Small-scale turbulence dynamo and characteristic scales of MHD turbulence

Jungyeon Cho

Chungnam National University, Korea

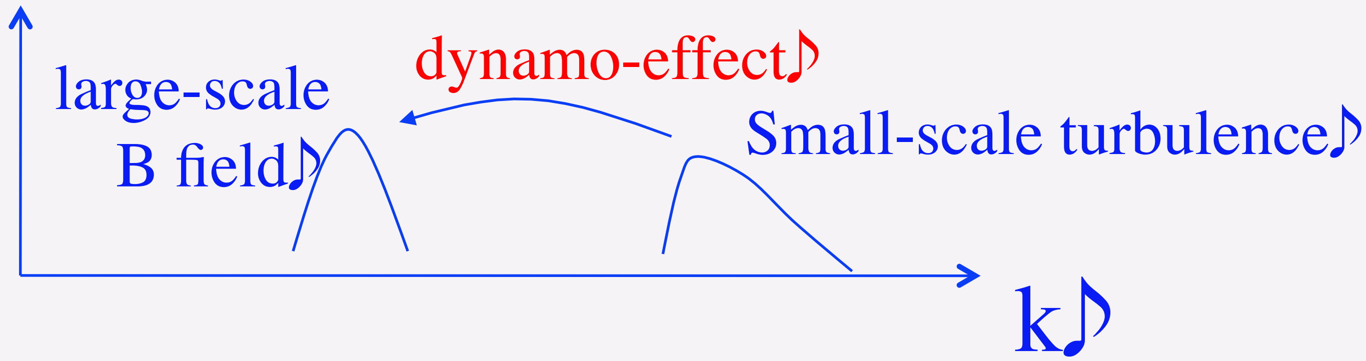
Cho & Vishniac (2000, ApJ)
Cho, Vishniac, Beresnyak, Lazarian & Ryu (2009, ApJ)♪
Cho & Ryu (2009, ApJL)

Outline♪

- Growth of B
- Characteristic scales of B♪

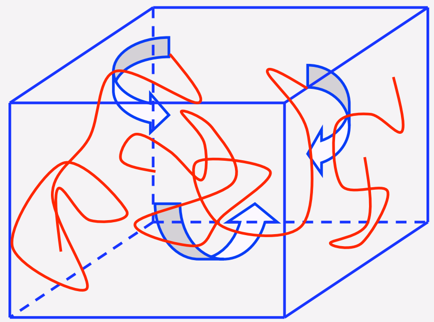
Two types of dynamos♪

- Mean field dynamo

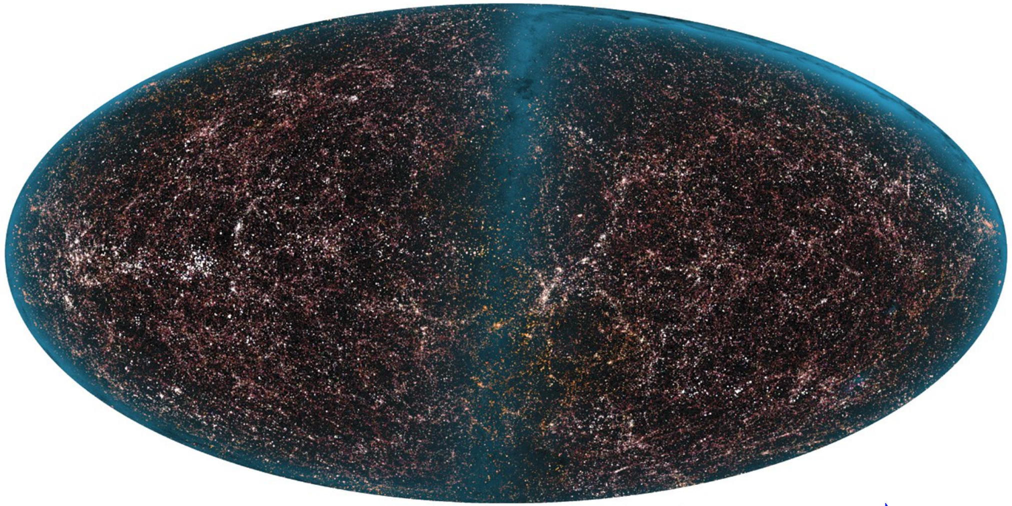


- Small-scale dynamo (=turbulence dynamo):

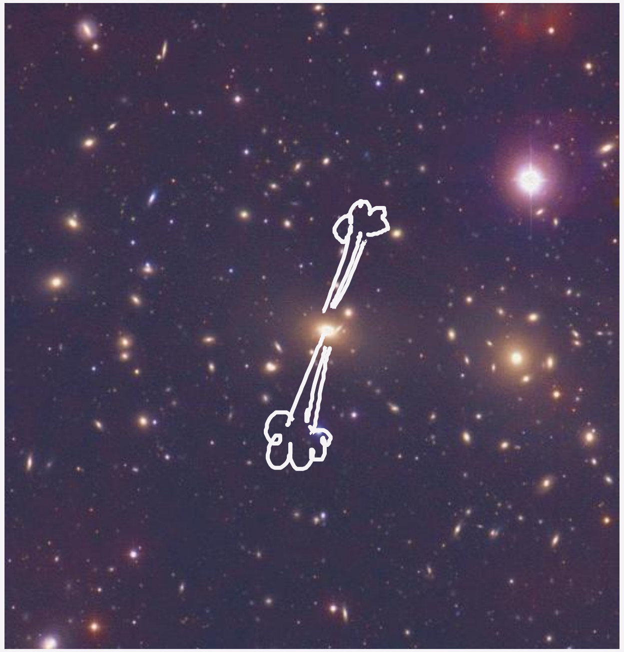
  growth of small-scale B by small-scale turbulence♪



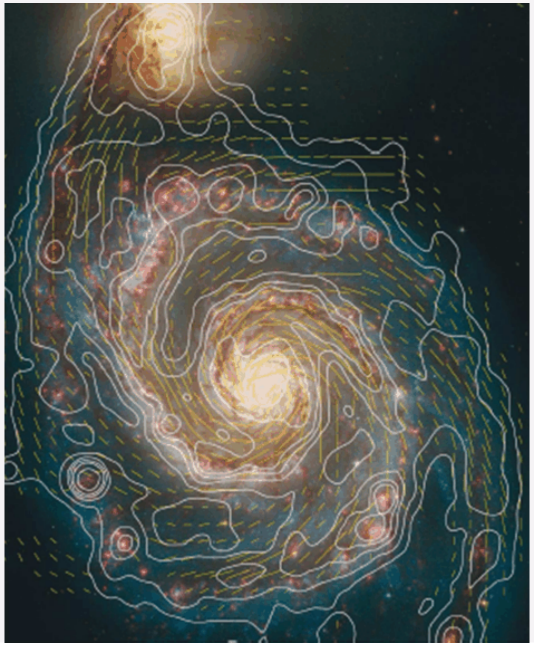
Small-scale dynamo is important for many astrophysical fluids



Large-scale structure of the universe♪



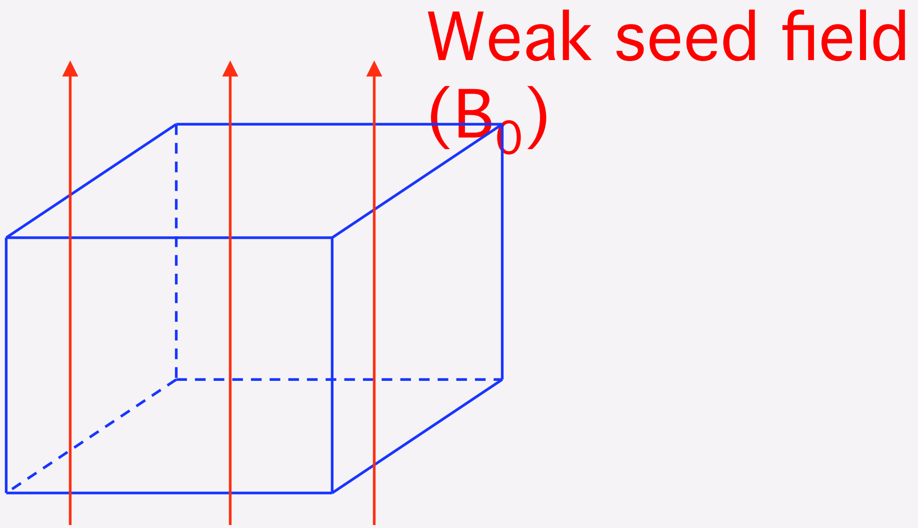
clusters♪



galaxies♪

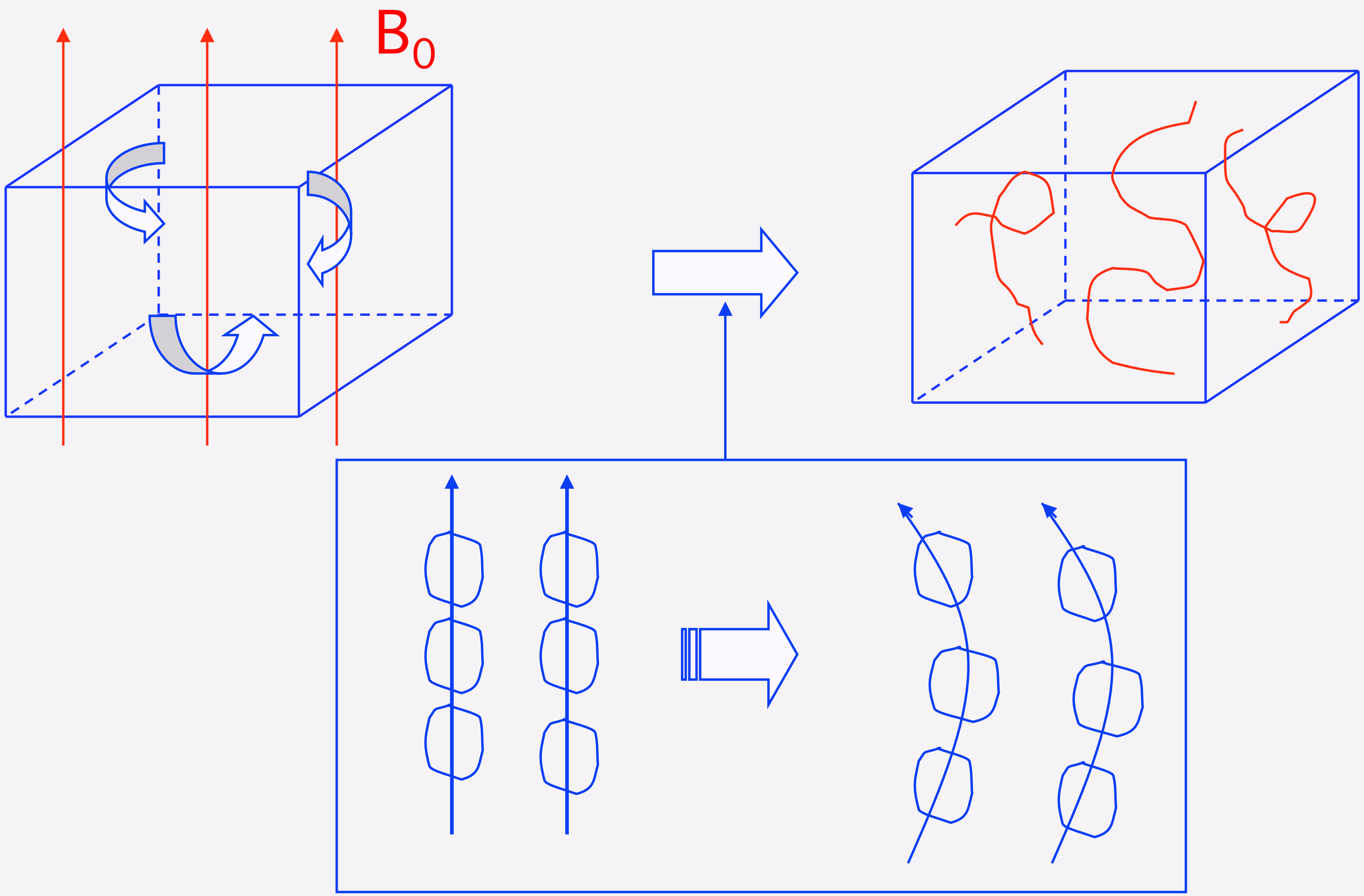
Topic 1. Amplification of B fields in turbulence

- How can MHD turbulence amplify B fields?



