Effective Spectral Resolution of the ATNF Correlator

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This note describes the effective spectral resolution of the ATCA correlator in a variety of different configurations (in 2-bit mode). The resolution is given before and after Hanning smoothing (i.e. $z_i = 0.25z_{i-1} + 0.5z_i + 0.25z_{i+1}$).

The basic correlator unit is the module (see Wilson et al. 1992), which defines a number of lags for a given bandwidth. Extra channels are created by appropriately adding modules together. There are eight modules available per baseline and these are distributed amongst the required polarizations and frequencies. For example, for frequency 1, one could use one module for each of the XX, YY, XY, and YX correlations. For frequency 2, one could have more channels by adding 2 modules together, but then there are only enough modules left for 2 polarization pairs, say XX and YY.

The number of lags that results from a particular configuration (collection of modules), is unlikely to be a power of 2. Thus, for the FFT in the correlator, lags (with zero weight) are added up to the next power of 2, or are discarded down to the next power of 2, depending on which is closest. If a large number of lags were zero padded, the spectrum would be substantially oversampled, as the frequency spectrum is sampled at the Nyquist rate appropriate to the number of channels, which is Fourier transformed (power of 2). However, it rarely happens that a large number of lags are zero padded (see Table). On the other hand, in some configurations a significant number of lags are dropped, causing worse spectral sidelobes than would otherwise occur.

Each row of the figure shows 4 plots per correlator configuration; the left-most is the lag-weighting function. The title indicates the natural number of lags and the number of lags that were Fourier transformed to the frequency domain (by zero padding or dropping) resulting in the second figure. The solid line shows just the inner part of the spectral response and is substantially oversampled. The filled circles show the Nyquist rate samples which the correlator actually produces. The crosses mark the FWHM of the main spectral lobe. The next plot shows the lag-weighting function again, but this time after it has been multiplied by a raised cosine curve, which is the Fourier transform of the Hanning kernel used to smooth the Nyquist-sampled frequency spectrum. The Fourier transform is again taken to produce the last plot, the Hanning smoothed spectral response. On each spectral response plot, the FWHM of the main lobe is given in channels.

It is possible to use on-line "Hanning" smoothing with the ATNF correlator, although this option is rarely used. However, it should be noted that this smoothing is implemented somewhat differently from that which I have presented here. I have done what most users do with off-line software, and that is to Hanning smooth the sampled frequency spectrum (although I did it by multiplying the lag spectrum by the Fourier transform of the Hanning kernel). In the correlator, the lag-weighting function is replaced by the raised cosine function rather than being multiplied by it. This means that you are guaranteed to have the nice spectral-sidelobe characteristics of the raised cosine function. We can see the spectral response of the raised cosine function by looking at one of the plots. Consider the plots for the 8-MHz bandwidth configuration with the natural number of lags equal to 512 (third page and third row). One can see that the natural lag function is exactly a top hat, so that multiplying by the raised cosine function is the same as replacing it by this function. Thus, the last plot of that row shows the spectral response of the raised cosine alone. This it what one always gets when "Hanning" smoothing with the correlator software.

The figure is summarized in the table. Column 1 gives the bandwidth, column 2 the natural number of lags, column 3 the number of lags actually Fourier transformed, column 4 the number of modules required to make this configuration (for one polarization and one frequency), column 5 the channel separation in KHz (the channels are sampled at the Nyquist rate), column 6 the FWHM of the main spectral lobe in channels, column 7 the FWHM of the main spectral lobes after Hanning smoothing (again in channels) and column 8 the name of a current ATNF correlator configuration in which this module configuration can be found. Because it depends upon how many polarization products you want and what you do with the second frequency (and even if you want to discard baselines to shuffle modules about), more than one correlator configuration may actually use this module configuration.

The lag-weighting functions gradually change from a triangle (128 MHz) to a top hat (4 MHz). Thus, the spectral response changes from a SinC-squared function (zeros every other channel) to a SinC function (zeros every channel).

