Magnetic reconnection in coronal plasmas

I.J.D Craig

Department of Mathematics

University of Waikato

Hamilton

New Zealand

Why reconnection?

Reconnection is the only mechanism that can alter the magnetic field topology. It is required in two fundamental problems:

Dynamo theory; How to develop the strongs fields from a weak "seed field".

Coronal heating; How to heat the corona and account for explosive flare release.

Here we concentrate on developing exact anlytic models for 3D coronal reconnection.

The flare problem

In solar flares around 10^{30} ergs are released rapidly, in 100s or so. Magnetic reconnection—a resistive process involving the cutting and rejoining of field lines—is the accepted release mechanism.

But weak coronal resistivity generally leads to energy loss rates that are too slow to account for flare observations.

How can the rate of reconnection be speeded up?

Coronal parameters

In reconnection theory we are dealing with 3D magnetic and velocity fields. Scale the problem using the typical values

$$B_c = 10^2\mathrm{G} \quad l_c = 10^{9.5}\mathrm{cm} \quad n_c = 10^9\mathrm{cm}^{-3}$$

and employ Alfvénic units . Times are measured in units of $\tau_A = l_c/v_A$ where $v_A \simeq 10^9 \mathrm{cm\ s}^{-1}$ is the Alfvén speed.

Energy losses have the units

$$\frac{B^2}{8\pi} {l_c}^3/\tau_A = 4 \times 10^{30} \quad \mathrm{ergs/s}$$

Modest flares require around 10^{27} erg/s.

The coronal resistivity

In these units the resistivity is

$$\eta \simeq 10^{-14}$$

Contrast this with the viscosity coefficient $\nu \simeq 10^{-3}$.
However η multiplies the highest derivatives in the MHD equations and so cannot be neglected. This difficulty meant no exact reconnection solutions were discovered until the mid 1990's.

Now 3D solutions are available that cover spine, fan and separator (no-null) reconnection.
Even so, it is difficult to go beyond the slow Sweet-Parker (1958) rate $\eta^{1/2}$ of energy release.

Governing equations

These are the (MHD) momentum and induction equations

$$\partial_t \mathbf{V} + (\mathbf{V} \cdot \nabla)\mathbf{V} = \mathbf{J} \times \mathbf{B} - \nabla P + \nu \nabla \cdot \mathcal{S}, \tag{1}$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \eta \nabla \times \mathbf{J}, \tag{2}$$

plus constraints

$$\nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{V} = 0. \tag{3}$$

Here P is the plasma pressure, $\nabla \cdot \mathcal{S}$ is the viscous force and

$$\mathbf{J} = \nabla \times \mathbf{B}$$

the current density.

Resistive effects require huge $\mathbf{J}$ gradients.

Key questions

Can analytic solutions valid for arbitrary η be constructed?

Are the models physically realistic?

Can resistive scaling laws be deduced for the model?

Can the spine and fan geometry of the null be exploited, as kinematic studies would suggest?

Fig.1: Field skeleton

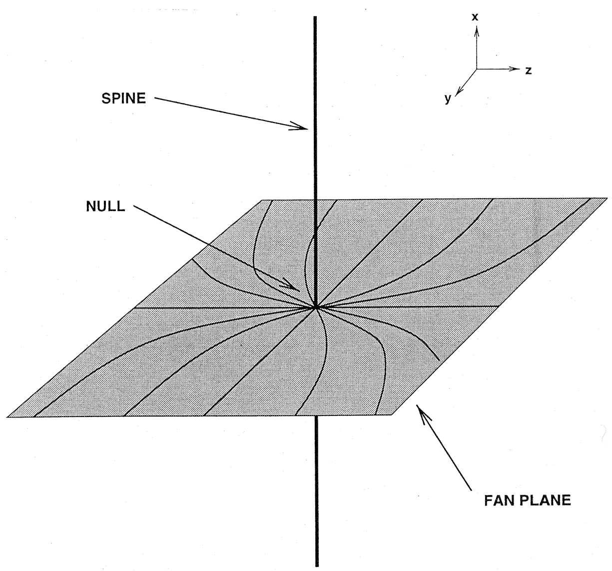


Figure 5.2: Schematic of the separatrix structure of a simple three dimensional X-type null

The field skeleton

The skeleton defines the eigenstructure of $\nabla \mathbf{B}$ close to a null. So a current free X-point, say

$$\mathbf{P} = (x, y, -2z), \quad \Rightarrow \lambda_i = (1, 1, -2).$$

Positive eigenvalues corresponding to outflow (say) must be balanced by inflow along the spine (the z-axis).

