Magnetic pressure driven jet flow in young stellar systems

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Meteorites: Foundation Stones of the Planets

Cross section of the Allende carbonaceous chondrite

Chondrules (melt objects)
1800 K

Calcium Aluminium Inclusion (CAI) (condensates)
2000 K

Surrounding matrix (contains high temperature condensates)
500 K

CAIs – some of the first solids to form in the solar system ~ 4.57 billion years ago

Chondrules formed ~ million years after CAIs

Igneous rocks surrounded by “cold” sedimentary material.
“Hot” rocks surrounded by “cold” material
Formation: local or non-local process?

Chondrules – first noted in 1802; first formation theory in 1877 approximately 20 formation theories developed since 1877

Meteoriticists – various lines of evidence (e.g. meteorites are from the asteroid belt: ~ 3 AU from the Sun) suggest local formation

Astrophysicists – most energy at or near the inner rim of the disk: non-local
What is the Jet Flow Model?


Nuth III J. A., 2001 American Scientist, 89, 228-235
Jet Processing of an Accretion Disk

- Lifetime $10^6$ - $10^7$ years
- Lifetime outflow mass ejected $\approx 0.1$ M$_\odot$
- Outflow “rock” mass ejected $10^{-3}$ - $10^{-4}$ M$_\odot$
- Total rock mass of the planets $\approx 10^{-4}$ M$_\odot$

10% fall back implies $10^{-4}$ - $10^{-5}$ M$_\odot$

returned to the solar nebula

Predictive Theory:
Chondrules/CAI formed over a $10^7$ year period. (Liffman 1992)
Chondrules/CAI to be found in comets (Skinner 1990, Liffman, Shu et al.)
Dust particles obtained from a Kuiper Belt (i.e. ~ 40 AU) comet

Same pattern as seen in meteorites:
  High formation temperature
  (> 1400K) rocks surrounded by cold material – in this case, ice.

Chondrules and CAIs also found in Comet Wild 2
  Nakamura et al. (2008)

strongly suggest non-local formation.
At 40 AU high temperature heating is difficult to understand
Observations - forsterite dust formed in the interior disk regions surrounding Sun-like stars

“star burst” produces forsterite ($\text{Mg}_2\text{SiO}_4$) grains from amorphous dust within or around $\sim 0.5$ AU from the star.

These type of grains are observed in comets. Non-local formation!

Abraham et al. Nature 2009
What is the jet flow mechanism?

Will consider magnetic pressure, instead of centrifugally driven.
Protostellar/T Tauri Jets

Two basic models:

MPD model

CWD Model

Why the dichotomy?

Flow $\parallel$ to $\mathbf{B}$

Flow $\perp$ to $\mathbf{B}$
Relative motion between the disk and stellar magnetic field generates an electric field.

\( \Omega_* \) – stellar angular velocity
\( \mu_* \) – stellar mass
\( B_* \) – stellar magnetic field
\( j \) – current density

\( R_t \) – truncation radius
\( R_{co} \) – co-rotation radius

Nothing wrong with the MHD description, but the J, B, E description may provide some different insights.
Star-Disk Electric Circuit

Toroidal magnetic field induced within the disk
Magnetic Scale Height

The $z$ component of the steady state momentum equation

$$\rho (v \cdot \nabla) v = -\nabla p + \rho g + j \times B$$

hydrostatic ($v = 0$), isothermal form:

$$\frac{\partial \rho}{\partial z} + \kappa z \rho + \xi z = 0$$

which has the solution

$$\rho (r, z) = \rho_c (r) \exp \left[ - \left( \frac{z}{h} \right)^2 \right] - \rho_\infty \left( 1 - \exp \left[ - \left( \frac{z}{h} \right)^2 \right] \right)$$

where

$$\rho_\infty = (\mu_0 \sigma r)^2 \left( \frac{B_z^2}{\mu_0} \right) \left( \frac{\Omega^*}{\Omega_K} - 1 \right)^2$$
Magnetic Scale Height

For the magnetically confined disk there is a distance from the central plane of the disk, $H_B$, where the density of the disk goes to zero:

$$\rho(r,z) = 0 \Rightarrow z = H_B = h\sqrt{\ln\left[1 + \frac{\rho_c(r)}{\rho_\infty}\right]}$$

