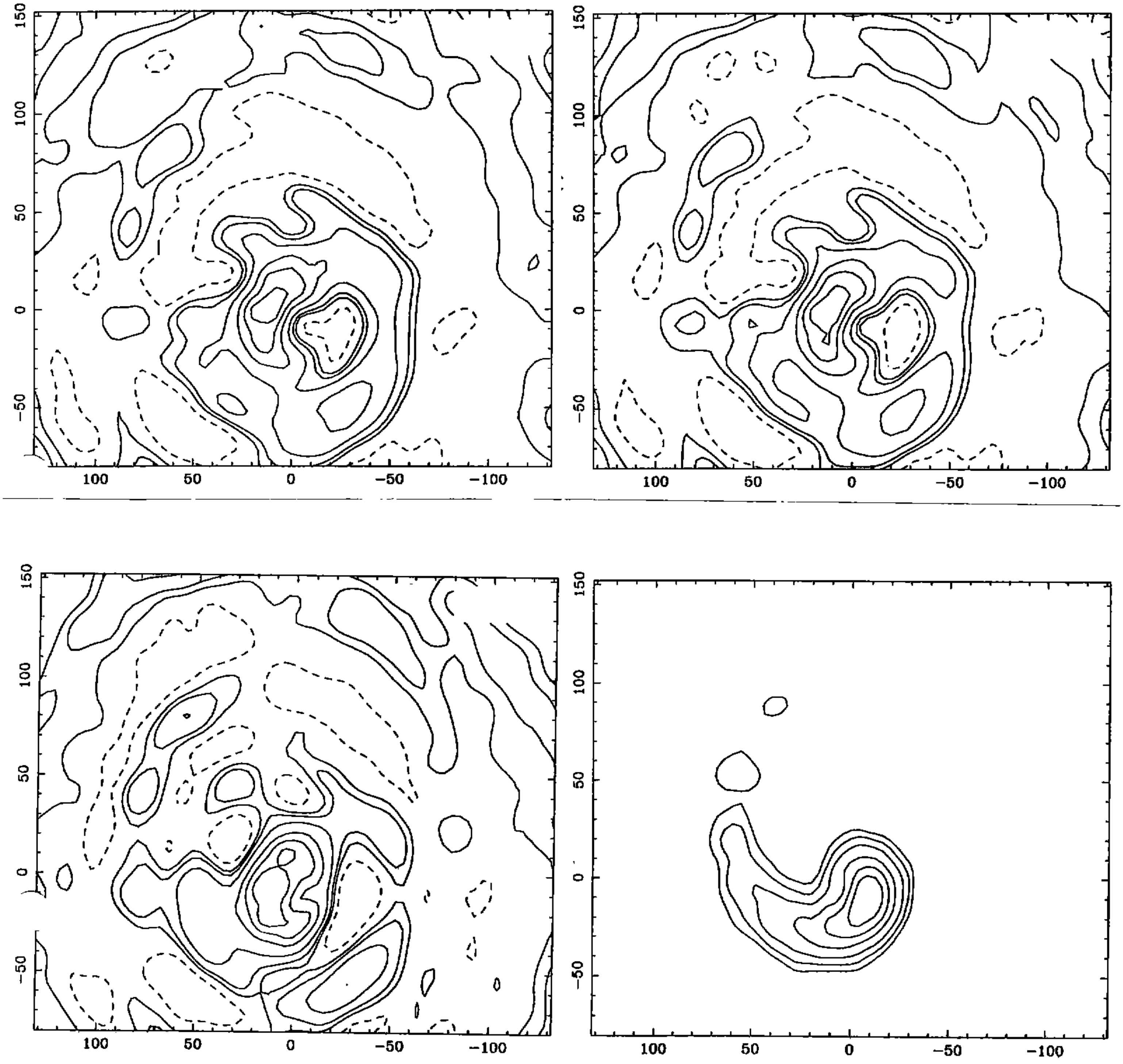

Figure 8: Selfcal iterates for the random array, using a reverse spiral as the initial model. SNR=10, Interval=5, PError=0°. Iterations 0, 1, 10 and 20 are shown.

