The Effect of Synchrotron Losses on Multiple Diffusive Shock Acceleration

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Abstract

The effect of synchrotron losses on diffusive shock acceleration (DSA) at many shocks is treated numerically. Synchrotron losses determine a maximum energy to which electrons can be accelerated through DSA, and this is referred to as the synchrotron cutoff, p_c. The distribution of accelerated electrons after many shocks is found (a) for a distribution injected at the initial shock, to tend a plateau ($f(p)$ independent of $p \lesssim 0.1p_c$), and (b) for the cumulative distribution from injection at each shock to tend to $f(p) \propto p^{-b}$ with $b \approx 3$ well below the synchrotron cutoff with a peak in the slope ($b_{\rm min} \approx 2$) at $p \lesssim 0.1p_c$. It is suggested that the latter result might account for the flat synchrotron spectra observed in some Galactic Centre sources.

Keywords: Particle acceleration, flat synchrotron spectra, shock waves

1 Introduction

Diffusive shock acceleration (DSA) is the favored acceleration mechanism for relativistic electrons in most synchrotron sources (e.g., the reviews by Drury 1983; Blandford & Eichler 1987). DSA results naturally in a power law electron energy or momentum distribution, and hence a power law synchrotron spectrum. In terms of the momentum distribution function, a power law distribution is of the form $f(p) \propto p^{-b}$, where p is the momentum and b is the power law index. A power law synchrotron spectrum of the form $I_\nu \propto \nu^{-\alpha}$, where ν is the frequency, corresponds to electrons with $b = 2\alpha + 3$. DSA at a single shock gives a power law distribution with $b = 3r/(r-1)$, where r is the compression ratio of the shock, which has a maximum value $r = 4$ for strong shocks in a nonrelativistic gas with ratio of specfic heats 5/3. Hence the flattest distribution that can be produced by a single shock has $b = 4$ and hence $\alpha = 0.5$.

Flat radio spectra ($\alpha \approx 0$) in some extragalactic sources are usually attributed to self absorption (e.g., Kellermann & Pauliny-Toth 1969). For the self-absorption to produce a flat spectrum over at least a decade in frequency (as often observed) requires specific geometric properties in the source (e.g., Marscher 1977), leading to description of the self-absorption interpretation as a 'cosmic conspiracy' model (Cotton et al. 1980). There are nonthermal sources with flat spectra in the Galactic Centre region (e.g., Yusef-Zadeh 1989) which are not plausibly self-absorbed. Thus, at least for the galactic sources with flat synchrotron spectra, a more plausible explanation is that the the electron distribution is flatter than $b = 4$ (e.g., Melrose 1996). It is known that DSA at a sequence of shocks ("multiple DSA") tends to a distribution $f(p) \propto p^{-3}$ after an arbitrarily large number of shocks (e.g., White 1985; Achterberg 1990; Schneider 1993; Melrose & Pope 1993; Pope & Melrose 1994), and this does imply a flat synchrotron spectrum, $\alpha \approx 0$, as required. However, an explanation of flat spectra in terms of multiple DSA in this way has some unsatisfactory features: the approach to the asymptotic distribution $f(p) \propto p^{-3}$ is slow and requires that a large number of shocks propagate across the acceleration region; the particles must escape from the system before they have time to cool due to synchroton losses; and multiple DSA cannot account for even weakly inverted spectra, $\alpha < 0$. However, the relation between the actual particle distribution (which in inhomogeneous due to the shocks, and only approaches $f(p) \propto p^{-3}$ asymptotically) and the synchrotron spectrum involves an integral over the entire source, and this needs to be modelled in detail to determine the actual synchrotron spectrum.