Although this is the true height of the disk, it has a problem:

$$B_z \rightarrow 0 \Rightarrow \rho_\infty \rightarrow 0 \Rightarrow H_B \rightarrow \infty$$

A more consistent definition gives the e-folding magnetic scale height, $h_B$.

$$\rho(r,z) = \frac{\rho_c}{e} \Rightarrow h_B = h\sqrt{\ln\left[1 + \frac{1 - 1/e}{1/e + \rho_\infty / \rho_c(r)}\right]}$$

Which has the desired property that

$$B_z \rightarrow 0 \Rightarrow h_B \rightarrow h$$
Example: X-ray Binary

\[ \sigma = \frac{1}{\mu_0 \eta_*} \approx \frac{1}{\mu_0 \nu_*} \]

\[ r_* = 10^4 \text{ m} \]
\[ M_* = M_{\text{sol}} \]
\[ \dot{M} = 10^{-8} \text{ M}_{\text{sol}} / \text{yr} \]
\[ r_c = 2r_* \]

\[ \nu_* = \alpha_{ss} C_s h_B \]

Liffman and Bardou (1999)
Star-Disk Torque

Suppose there is a trans-field, short-circuit – perhaps due to Gravity Drift

The $J, B, E$ model - same as the MHD answer for the torque on the disk from the star

$$\frac{d\tau}{dr} = \frac{4\pi r^2 B_\phi B_{*z}}{\mu_0}$$

Suppose there is a trans-field, short-circuit – perhaps due to Gravity Drift
Gravitational Drift drives a radial current that short circuits the star-disk current

\[ V_D = \frac{F_{\perp} \times B}{qB^2} \]

**Drift Velocity**

\[ j_{\perp r} \approx n_i G M m_i z B_\phi \left( \frac{r^2 + z^2}{(r^2 + z^2)^{3/2}} \right) \frac{B_{\rho}^2 + B_z^2}{R_t} \]

**Gravitational Drift Current**

Liffman (2007)
The radial transfield currents produce toroidal fields above and below the disk. $j \times B$ flow points away from the disk.

acceleration region starts near the top of the disk ($z = z_0$)

acceleration region finishes when the radial current is exhausted ($z = z_T$)

Assume $j_r$ is a constant with $z$

\[
B_\phi(r,z) = \frac{\mu_0 I_r(r)}{2\pi r_0} \left(1 - \frac{z - z_0}{z_T - z_0}\right) = 2.7 \times 10^{-4} \frac{(I_r/10^{13} \text{A})}{(r/0.05\text{AU})} \left(1 - \frac{z - z_0}{z_T - z_0}\right) \text{T}
\]

Liffman (2007)
Integrate $j_r \times B_\phi$ over volume – total force is independent of $j_r$ variation with $z$

$$F = \frac{\mu_0 I_M I_r}{4\pi} \ln \left( \frac{r_o}{r_i} \right) \left( 2 - \frac{I_r}{I_M} \right)$$

This implies that the ejection speed of the gas is independent of $j_r$ variation with $z$
Potentially high exhaust speed

\[ \vec{v}_e \approx \sqrt{\frac{\mu_0}{2\rho_0}} \frac{l_r(r_0)}{\pi r_0} = 337 \sqrt{10^{-12} \text{kg m}^{-3}} \left( \frac{l_r}{10^{13} \text{A}} \right) \left( \frac{0.05 \text{AU}}{r_0} \right) \text{km s}^{-1} \]
The $z$ component of the steady state momentum equation

$$\rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \rho \mathbf{g} + \mathbf{j} \times \mathbf{B}$$

We can obtain a general flow speed “Bernoulli-like” equation

$$V_z^2(z_T) = V_z^2(z_0) + 2GM \left( \frac{1}{\sqrt{r^2 + z_T^2}} - \frac{1}{\sqrt{r^2 + z_0^2}} \right) + \frac{\mu_0 I_M I_r}{2\pi^2 \rho_0 r_0^2}$$

This allows us to deduce solutions for distance, speed, density of the flow as a function of time

