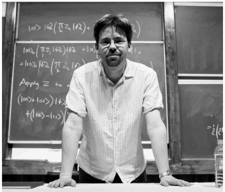
Sub-wavelength quantum imaging for astronomy and LIDAR detection

Zixin Huang[1,2], Cosmo Lupo[2], Pieter Kok[2]

1. Center for Engineered Quantum Systems, Department of Physics and Astronomy, Macquarie University
2. Department of Physics & Astronomy, University of Sheffield, UK



Outline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Outline

Motivation: surpassing Rayleigh's criterion

N-source generalisation

Quantum hypothesis testing for exoplanet detection

To the future: Large Baseline Quantum-Enhanced Imaging Networks

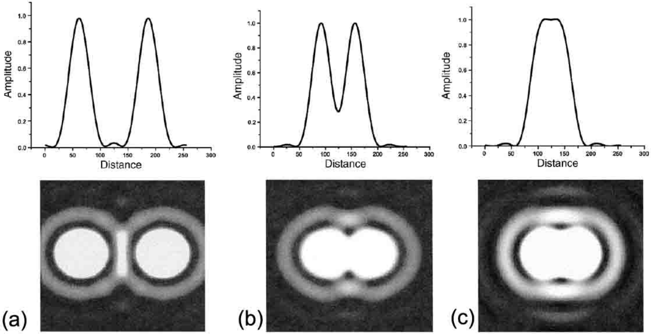


Figure: taken from www.globalsino.com

Minimum resolvable angular separation

$$\theta \approx 1.22\frac{\lambda}{D}$$

Task: estimate θ_2

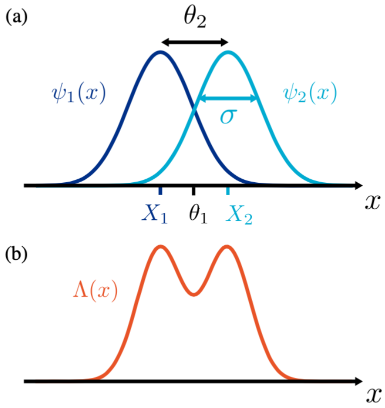


Figure: [1] Tsang et al., Phys. Rev. X 6, 031033 (2016)

Cramer-Rao bound:

$$\Delta^2\theta = \langle\theta^2\rangle - \langle\theta\rangle^2 \geq \frac{1}{\nu I(\theta)}$$

Fisher information:

$$I(\theta) = \sum_i p(i|\theta)\left(\frac{\partial \log[p(i|\theta)]}{\partial\theta}\right)^2$$

Quantum state:

$$\rho_\varphi = \sum_j \lambda_j |j\rangle\langle j|$$

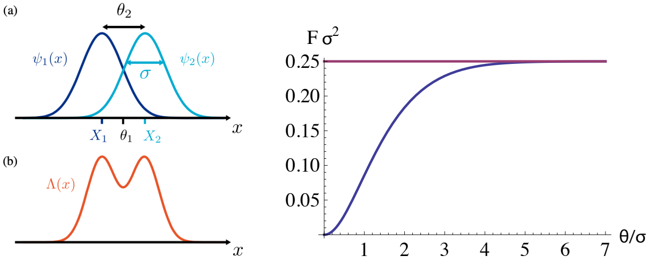
Quantum Fisher information

$$F(\rho_\theta) = \sum_{\lambda_j+\lambda_k\neq 0} 2\frac{|\langle j|\frac{\partial\rho}{\partial\theta}|k\rangle|^2}{\lambda_j+\lambda_k},$$