In this paper we show that synchrotron losses combined with multiple DSA can be efficient in forming flat and inverted synchrotron spectra. The underlying idea is threefold. First, it is known that the effect of synchrotron losses on a power law distribution is to steepen a distribution with $b > 4$, and to cause a pile up (an integrable divergence) for $b < 4$, as we show explicitly in §5 below. Second, multiple DSA is known to form a curved distribution with the flatter portion ($b < 4$) moving to higher energies as the number of shocks increases. Third, synchrotron losses, which are most important in the compressed-B region just downstream of the shock, limit the maximum p to which the particles can be accelerated. We refer to this maximum p as the *synchrotron cutoff*, which is defined in a more formal manner below. Combining these ideas, when multiple DSA produces a slope $b < 4$ just below the synchrotron cutoff energy, synchrotron losses tend to flatten the distribution even further. Our objective in this paper is to describe this flattening in detail using a simple numerical model. In this model all processes (DSA, synchrotron losses and adiabatic decompression here) are treated as independent, described by relatively simple operators, and a combination of processes is treated by applying the appropriate operators sequentially.

The model is described in §2. For the combination of DSA and synchrotron losses the assumption that the processes may be treated sequentially is justified in §3. Our results are presented in §4, and their interpretation is discussed in §5. Our conclusions are summarized in §6.

2 Method

We treat the effects of DSA at a single shock, adiabatic decompression and synchrotron losses in terms of operators, $\hat{L}_{\rm dsa}(p)$, $\hat{L}_{\rm dec}(p)$, $\hat{L}_{\rm loss}(p)$, respectively, operating on an initial distribution function to produce the final distribution function. DSA is described by (e.g., Melrose 1986, p. 249)

$$\hat{L}_{\rm dsa}(p)f(p) = bp^{-b}\int_0^p dp'\, p'^{(b-1)}\, f(p'), \tag{1}$$

with $b = 3r/(r-1)$, where r $(1 < r < 4)$ is the compression ratio for the shock. One has $b > 4$ with $b \to 4$ for $r \to 4$, corresponding to the strongest possible shock in a nonrelativistic plasma. Decompression is required to reduce the magnetic field, B, from its compressed value $B = rB_0$ (which applies strictly only to a perpendicular shock) just behind the shock to its ambient value, $B = B_0$, before the arrival of the next shock. Then $p^2 \propto B$ implies

$$\hat{L}_{\rm dec}(p)f(p) = f(pr^{1/3}). \tag{2}$$

Synchrotron losses are described by $dp/dt = -Ap^2$, $A = (32\pi/9)(r_e^2/m_e^2c^2)(B^2/2\mu_0)$, where r_e is the classical radius of the electron, and m_e is the mass of the electron. Let p' be the solution of $dp/dt = -Ap^2$ at t that produces p at $t+\Delta t$, so that one has $p' = p/(1-pA\Delta t)$. Then, if the synchrotron losses are allowed to operate for a time Δt, Liouville's theorem implies

$$\hat{L}_{\rm loss}(p)f(p) = (1-pA\Delta t)^{-4}\, f(p/[1-pA\Delta t]). \tag{3}$$

The dependence of the synchrotron loss rate on the strength of the shock, $A \propto r^2$, implies that for a strong shock ($r \gg 1$) the losses are most rapid in the compressed-B region just downstream of the shock. Assuming that the losses are important only in this region, Δt may be identified as the time spent by the electron there. However, for our numerical calculations only the combination $A\Delta t$ need be specified, and this is a free parameter.

In our calculations, we inject an initial δ-function distribution, $\propto \delta(p-p_0)$, and subject it sequentially to DSA, synchrotron losses and decompression. In multiple DSA, the resulting output distribution is the input into a second identical sequence, and this sequence is repeated as many times as desired. We consider both the case where there is a single initial injection and the case where there is an injection at each shock. In the absence of synchrotron losses it is necessary to extend the calculations to very much higher p-values than are ultimately of interest to avoid rounding errors. A rounding error at $p_{\rm max}$ after the first shock propagates down to $r^{-(N-1)/3}p_{\rm max}$ after N shocks, and for large N this leads to errors in conservation of particles. We chose a sufficiently high $p_{\rm max}$ such that particles are conserved to an accuracy of $\leq 0.5\%$ for $N = 50$.

Another feature of the model is the neglect of the spatial coordinates, which assumption is usually made in this context. The underlying idea is that spatial inhomogeneities are taken into account through implicit coupling between separate regions of space. Here such couplings are implicit in the operators themselves. For example, $\hat{L}_{\rm dsa}$ couples the downstream to the upstream region, and $\hat{L}_{\rm dec}$ couples the region immediately behind the shock to the decompressed region further downstream.