Stretching of field lines

t=0:

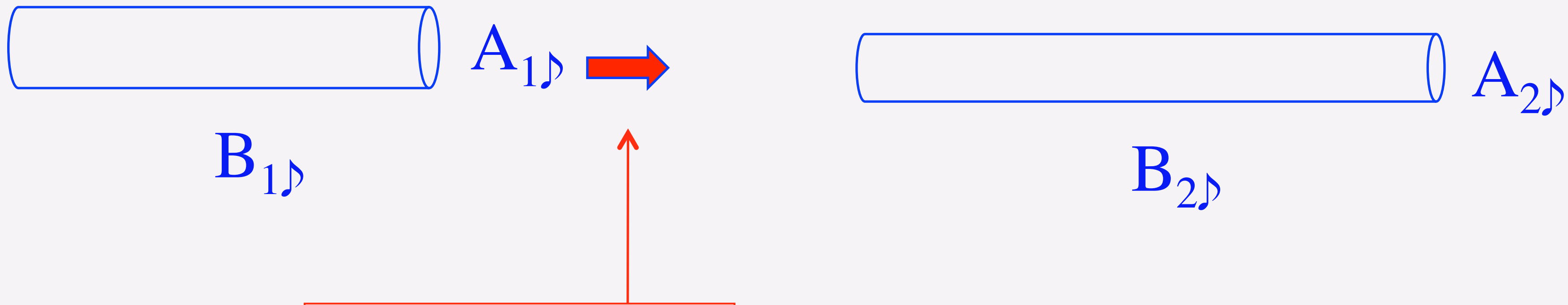


Fluid elements and field lines move together

*Back reactions are negligible if $E_{mag}<E_{kin}$

Stretching ➔ Increase of E_B♪

(Suppose that the fluid is incompressible)♪



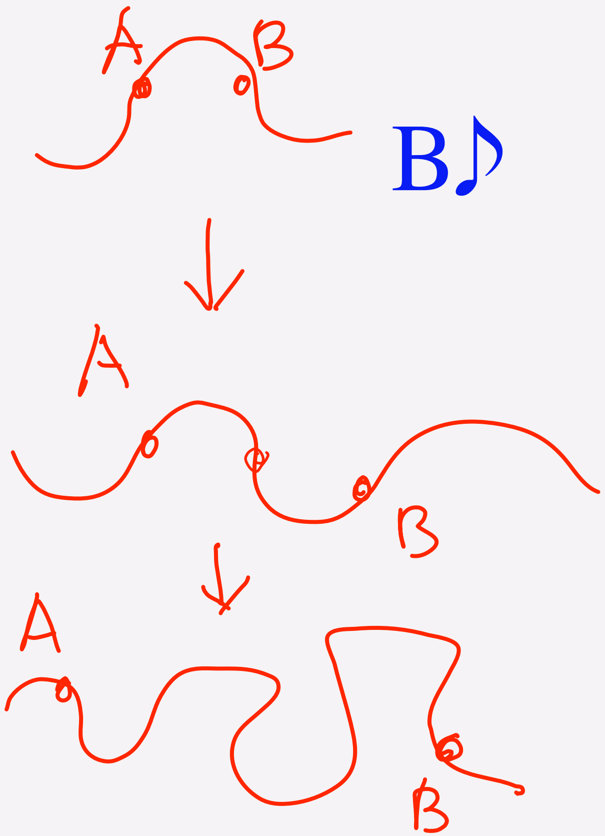
$\boxed{B_1 A_1 = B_2 A_2}$♪

-$B_2 = (A_1/A_2)B_1 > B_1$

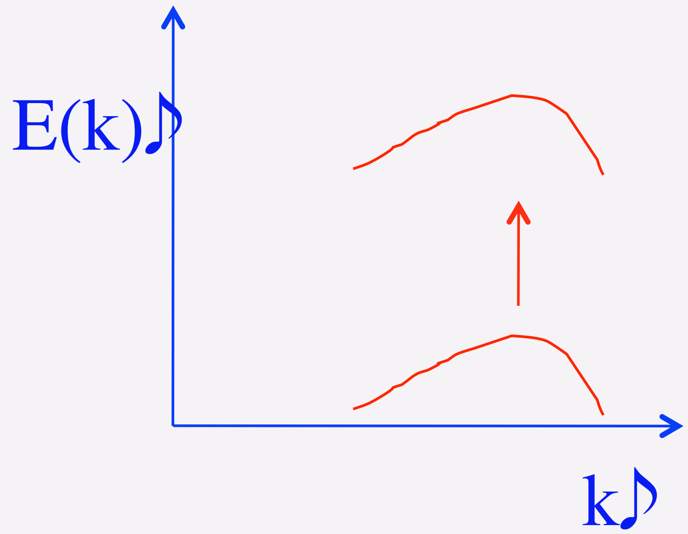
-$B_2^2/8\pi > B_1^2/8\pi$♪

Expectations:

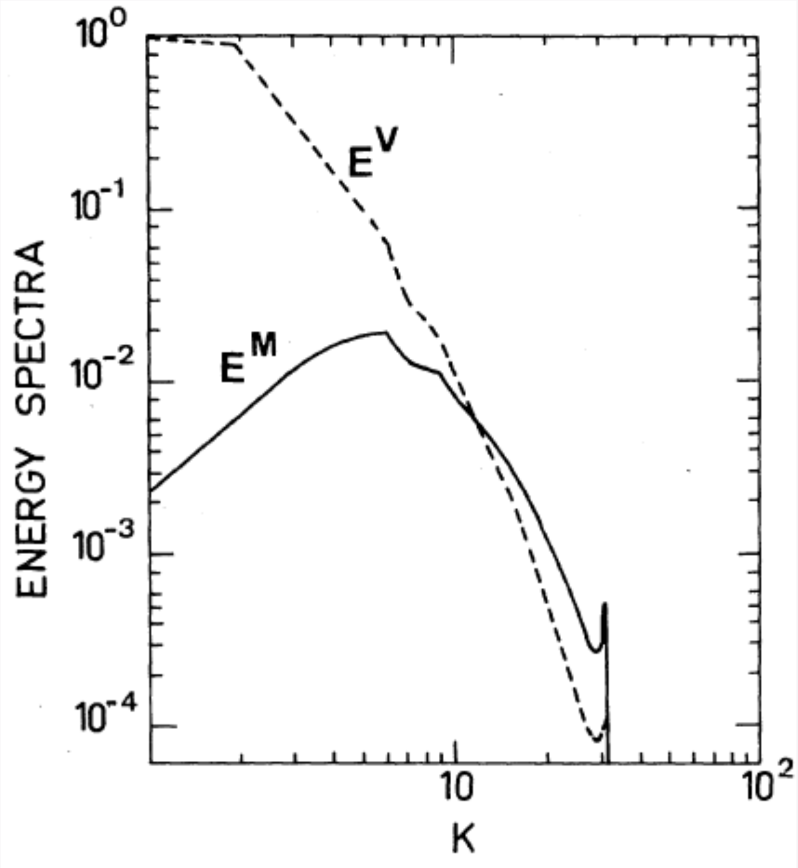
Stretching on the dissipation scale will occur first because eddy turnover time is shortest there ♪

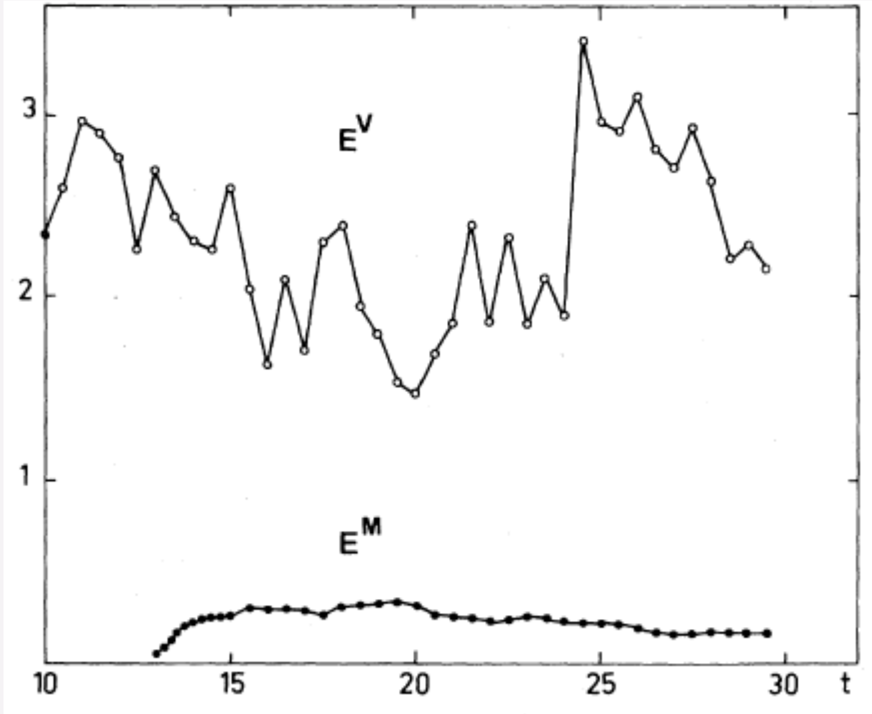


B♪



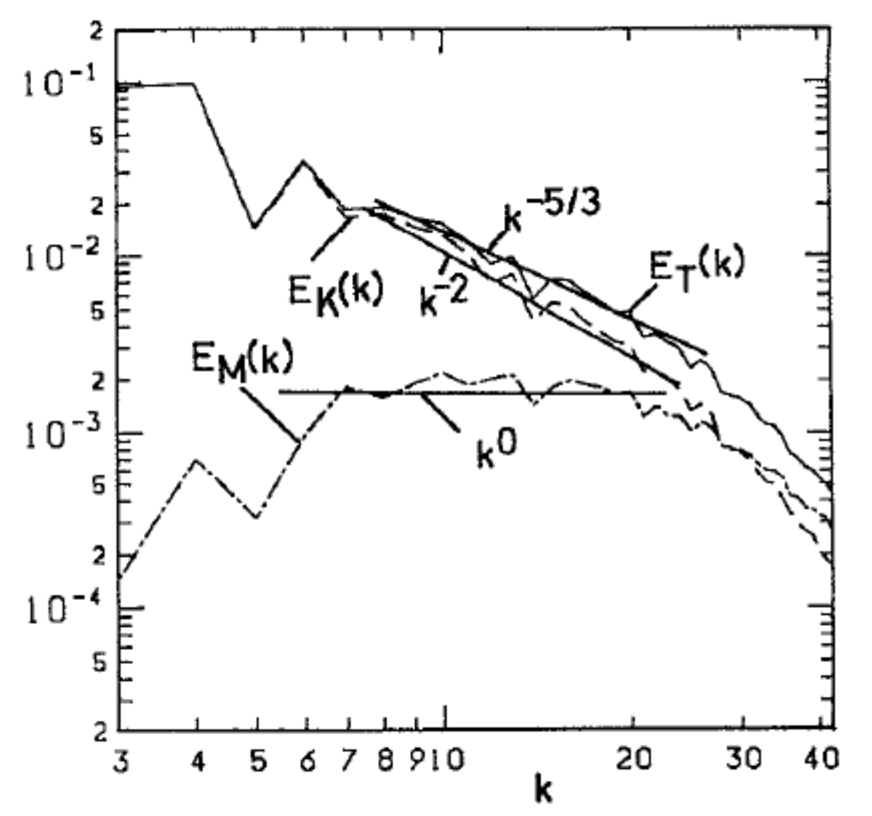
Exponential growth
(Batchelor 1950)♪

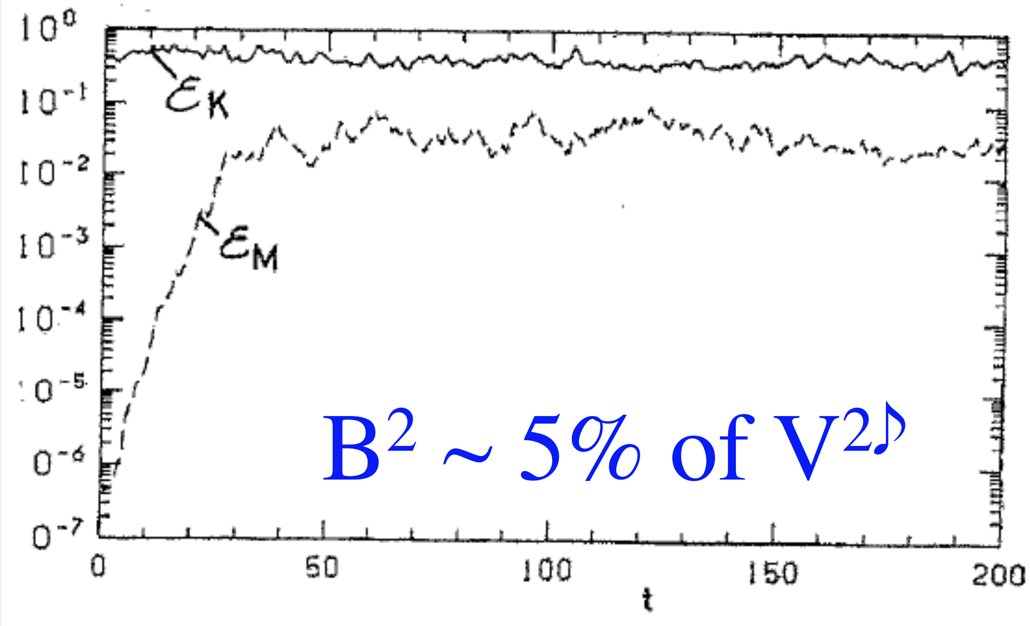




$B^2 < 10\%$ of V^2♪

Meneguzzi et al. 1981
(Resolution = 64^3)♪

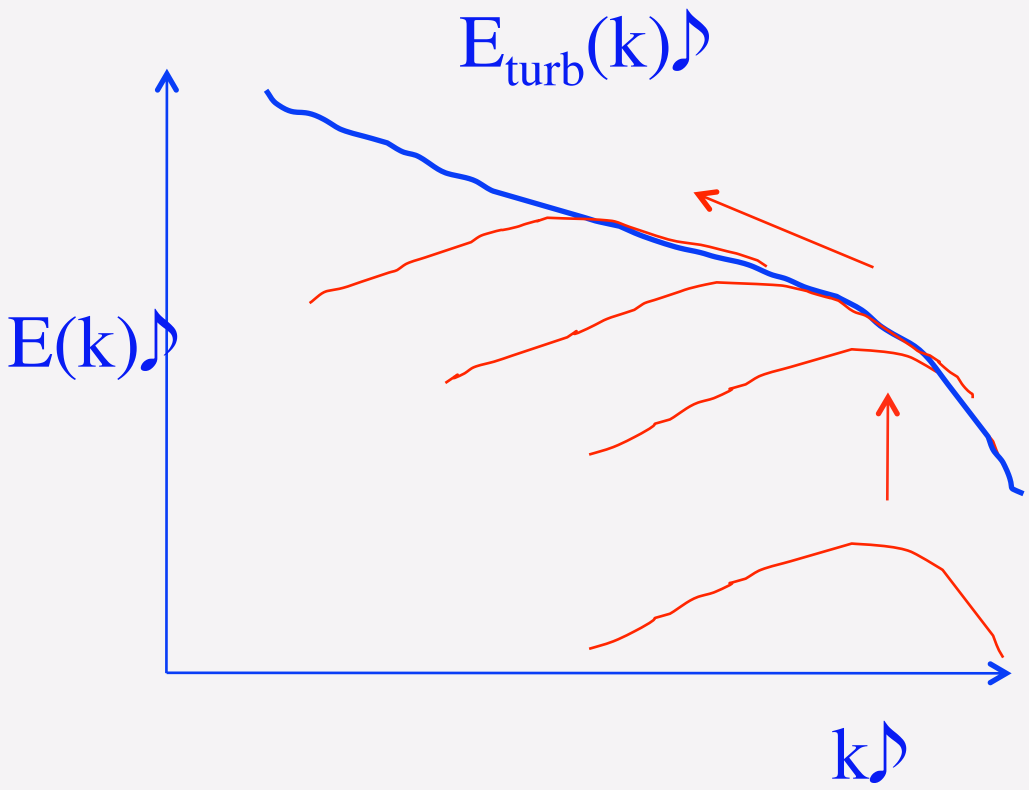




$B^2 \sim 5\%$ of V^2

Kida et al. 1991
(Resolution ~ 85^3)

