The Sign of Stokes V, *et cetera*

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Here we review the sign conventions used by the ATCA, for data within an RPFITS file and within the *MIRIAD* system.

RPFITS and *AIPS* conventions and ATCA characteristics

AIPS and the on-line software that writes the RPFITS files follow the same conventions. Here we call these conventions the *AIPS*' ones.

AIPS use the convention that the baseline number, for baseline i–j, as

$$bl = 256 \times i + j.$$

This labels the correlation,

$$C = E_i E_j^*$$

where E_i and E_j are the electric fields at the two antennas. Using this convention, the $u - v$ coordinates are calculated as the position of antenna i minus the position of antenna j. That is, if (x, y, z) are CIO coordinates of the two antennas and H is the hour angle on the Greenwich meridian, then

$$u = (x_i - x_j)\sin H + (y_i - y_j)\cos H$$

Convenience also dictates that the data are always stored in a "normal form" where $i \leq j$. If this is not so, the data can be converted to this form by interchanging the antenna numbers, conjugating the correlations and negating the $u - v - w$ coordinates (we consider the effect of this operation on polarisation labelling below).

Polarimetric correlations

Each ATCA antenna has two orthogonal linear feeds. The two feeds are labelled `X` and `Y` in the RPFITS file, or as `A` and `B`,' respectively, by the on-line software. The position angle of the `X` feed to the source is the parallactic angle plus 45° (45° to the celestial east), whereas the position angle of the `Y` feed is the parallactic angle minus 45° degrees. In many correlation configurations, all four possible polarisation correlations are measured – that is

$$X_i X_j^*,\ Y_i Y_j^*,\ X_i Y_j^*,\ Y_i X_j^*$$

These correlations are "Stokes axis" values of -5, -6, -7 and -8, respectively, and labels `XX`, `YY`, `XY` and `YX` respectively.

If one were to conjugate polarimetric correlation coefficients (e.g. if this was needed to convert the correlations to normal form), the polarimetric labelling is affected. For example, conjugating

$$C = X_i Y_j^*$$

would result in the correlation

$$C = Y_j X_i^*.$$

That is, conjugating the baseline i–j, `XY` correlation converts it into the baseline j–i, `YX` correlation.

XY phase convention

For many correlator configurations, the on-line system measures the XY phase, and saves this in the SYSCAL records of an RPFITS file. As far as sign conventions go, the value stored in the SYSCAL record is simply the phase of the XY autocorrelation. That is

$$\phi_i = \arg(X_i Y_i^*).$$

This is the value reported by *AIPS* ATLOD.

So far we have ignored the "sideband indicator" (or IF_INVERT switch). If this is -1, then the correlation data stored in the RPFITS file has to be conjugated, and the XY phase measurement negated. Note, however, that the XY phase value reported by *AIPS* does not include this possible negation. Also these negations and conjugations do not affect baseline numbering or polarisation correlation labelling – it is purely a correction to the data.

The *MIRIAD* convention

MIRIAD's labelling conventions for labelling baselines and polarisation correlations are superficially very similar to the *AIPS* convention. However by historical accident, they are subtly distinct. Particularly with polarimetric data, attention must be paid to avoid sign errors. As will be clear from the final section of this memo, we had insufficient understanding of the sign conventions of the data, or even the orientation of the feeds, when we developed some polarimetric software in *MIRIAD*.

Like *AIPS*, *MIRIAD*'s baseline numbering scheme is

$$bl = 256 \times i + j.$$

Again the normal form is enforced, with $i \leq j$. However *MIRIAD* treats the *conjugated antenna* as the first antenna, whereas the *AIPS* convention is that the non-conjugated antenna is considered the first antenna. That is, in *MIRIAD*'s convention, baseline i–j corresponds to the correlation

$$C = E_i^* E_j.$$

For this convention, the $u-v-w$ coordinate is computed as the location of antenna j minus the location of antenna i (i.e. the reverse of *AIPS*).

As with *AIPS*, Stokes axis values are used, with values -5, -6, -7 and -8 corresponding to the labels XX, YY, XY and YX respectively. However the label XY for baseline i–j corresponds to the correlation

$$C = X_i^* Y_j.$$

That is, again the conjugated antenna is considered the first antenna.