3 Alternative treatment of DSA with synchrotron losses

Our sequential procedure for treating synchrotron losses involves neglecting the synchrotron losses so that DSA forms a distribution that extends well beyond the synchrotron cutoff, and then allowing synchrotron losses to modify this distribution. In order to check the validity of this sequential procedure we treat DSA in another manner that allows one to include the acceleration and the synchrotron losses at the same time, rather than sequentially. Our numerical results show that the two procedures produce indistinguishable results. Here we summarize the alternative procedure and outline an analytic proof that the two procedures are equivalent.

The alternative method is an adaptation of that explained by Achterberg (1990). An individual electron gains $\Delta(p)$ in one cycle time, t_c, where a cycle involves crossing the shock from upstream to downstream and back to upstream again. There is a probability, $P(p)$, of an electron escaping downstream in each cycle. For a highly relativistic particle, the theory implies

$$\Delta(p) = \frac{4(r-1)}{3r}\,\frac{u_1}{c}\,p, \qquad P(p) = \frac{4u_1}{cr}, \tag{4}$$

where u_1 is the shock speed; t_c depends on the mean free paths of the electrons in the upstream and downstream regions. Then the distribution of electrons escaping downstream is

$$f(p) \propto \frac{P(p)}{p^2\Delta(p)}\,\exp\left[-\int_{p_0}^{p} dp'\,\frac{P(p')}{\Delta(p')}\right], \tag{5}$$

where the normalization of $f(p)$ depends on an arbitrary injection rate. The integral in (5) with (4) is elementary, and the result reproduces (1) for an initial distribution $\propto \delta(p-p_0)$.

Synchrotron losses may be included in (4) and (5) by including the synchrotron losses in the calculation of $\Delta(p)$ in each cycle. The average acceleration rate over one cycle, $dp/dt = \Delta(p)/t_c = Cp$, is then modified to

$$dp/dt = Cp - Ap^2, \tag{6}$$

where the additional term describes the synchrotron losses. The synchrotron cutoff corresponds to $p_c = C/A$, and no electron can be accelerated to beyond this cutoff ($f(p) = 0$ for $p \geq C/A$). The resulting distribution of electrons then follows from (5) with $\Delta(p)$ reinterpreted in this way, that is, with the replacement $\Delta(p) \to \Delta(p) - Ap^2 t_c$, where $Ap^2 t_c$ is the synchrotron loss in one cycle. The integral can be performed analytically and the result is the same as is obtained using the sequential procedure (3). The effective synchrotron loss time, Δt in (3), found by equating the two results, is

$$\Delta t = \frac{c}{u_1}\,\frac{r-1}{3r}\,t_c. \tag{7}$$

Thus, except for very weak shocks ($r \approx 1$), (7) implies $\Delta t \gg t_c$ and the change over any one cycle is small, justifying our use of the discrete change in one cycle to determine a continuous changes over many cycles.

The foregoing proof of equivalence of the two treatments of DSA and synchrotron losses applies for an initial distribution $f(p) \propto \delta(p - p_0)$. The generalization to an arbitrary initial distribution, $f_0(p)$ say, follows simply by applying the operation $\int dp_0\, f_0(p_0)$. (In effect the solution for an initial δ-function distribution is a Green's function for the general case.) This establishes the equivalence of the two procedures: the sequential procedure is exact within the framework of this alternative treatment of DSA. Thus we are well justified in using the sequential procedure in our numerical calculations.