From the table, one can see that in the standard 128-MHz, 32-channel continuum configuration, the FWHM of the effective spectral resolution is 1.77 channels; the spectrum is effectively oversampled. The second 128-MHz correlator configuration is the same as the first, except that the lag-weighting function has been altered to eliminate Gibbs phenomena (which cause ghost images reflected about the phase centre at high dynamic range – see Sault 1995). This has the additional effect that it substantially suppresses the spectral sidelobes as well. The third 128-MHz configuration is also substantially oversampled because the lag-weighting function has significant zero padding (lags 48 to 64). However, as the bandwidth is successively halved, the spectral resolution quickly converges on 1.2 channels. Therefore, the resolution after Hanning smoothing converges as the bandwidth is halved to 2 channels and it is usual to drop every other channel after Hanning smoothing narrowband data.

A brief discussion of "birdies" is worth while in the context of these spectral responses. Birdies are internally generated, very narrow bandsignals that occur at multiples of 128 MHz and corrupt our spectra.

Consider the first 128 MHz configuration. This is the standard continuum mode. One can see that the sidelobes and zeros of the SinC-squared function (second plot in row) fall exactly on channels. If the birdie is exactly centred on a channel (which we try to arrange), say channel j, it will ring through the spectrum according to this spectral response. However, it can be eliminated by discarding channels j, and $j \pm (2i-1)$ where $i = 1, 2, 3....$ Because the effective spectral resolution is 1.8 channels, you lose very little in the signal-to-noise ratio, even though you have discarded half of the channels. This tactic is routinely used in processing ATCA data (see Killeen 1995).

In the 4-MHz (and effectively 8-MHz) modes, a birdie does not ring through the spectrum, because the positive and negative sidelobes are "between" the sampled channels, which sit exactly on the zeros. However, for intermediate configurations such as 32 MHz, a birdie will ring most unpleasantly. In this case, the sampled channels don't fall exactly on the zeros of the spectral response (see the plot), so that the response to a birdie cannot be eliminated by discarding channels. It is only at the extremes of bandwidth that we can deal fully with birdies.

References

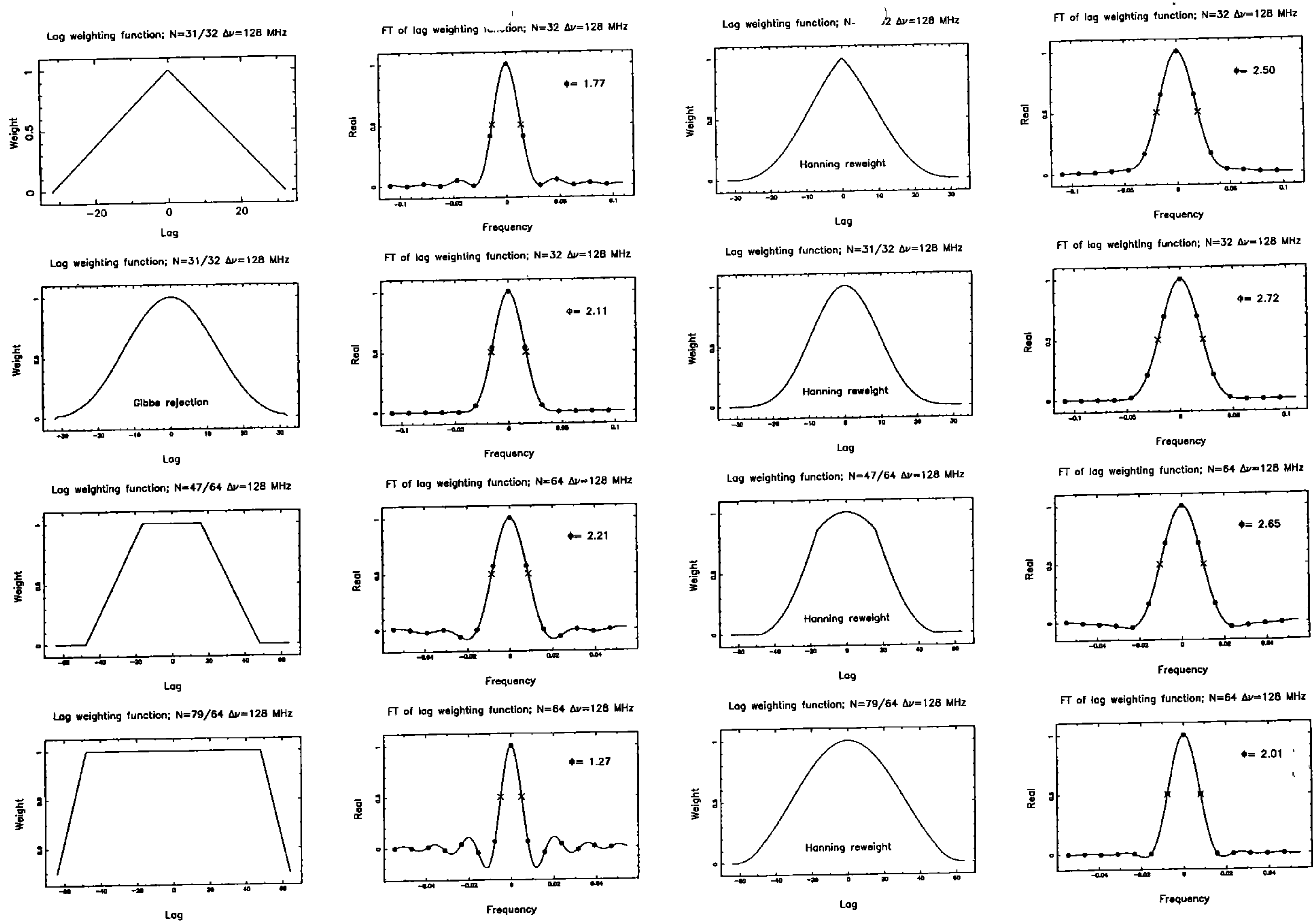
Killeen, N.E.B., 1995, ATNF ATCA Data Information Report # 17, available on the WWW at http://wwwatnf.atnf.csiro.au/Documentation/at_bugs.html