Now superpose a disturbance field $\mathbf{Q}$ onto skeleton $\mathbf{P}$. Identically we must have

$$\nabla \times [\nabla \times \mathbf{Q}) \times \mathbf{Q}] = 0. \tag{4}$$

for consistency with momentum equation (1).

If $\mathbf{Q}$ bends the spine fan currents appear; $\mathbf{Q}$ distorting the fan implies tubular currents along the spine.

Fig.2a: Fan current reconnection

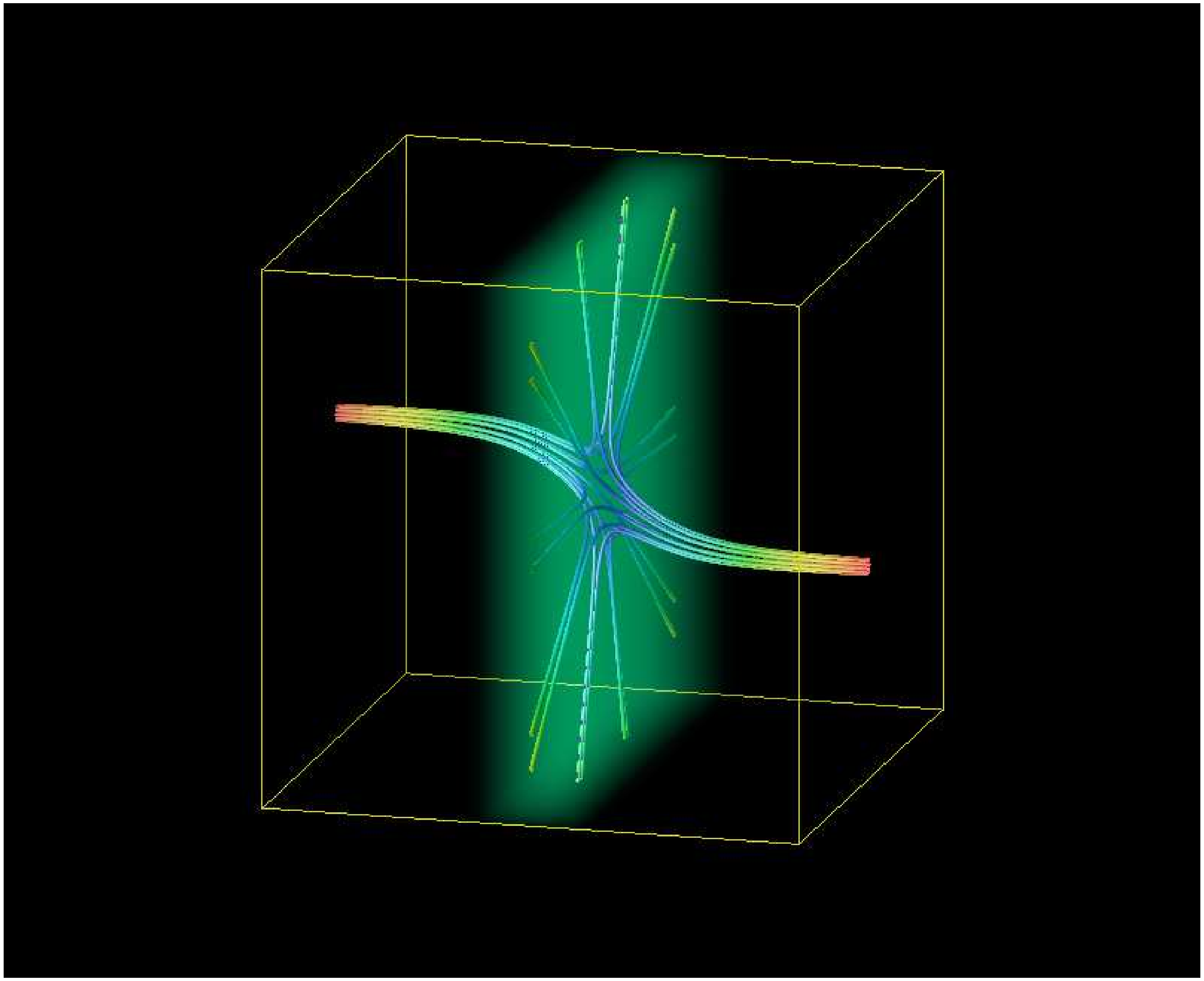
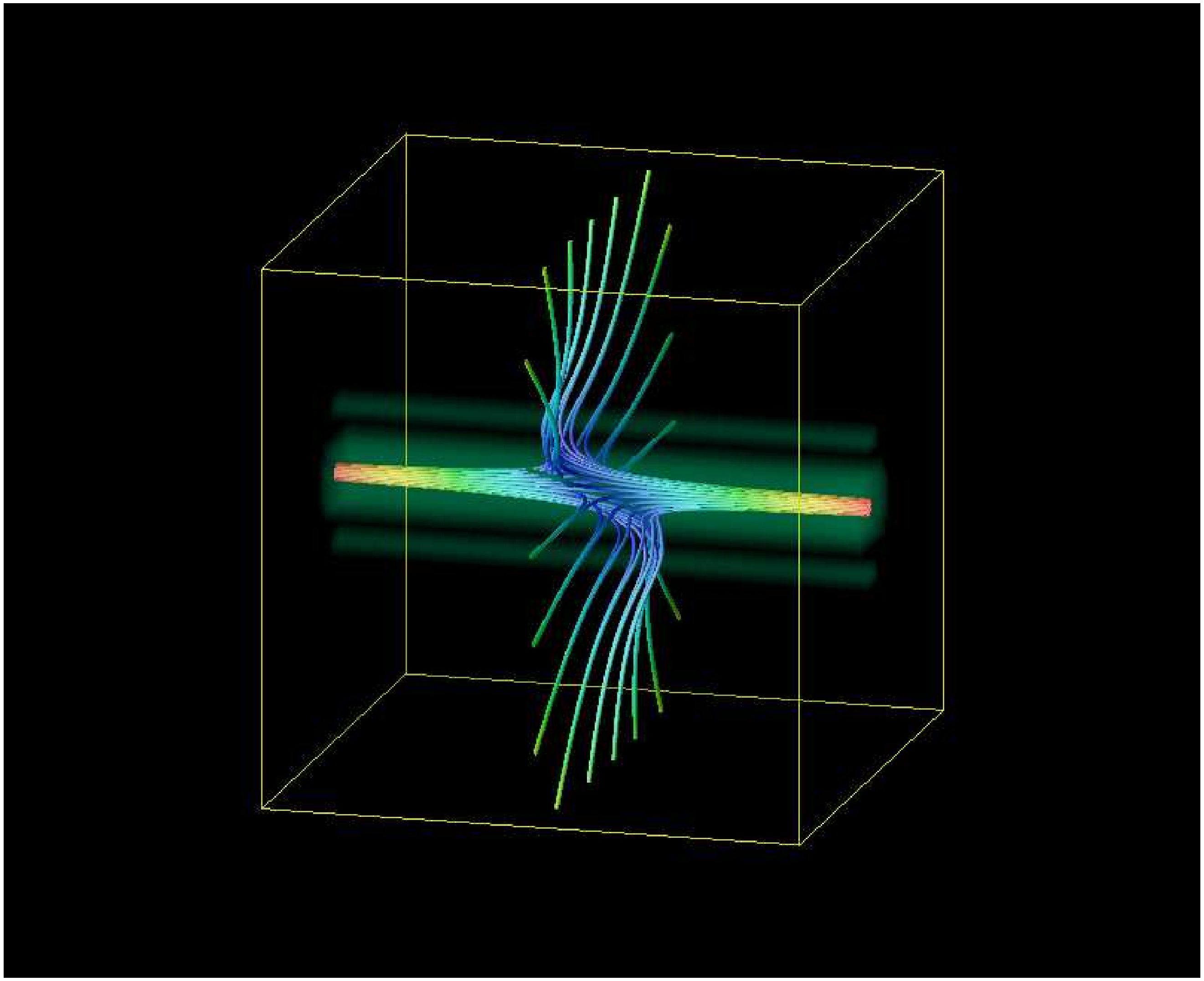