Superresolution of two sources

Model: two incoherent, quasi-monochromatic point sources

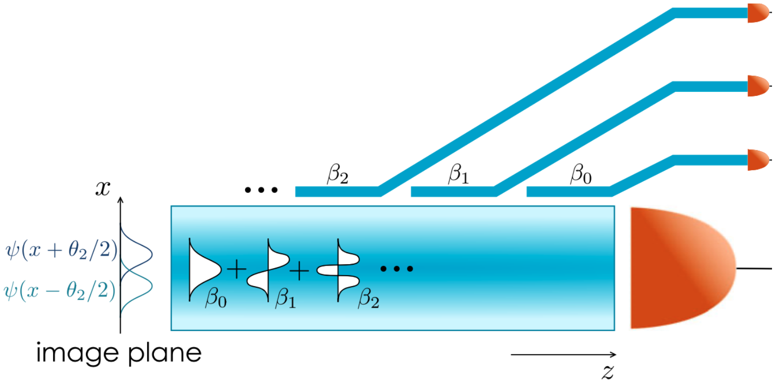
$$\rho \approx \frac{1}{2}\left(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|\right), \qquad |\psi_i\rangle = \int_{-\infty}^{\infty} \psi_i(x)\,|x\rangle$$



$$F(\theta_2) = \frac{1}{4\sigma^2}$$

[1] Tsang et al., Phys. Rev. X 6, 031033 (2016)

Optimal measurement: FI = QFI



[1] Tsang et al., Phys. Rev. X 6, 031033 (2016)
also F. Tamburini, PRL 97, 163903 (2006)

Quantum metrology for superresolution

PRL **117,** 190802 (2016) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2016

Ultimate Precision Bound of Quantum and Subwavelength Imaging

Cosmo Lupo[1] and Stefano Pirandola[1,2]
[1]*York Centre for Quantum Technologies (YCQT), University of York, York YO10 5GH, United Kingdom*
[2]*Computer Science, University of York, York YO10 5GH, United Kingdom*
(Received 6 July 2016; published 4 November 2016)

PRL **118,** 070801 (2017) PHYSICAL REVIEW LETTERS week ending 17 FEBRUARY 2017

Beating Rayleigh's Curse by Imaging Using Phase Information

Weng-Kian Tham,[1] Hugo Ferretti,[1] and Aephraim M. Steinberg[1,2]
[1]*Centre for Quantum Information & Quantum Control and Institute for Optical Sciences, Department of Physics, University of Toronto, 60 St. George St, Toronto, Ontario, Canada, M5S 1A7*
[2]*Canadian Institute For Advanced Research, 180 Dundas St. W., Toronto, Ontario, Canada, M5G 1Z8*
(Received 21 July 2016; published 15 February 2017)

PHYSICAL REVIEW LETTERS **122,** 140505 (2019)

Towards Superresolution Surface Metrology: Quantum Estimation of Angular and Axial Separations

Carmine Napoli,[1,2,*] Samanta Piano,[2,†] Richard Leach,[2,‡] Gerardo Adesso,[1,§] and Tommaso Tufarelli[1,‖]
[1]*School of Mathematical Sciences and Centre for the Mathematics and Theoretical Physics of Quantum Non-Equilibrium Systems, University of Nottingham, University Park Campus, Nottingham NG7 2RD, United Kingdom*
[2]*Manufacturing Metrology Team, Faculty of Engineering, University of Nottingham, Jubilee Campus, Nottingham NG8 1BB, United Kingdom*

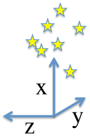
PHYSICAL REVIEW LETTERS **121,** 180504 (2018)

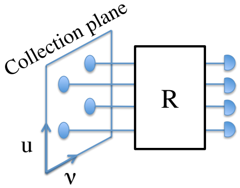
Quantum Limited Superresolution of an Incoherent Source Pair in Three Dimensions

Zhixian Yu and Sudhakar Prasad[*]
Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131, USA
(Received 26 May 2018; published 31 October 2018)

Outline

Arbitrary number of sources





N_s incoherent sources, quasi-monochromatic, coordinate $\vec{r_s}$.
N_c collectors, position $\vec{\omega_j}$. Photon impinging $\rightarrow |j\rangle$

$$|\psi(r_s)\rangle = \frac{1}{\sqrt{N_c}} \sum_j^{N_c} e^{i\phi(\vec{r_s},\vec{\omega_j})} |j\rangle ,$$

$$\rho = \sum_s^{N_s} p(s) |\psi(r_s)\rangle \langle\psi(r_s)|$$

C Lupo, Z Huang, P Kok, Phys. Rev. Lett. 124, 080503 (2020)