Although *MIRIAD* treats the conjugated antenna as the first antenna, no account is made for this when gains and polarisation leakages are determined. Instead the software treats the second antenna as if it were the conjugated one. The net result of this is that the gains and polarisation leakage solutions that *MIRIAD* solves for and applies are the conjugate of "reality". This is rarely (if ever) significant, as we determine calibration parameters astronomically. Because the software that determines the calibration parameters and the software that applies them use the same convention, the conjugated nature of the parameters comes out in the wash. The net result is correct. However, if the calibration parameters were determined by some way other than astronomically (e.g. we were to estimate atmospheric phase based on total power measurements), then we would have to be aware that *MIRIAD* convention would introduce an unexpected extra minus sign.

The memo "AT Polarisation Calibration" (Sault, Killeen and Kesteven) (ATNF document 39.3/015) forms the basis for *MIRIAD*'s conversion between raw polarisation and Stokes correlations. The convention used for Stokes-V is the IEEE definition, which has been adopted by the IAU, and used by many observatories such as the VLA. It is based Thompson, Moran and Swenson. Note, however, that this memo uses the convention that "XY" corresponds to the correlation XY^* (i.e. the $\mathcal{AIPS}$/RPFITS labelling convention, *not* the $\mathcal{MIRIAD}$ convention), and so there are some sign differences for Stokes V between the equations given by Sault, Killeen and Kesteven, and those actually implemented in $\mathcal{MIRIAD}$.

As an aside, the $\mathcal{MIRIAD}$ uv variable `chi` gives the position angle of the `X` feed as the rotation east of celestial north. That is, the the variable is the parallactic angle plus 45°.

Net effect of convention difference

When $\mathcal{MIRIAD}$ loads data (either task `fits` or `atlod`), it must convert the data and labelling convention from the $\mathcal{AIPS}$/FITS/RPFITS convention to its own. Assuming the data is in normal form (i.e. for baseline i–j, $i \leq j$) then the net result is to conjugate the data and negate the $u - v$ coordinates. For task `atlod`, this conjugation is in addition the conjugation required if the sideband indicator is -1. The baseline number and polarisation label remains unchanged by the convention change.

Conversely, when $\mathcal{MIRIAD}$ writes out a FITS file, it conjugates the data and negates $u - v - w$ coordinates, but does not change the baseline number or polarisation label.

XY phase convention

For baseline i–j, polarisation `YY` (which is the correlation $Y_i^* Y_j$), $\mathcal{MIRIAD}$ defines the XY phase in terms of the equation

$$Y_i^* Y_{j,\mathrm{corrected}} = Y_i^* Y_{j,\mathrm{raw}} / (\exp(i\phi_{i,\mathrm{miriad}}) \exp(-i\phi_{j,\mathrm{miriad}}))$$

Using this convention, the $\mathcal{MIRIAD}$ value for XY phase will be the same as that in the SYSCAL record (and as reported by $\mathcal{AIPS}$ `ATLOD`). However, there are two more issues that affect $\mathcal{MIRIAD}$'s XY phase convention:

- The convention used by Sault, Killeen and Kesteven was that the `Y` feed was +90° (i.e. 90° to the celestial east) relative to the `X` feed. This is the convention used in Thompson, Moran and Swenson. For the data written by the on-line system, the `Y` feed is 90° to the west of the `X` feed. This convention difference can be eliminated by adding 180° to the XY phase. Equivalently we could negate the `XY` and `YX` correlations as they are loaded. However is it easier to treat it as a XY phase convention problem, to ensure that the correction is not applied twice.
- The raw XY phase measurement stored in the SYSCAL record takes no account of the sideband indicator. If the sideband indicator is -1, the XY phase needs to be negated, and the correlation data conjugated.

The net result of these two issues means that the $\mathcal{AIPS}$ and $\mathcal{MIRIAD}$ XY phase values are related by:

$$\phi_{miriad} = \begin{cases} \phi_{aips} + 180^\circ & \text{if sideband indicator is 1} \\ -\phi_{aips} + 180^\circ & \text{if sideband indicator is -1} \end{cases}$$

$\mathcal{MIRIAD}$ convention for circular feeds

The conventions used by $\mathcal{MIRIAD}$ for circular feeds are consistent with those use for linear feeds. That is, the conjugated antenna is considered the first antenna and, for baseline i–j, the Stokes axis values of -1, -2, -3 and -4 correspond to the correlations

$$R_i^* R_j,\ L_i^* L_j,\ R_i^* L_j,\ L_i^* R_j$$

respectively.

