YUVAL SANDERS, MACQUARIE UNI

A BRIEF INTRODUCTION TO QUANTUM COMPUTING

HYPOTHETICAL COMPUTER, MAYBE BEATS CLASSICAL

- **Classical** computer: logic gates like AND, OR.
- Laws of physics are **time-reversible***.
  Best possible computer should be also.
- Reversible classical computation = **permutation** matrix.
  Quantum computation = **unitary** matrix.
- **Evidence for more computational power:**
  searching quantum database (**quadratic** speedup),
  factoring numbers (**superpolynomial** speedup?)

SIMULATIONS, DATA PROCESSING

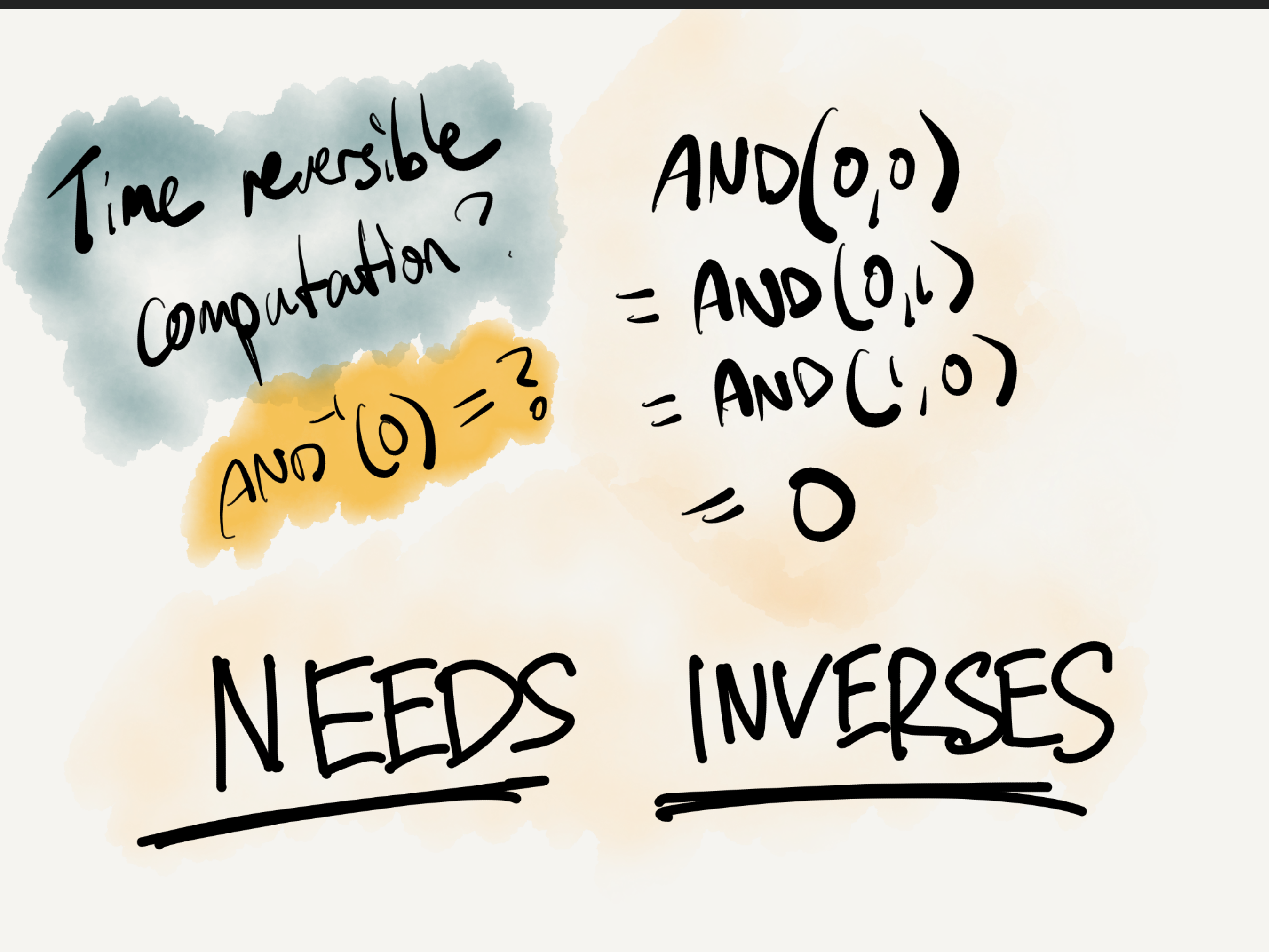
- **Search:** provable quadratic speedup given oracle model.
  Generalisation: quantum walk frameworks.
  Uses: database operations, statistical analysis / ML.
  Main challenge: memory access model.