Expectations
(for high resolution):

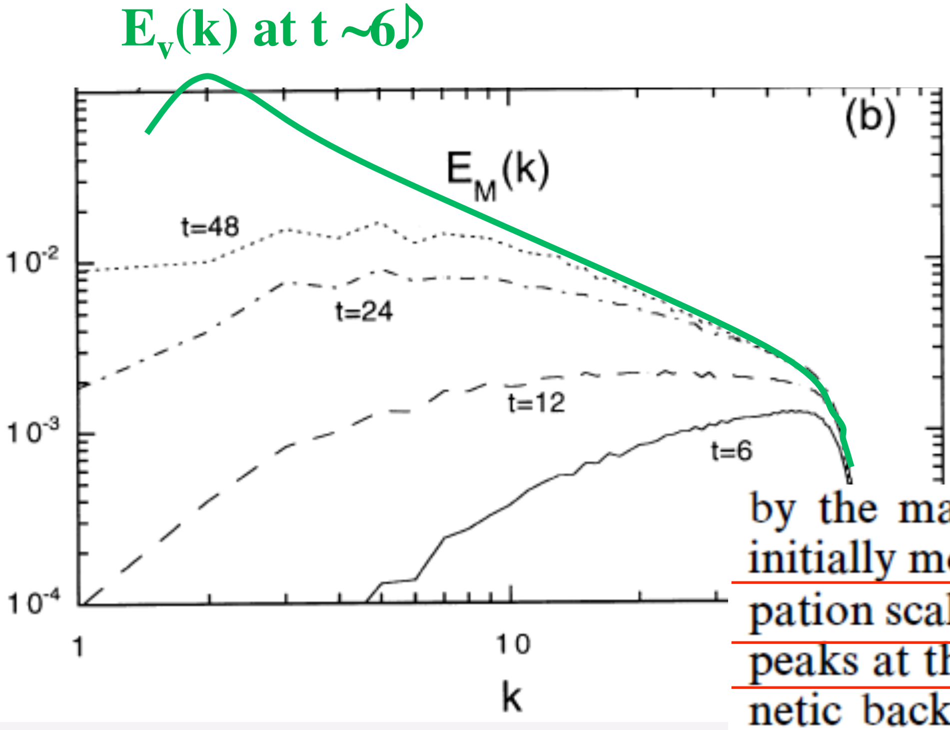


What will happen when $E_{turb} \sim E_{mag}$ on the dissipation scale?♪

➔ Exponential growth stage will end!♪

➔ Stretching scale gradually moves to larger scales.
♪

Cho & Vishniac 2000

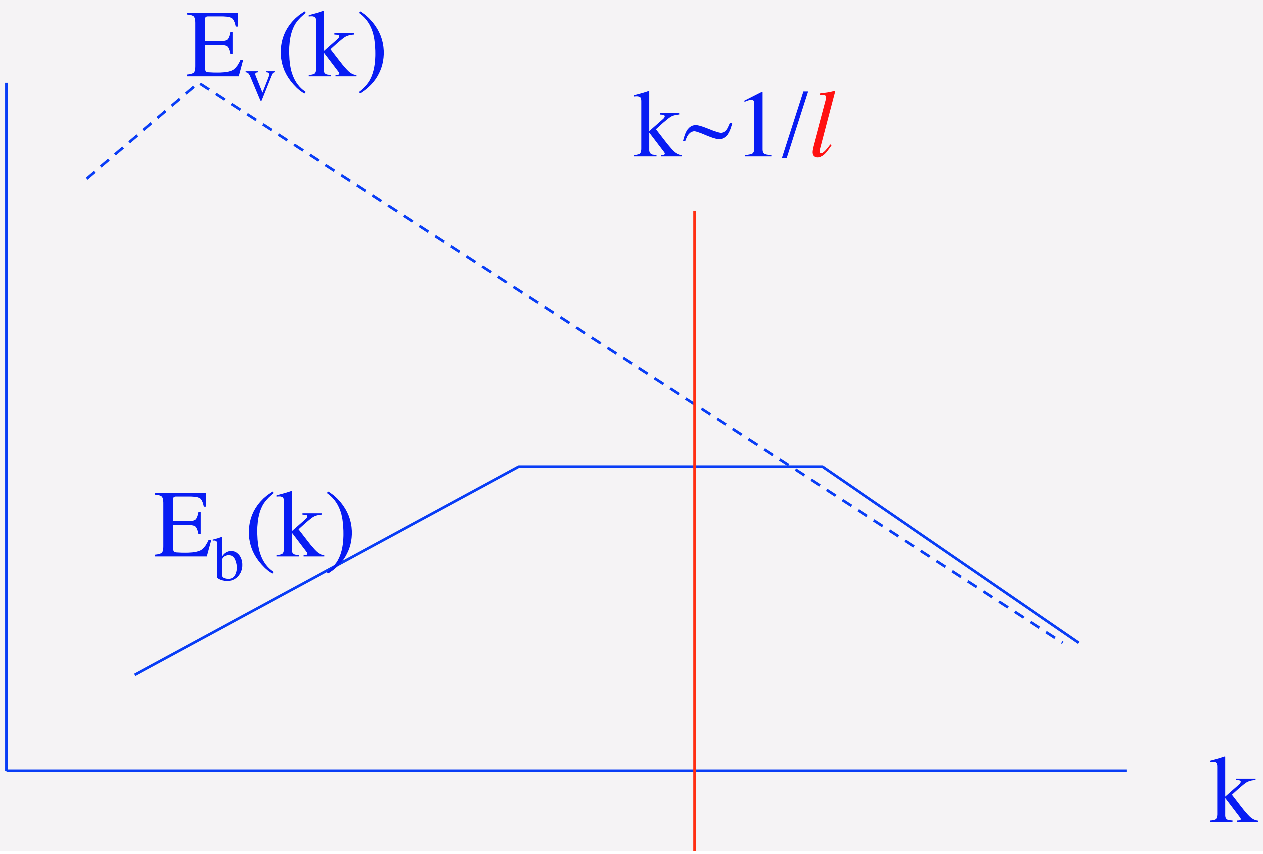


by the magnetic fields. Figure 3 shows that stretching is initially most active near or somewhat larger than the dissipation scale (cutoff scale) and the magnetic energy spectrum peaks at this scale. As the magnetic energy grows, the magnetic back reaction becomes important at the dissipation scale. When energy equipartition is reached at this scale, the stretching rate slows down and a second stage of slower growth begins. Figure 3 shows that during this stage the peak of the magnetic power spectrum moves to larger scales. Figure 2 shows that the second stage ends at

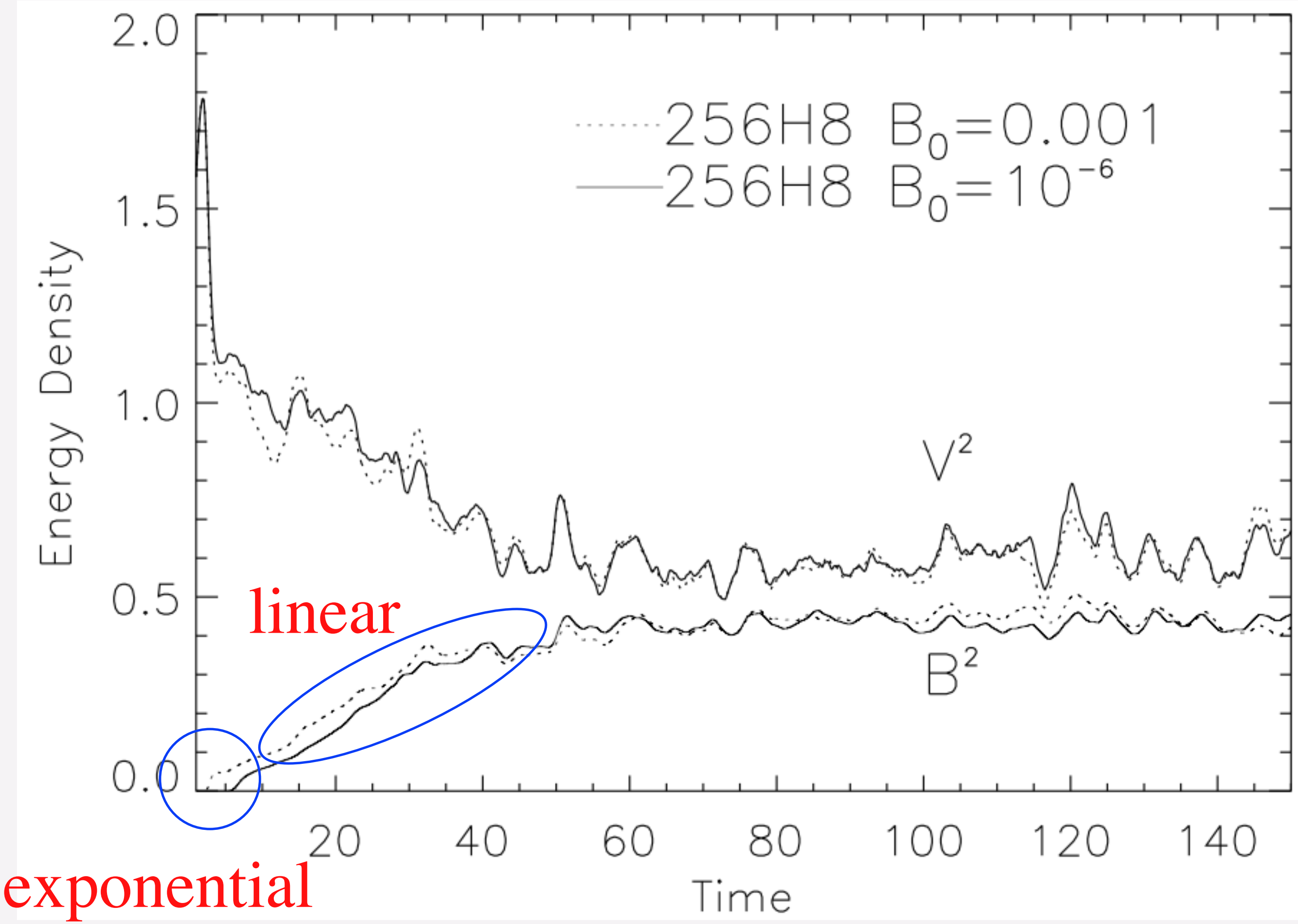
Growth rate during this stage is linear

(Schekochihin et al 2006)

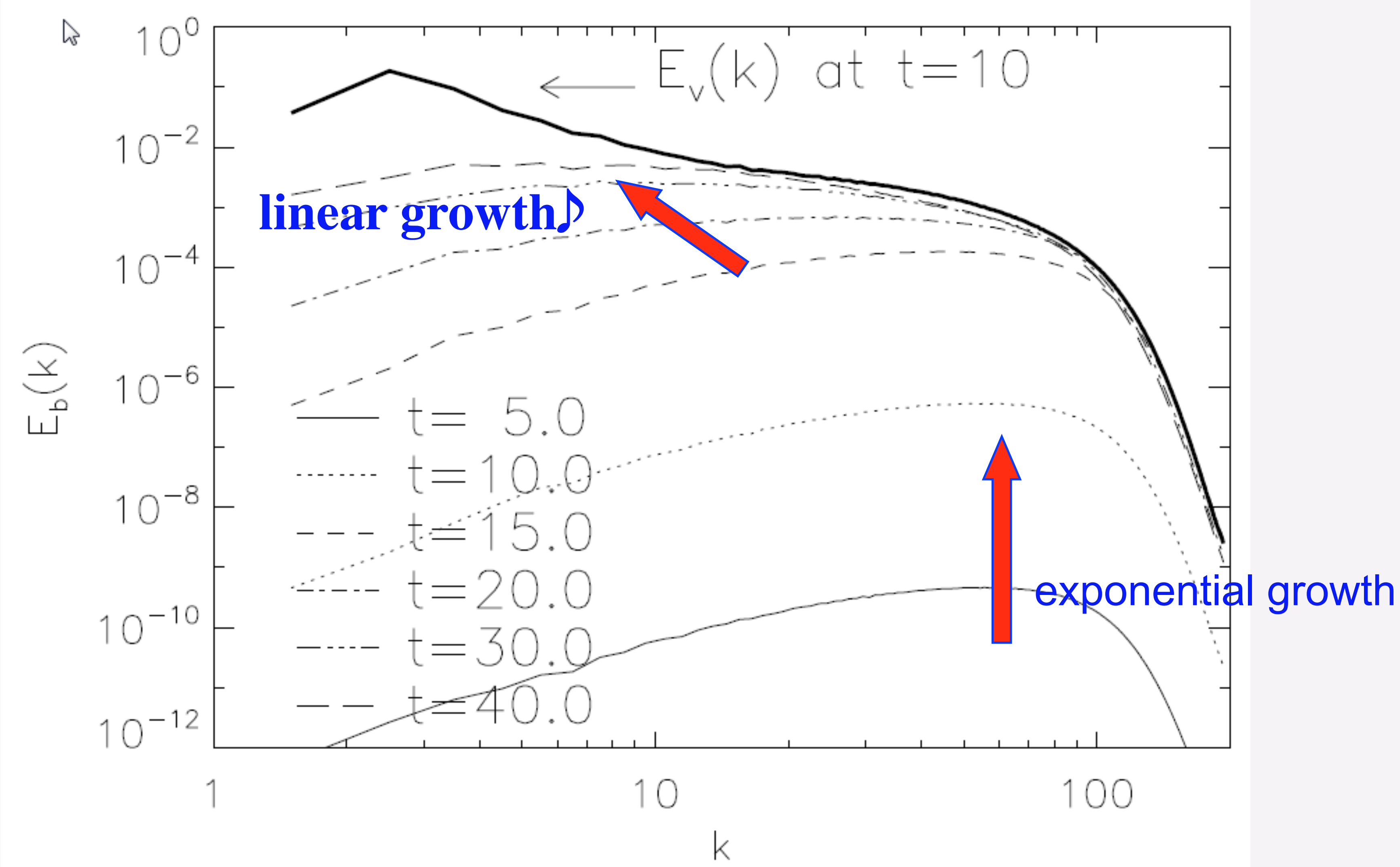
$$db^2(t)/dt \;\sim b_l^{\,2}/(l/v_l) \quad \sim v_l^{\,3}/l \quad \sim \varepsilon \sim \text{const}$$