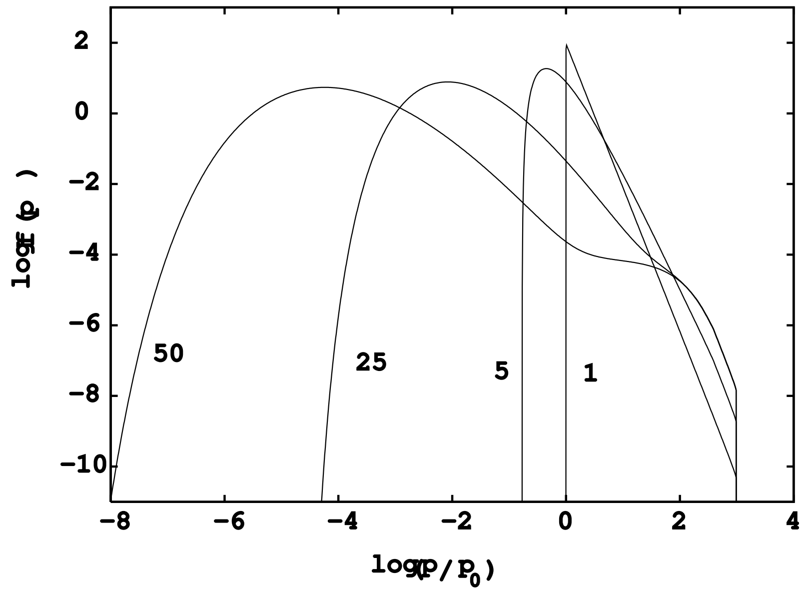

Figure 1: The effect of multiple DSA and synchrotron losses is illustrated for the distribution resulting from a single initial injection: $r = 3.8$, $p_c/p_0 = 10^3$, $N = 1$, 5, 25, 50.

4 Results

Our numerical results are illustrated in Figures 1–5. In these figures the logarithm of the distribution function is plotted as a function of $\log(p/p_0)$, so that a power law distribution corresponds to a straight line. The absolute values of $f(p)$ and of p are unimportant. The synchrotron cutoff momentum, p_c, is a free parameter and is chosen to be either three ($p_c/p_0 = 10^3$) or six ($p_c/p_0 = 10^6$) orders of magnitude above the injection momentum. All the shocks have the same strength, specified by the value of r, and the calculations are performed both for strong shocks with $r = 3.8$ and for shocks with $r = 2.0$. The adiabatic decompression after each shock moves the curve to the left (by $-(\log r)/3$), without changing its shape, so that after N shocks the lowest energy particle in the distribution has $\log p = -N(\log r)/3$.

In Figure 1 the evolution of a single distribution injected at the first shock is shown after 1, 5, 25 and 50 shocks, all with $r = 3.8$ and with the synchrotron cutoff $p_c = 10^3 p_0$. The sharply-peaked distribution is immediately after the first shock, before adiabatic decompression, and the other curves peaking to the left of the first are after 5, 25 and 50 shocks, respectively. The formation of a plateau (the nearly horizontal portion of the curve) is evident after 50 shocks.

In Figure 2 the distribution shown in Figure 1 for $N = 50$ is compared for the same case without synchrotron losses. In effect all the particles that would be above the synchrotron cutoff in the absence of synchrotron losses (dashed curve) appear in a hump just below the synchrotron cutoff when synchrotron losses are included (solid curve). This hump is a manifestation of the 'pile up' effect due to synchrotron losses, as discussed in §5 below.

In the theory of DSA it is usually assumed that there is injection at every shock. Hence the cases shown in Figure 1 and 2, where there is only a single initial injection with this distribution subjected to many shocks, is not realistic in practice. One expects the