Sault, R.J., 1995, ATNF ATCA Data Information Report #18, available on the WWW at http://wwwatnf.atnf.csiro.au/Documentation/at_bugs.html

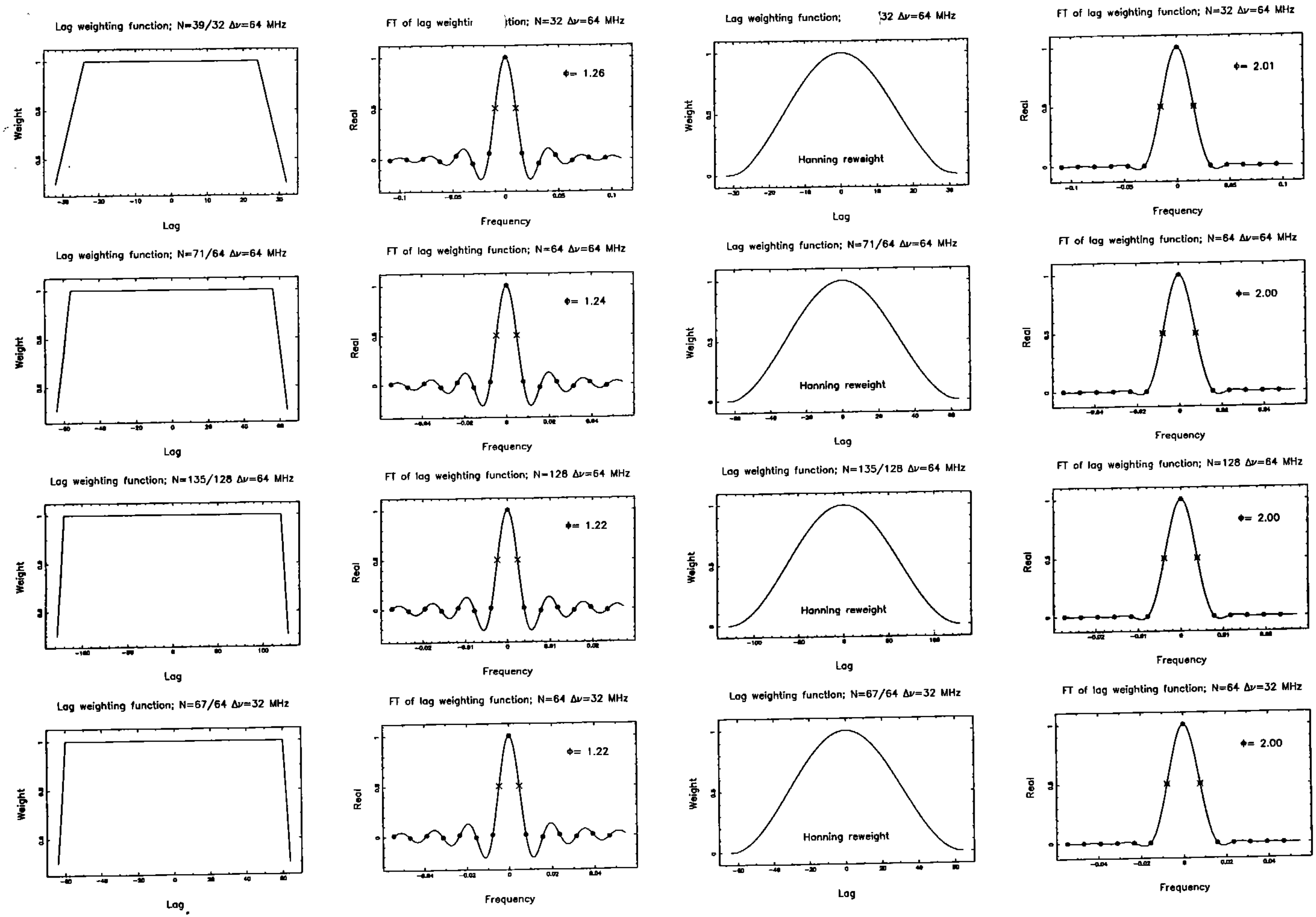
Wilson, W.E., Davis, E.R., Loone, D.G., and Brown, D.R., 1992, in Journal of Electrical and Electronics Engineering, Australia, **12**, no.2, p187