Results of simulations

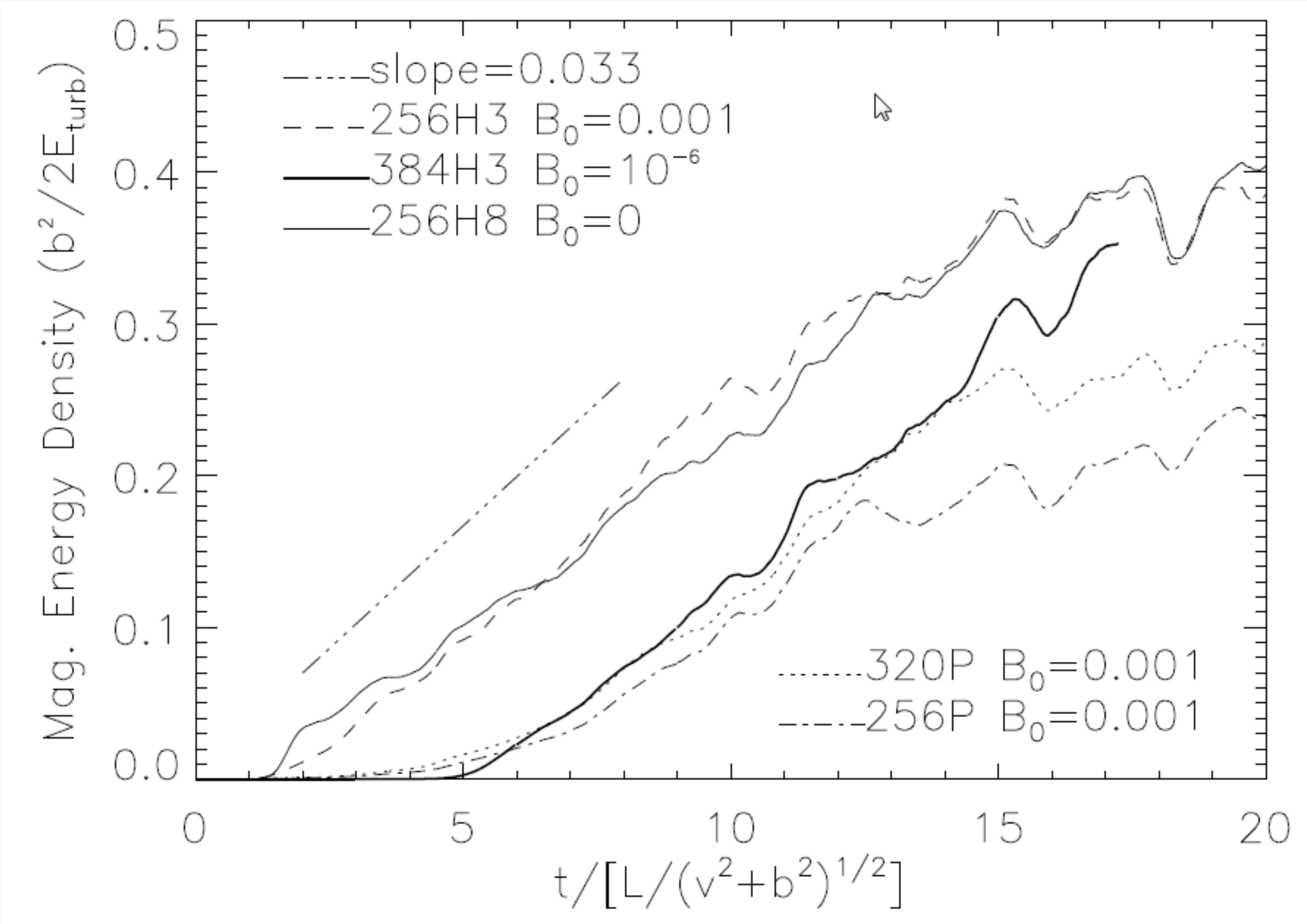


Cho, Vishniac, Beresnyak, Lazarian, Ryu (2009)

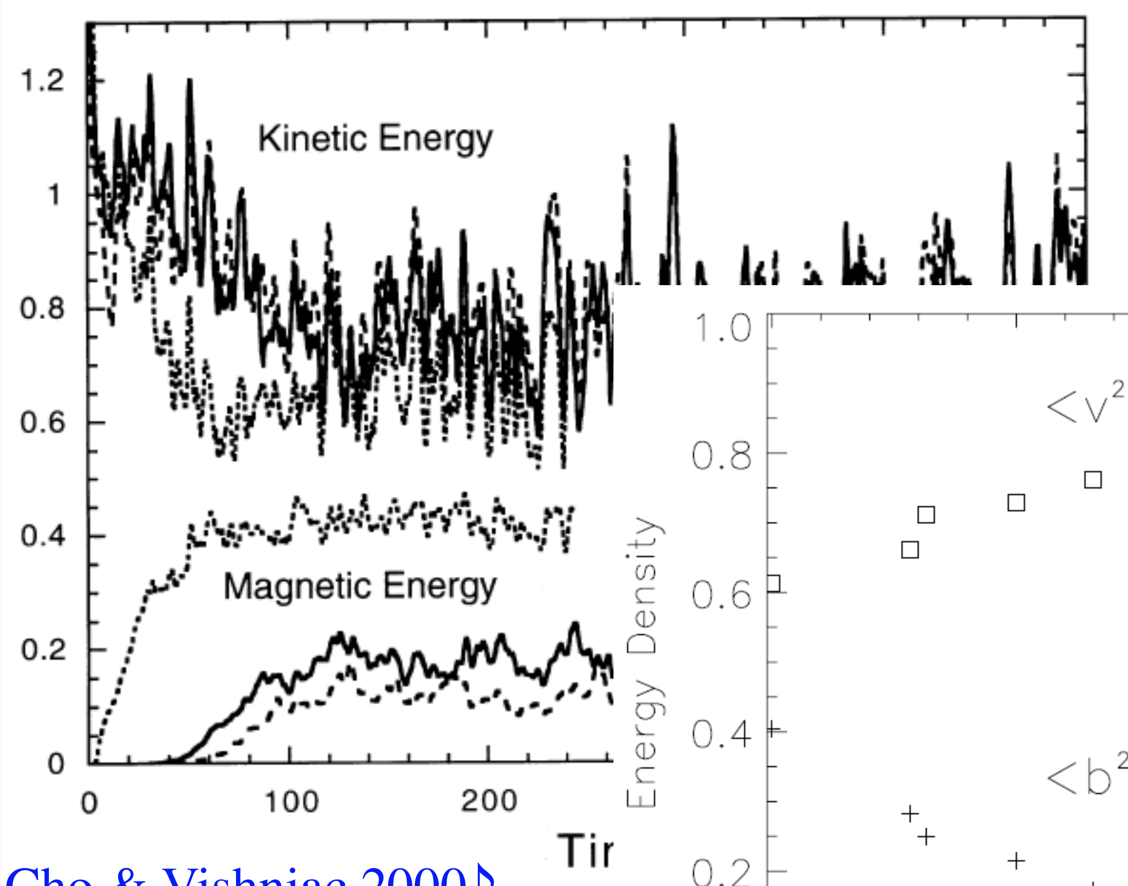


$E_v(k)$ at t=10
linear growth♪
exponential growth
t= 5.0
t=10.0
t=15.0
t=20.0
t=30.0
t=40.0
$E_b(k)$
k