Fig. 2b: Spine current reconnection



Constructing 3D reconnection solutions

For $\eta > 0, \nu = 0$ a typical construction is

$$\mathbf{V} = \alpha\mathbf{P}(\mathbf{x}) + \mathbf{v}(\mathbf{x}, t), \qquad \mathbf{B} = \beta\mathbf{P}(\mathbf{x}) + \mathbf{b}(\mathbf{x}, t),$$

with $\alpha > \beta \geq 0$. Note that β defines “shear” (Craig & Henton 1995).

The prototype fan and spine forms (Craig & Fabling 1996)

$$\mathbf{b}_S = Z(x, y, t)\hat{\mathbf{z}},$$

$$\mathbf{b}_F = X(x, t)\hat{\mathbf{x}} + Y(x, t)\hat{\mathbf{y}},$$

have reduced dimensionality due to condition (4).

Cylindrical models also follow this scheme (Watson & Craig 2002, Tassi et al 2002, Pontin & Craig 2006).

Steady fan solution

The simplest model is the two dimensional fan solution:

$$\mathbf{Q} = [0, Y(x), 0] \quad \mathbf{P} = [-x, y, 0]. \tag{2}$$

The formal solution is

$$\mathbf{B} = \beta\mathbf{P} + \frac{E}{\eta\mu}\mathrm{Daw}(\mu x)\hat{\mathbf{y}}$$

$$\mathbf{V} = \alpha\mathbf{P} + \frac{\beta}{\alpha}\frac{E}{\eta\mu}\mathrm{Daw}(\mu x)\hat{\mathbf{y}}$$

where E (the flux transfer rate) is constant and

$$\mu = \frac{\alpha^2 - \beta^2}{2\alpha\eta} = \frac{\bar{\alpha}}{2\eta} > 0.$$