Correlator Spectral Resolution							
$\Delta\nu$ MHz	lag_{max}	N_{chan}	$N_{modules}$	$\delta\nu$ KHz	FWHM Channels	$FWHM_{Hanning}$ Channels	Configuration
128	31	32	1	4000	1.77	2.50	FULL_128_2
128[a]	31	32	1	4000	2.11	2.72	FULL_128_2
128	47	64	2	2000	2.21	2.65	FULL_128_64_2
128	79	64	4	2000	1.27	2.01	-
64	39	32	1	2000	1.26	2.01	-
64	71	64	2	1000	1.24	2.00	FULL_64_64-128
64	135	128	4	500	1.22	2.00	-
32	67	64	1	500	1.22	2.00	-
32	131	128	2	250	1.21	2.00	FULL_32_128-128
32	259	256	4	125	1.21	2.00	FULL_32_256
16	129	128	1	125	1.21	2.00	-
16	257	256	2	62.5	1.21	2.00	FULL_16_256-128
16	513	512	4	31.25	1.21	2.00	FULL_16_512
8	256	256	1	31.25	1.21	2.00	-
8	512	512	2	15.625	1.21	2.00	FULL_8_512
8	1024	1024	4	7.8125	1.21	2.00	FULL_8_1024
8	2048	2048	8	3.90625	1.21	2.00	FULL_8_2048
4	511	512	1	7.8125	1.21	2.00	-
4	1023	1024	2	3.90625	1.21	2.00	FULL_4_1024
4	2047	2048	4	1.953125	1.21	2.00	FULL_4_2048