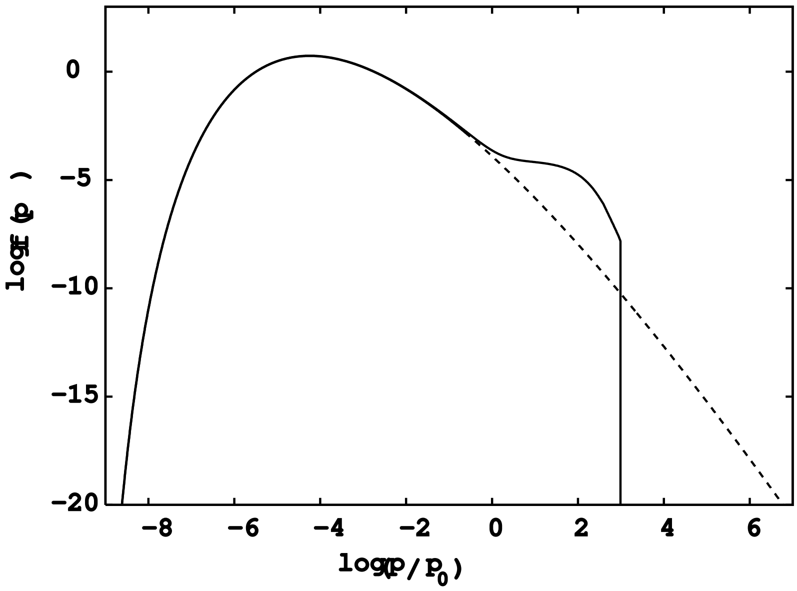

Figure 2: Comparison of the distributions with (solid curve) and without (dashed curve) synchrotron losses for injection only at the initial shock: $r = 3.8$, $p_c/p_0 = 10^3$, $N = 50$.

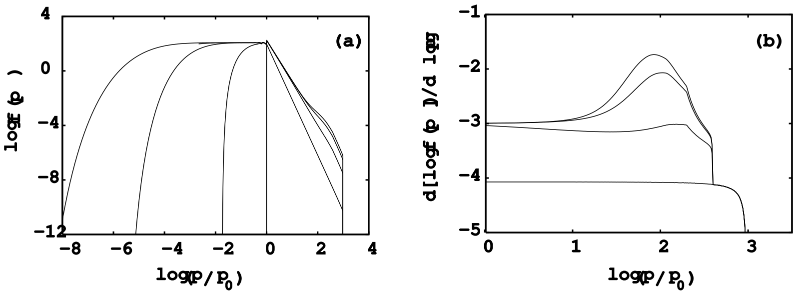

Figure 3: The cumulative effect after many shocks with injection at each shock; (a) the distribution, (b) the slope of the distribution for $p > p_0$: $r = 3.8$, $p_c/p_0 = 10^3$, $N = 1$, 10, 30, 50.

distribution after N shocks to consist of the sum over the distribution injected at the first shock subjected to N shocks, the distribution injected at the second shock subjected to $N - 1$ shocks, and so on to the distribution injected at the Nth shock subjected to only one shock. This sum is performed in evaluating the distributions shown in Figure 3. In Figure 3a the curves correspond to the sums after $N = 1$, 10, 30, 50, with the innermost distribution being for $N = 1$ (which is the same as the $N = 1$ case in Figure 1) and the outermost curves being for $N = 50$. An inflection develops in the distribution just below the synchrotron cutoff. To illustrate this more clearly, the slope of the distributions are plotted in Figure 3b. The lowermost curve is for $N = 1$; at low $p \gtrsim p_0$ it shows a power law with index $b = 4.07$, corresponding to $b = 3r/(r-1)$ with $r = 3.8$, cf. (1); the slope steepens as the synchrotron cutoff is approached. As N is increased the slope decreases monotonically and approaches $b = 3$ at low $p \gtrsim p_0$, in accord with theoretical predictions (White 1985; Achterberg 1990; Schneider 1993; Melrose & Pope 1993; Pope & Melrose 1994). Nearer the synchrotron cutoff, after about 10 shocks, a peak in the slope starts

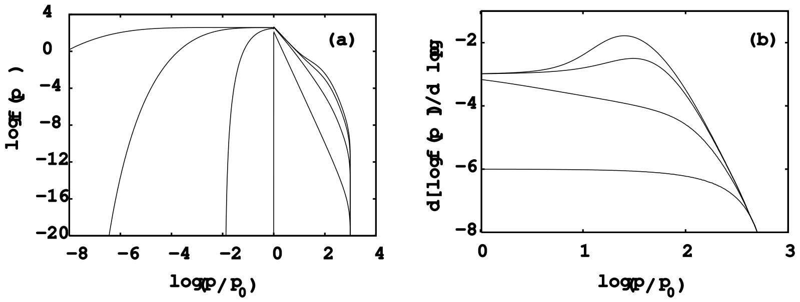

Figure 4: As for Figure 3, except for $r = 2.0$ and $N = 1$, 20, 80, 200.