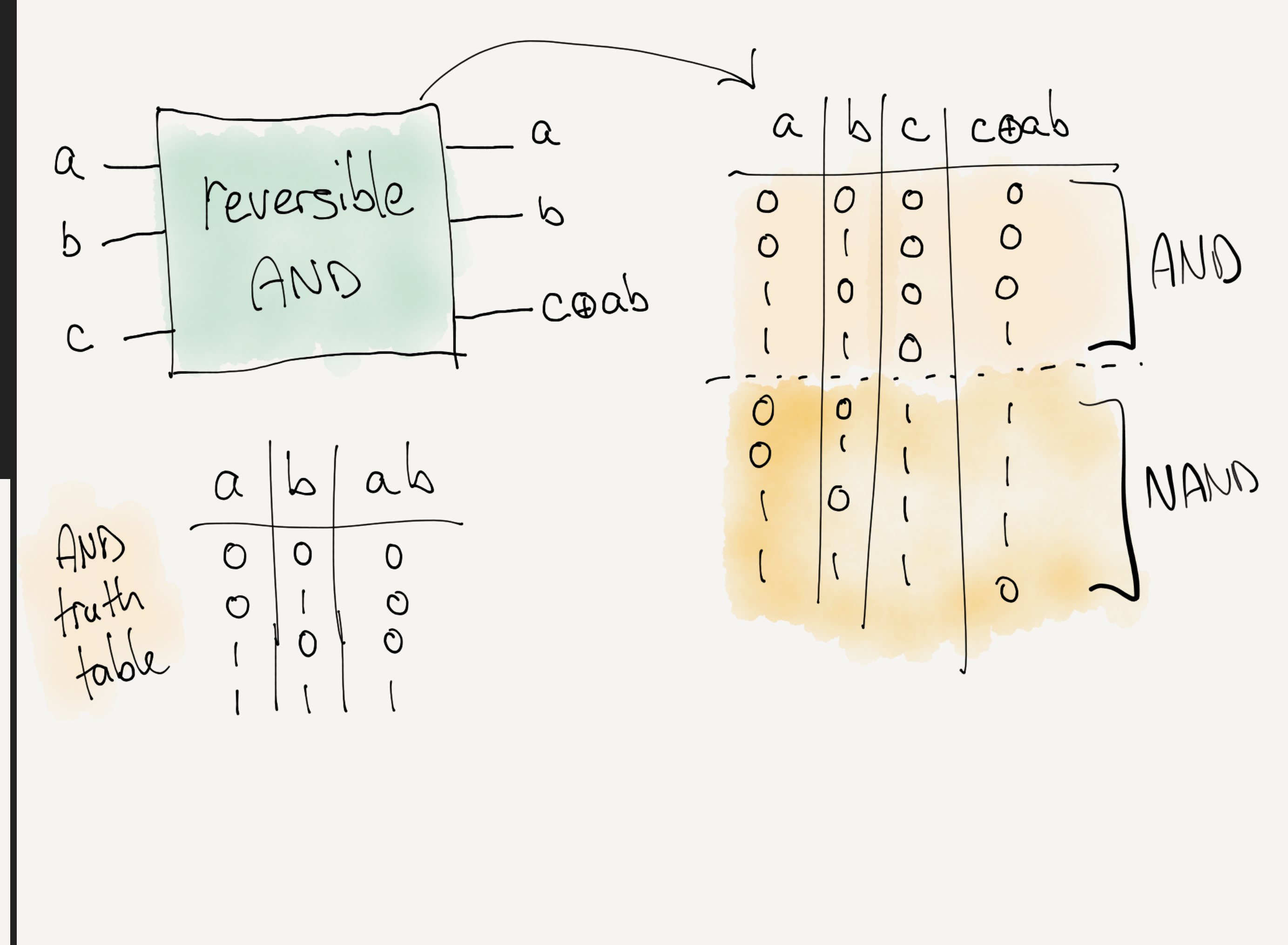
- **Factoring:** superpolynomial speedup over best classical.
  Generalisation: quantum phase estimation.
  Uses: simulation (replace function with time-evolution).
  Main challenge: comparison to classical, dequantisation.