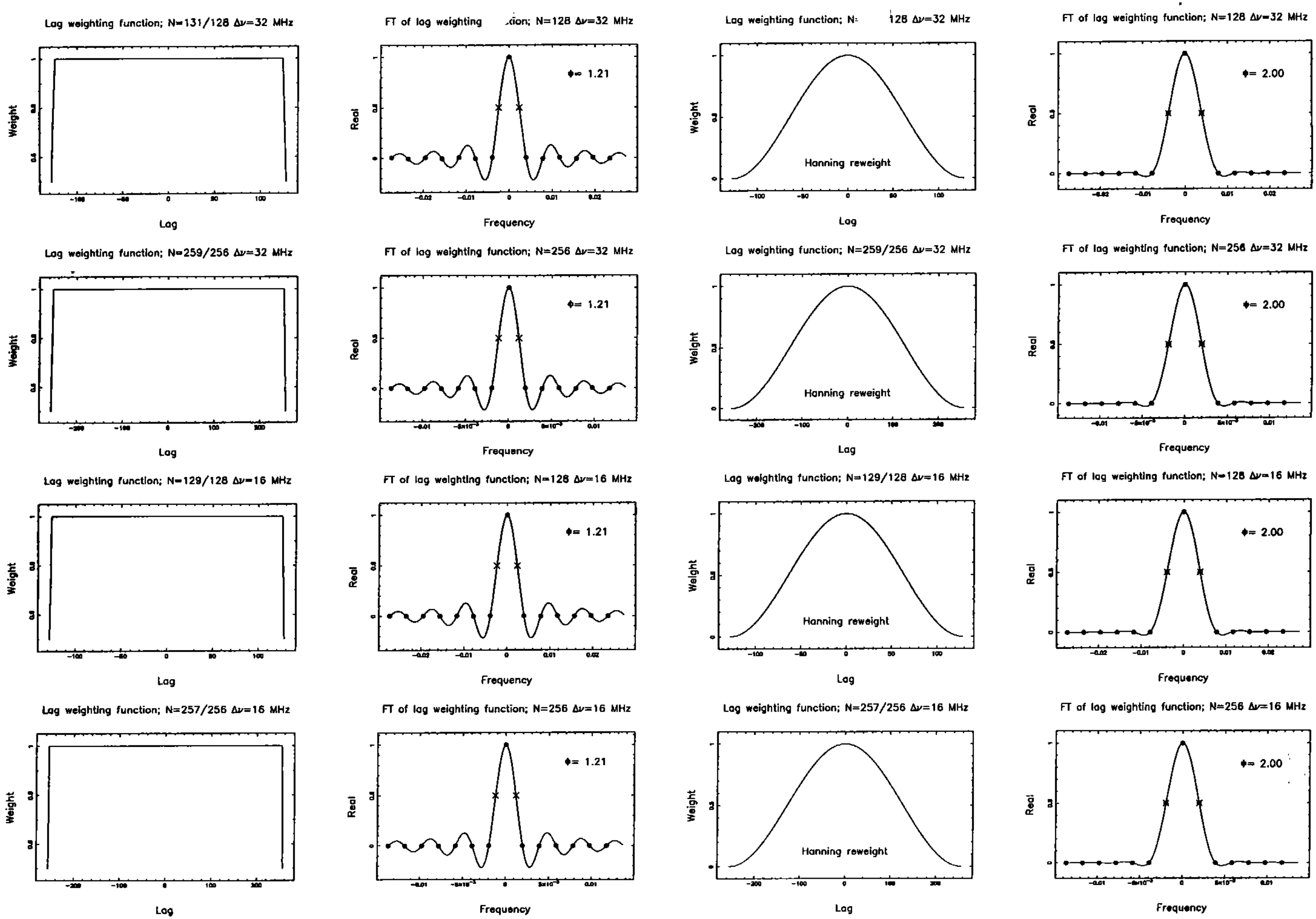
[a]Lag spectrum reweighted for Gibbs rejection (options=reweight in Miriad atlod)