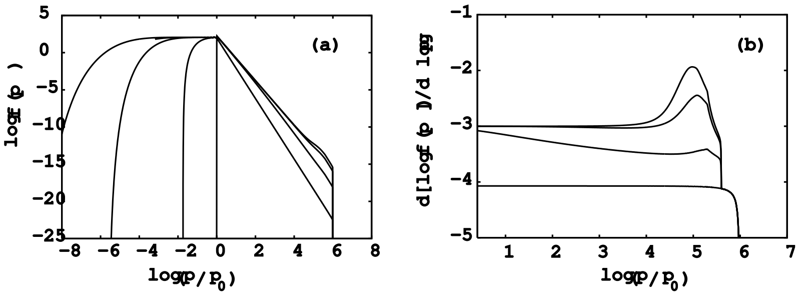

Figure 5: As for Figure 3, except for $p_c/p_0 = 10^6$.

to develop and becomes increasingly prominent with increasing N. This peak may be attributed to the contribution from the plateau-like portions of the distributions resulting from injection at the earliest shocks.

In order to illustrate that the effects shown in Figure 3 are not unique to very strong shocks, we performed calculations for weaker shocks with $r = 2.0$. The results are shown in Figure 4. As in Figure 3, an inflection in the distribution develops just below the synchrotron cutoff after many shocks. The development of this feature is slower for weaker shocks, and more shocks (up to $N = 200$) are included in Figure 4 than in Figure 3 (up to $N = 50$). The slope of the distribution for $N = 1$ is $b = 6.0$ at $p \gtrsim p_0$, corresponding to $b = 3r/(r-1)$ with $r = 2.0$, and it increases towards $b = 3$ with increasing N. The peak in the slope (Figure 4b) just below the synchrotron cutoff is somewhat broader, with the peak at a somewhat lower momentum, than for the stronger shocks. We also performed calculations for 100 shocks of random strength, with r chosen as a random variable between 1.5 and 4.0, and the results are similar to those shown in Figure 2–4.

To study the effect of increasing the range between the injection and the synchrotron cutoff, we repeated the calculations in Figure 3 for $p_c/p_0 = 10^6$. The results are plotted in Figure 5. Compared with Figure 3, Figure 5 shows that the portion of the distribution with $b \approx 3$ (corresponding to a flat synchrotron spectrum) extends over most of the wider range $p_0 < p < p_c$, with the peak in the slope remaining essentially unchanged at $p \approx 0.1 p_c$.

5 Interpretation

The foregoing results show three notable effects of synchrotron losses on multiple DSA: (a) it provides a high-p synchrotron cutoff (denoted p_c) beyond which no particle can be accelerated by DSA; (b) for a single initial injection, a plateau distribution, $f(p) = \text{const.}$, develops at $p \lesssim 0.1p_c$; (c) the cumulative effect of injection at every shock leads to a distribution $f(p) \propto p^{-3}$ for $p \ll p_c$; and (d) the distribution in (c) has a slope that rises gradually to a peak (with $b_{\rm min} \sim 2$ at $p \sim 0.1p_c$ in Figure 3).