Fan solution 2D

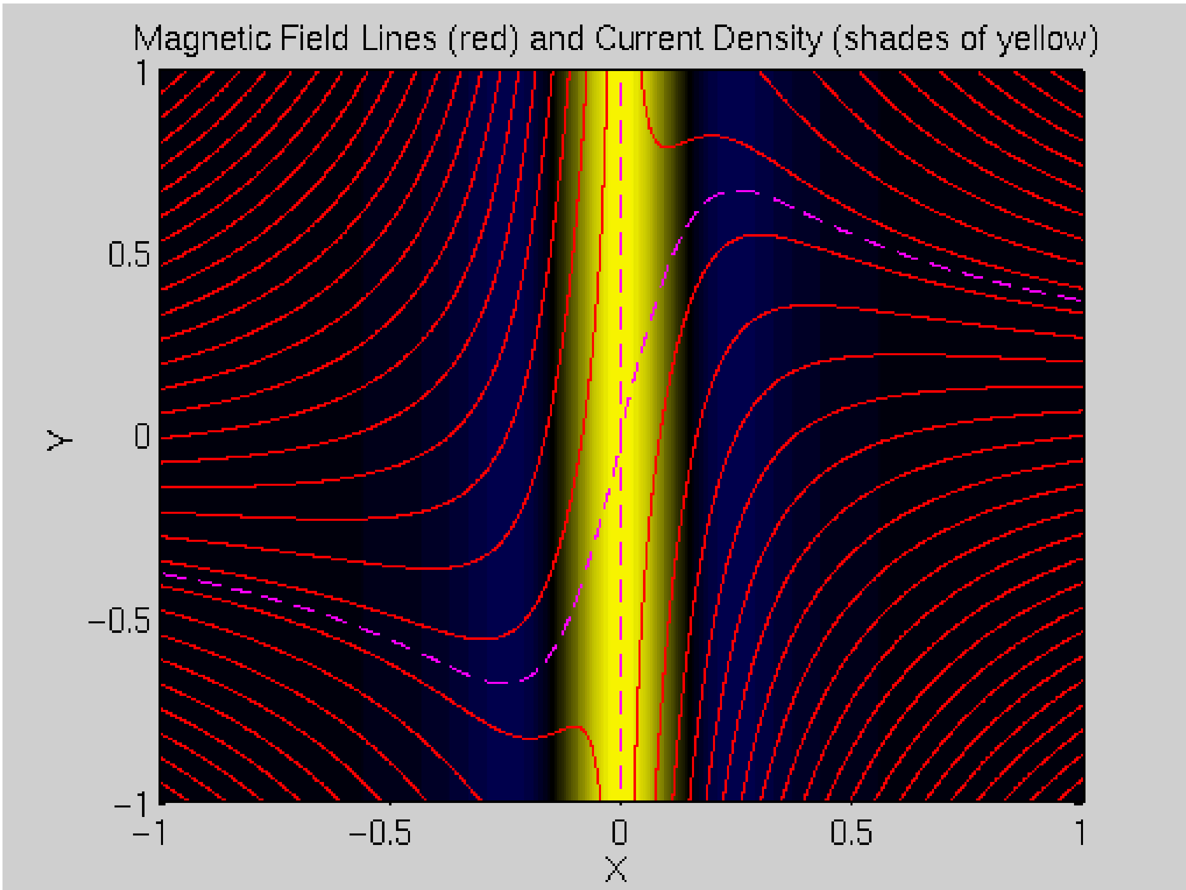


Figure 3: fan solution

Resistive scalings

The Dawson function identifies $x_s = \sqrt{\eta/\bar{\alpha}}$ as the current sheet thickness. Less formally, since the disturbance field satisfies

$$E - \bar{\alpha} x Y = \eta Y_x$$

we can equate outer and inner approximations, namely

$$Y_{out} \simeq \frac{E}{\bar{\alpha} x} \quad \text{and} \quad Y_{in} = \frac{E}{\eta} x$$

to get the same result for x_s. The field in the sheet

$$Y_s \simeq \frac{E}{\sqrt{(\bar{\alpha}\eta)}}.$$

therefore increases with η for fixed E !

Ohmic losses

This leads to Ohmic losses that diverge with η

$$W_\eta = \int \eta J^2 dV \simeq \eta \frac{Y_s^2}{x_s} \sim \eta^{-1/2}.$$

The problem—common to all exact analytic solutions—is that the flow can only maintain sheets with $Y_s \le \alpha$. Therefore Y_s has to be limited to the flow amplitude $Y_s^* \simeq \alpha$. When this is done

$$W_\eta \to \eta^{1/2}\, {Y_s^*}^{5/2}. \tag{5}$$

The rate differs from Sweet-Parker by the flux pile-up factor ${Y_s^*}^{5/2}$ which could exceed 10^2 $(600$ Gauss fields).

This result is found to hold for all fan-reconnection solutions.

Summary

Exact reconnection models can be constructed in 2D and 3D.

To enhance the Ohmic dissipation rate (5) can invoke a current limiting resistivity, $\eta_{eff} \rightarrow 10^6\eta \simeq 10^{-8}$. Then $W_\eta \rightarrow 10^{-2}$ which equates to 4×10^{28} erg/s for a flux pile-up factor of one hundred.

Other possible enhancements include:

Multiple null solutions;
Inclusion of Hall and viscous effects;
Using 3D turbulence models.

Flare-like release rates can be approached using these modifications but no model is yet accepted!