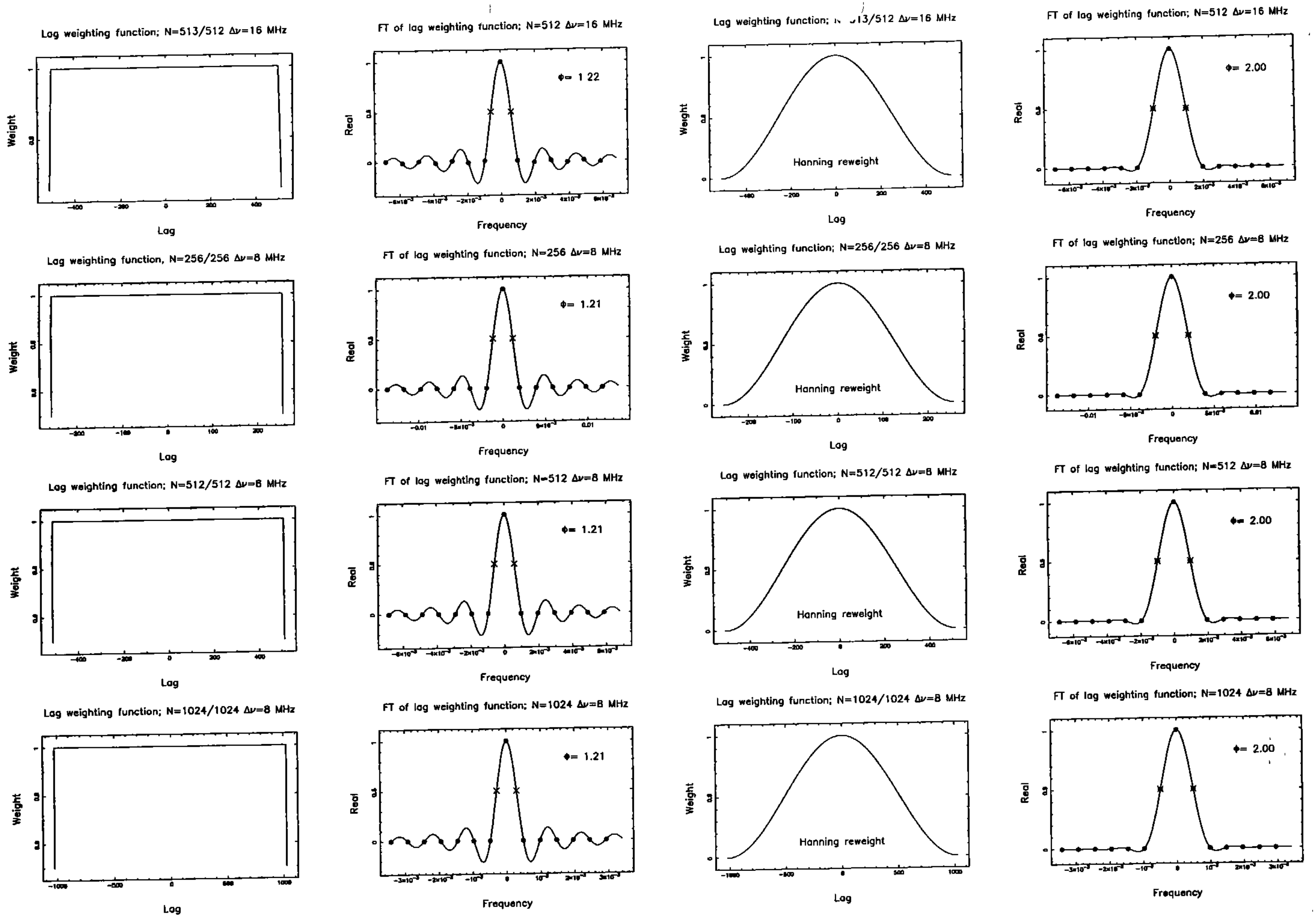


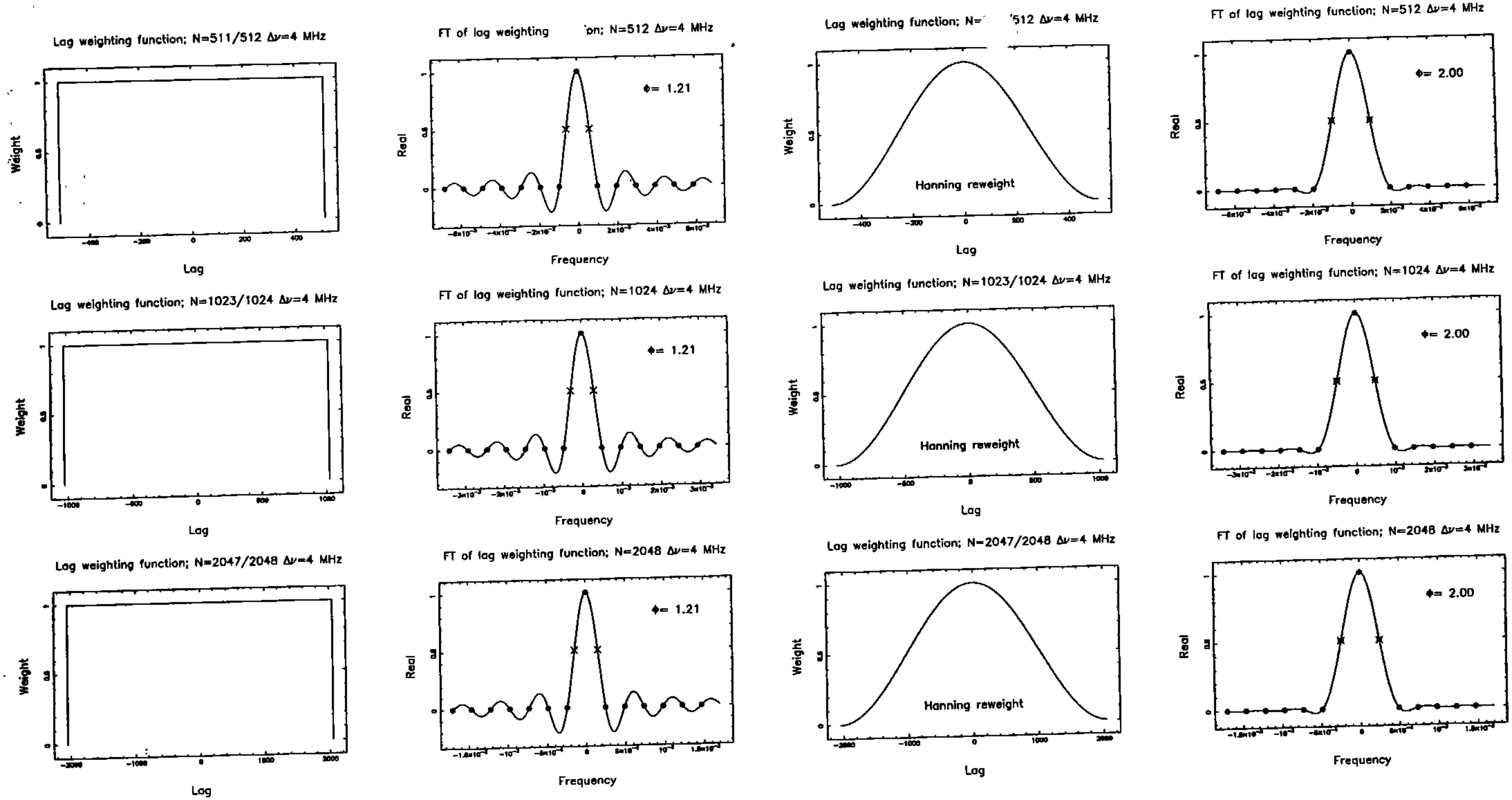
Lag weighting function; N=31/32 Δν=128 MHz
Weight
Lag
FT of lag weighting function; N=32 Δν=128 MHz
Real
Frequency
ϕ= 1.77
Lag weighting function; N=31/32 Δν=128 MHz
Hanning reweight
FT of lag weighting function; N=32 Δν=128 MHz
ϕ= 2.50
Lag weighting function; N=31/32 Δν=128 MHz
Gibbs rejection
FT of lag weighting function; N=32 Δν=128 MHz
ϕ= 2.11
Lag weighting function; N=31/32 Δν=128 MHz
Hanning reweight
FT of lag weighting function; N=32 Δν=128 MHz
ϕ= 2.72
Lag weighting function; N=47/64 Δν=128 MHz
FT of lag weighting function; N=64 Δν=128 MHz
ϕ= 2.21
Lag weighting function; N=47/64 Δν=128 MHz
Hanning reweight
FT of lag weighting function; N=64 Δν=128 MHz
ϕ= 2.65
Lag weighting function; N=79/64 Δν=128 MHz
FT of lag weighting function; N=64 Δν=128 MHz
ϕ= 1.27
Lag weighting function; N=79/64 Δν=128 MHz
Hanning reweight
FT of lag weighting function; N=64 Δν=128 MHz
ϕ= 2.01



Lag weighting function; N=39/32 Δν=64 MHz