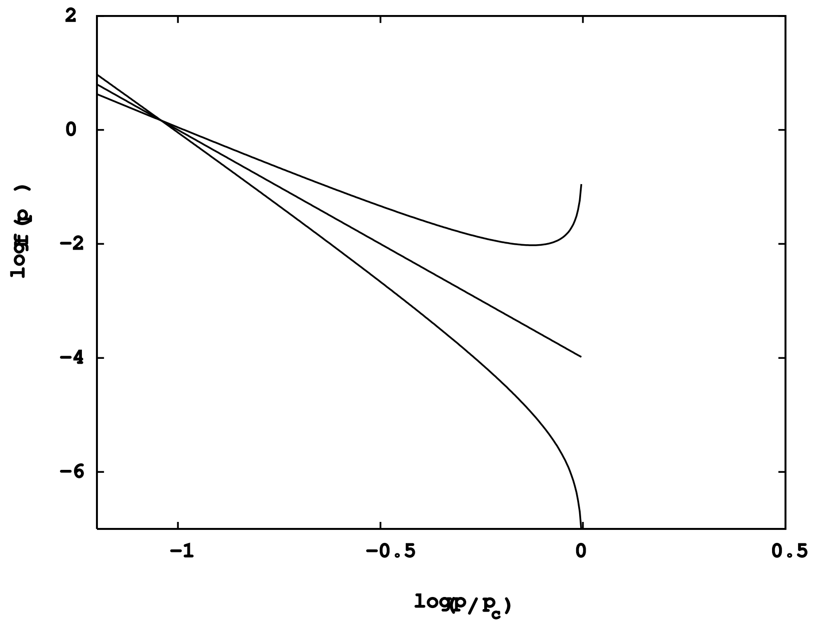

Figure 6: The effect of synchrotron losses on initial power law distributions with $b = 3$ (upper), $b = 4$ (middle) and $b = 5$ (lower).

In the following discussion an important (and long-known) effect of synchrotron losses plays a central role: synchrotron losses tend to steepen a distribution with $b > 4$ and to cause a turn-up in a distribution with $b < 4$. This is illustrated in Figure 6 where the initial distribution is a power law ($\propto p^{-b}$) that extends to $p = \infty$. After a time t the particles initially with $p = \infty$ have $p = p_c = 1/At$, which is the synchrotron cutoff in this case. A distribution with $b > 4$ initially becomes steeper (both with increasing p and increasing t) with $f(p) \to 0$ for $p \to p_c$. The distribution with $b = 4$ does not change in shape and cuts off abruptly at $p = p_c$. A distributions with $b < 4$ initially develops a pile up with $f(p) \to \infty$ for $p \to p_c$.

The formation of a plateau distribution for a single initial injection subjected to many shocks can be understood in terms of two effects. One effect is that multiple DSA tends to flatten the distribution towards the asymptotic distribution $f(p) \propto p^{-3}$. Thus, although DSA at a single shock (in a plasma with ratio of specific heats 5/3) cannot produce a distribution flatter than $b = 4$, and $b = 4$ only for the strongest possible shock with $r = 4$, multiple DSA can produce a distribution with $b < 4$. The other effect is that once a distribution with $b < 4$ forms, synchrotron losses tend to cause electrons to pile up just below the synchrotron cutoff, cf. Figure 6. Together these effects account for the distributions in Figure 3–5, with b close to 3 well below p_c and a peak in the slope just below the cutoff at p_c.

Schlickeiser (1984) showed that the combination of (second-order) Fermi acceleration and synchrotron losses causes a 'pile up' just below synchrotron cutoff, and our result is related to Schlickeiser's result. The combination of DSA and decompression should lead to a Fermi-like acceleration mechanism, in the sense that the combination may be described by a diffusion equation in momentum space. Hence, the asymptotic solution for multiple DSA should approach the asymptotic solution for Fermi acceleration: for constant injection at $p = p_0$ this is a plateau ($b = 0$) for $p < p_0$ and is $b = 3$ for $p > p_0$. The synchrotron losses provide a high-p barrier that prevents particles from diffusing to very high p, and this may be regarded as a reflecting boundary in momentum space. This reflection acts like a source of particles at the synchrotron cutoff so that one expects the asymptotic spectrum to approach a plateau just below the cutoff. The tendency to form a plateau distribution for a single initial injection, cf. Figure 1, is a manifestation of this effect. When expressed in terms of the energy spectrum of the electrons, $N(\varepsilon)$, with $\varepsilon \approx pc$ for highly relativistic particles, and hence

$$N(\varepsilon)d\varepsilon = 4\pi f(p)p^2dp, \tag{8}$$

a plateau momentum distribution implies an energy spectrum $N(\varepsilon) \propto \varepsilon^2$. Thus our results show that DSA combined with synchrotron losses produces a pile up similar to that found by Schlickeiser (1984) for Fermi acceleration combined with synchrotron losses. However, in the more realistic case where there is injection at each shock (which would be simulated by constant injection in Fermi acceleration) the asymptotic distribution is $f(p) \propto p^{-3}$ or $N(\varepsilon) \propto \varepsilon^{-1}$, becoming somewhat flatter just below the synchrotron cutoff. This portion of the distribution with $b < 3$ implies that it is possible in principle for the model to account for weakly inverted spectra ($\alpha = (b-3)/2 \lesssim 0$), but only at relatively high frequencies, corresponding to emission by electrons with momenta just below p_c (around $0.1p_c$ according to Figure 3).