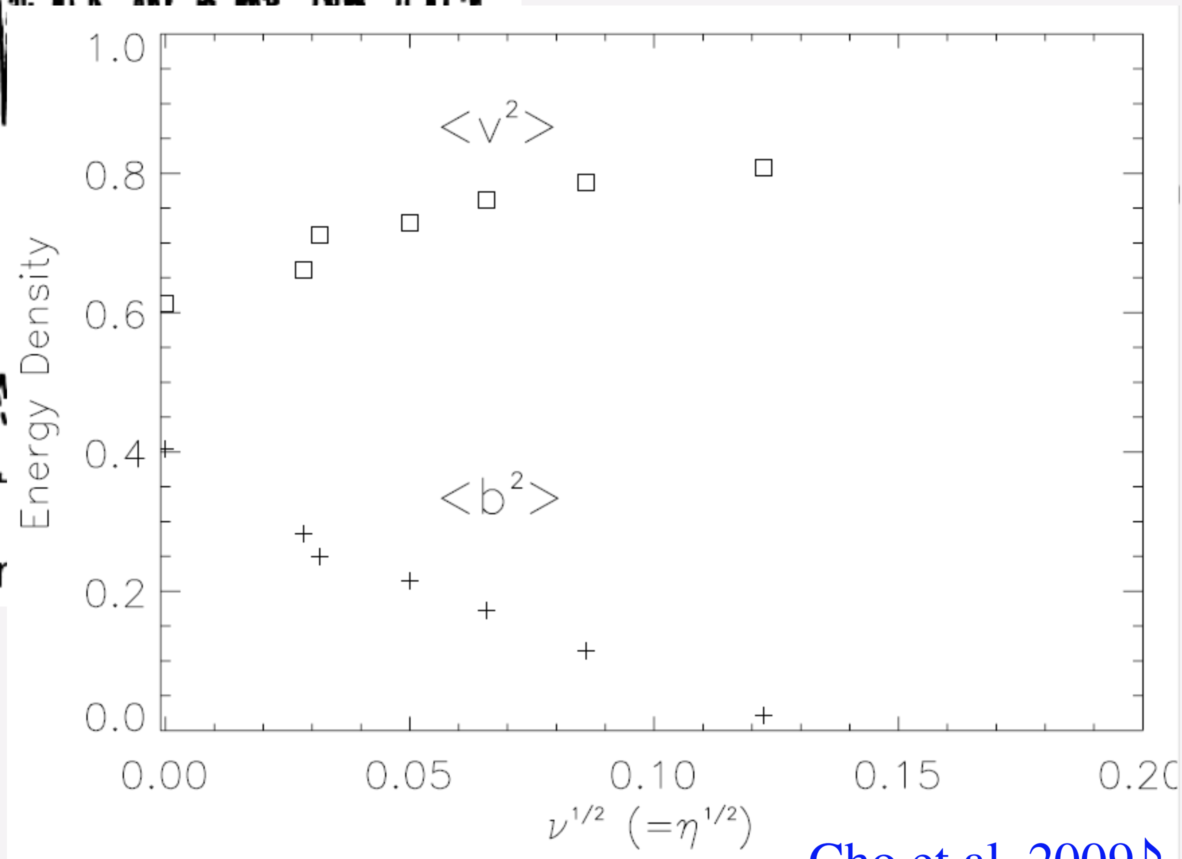
The growth rate seems to be universal



Saturation level depends on ν (=η)♪

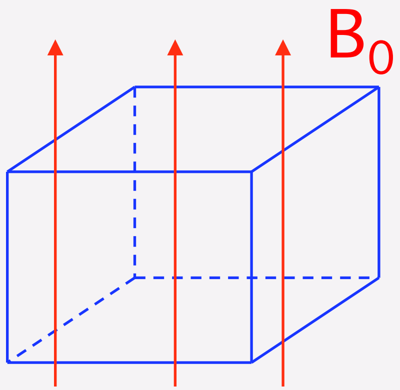


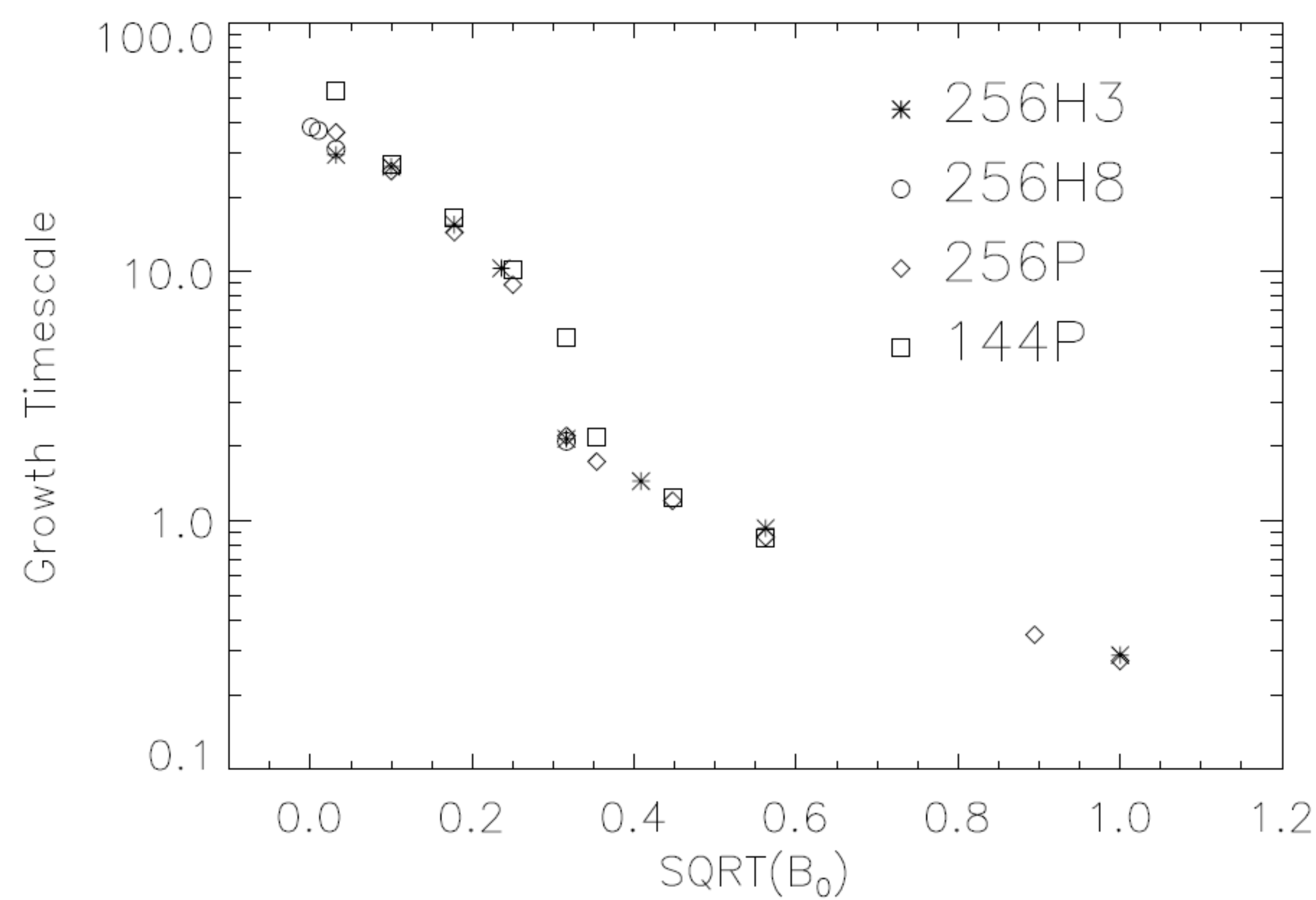
Cho & Vishniac 2000♪

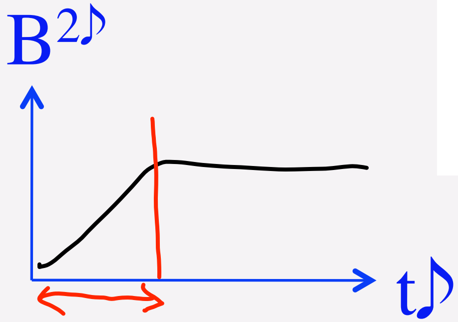


Cho et al. 2009♪

Growth time scale vs. B_0

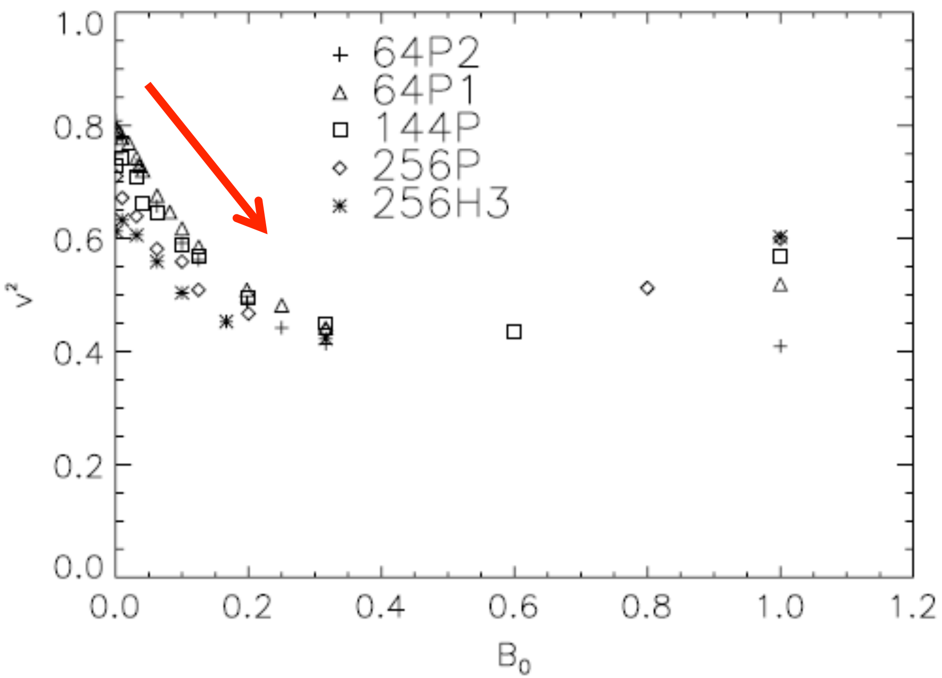


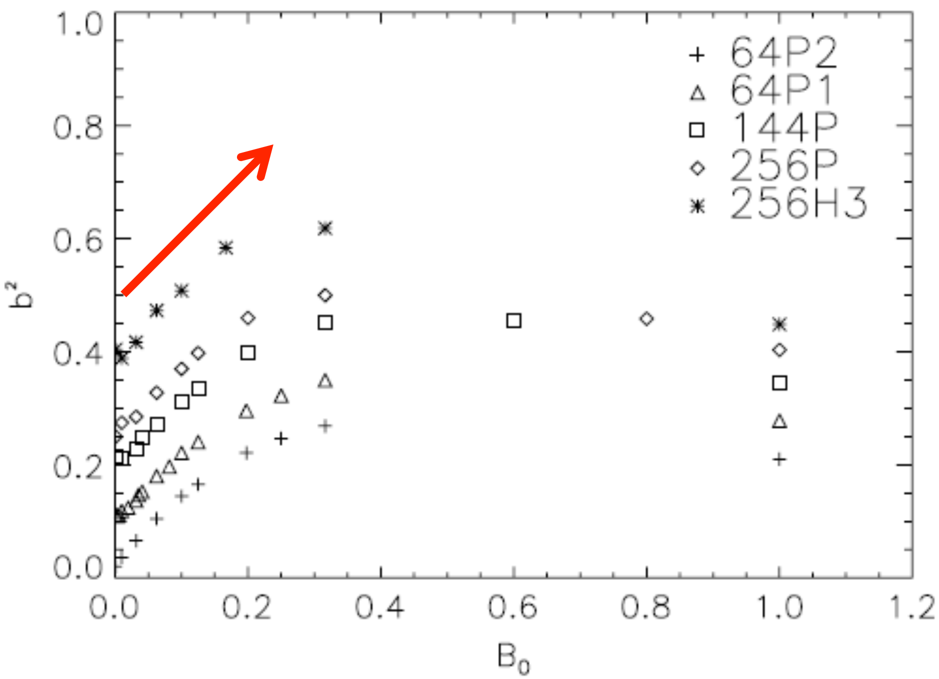


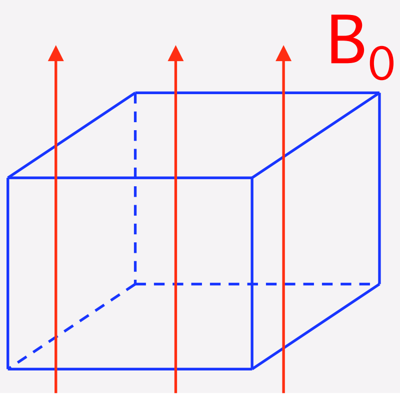


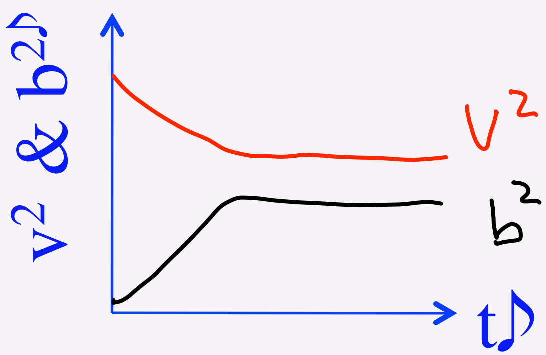
B_0 ↑ ➔ growth timescale

Saturation level vs. B_0







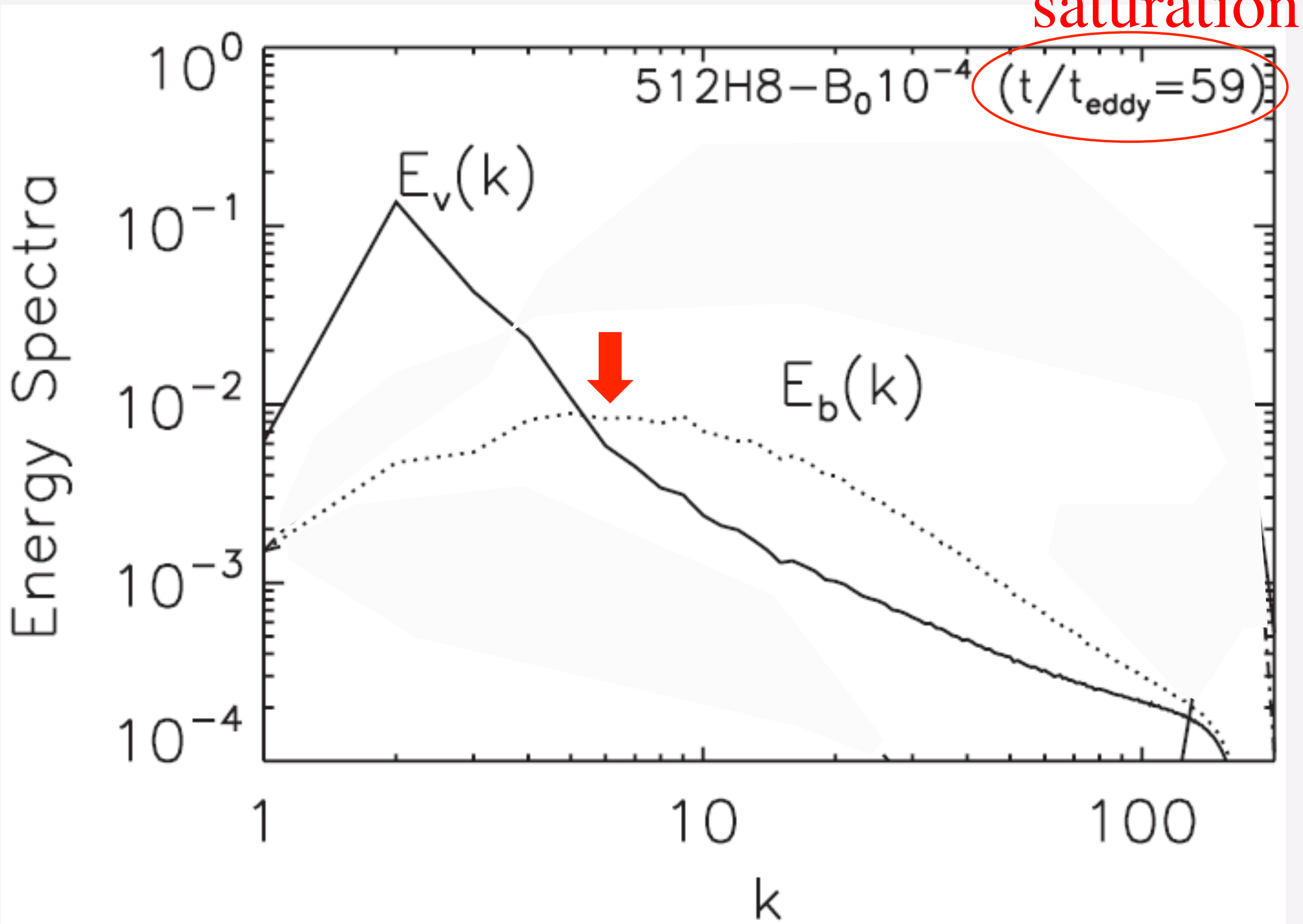


Conclusions for Topic 1

-Turbulence can amplify weak seed B fields
-Two stages of amplification: exp. and linear
-$\langle B^2 \rangle \sim \langle V^2 \rangle$ at saturation