Liffman et al. (2011)
How can toroidal fields drive the flow? Intuitive model

Step 1
- Star
- Coronal field $j_M$
- Disk field $j_D$
- Corona

Step 2
- Stellar wind
- Current loops
- Jet

Step 3
- Stellar wind
- Jet
- Disk
Predictions

This simple theory leads to a number of, potentially, observable predictions

(1) The ejection speed of the flow increases as one approaches the star

\[ v_e = \sqrt{\frac{\mu_0}{2\rho_0}} \frac{I_r(r_0)}{\pi r_0} = 337 \sqrt{\frac{10^{-12} \text{ kg m}^{-3}}{\rho_0}} \left( \frac{I_r}{10^{13} \text{ A}} \right) \left( \frac{0.05 \text{AU}}{r_0} \right) \text{ km s}^{-1} \]

(2) Stellar rotation period may be a fundamental period of the flow

(3) The flow starts at \( R_t \) and ends at \( R_o \) when the transfield current runs out

\[
\left( \frac{R_t}{R_{co}} \right)^{3/2} - 1 = 2 \left( \frac{R_t}{R_{co}} \right)^{3/2} \left( 1 - \left( \frac{R_t}{R_o} \right)^{1/2} \right) - \frac{5}{4} \left( 1 - \left( \frac{R_t}{R_o} \right)^2 \right) \]

Current balance equation
Predictions

Mass ejection is proportional to mass accretion

\[ \dot{M}_w = \frac{\dot{M}_a}{4} \left( \frac{R_o^2}{R_t^2} - 1 \right) \]

Mass ejection rate and speed increases as the inner edge approaches the star.

Star

Jet Flow

\( R_{co} \)

\( R_t \)
Predictions

Mass ejection is proportional to mass accretion

\[ \dot{M}_w = \frac{\dot{M}_a}{4} \left( \left( \frac{R_o}{R_t} \right)^2 - 1 \right) \]

Mass ejection rate and speed increases as the inner edge approaches the star.

![Graph showing the relationship between mass ejection and the ratio of the inner edge to the critical radius.](image)
Predictions

Mass ejection is proportional to mass accretion

\[ \dot{M}_w = \frac{\dot{M}_a}{4} \left( \left( \frac{R_o}{R_t} \right)^2 - 1 \right) \]
Laboratory Magnetic Jet Flows

Fastest speed listed in the literature ~ 200 km/s
Professor Aleksej Ivanovich Morozov
Conclusions

(1) Much of the solid material in the solar accretion disk underwent thermal processing with radial transport

(2) A solar bipolar jet flow/rim wind could have provided the formation and transport mechanism for this material

(3) Mass ejection is proportional to mass accretion

(4) Toroidal fields may power the jet flows.

(5) Jet flows produced at the inner rim of the disk

(6) Mass ejection rate increases as the inner edge of the disk approaches the star and decreases as the inner edge approaches the co-rotation radius

(7) Inner disk is compressed by the jet flow

Meteorites may offer a way of deducing the mechanism for magnetically-driven outflows
Thank you

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MHD Equations

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]  
Mass Conservation

\[ \frac{\partial \left( \rho \mathbf{v} + \varepsilon_0 \mathbf{E} \times \mathbf{B} \right)}{\partial t} + \nabla \cdot \left( \mathbf{P} + \rho \mathbf{vv} - \tilde{\mathbf{M}} \right) = 0 \]  
Momentum Conservation

\[ \tilde{\mathbf{M}} = \frac{\mathbf{B} \mathbf{B}}{\mu_0} + \varepsilon_0 \mathbf{E} \mathbf{E} - \mathbf{i} \left( \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2 \mu_0} \right) \]  
Maxwell Stress Tensor

\[ \frac{\partial \left( u + \frac{1}{2} \rho v^2 + \frac{1}{2} \varepsilon_0 \mathbf{E}^2 + \frac{B}{2 \mu_0} \right)}{\partial t} + \nabla \cdot \left( \Phi_{ch} + \mathbf{v} \left( u + \frac{1}{2} \rho v^2 \right) + \mathbf{v} \cdot \mathbf{P} + \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \right) = 0 \]  
Energy Conservation

\[ u = \frac{p}{\Gamma - 1} \]  
Equation of State

\[ j = \sigma \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \]  
Ohm’s Law

And Maxwell’s Equations
Energy Conservation

We want the conservative form: \[ \nabla \cdot (vQ) = 0 \]
which \[ Q \] is a flow constant.