6 Conclusions

We present the results of numerical calculations that show the effect of synchrotron losses on diffusive shock acceleration (followed by adiabatic decompression) at multiple shocks. Our main results can be summarized as follows.

- Synchrotron losses are most important, during the acceleration process, when the electrons are in the compressed-B region just downstream from the shock. Synchrotron losses imply a synchrotron cutoff, $p = p_c$, to the distribution of accelerated particles; DSA cannot cause any particle to be accelerated to $p > p_c$.

- It is shown analytically that two procedures for treating the combination of synchrotron losses and DSA are equivalent. In one treatment, the effects of synchrotron losses are included in the momentum change in each cycle of a particle crossing the shock from upstream to downstream and back. In the other procedure, used in our numerical calculations, synchrotron losses are first neglected to find the distribution of electrons resulting from DSA alone, and then the synchrotron losses are allowed to modify this distribution. The two procedures are equivalent provided that the time for which the synchrotron losses are allowed to operate in the latter 'sequential' procedure is identified as the time (7).

- Just below the synchrotron cutoff, the distribution of particles injected at an initial shock and subjected to DSA at many shocks without further injection tends to form a plateau distribution ($f(p)$ independent of p), which corresponds to an energy spectrum $N(\varepsilon) \propto \varepsilon^2$.

- The distribution below the synchrotron cutoff due to the cumulative effect of injection at every shock tends to a distribution $f(p) \propto p^{-b}$ with $b \approx 3$ at $p \ll p_c$, with the distribution becoming somewhat flatter such that the slope has a peak (with $b \approx 2$) just below p_c (at $\approx 0.1 p_c$ for strong shocks). Such a distribution, if the source were homogeneous (which it is not due to the shocks), would correspond to a flat synchrotron spectrum ($\alpha = (b-3)/2 \approx 0$) becoming a weakly inverted spectrum ($\alpha \approx -0.5$) with a peak just below a sharp cutoff due to synchrotron losses.

We conclude that it is possible in principle for multiple DSA coupled with synchrotron losses to account for a flat synchrotron spectrum. This may be a viable explanation for the flat synchrotron spectra observed in some Galactic Centre sources. A more detailed investigation of this possibility is warranted.

Acknowledgments

We thank Lewis Ball and Stephen Hardy for helpful comments on the manuscript. This investigation was carried out while one of us (A.C.) was a summer Vacation Scholar in the Special Research Centre for Theoretical Astrophysics.

References

Achterberg, A. 1990, in Brinkman, W., Fabian, A.C., & Giovanelli, F. (eds), *Physical Processes in Hot Cosmic Plasmas*, Kluwer Academic Publishers (Dordrecht), p. 67

Blandford, R.D., & Eichler, D. 1987, Phys. Rep. 154, 1

Cotton, W.D., *et al.* 1980, ApJ 238, L123

Drury, L.O'C. 1983, Rep. Prog. Phys. 46, 97

Kellermann, K.I., & Pauliny-Toth, I.I.K. 1977, ApJ 155, L71

Marscher, A.P. 1977, AJ 82, 781

Melrose, D.B. 1986, *Instabilities in Space and Laboratory Plasmas*, Cambridge University Press

Melrose, D.B. 1996, in Ekers, R., Fanti, C., & Padrielli, L., (eds) *Extragalactic Radio Sources*, Kluwer, Dordrecht, p. 423

Melrose, D.B., & Pope, M.H. 1993, Proc. Astron. Soc. Australia 10, 222

Pope, M.H., & Melrose, D.B. 1994, Proc. Astron. Soc. Australia 11, 175

Schlickeiser, R. 1984, AA 136, 227

Schneider, P. 1993, AA 278, 315

White, R.L. 1985, ApJ 289, 698
Yusef-Zadeh, F. 1989, in *The center of the Galaxy IAU Symp. 136*, ed. Morris, M., D. Reidel (Dordrecht), p. 243