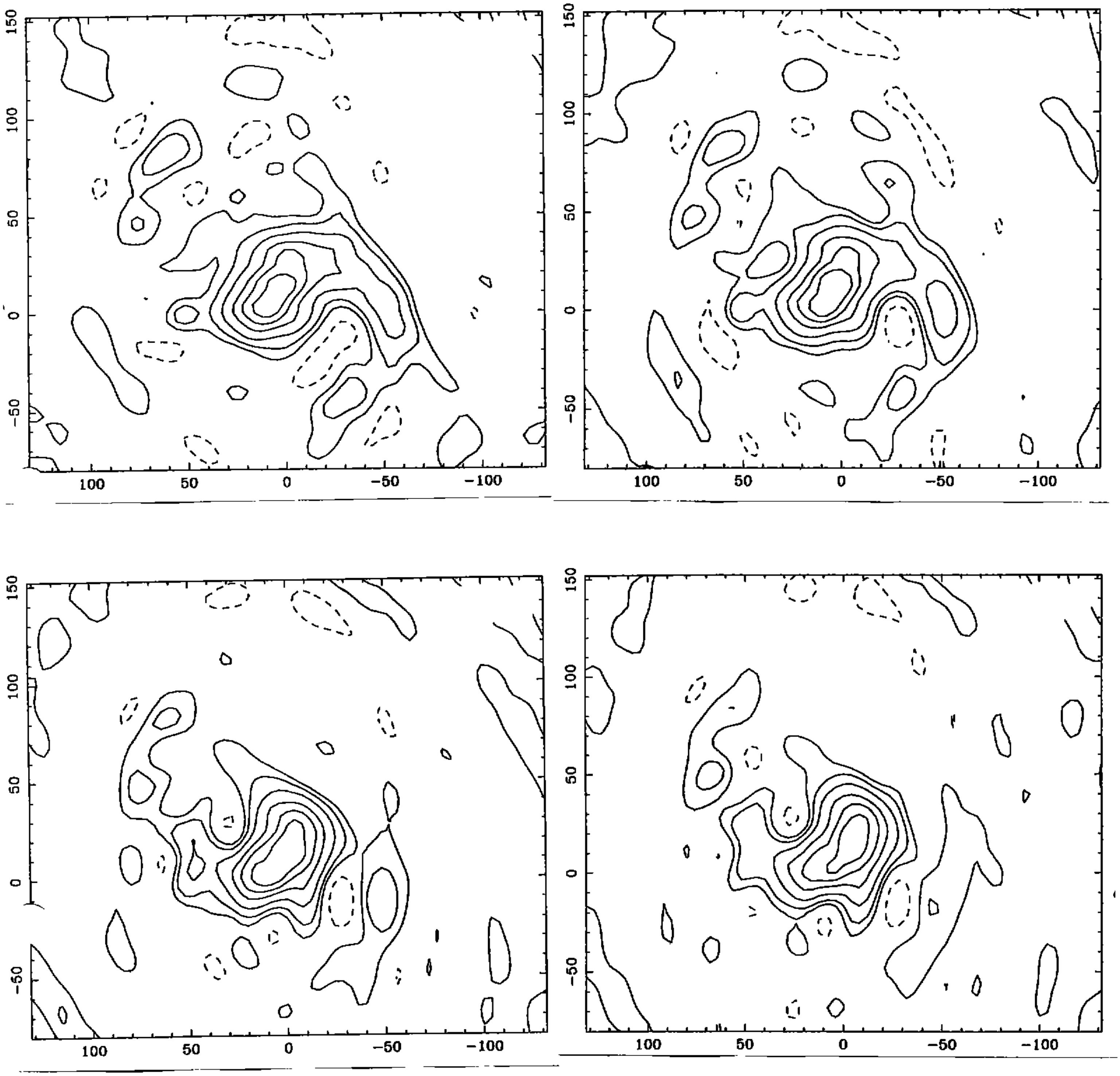

Figure 9: Selfcal iterates for an east-west array, using a reverse spiral as the initial model. SNR=10, Interval=5, PError=0°. Iterations 0, 1, 10 and 20 are shown. Compare with Figure 7, to note that the east-west array again takes significantly more iterations to reject the bad initial model. This run converged after about 30 iterations.