Steady state, isotropic pressure
\[ \nabla \cdot \left( v\left( u + \frac{1}{2} \rho \mathbf{v}^2 + p \right) + \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \right) = 0 \]

Infinite conductivity
\[ \mathbf{E} + v \times \mathbf{B} = 0 \]

Equation of State
\[ u = \frac{p}{\Gamma - 1} \]

\[ \nabla \cdot \left( v\left( u + \frac{1}{2} \rho \mathbf{v}^2 + p + \frac{\mathbf{B}^2}{\mu_0} \right) - \frac{\mathbf{B}}{\mu_0} v \cdot \mathbf{B} \right) = 0 \]
\[ \nabla \cdot \left( \mathbf{v} \left( \mathbf{u} + \frac{1}{2} \rho \mathbf{v}^2 + \mathbf{p} \right) \right) = 0 \quad + \text{other flow constants} \]

\[ \quad \downarrow \]

\[ \left( \frac{\frac{\mathbf{v}}{C_s}}{\mathbf{v}} - 1 \right) \frac{d\mathbf{v}}{\mathbf{v}} = \frac{dA}{A} \quad \text{where} \quad C_s^2 = \frac{\Gamma \rho}{\rho} \]

De Laval Nozzle Equation
Converging/Diverging Nozzle

\[ \left( \frac{v}{C_s} \right)^2 - 1 \frac{dv}{v} = \frac{dA}{A} \]

\( \nu = 0, \; dv > 0, \Rightarrow dA < 0; \)

\( \nu = C_s, \; dv > 0, \Rightarrow dA = 0; \)

\( \nu > C_s, \; dv > 0, \Rightarrow dA > 0. \)
\[ \nabla \cdot \left( u + \frac{1}{2} \rho v^2 + p + \frac{B^2}{\mu_0} \right) - \frac{B}{\mu_0} v \cdot B = 0 \]

\[ \nabla \cdot \left( v \left( u + \frac{1}{2} \rho v^2 + p + \frac{B^2}{\mu_0} \right) \right) = 0 \]

\[ \left( \frac{v}{C_F} \right)^2 - 1 \frac{dv}{v} = \frac{dA}{A} \quad \text{Magnetic Nozzle} \]

where

\[ C_F^2 = C_A^2 + C_s^2 \]

Fast Magnetosonic Speed

\[ C_A^2 = \frac{B^2}{\mu_0 \rho} \]

Alfvén Speed
C_A and C_S

\[ C_S = 11.8 \left( \frac{T}{10^4 \text{ K}} \right)^{1/2} \text{ km s}^{-1} \]

and

\[ C_A = 282 \left( \frac{B}{100 \text{ G}} \right) \left( \frac{\rho}{10^{-12} \text{ g cm}^{-3}} \right)^{-1/2} \text{ km s}^{-1} \]

So there exists the possibility of high speed flow, relatively low temperature flow in a magnetic jet.
Radial Transport of Processed Material in Circumstellar Disks

The dust in the ISM has an amorphous structure.

Inner disk dust ~ 90% crystalline silicate. Outer disk dust ~40% crystalline silicate

The dust in inner disks is more processed. Evidence suggests that the dust is processed in the centre of the disk and then moves radially outwards.

Van Boekell et al. 2004
Transport mechanisms from the inner to outer sections of the accretion disk

transport mechanism from the inner to outer regions of YSO accretion disks

Outflow Transport
(Skinner, Liffman, Shu &c)

Turbulent Eddy Advection (Morfill and Volk 1984)

http://amesteam.arc.nasa.gov/Research/disks_science.html
Photo-evaporation and YSO disks

Photo-evaporation gives the gravitational radius: $R_g \sim 1$ AU (Liffman 2003)

Photoevaporation splits the disk into an inner and outer disk (Gorti et al 2009)

If there is not disk then turbulent convective transfer might have a problem