Characteristic scales of B are different from the outer scale

saturation stage

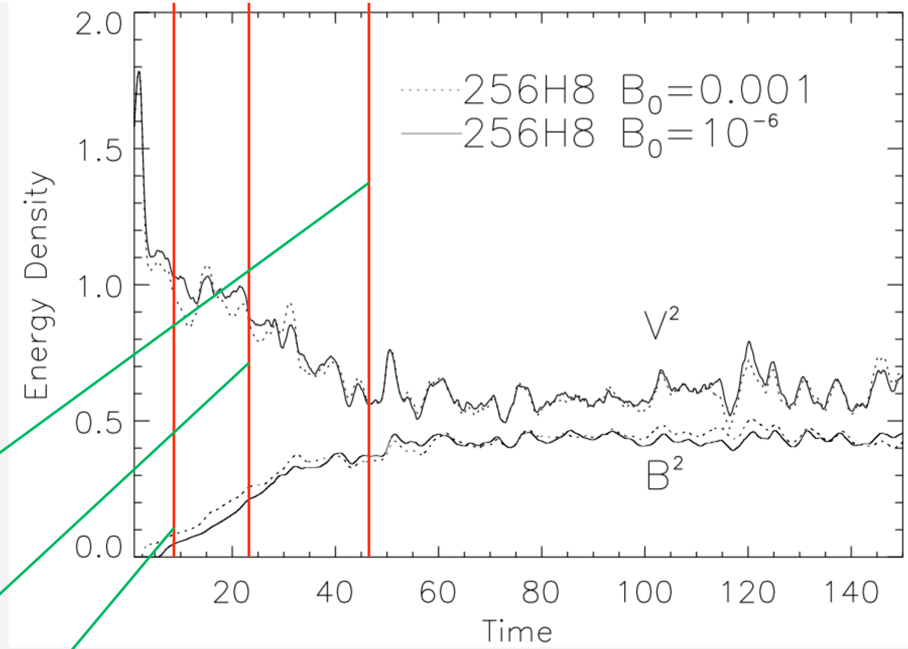


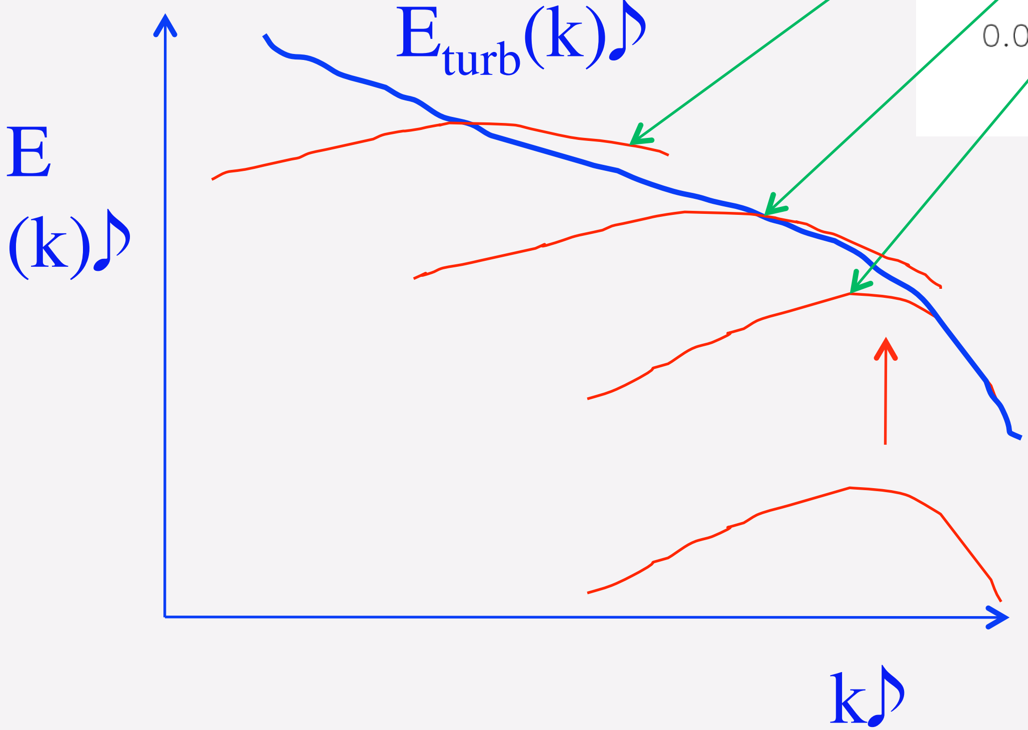
The peak of magnetic energy spectrum

~ 2 or 3 times smaller than the outer scale

(Cho & Vishniac 2000; Cho & Ryu 2009)

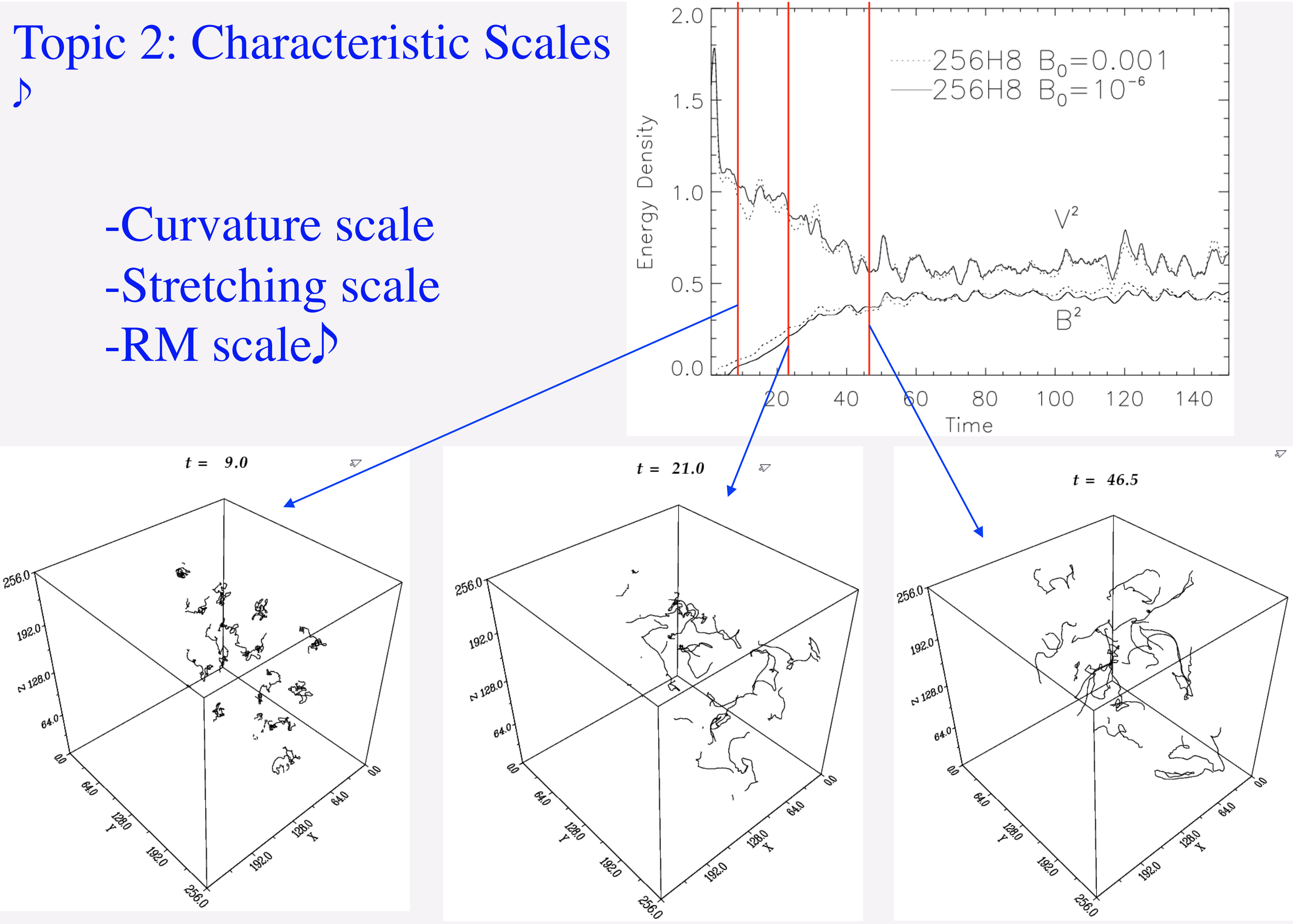
Topic 2: Characteristic Scales
(Cho & Ryu 2009)♪