Transforming between visibility and image

The $\mathcal{AIPS}$and $\mathcal{MIRIAD}$conventions for computing (u, v, w) coordinates is the opposite of that in Thompson, Moran and Swenson. This convention difference results in the sign in the exponent of the Fourier transform relationship (between visibility and image) also being opposite. In particular, $\mathcal{AIPS}$and $\mathcal{MIRIAD}$use the convention that

$$V(u,v) = \int I(\ell,m)\exp(i2\pi(u\ell + vm))dl\,dm \qquad (1)$$

This convention difference has no implications as far as polarimetric conversion is concerned.

A history of $\mathcal{MIRIAD}$ sign convention errors

In the lifetime of $\mathcal{MIRIAD}$'s polarimetry software, the conventions of RPFITS and $\mathcal{MIRIAD}$ itself, have been misunderstood, which have led in turn to sign errors in the converted Stokes parameters. The good news is that the convention errors did not affect the calibration. Re-loading the data and redoing the calibration will not lead to any improvement in the calibration. The only thing that would change is the sign of some of the Stokes parameters and some of the calibration parameters.

Another caveat is that data loaded (with either `fits` or `atlod`) before 9-Sep-92 had baseline numbers incorrectly stored – the antenna numbers were stored in the reverse order. The $u-v$ coordinates stored with these data were correct. However because the antenna numbers have been reversed, if the $u-v$ coordinates were recomputed (with `uvedit`), the resultant coordinates would be the negative of the correct values. Note that this problem does not apply to Hat Creek data loaded with either `hcconv` or `uvhat`.

Below we list the sign errors that will result for data "loaded" and "processed" at different times. By "loaded", we mean data, which are raw polarisation correlations (linears or circulars), that are converted to $\mathcal{MIRIAD}$ format using either of the $\mathcal{MIRIAD}$ tasks `fits` or `atlod`. By "processed" we mean either calibrated or converted to Stokes parameters.

The dates given are the date when a change was made in the Marsfield copy of $\mathcal{MIRIAD}$. The change will have reached other sites at a later stage. The version date given by $\mathcal{MIRIAD}$ tasks (which is usually saved in the history file of a dataset) should be helpful here.

Linear feeds

- Loaded before 9-Sep-92, processed before 13-Dec-93: The sign of Stokes V is correct. Leakages will be the conjugate of data loaded after 9-Sep-92. Recomputed $u-v$ coordinates are will be the negative of the correct values.
- Loaded before 9-Sep-92, processed after 13-Dec-93: Stokes V will be the negative of the correct value. Leakages will be the conjugate of data loaded after 9-Sep-92. Recomputed $u-v$ coordinates will be the negative of the correct values.
- Loaded after 9-Sep-92, processed 9-Sep-92 – 13-Dec-93: Stokes V will be the negative of the correct value. $U-V$ coordinates will be correctly recomputed.
- Loaded after 9-Sep-92, processed after 13-Dec-93: All is correct.

Circular feeds

- Loaded before 9-Sep-92, processed before 9-Sep-92: Sign of Stokes U is correct. Recomputed $u-v$ coordinates will be the negative of the correct value.

- Loaded 9-Sep-92 – 29-Sep-92, processed 9-Sep-92 – 13-Dec-93: Stokes U will be the negative of the correct value. $U - V$ coordinates will be correctly recomputed.
- Loaded 9-Sep-92 – 29-Sep-92, processed after 13-Dec-93: All is correct.
- Loaded 29-Sep-92 – 13-Dec-93, processed 29-Sep-92 – 13-Dec-93: All is correct.
- Loaded 29-Sep-92 – 13-Dec-93, processed after 13-Dec-93: Stokes U will be the negative of the correct value. $U - V$ coordinates will be correctly recomputed.
- Loaded after 13-Dec-93, processed after 13-Dec-93: All is correct.

Other sign problems

In 1990 there were some sign errors in the RPFITS files written by the on-line system.

- Prior to 19-Jun-90, the sideband indicator was stuck at +1. Between 19-Jun-90 and 1-Aug-90, the sideband indicator was stuck at -1. Neither *AIPS* nor *MIRIAD* `atlod` accounts for this error, although *AIPS* `atlod` has the ability to enter the sideband indicator by hand. If data are affected by these errors, images formed from them will be rotated by 180° relative to the correct images, and Stokes V will be the negative of the correct value.
- Prior to 9-Jul-90, the baseline numbering convention was incorrect. It is unclear how this actually affected the data.