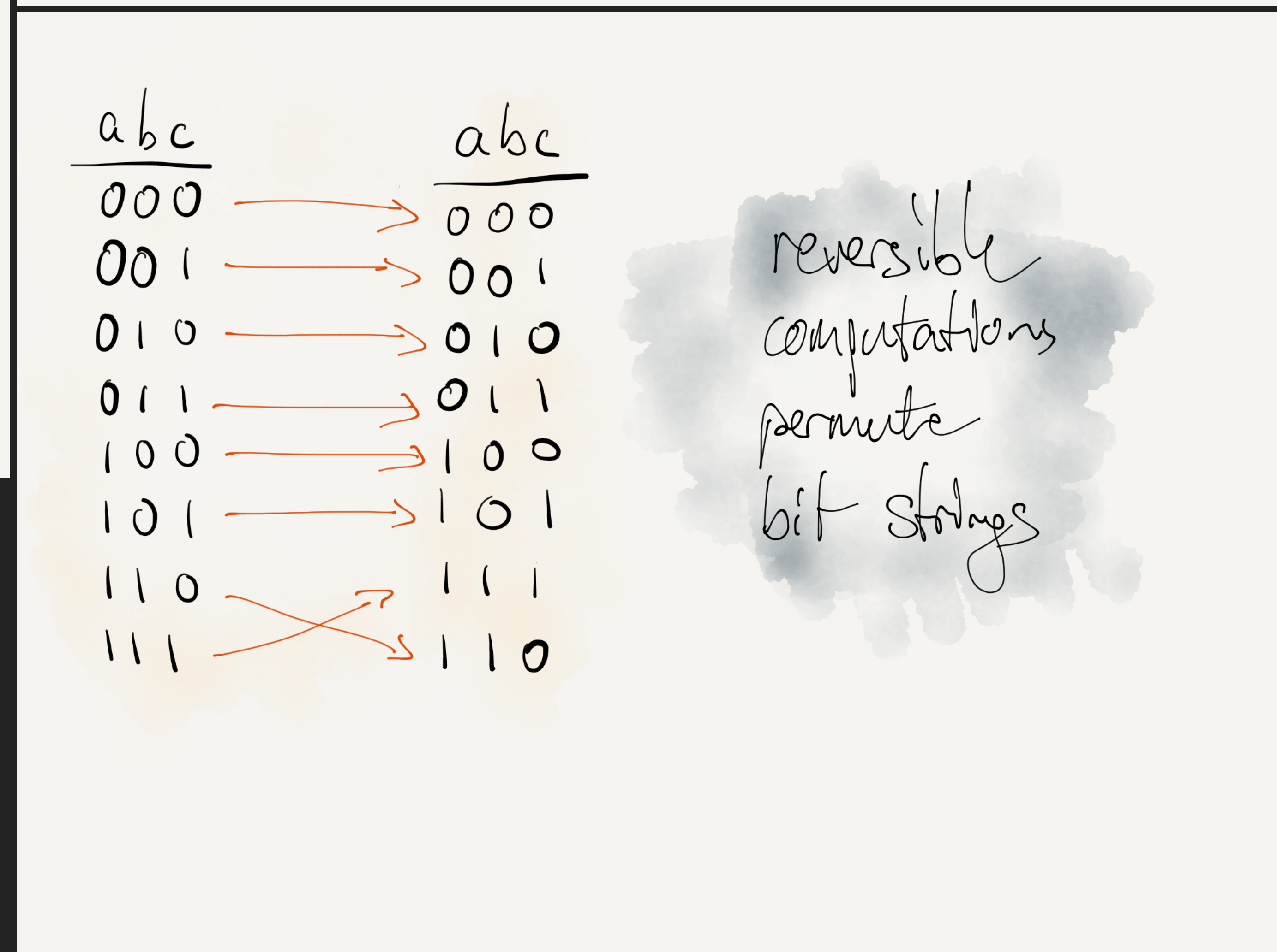
QUANTUM COMPUTERS ARE HYPOTHETICAL. THEIR USES STILL AREN'T CLEAR.

The Quantum Computing Community

BUT WHAT ARE THEY?
HOW DO THEY WORK?


Time reversible computation?
AND⁻¹(0) = ?
AND(0,0)
= AND(0,1)
= AND(1,0)
= 0
NEEDS INVERSES


a
b
c
reversible AND
a
b
c⊕ab
AND truth table
a | b | ab
0 0 0
0 1 0
1 0 0
1 1 1
a | b | c | c⊕ab
0 0 0 0
0 1 0 0
1 0 0 0
1 1 0 1
0 0 1 1
0 1 1 1
1 0 1 1
1 1 1 0
AND
NAND


abc
000
001
010
011
100
101
110
111
abc
000
001
010
011
100
101
111
110
reversible computations permute bit strings

MATRIX FORM

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
p_{000} \\ p_{001} \\ p_{010} \\ p_{011} \\ p_{100} \\ p_{101} \\ p_{110} \\ p_{111}
\end{bmatrix}
=
\begin{bmatrix}
p_{000} \\ p_{001} \\ p_{010} \\ p_{011} \\ p_{100} \\ p_{101} \\ p_{111} \\ p_{110}
\end{bmatrix}
$$

permutation matrix

input is PROBABILISTIC

how about QUANTUM?

$$|0\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\text{NOT} \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\text{NOT}|0\rangle = |1\rangle$$

$$\text{NOT}|1\rangle = |0\rangle$$

$$\text{SWAP} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{SWAP}|ab\rangle = |ba\rangle$$

$$|ab\rangle \equiv |a\rangle \otimes |b\rangle$$

PERMUTATION MATRIX $\equiv$ CLASSICAL COMPUTATION

$$i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle$$

$\hbar \equiv 1$

HOW TO MAKE A COMPUTER

(in the mind of a physicist)

$$H = \text{NOT} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle = \left(\cos(t)\mathbb{1} - i\sin(t)H\right)|\psi(0)\rangle$$

$t = \pi/2 \Rightarrow$ execute NOT (phase is unphysical)

$$t = \pi/4 \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

[This **IS** physical!]

The laws of physics
allow us to
CREATE & MANIPULATE
SUPERPOSITIONS
of computer states

JOHN VON NEUMANN

It would appear that **we have reached the limits** of what is possible to achieve with computer technology, although **one should be careful with such statements**, as they tend to **sound pretty silly in five years**.