SNR	DRange	Interval	PError	Initial	Final	Iters	C?
1	31.3	1	0	1.02	1.45	3	
			20	1.31	1.48	3	
			40	2.53	1.38	1	
			80	4.91	1.47	4	
		5	0	1.02	1.16	1	
			20	1.46	1.16	1	
			40	2.67	1.17	1	
			80	4.92	1.17	11	
		10	0	1.02	1.19	1	
			20	1.44	1.20	1	
			40	2.75	1.18	1	
			80	5.02	–	–	•
		50	0	1.02	1.20	1	
			20	1.68	1.20	1	
			40	3.12	1.20	2	
			80	4.44	1.71	3	•
		100	0	1.02	1.29	1	
			20	1.91	1.20	1	
			40	3.29	1.19	1	
			80	–	–	–	•
5	157	1	0	1.30	2.14	6	
			20	3.87	2.13	1	
			40	11.4	2.13	3	
			80	24.1	2.10	6	
		5	0	1.30	2.05	5	
			20	4.95	2.06	1	
			40	12.1	2.05	10	
			80	24.2	2.05	18	
		10	0	1.30	2.01	4	
			20	5.13	2.01	1	
			40	13.1	2.00	2	
			80	25.0	2.00	17	
		50	0	1.30	1.92	3	
			20	6.63	1.91	8	
			40	14.8	1.92	15	
			80	21.6	7.30	3	•
		100	0	1.30	1.81	3	
			20	8.11	1.83	7	
			40	15.8	1.81	8	
			80	–	–	–	•

Table 1: Selfcal Statistics for a 1 Day Synthesis for a East-West Array

SNR	DRange	Interval	PError	Initial	Final	Iters	C?
10	313	1	0	1.64	2.49	5	
			20	7.43	2.53	1	
			40	22.4	2.57	17	
			80	48.2	2.53	13	
		5	0	1.64	2.57	6	
			20	9.55	2.48	12	
			40	24.2	2.49	26	
			80	48.6	2.56	40	
		10	0	1.64	2.49	5	
			20	10.2	2.44	7	
			40	26.4	2.45	10	
			80	49.9	12.3	4	●
		50	0	1.64	2.43	5	
			20	13.1	2.40	23	
			40	29.5	2.41	28	
			80	43.2	14.6	3	●
		100	0	1.64	2.26	3	
			20	16.0	2.29	19	
			40	31.5	2.26	18	
			80	–	–	–	●
50	1565	1	0	4.62	4.09	1	
			20	36.6	4.64	22	
			40	111	4.35	41	
			80	241	4.51	45	
		5	0	4.62	4.71	0	
			20	47.8	4.77	36	
			40	120	5.42	39	
			80	242	6.71	43	
		10	0	4.62	4.37	0	
			20	50.9	4.13	29	
			40	132	4.22	35	
			80	249	65.0	4	●
		50	0	4.62	4.93	0	
			20	64.9	4.41	41	
			40	–	–	–	●
			80	215	74.3	3	●
		100	0	4.62	4.26	0	
			20	79.6	4.77	28	
			40	157	4.44	26	
			80	????	< 6.85	> 50	

Table 1: Continued ...

SNR	DRange	Interval	PError	Initial	Final	Iters	C?
100	3130	1	0	6.30	9.88	1	
			20	72.7	10.0	19	
			40	223	8.92	32	
			80	482	9.02	34	
		5	0	6.30	10.1	8	
			20	95.3	10.6	29	
			40	240	9.28	41	
			80	484	14.1	42	
		10	0	6.30	7.65	1	
			20	101	10.5	22	
			40	265	10.2	27	
			80	500	–	–	•
		50	0	6.30	7.97	2	
			20	130	9.36	35	
			40	294	8.32	47	
			80	431	148	3	•
		100	0	6.30	9.88	13	
			20	160	8.95	29	
			40	314	8.82	30	
			80	–	–	–	•

Table 1: Continued ...