Weight
Lag
FT of lag weightir)tion; N=32 Δν=64 MHz
Real
Frequency
ϕ= 1.26
Lag weighting function; '32 Δν=64 MHz
Weight
Lag
Hanning reweight
FT of lag weighting function; N=32 Δν=64 MHz
Real
Frequency
ϕ= 2.01
Lag weighting function; N=71/64 Δν=64 MHz
Weight
Lag
FT of lag weighting function; N=64 Δν=64 MHz
Real
Frequency
ϕ= 1.24
Lag weighting function; N=71/64 Δν=64 MHz
Weight
Lag
Hanning reweight
FT of lag weighting function; N=64 Δν=64 MHz
Real
Frequency
ϕ= 2.00
Lag weighting function; N=135/128 Δν=64 MHz
Weight
Lag
FT of lag weighting function; N=128 Δν=64 MHz
Real
Frequency
ϕ= 1.22
Lag weighting function; N=135/128 Δν=64 MHz
Weight
Lag
Hanning reweight
FT of lag weighting function; N=128 Δν=64 MHz
Real
Frequency
ϕ= 2.00
Lag weighting function; N=67/64 Δν=32 MHz
Weight
Lag
FT of lag weighting function; N=64 Δν=32 MHz
Real
Frequency
ϕ= 1.22
Lag weighting function; N=67/64 Δν=32 MHz
Weight
Lag
Hanning reweight
FT of lag weighting function; N=64 Δν=32 MHz
Real
Frequency
ϕ= 2.00