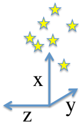
Coordinates of the sources: $\vec{r} = (x_1, y_1, z_1, ..., x_{N_s}, y_{N_s}, z_{N_s})$
A unit vector with $3N_s$ components: $\vec{a} = (a_1, a_2, ...a_{3N_s})$
A generalised coordinate $\theta = \vec{a} \cdot \vec{r}$,

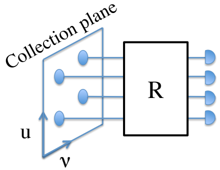
$$\rho = \sum_{s}^{N_s} p(s) \left|\psi(r_s)\right\rangle \left\langle\psi(r_s)\right|$$

Quantum Fisher information

$$F(\rho_\theta) = \sum_{\lambda_j+\lambda_k\neq 0} 2\frac{|\langle j|\frac{\partial\rho}{\partial\theta}|k\rangle|^2}{\lambda_j+\lambda_k}, \qquad \rho_\theta = \sum_j \lambda_j \left|j\right\rangle \left\langle j\right|$$

Arbitrary number of sources





Purification: $\rho \to |\Psi(\vec{r_s})\rangle = \sum_{j,s} \sqrt{p(s)} e^{i\phi(\vec{r_s},\vec{\omega}_j)} |j\rangle |s\rangle$

$$\vec{a} = (a_1, a_2, ...a_{3N_s}), \quad \vec{r} = (x_1, y_1, z_1, ..., x_{N_s}, y_{N_s}, z_{N_s})$$

$$\theta = \vec{a} \cdot \vec{r}, \quad \delta\theta = \vec{a} \cdot (r' - r)$$

$$\mathrm{QFI}(\theta) = \lim_{\delta\theta \to 0} \frac{8(1 - f_{r,r'})}{\delta\theta^2}$$

$$f_{r,r'} = \max_V |\langle \Psi(r)| I \otimes V |\Psi(r')\rangle|$$

C Lupo, Z Huang, P Kok, Phys. Rev. Lett. 124, 080503 (2020)

Achieving the QFI

- QFI for θ reduces down to a matrix trace norm $||M||_1 = \mathrm{Tr}(\sqrt{M^\dagger M})$, M depends on $p(s)$ and the optical paths
- QFI can be achived with linear optical unitary + photon counting.

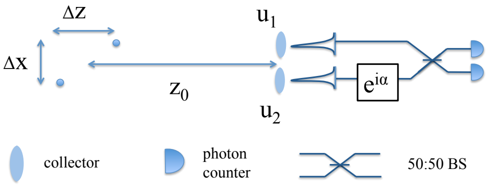
$$|\psi(r_s)\rangle = \frac{1}{\sqrt{N_c}} \sum_{j}^{N_c} e^{i\phi(\vec{r_s},\vec{\omega}_j)} |j\rangle$$

Define

$$U(r_s) = \exp\left[-i(\hat{g}_x x_s + \hat{g}_y y_s + \hat{g}_z z_s)\right]$$
$$|\psi(r_s)\rangle = U(r_s) |\psi(0)\rangle$$

$\hat{g}_x, \hat{g}_y, \hat{g}_z \propto$ the positions of the collectors

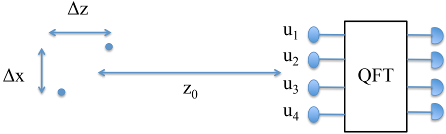
An example: two sources



To estimate Δx, $u = (u_1 - u_2)$

$$F\left(\frac{\Delta x}{z_0}\right) \propto (\langle g_x^2 \rangle - \langle g_x \rangle^2) = \frac{1}{4}(\langle u^2 \rangle - \langle u \rangle^2) \qquad (1)$$

Precision is characterized by the variance of the spatial distribution of the collectors.