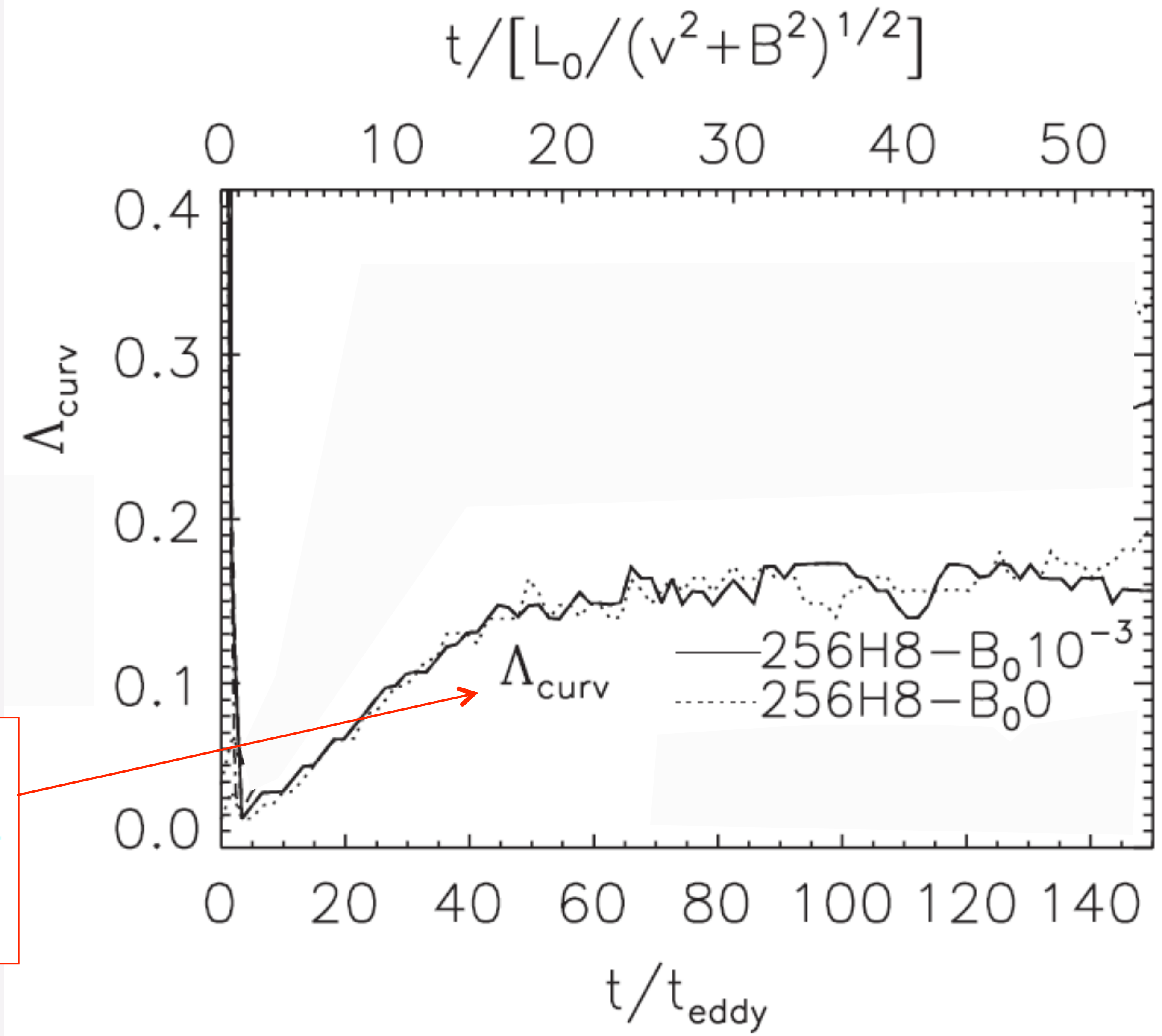


Topic 2: Characteristic Scales ♪

-Curvature scale
-Stretching scale
-RM scale♪

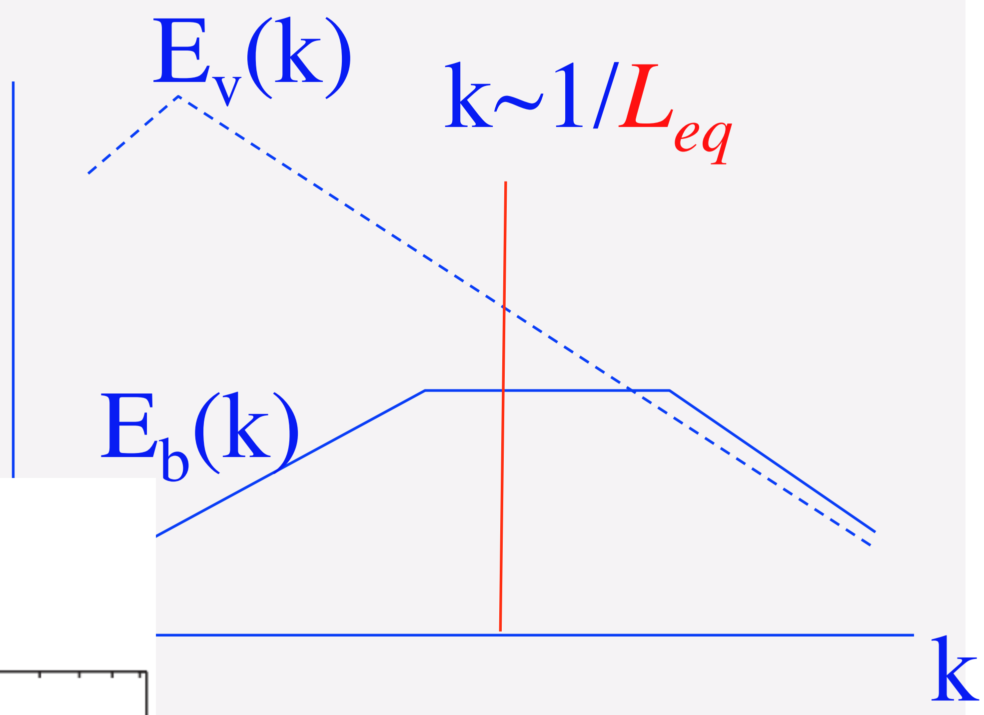


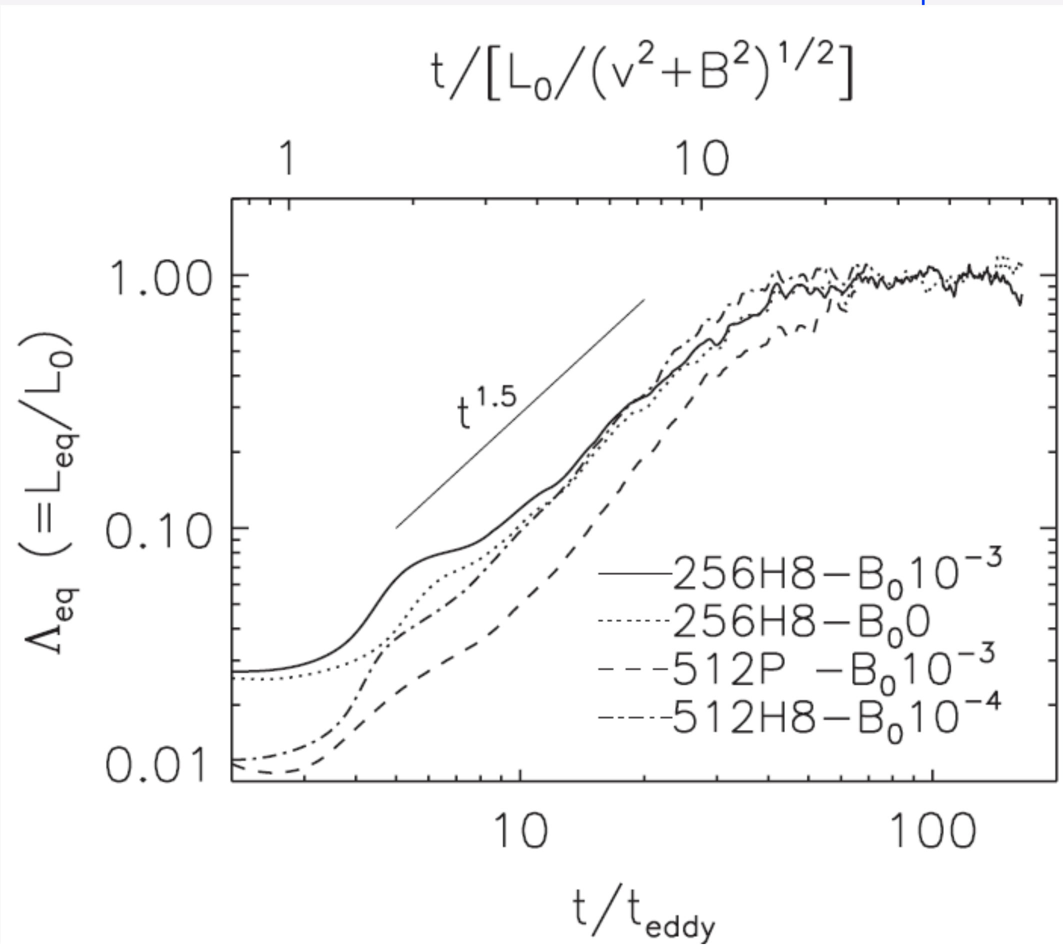
Curvature Scale ~ 15% of the outer scale♪