Lag weighting function; N=131/128 Δν=32 MHz
Weight
Lag
FT of lag weighting function; N=128 Δν=32 MHz
Real
Frequency
ϕ= 1.21
Lag weighting function; N=128 Δν=32 MHz
Hanning reweight
FT of lag weighting function; N=128 Δν=32 MHz
ϕ= 2.00
Lag weighting function; N=259/256 Δν=32 MHz
FT of lag weighting function; N=256 Δν=32 MHz
Lag weighting function; N=259/256 Δν=32 MHz
FT of lag weighting function; N=256 Δν=32 MHz
Lag weighting function; N=129/128 Δν=16 MHz
FT of lag weighting function; N=128 Δν=16 MHz
Lag weighting function; N=129/128 Δν=16 MHz
FT of lag weighting function; N=128 Δν=16 MHz
Lag weighting function; N=257/256 Δν=16 MHz
FT of lag weighting function; N=256 Δν=16 MHz
Lag weighting function; N=257/256 Δν=16 MHz
FT of lag weighting function; N=256 Δν=16 MHz


Lag weighting function; N=513/512 Δν=16 MHz
Weight
Lag
FT of lag weighting function; N=512 Δν=16 MHz
φ= 1 22
Real
Frequency
Lag weighting function; N=513/512 Δν=16 MHz
Hanning reweight
Weight
Lag
FT of lag weighting function; N=512 Δν=16 MHz
φ= 2.00
Real
Frequency
Lag weighting function, N=256/256 Δν=8 MHz
Weight
Lag
FT of lag weighting function; N=256 Δν=8 MHz
φ= 1.21
Real
Frequency
Lag weighting function; N=256/256 Δν=8 MHz
Hanning reweight
Weight
Lag
FT of lag weighting function; N=256 Δν=8 MHz
φ= 2.00
Real
Frequency
Lag weighting function; N=512/512 Δν=8 MHz
Weight
Lag
FT of lag weighting function; N=512 Δν=8 MHz
φ= 1.21
Real
Frequency
Lag weighting function; N=512/512 Δν=8 MHz
Hanning reweight
Weight
Lag
FT of lag weighting function; N=512 Δν=8 MHz
φ= 2.00
Real
Frequency
Lag weighting function; N=1024/1024 Δν=8 MHz
Weight
Lag
FT of lag weighting function; N=1024 Δν=8 MHz
φ= 1.21
Real
Frequency
Lag weighting function; N=1024/1024 Δν=8 MHz
Hanning reweight
Weight
Lag
FT of lag weighting function; N=1024 Δν=8 MHz
φ= 2.00
Real
Frequency



Lag weighting function; N=511/512 Δν=4 MHz
Weight
Lag
FT of lag weighting ʼon; N=512 Δν=4 MHz
ϕ= 1.21
Real
Frequency
Lag weighting function; N=ʼ ʼ512 Δν=4 MHz
Hanning reweight
Weight
Lag
FT of lag weighting function; N=512 Δν=4 MHz
ϕ= 2.00
Real
Frequency
Lag weighting function; N=1023/1024 Δν=4 MHz
Weight
Lag
FT of lag weighting function; N=1024 Δν=4 MHz
ϕ= 1.21
Real
Frequency
Lag weighting function; N=1023/1024 Δν=4 MHz
Hanning reweight
Weight
Lag
FT of lag weighting function; N=1024 Δν=4 MHz
ϕ= 2.00
Real
Frequency
Lag weighting function; N=2047/2048 Δν=4 MHz
Weight
Lag
FT of lag weighting function; N=2048 Δν=4 MHz
ϕ= 1.21
Real
Frequency
Lag weighting function; N=2047/2048 Δν=4 MHz
Hanning reweight
Weight
Lag
FT of lag weighting function; N=2048 Δν=4 MHz
ϕ= 2.00
Real
Frequency