Conclusions

- We solve the problem of determining a 3D position of an arbitrary number of sources
- Linear interferometry and photon counting are optimal
- Explicit construction of the interferometer
- We provide insight into why coherent detection overcomes the Rayleigh curse by recasting imaging as interferometry at the outset.

C Lupo, Z Huang, P Kok, Phys. Rev. Lett. 124, 080503 (2020)

Outline

Different methods

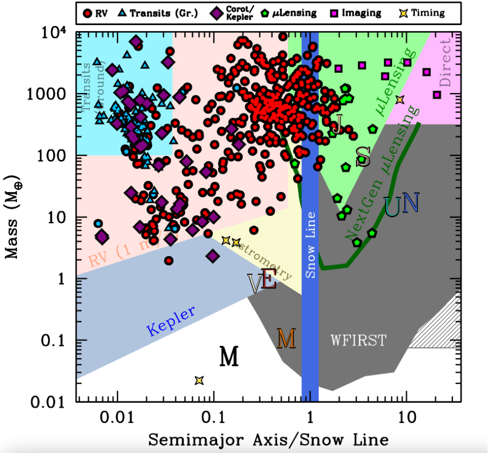


Figure: taken from Wright et al., arXiv:1210.2471 [astro-ph.EP]

1 or 2 sources?

Two hypotheses H_a, H_b
Classical: $p_a(x), p_b(x)$, quantum: ρ_a, ρ_b

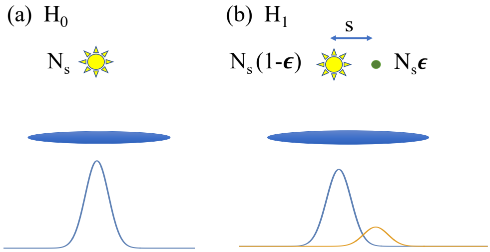


Figure: (a) The scenario where the there is only 1 source. (b) There are two near-by sources present.

arXiv:2106.00488

Quantum state discrimination

1. Symmetric discrimination: trace distance - quantum Chernoff bound [8], $P_e \sim \exp[-N\ f(T)]$

$$T_c(p_a, p_b) = 1/2 \int dx\ |p_a(x) - p_b(x)|$$

$$T_Q(\rho_a, \rho_b) = 1/2||\rho_a - \rho_b||_1$$

2. Asymmetric: relative entropy - quantum Stein lemma [9],

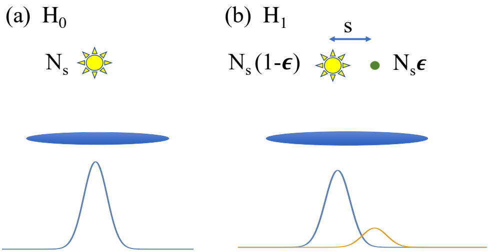
$$P_e \sim \exp[-NS + O(\epsilon^{-1}, \ln N)]$$

$$S_c(p_a||p_b) = \int\ dx\ p_a(x)(\log_2 p_a(x) - \log_2 p_b(x))$$

$$S_Q(\rho_a||\rho_b) = \mathrm{Tr}[\rho_a(\log_2 \rho_a - \log_2 \rho_b)]$$

[8] Audenaert et al., Phys. Rev. Lett. 98, 160501 (2007)
[9] F. Hiai, D. Petz, Commun. Math. Phys. 143, 99 (1991)