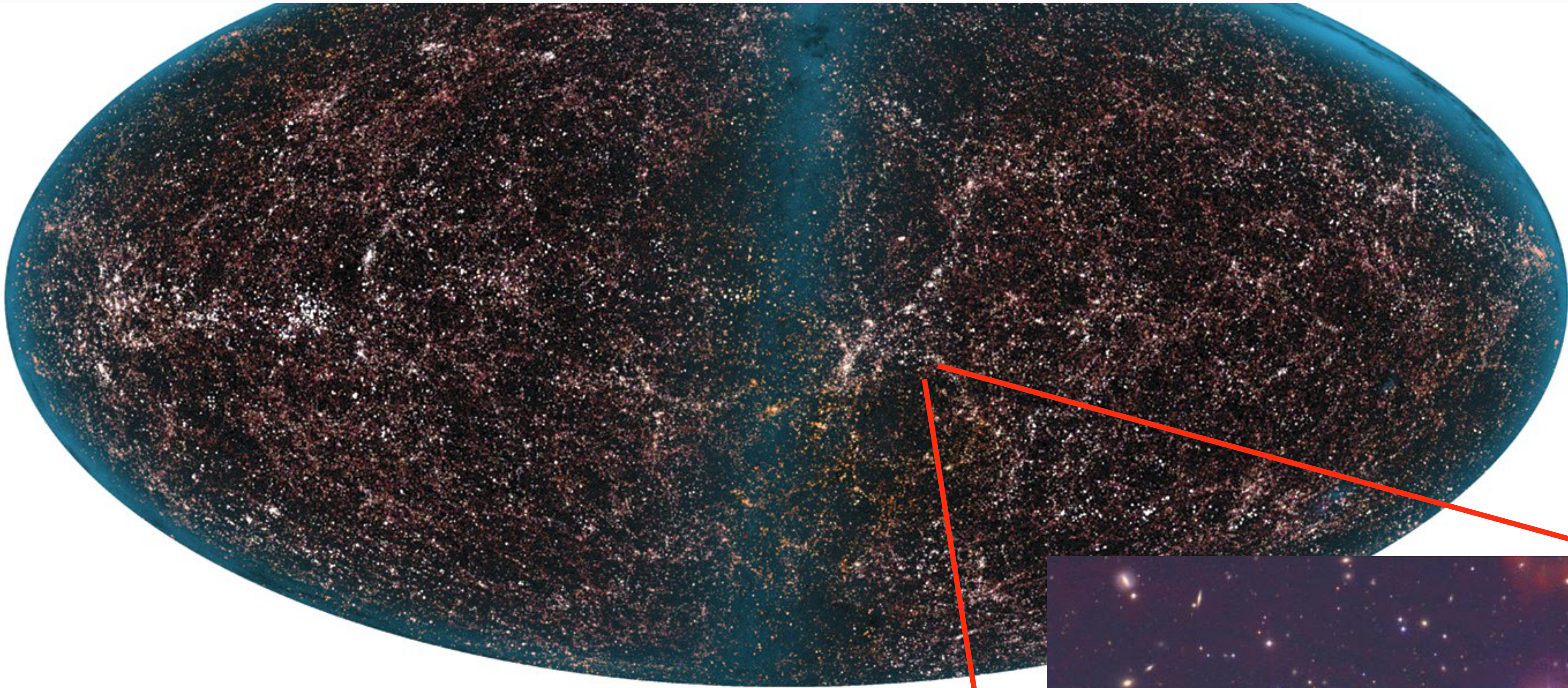
$$\frac{L_{curv}}{L_0}$$

Stretching Scale ∝ $t^{3/2}$♪



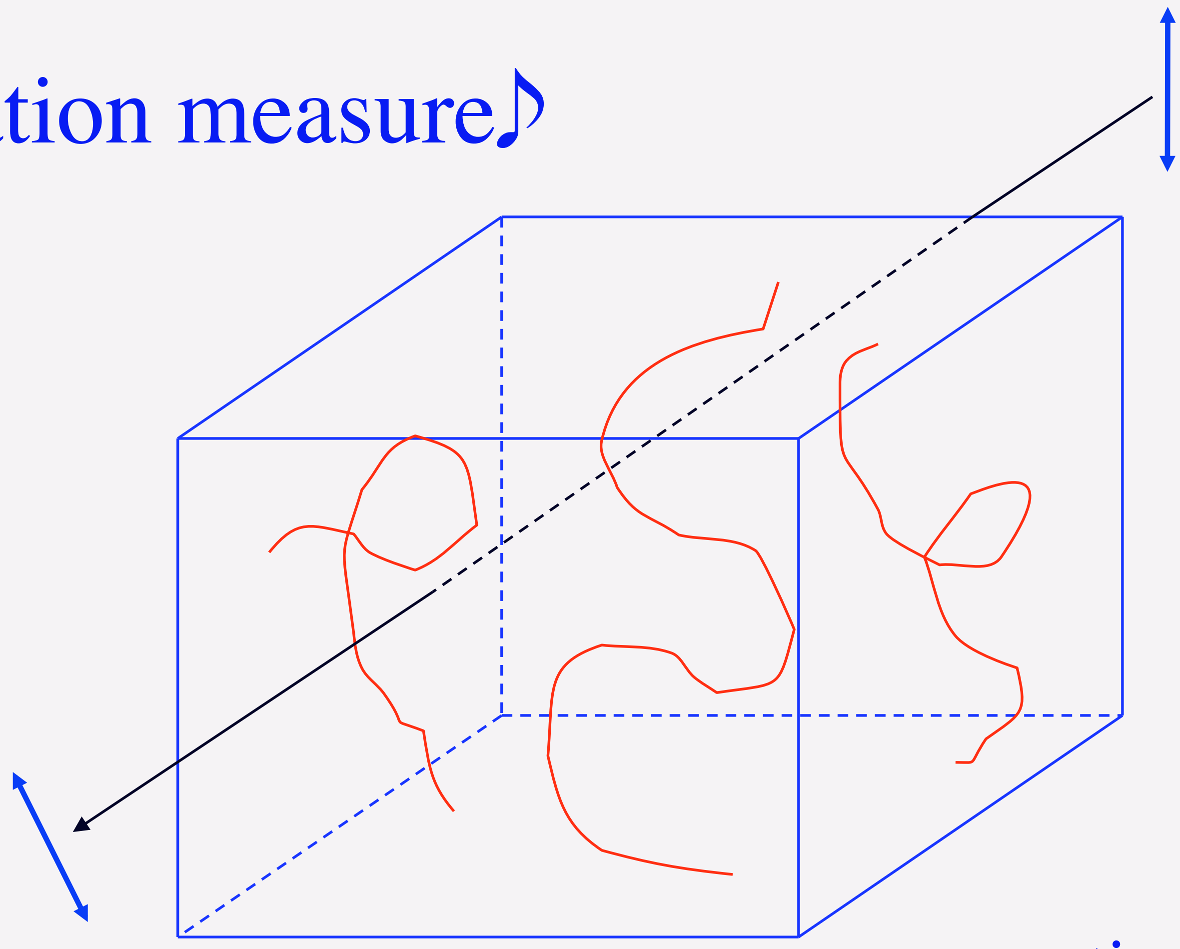


Scale for rotation measure (RM)=? ♪

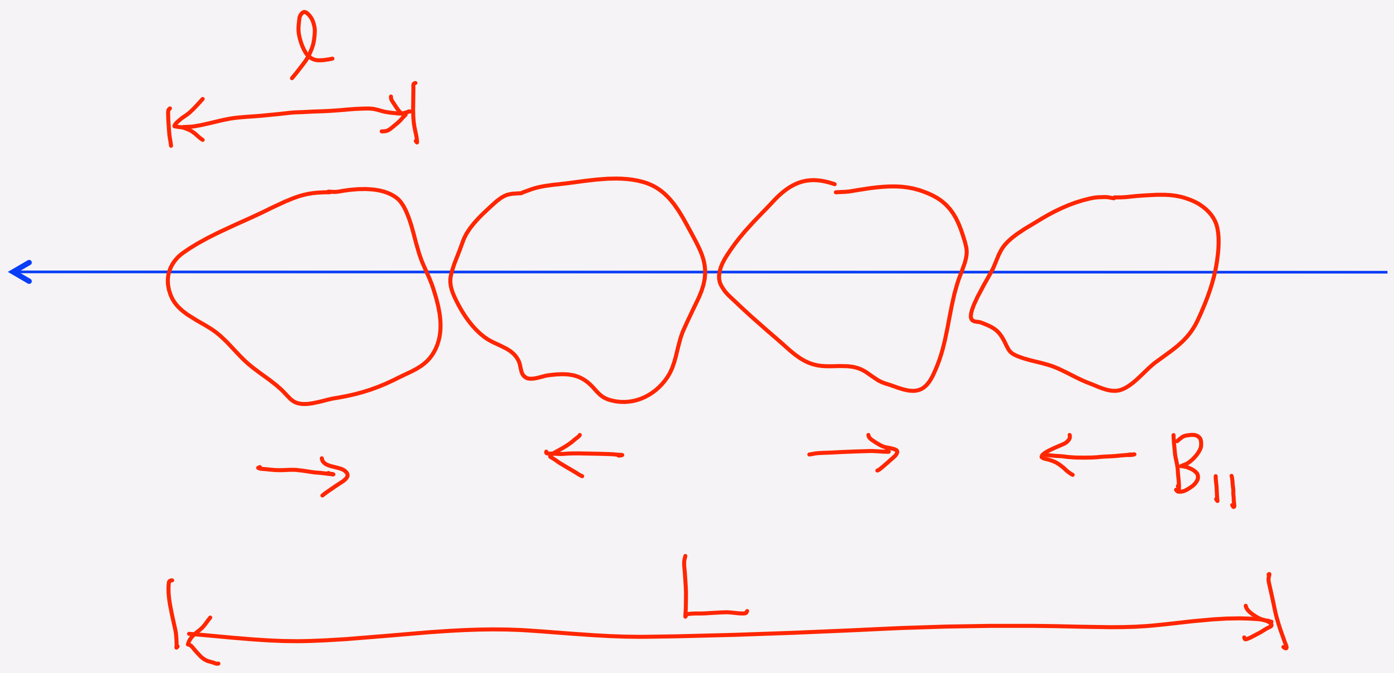


cf. T. Akahori's talk♪

Rotation measure♪



$$\text{angle} \propto \text{RM} \propto \int n_e\, B_{\|}\, dl$$

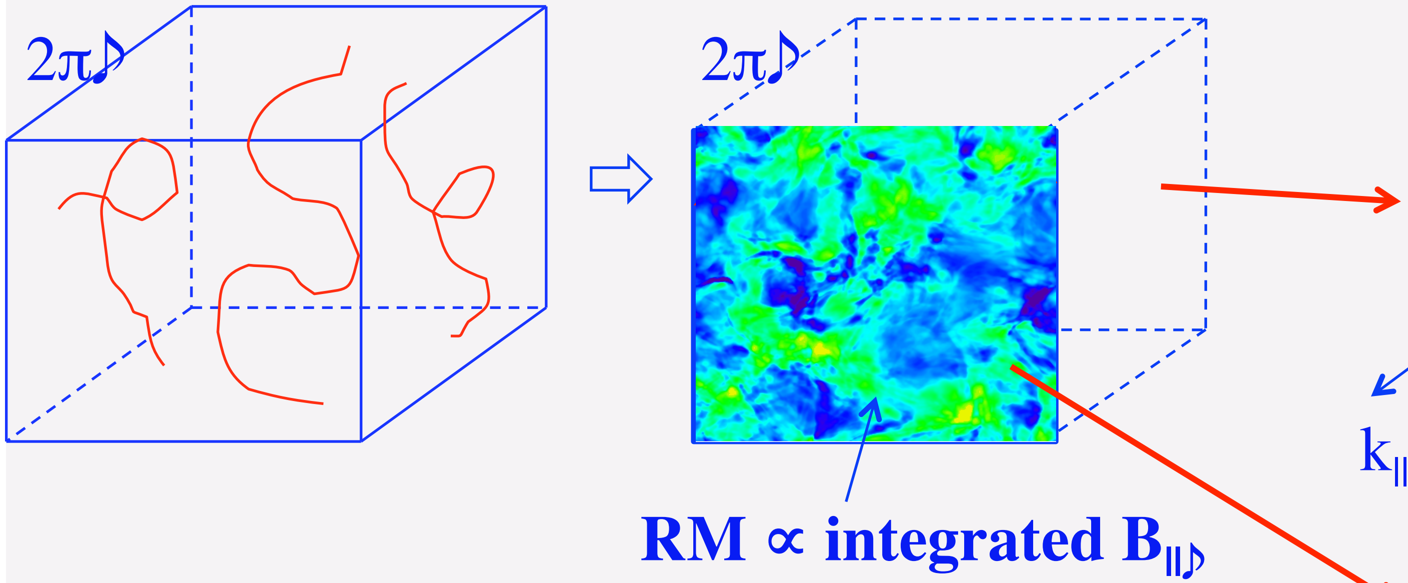


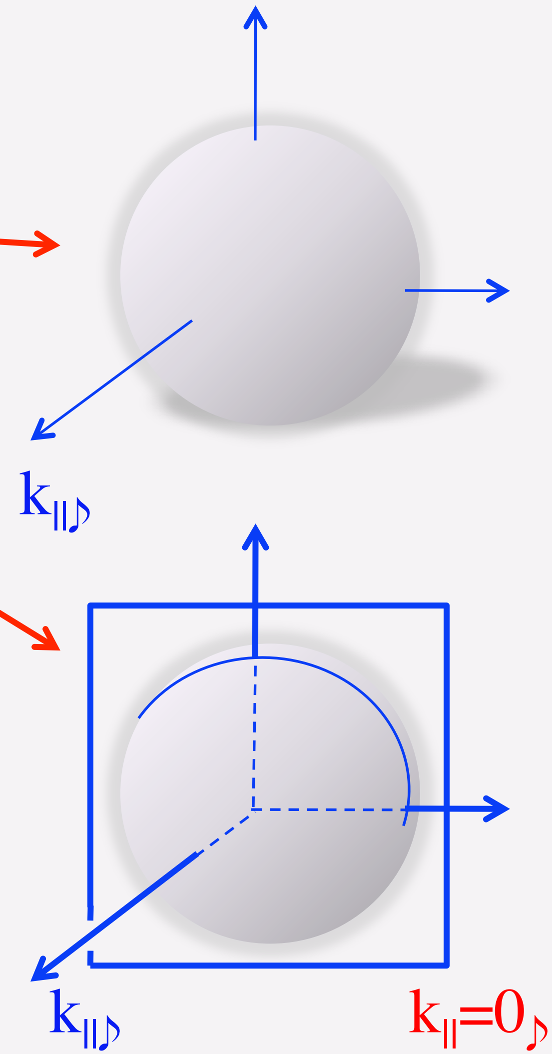
$$\sigma_{RM} \approx (\text{step size}) \cdot \sqrt{\#\text{ of cells}}$$

$$\approx \left(\overline{n_e B_{\parallel}}\,\ell\right) \cdot \sqrt{\frac{L}{\ell}}$$

(For simplicity, let's assume n_e=const.)

$$\rightarrow \text{RM} \propto \int_0^{2\pi} B_{||}\, ds$$





* **Fourier transform of RM**

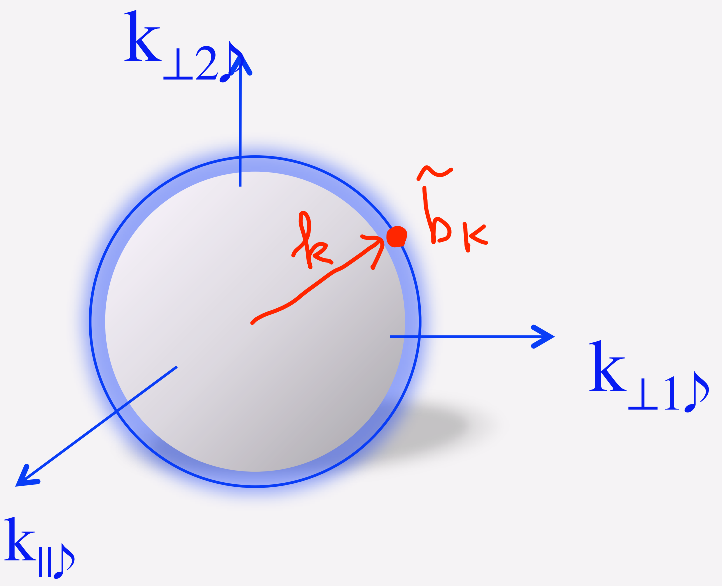
∝ $k_{||}=0$ plane in Fourier space

(× L) ♪

Bad news: I'll use more math.

Good news: I'll use only high school math.

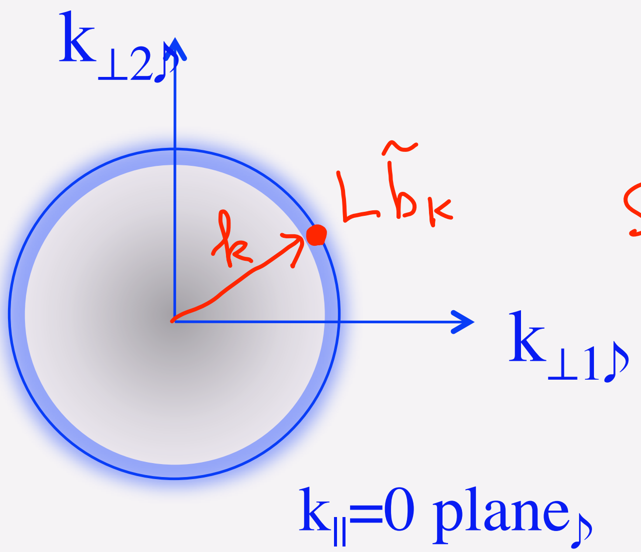
$\vec{B}$



average power of Fourier modes with $|\mathbf{k}|=k$

$$E(k) = 4\pi k^2 \left|\tilde{b}_k\right|^2$$

RM



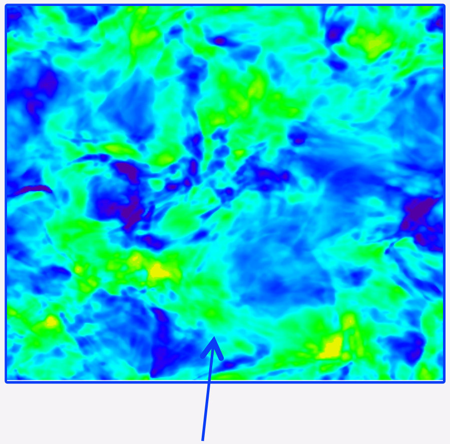
$$\text{Spectrum} = 2\pi k \left|L\tilde{b}_{k,\parallel}\right|^2 = \frac{L^2}{2\cdot 2}\,\frac{E(k)}{k}$$

\ spectrum of RM $=(L^2/4)\,E_B(k)/k$

See also Enβlin & Vogt (2003)

$\sigma_{RM}{}^2 = ?$♪

🡸 $\sigma_{RM}{}^2 = \langle (RM)^2 \rangle$♪



RM♪

Of course, we can calculate $\langle (RM)^2 \rangle$ directly.♪

Another way of getting $\sigma_{RM}{}^2$?♪

($\sigma_{RM}{}^2 = \int E_{RM}(k)\, dk \quad \propto \int E_B(k)/k\, dk$♪ ♪

Therefore we have

$$(\sigma_{RM})^2 = \langle (RM)^2 \rangle = (L^2/4) \int E_B(k)/k \, dk$$

L_{int}♪

From this, we have

$$\sigma^2_{RM} = \frac{L^2}{4} \frac{<B^2>}{\int E(k)dk} \int \frac{E(k)}{k} dk = \frac{L<B^2>}{4} \frac{L\int \frac{E(k)}{k} dk}{\int E(k)dk}$$

$$= \frac{<B^2> L L_{\rm int}}{4}$$

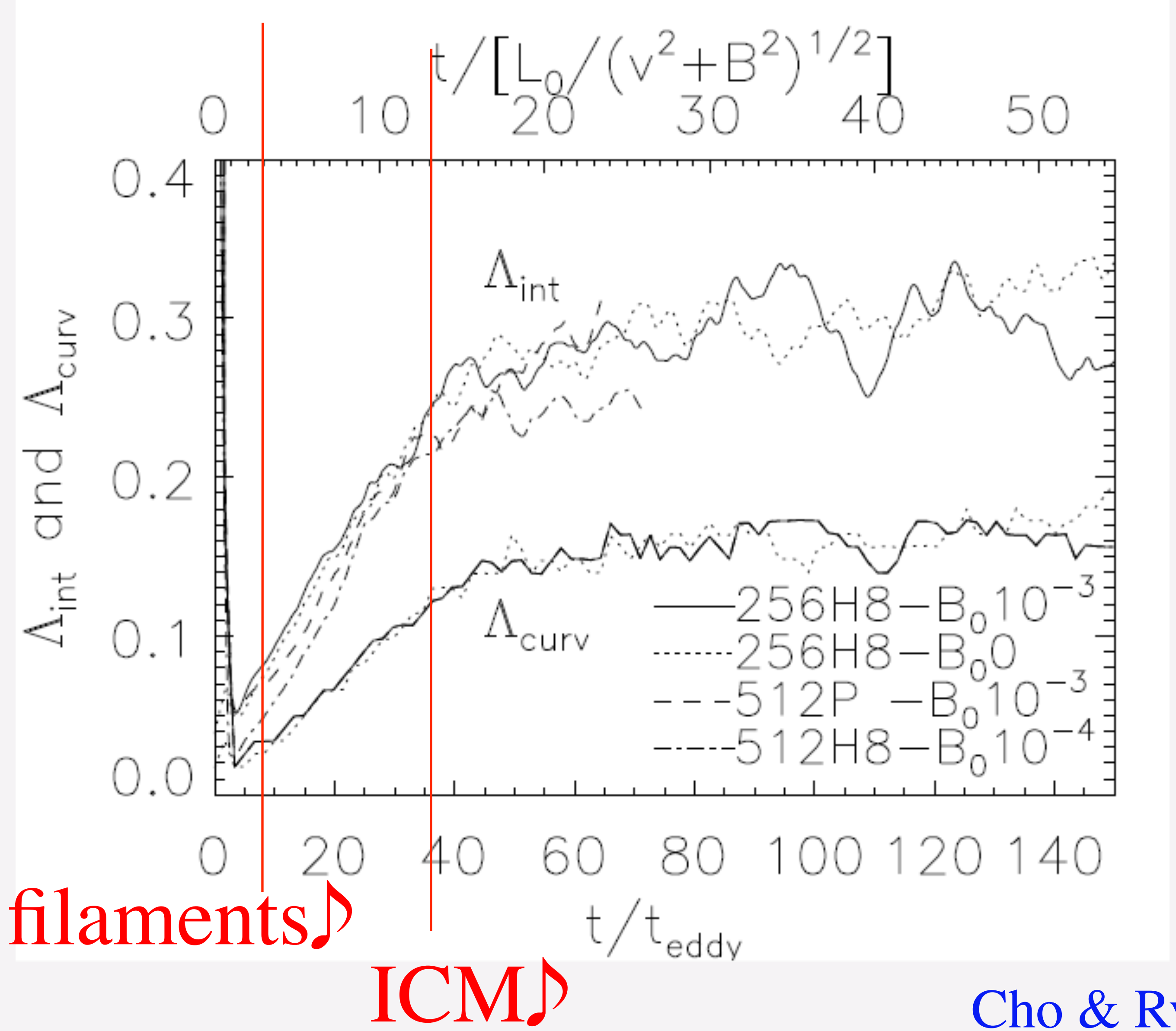
$$\Rightarrow \quad \sigma_{RM} = \frac{<B^2>^{1/2} \sqrt{L L_{int}}}{2} \quad \otimes \quad n_e$$

$$\Rightarrow \quad \boxed{\sigma_{RM} = 0.81 \ \bar{n}_e \frac{B_{rms}\sqrt{L_{int} L}}{2} \ \mathrm{rad \ m^{-2}}}$$

B=μG
L=pc
n_e=cm^{-3}♪

$\Lambda_{int} = L_{int} / L_{outer\ scale}$

$$L_{int} = 2\pi \frac{\int E_b(k)/k\ dk}{\int E_b(k)\ dk}$$



Cho & Ryu 2009

$$\sigma_{RM} = 0.81\ \bar{n}_e \frac{B_{rms}\sqrt{L_{int}L}}{2}\ \mathrm{rad\ m^{-2}}$$

B=μG
L=pc
n_e=cm^{-3}♪

$$\sigma_{RM} \sim 100\ \mathrm{rad\ m^{-2}\ for\ clusters}$$

*We used $L_{outer\ scale}$ ~ 100kpc, L~1Mpc,
n_e~10^{-3} cm^{-3}, …♪

$$\sigma_{RM} \sim 1\ \left(\frac{\bar{n}_e}{10^{-5}\mathrm{cm}^{-3}}\right)\left(\frac{B_{rms}}{0.3\ \mu\mathrm{G}}\right)\left(\frac{L_{int}}{300\mathrm{kpc}}\ \frac{L}{5\mathrm{Mpc}}\right)^{1/2}\ \mathrm{rad\ m^{-2}}$$

for filaments ⊥ to line-of-sights.

*But there are uncertainties…

Conclusions

-Turbulence can amplify weak seed B fields
-Two stages of amplification: exp. and linear

-Stretching scale $\propto t^{3/2}$
-We showed that the relevant length scale for RM is the integral scale L_{int}.

-We obtained estimates of σ_{RM} for clusters and filaments