SNR	DRange	Interval	PError	Initial	Final	Iters	C?
50	2537	1	0	6.43	7.05	0	
			20	64.9	6.47	14	
			40	212	6.57	17	
			80	463	6.65	13	
		5	20	68.9	6.74	6	
			80	475	5.78	19	
		10	20	75.8	6.66	8	
			40	231	6.63	18	
			80	483	6.59	21	
		50	20	94.8	5.79	16	
			40	253	6.68	11	
		100	20	113	6.62	13	
			40	291	6.67	8	
100	5074	1	0	11.6	13.2	1	
			20	130	14.0	7	
			40	425	13.0	10	
			80	926	12.6	14	
		5	20	139	12.7	7	
			40	438	13.7	9	
			80	950	13.5	18	
		10	20	151	12.2	10	
			40	462	12.5	12	
			80	966	12.7	16	

Table 2: Selfcal Statistics for a 4 Day Synthesis of the East-West Array

SNR	DRange	Interval	PError	Initial	Final	Iters	C?
1	31.3	1	0	1.17	1.41	2	
			20	1.40	1.41	0	
			40	2.60	1.41	1	
			80	5.05	1.42	3	
		5	0	1.17	1.21	0	
			20	1.36	1.32	1	
			40	2.49	1.22	1	
			80	4.84	1.21	3	
		10	0	1.17	1.24	0	
			20	1.43	1.24	1	
			40	2.71	1.23	1	
			80	4.98	1.23	3	
		50	0	1.17	1.27	0	
			20	1.46	1.27	1	
			40	2.60	1.27	1	
			80	4.96	1.26	4	
		100	0	1.17	1.26	0	
			20	1.50	1.26	1	
			40	2.66	1.26	1	
			80	4.78	–	–	●
5	157	1	0	1.47	1.66	1	
			20	3.77	1.69	1	
			40	11.2	1.67	1	
			80	24.4	1.64	2	
		5	0	1.47	1.63	0	
			20	3.85	1.62	1	
			40	11.5	1.63	3	
			80	24.0	1.63	4	
		10	0	1.47	1.64	0	
			20	4.71	1.62	1	
			40	12.7	1.62	3	
			80	25.0	1.63	5	
		50	0	1.47	1.64	0	
			20	4.65	1.66	1	
			40	12.1	1.65	4	
			80	25.0	1.64	9	
		100	0	1.47	1.65	1	
			20	5.28	1.64	5	
			40	12.4	1.60	7	
			80	23.6	1.67	5	

Table 3: Selfcal Statistics for a 1 Day Synthesis of a Random Array

SNR	DRange	Interval	PError	Initial	Final	Iters	C?
10	313	1	0	1.71	1.84	0	
			20	7.00	1.86	1	
			40	22.2	1.82	3	
			80	48.7	1.80	5	
		5	0	1.71	1.79	0	
			20	7.37	1.78	1	
			40	23.1	1.74	6	
			80	34.5	1.77	7	
		10	0	1.71	1.81	0	
			20	9.29	1.78	2	
			40	25.4	1.84	5	
			80	50.1	1.78	9	
		50	0	1.71	1.78	0	
			20	9.19	1.72	5	
			40	24.3	1.82	6	
			80	50.1	1.75	12	
		100	0	1.71	1.85	0	
			20	10.5	1.81	7	
			40	24.7	1.81	10	
			80	47.1	1.75	6	
50	1565	1	0	4.02	4.78	1	
			20	34.1	4.65	2	
			40	111	5.67	3	
			80	243	5.21	6	
		5	0	4.02	3.78	0	
			20	36.4	3.51	8	
			40	116	3.89	12	
			80	240	8.85	3	
		10	0	4.02	3.97	0	
			20	46.0	4.20	7	
			40	127	4.00	11	
			80	251	3.88	15	
		50	0	4.02	3.89	0	
			20	45.4	3.98	9	
			40	121	3.91	10	
			80	251	3.83	16	
		100	0	4.02	3.97	0	
			20	51.8	4.07	13	
			40	123	4.04	15	
			80	235	4.07	10	

Table 3: Continued ...

SNR	DRange	Interval	PError	Initial	Final	Iters	C?
100	3130	1	0	9.41	8.79	0	
			20	68.1	8.30	8	
			40	221	8.83	8	
			80	485	8.59	10	
		5	0	9.41	9.11	0	
			20	72.9	9.04	2	
			40	231	8.85	7	
			80	480	8.33	6	
		10	0	9.41	8.63	0	
			20	92.1	8.94	5	
			40	254	8.29	10	
			80	502	8.75	15	
		50	0	9.41	8.96	0	
			20	90.8	8.82	8	
			40	243	9.13	10	
			80	502	8.62	19	
		100	0	9.41	8.81	0	
			20	104	9.06	12	
			40	247	8.97	14	
			80	471	8.73	7	

Table 3: Continued ...