Classical probability distributions

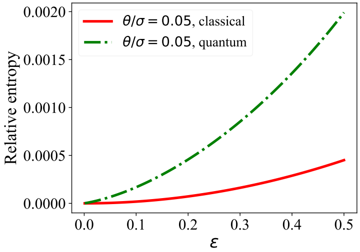


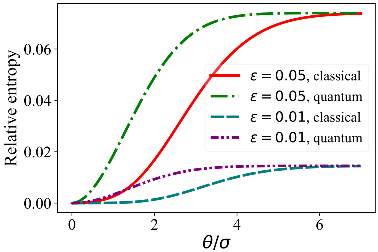
Classical relative entropy:

$$S_c(p_a||p_b) \approx \frac{2\theta^2\epsilon^2}{\sigma^2\log(16)} + \frac{\theta^4\epsilon^2}{\sigma^4\log(16)} - \frac{4\theta^4\epsilon^3}{\sigma^4\log(16)}$$

arXiv:2106.00488

Classical vs Quantum





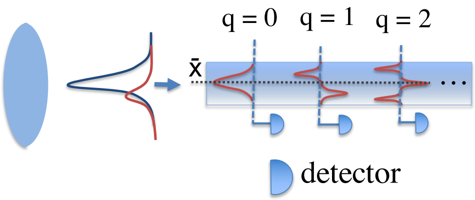
Classical relative entropy:

$$S_c(p_a||p_b) \approx \frac{2\theta^2\epsilon^2}{\sigma^2\log(16)}$$

Quantum relative entropy;

$$D(\rho_a||\rho_b) \approx \frac{\theta^2\epsilon}{4\sigma^2\log(2)} + O(\epsilon^2\theta^2).$$

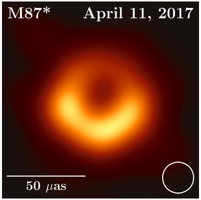
Optimal measurement and conclusions

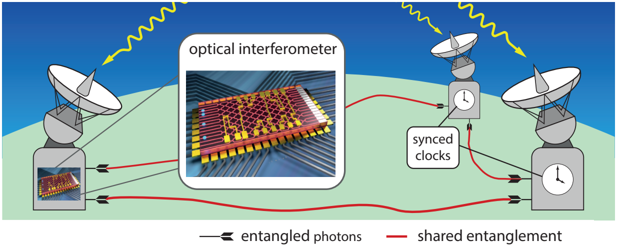


- We compute the type-II error probability exponent of discriminating between 1 or two sources with arbitrary intensity.
- in the limit that $\epsilon \ll 1$, the quantum relative entropy is larger than that of direct imaging by a factor of $1/\epsilon$.

arXiv:2106.00488

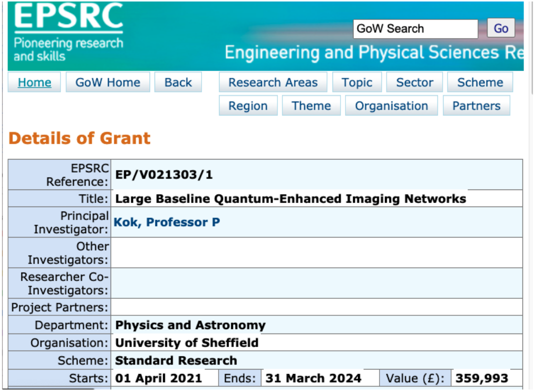
To the future





- Long-distance optical coherence, entanglement-assisted network
- Quantum error correction to combat to loss to decoherence
- Current collaborations: Bristol, Heriot-Watt, Erlangen

Large Baseline Quantum-Enhanced Imaging Networks
£359,993 ($\approx$ \$650,000 AUD) grant from the EPSRC
Two postdoc positions available



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Details of Grant

EPSRC Reference:	**EP/V021303/1**				
Title:	**Large Baseline Quantum-Enhanced Imaging Networks**				
Principal Investigator:	**Kok, Professor P**				
Other Investigators:					
Researcher Co-Investigators:					
Project Partners:					
Department:	**Physics and Astronomy**				
Organisation:	**University of Sheffield**				
Scheme:	**Standard Research**				
Starts:	**01 April 2021**	Ends:	**31 March 2024**	Value (£):	**359,993**

Questions?

Thank you for your attention.

Figure: (Left) my hamster; (right) one of my Indian ringneck parakeets