Orthogonally Polarized Modes: Superposition, Composition and Mutual Exclusivity

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Pulsar electrodynamics: an unsolved problem

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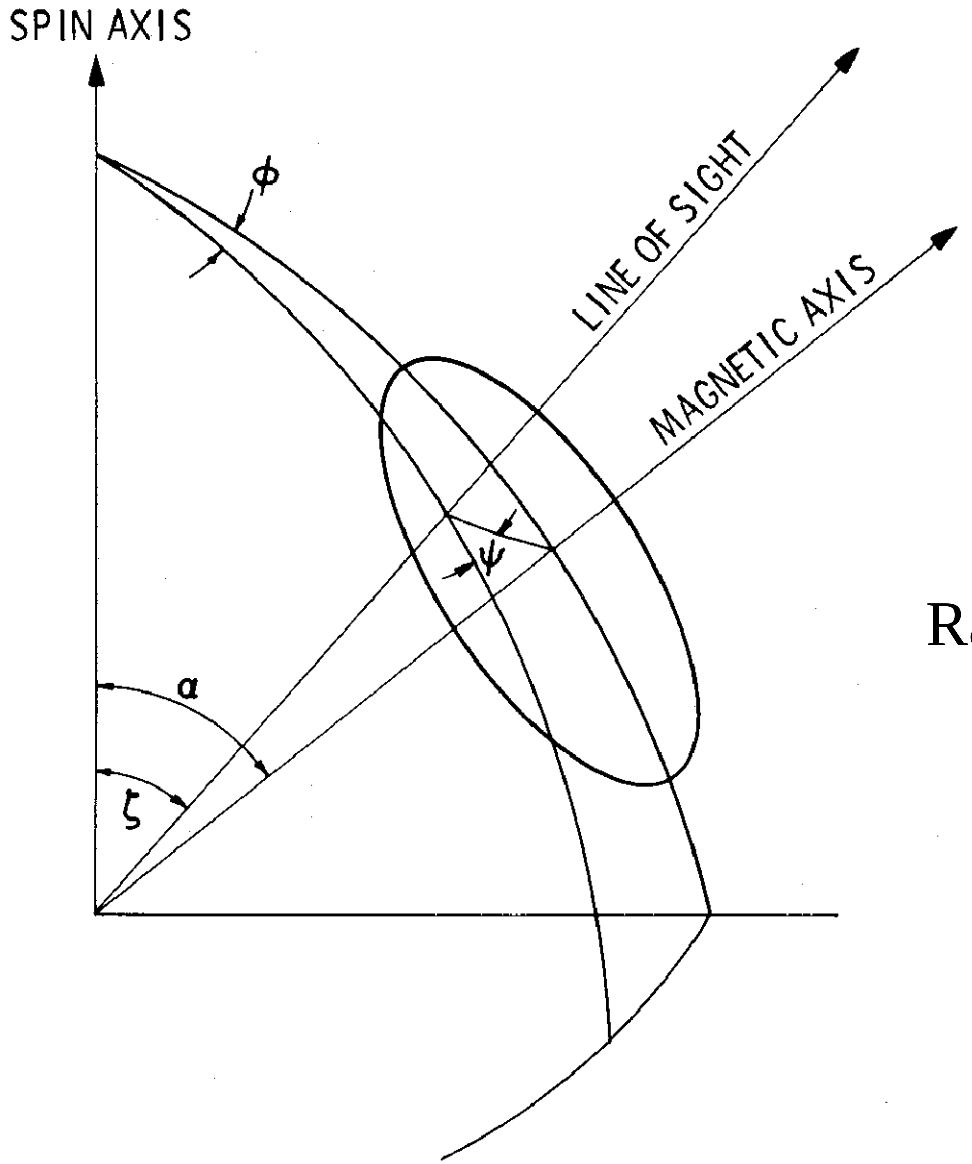
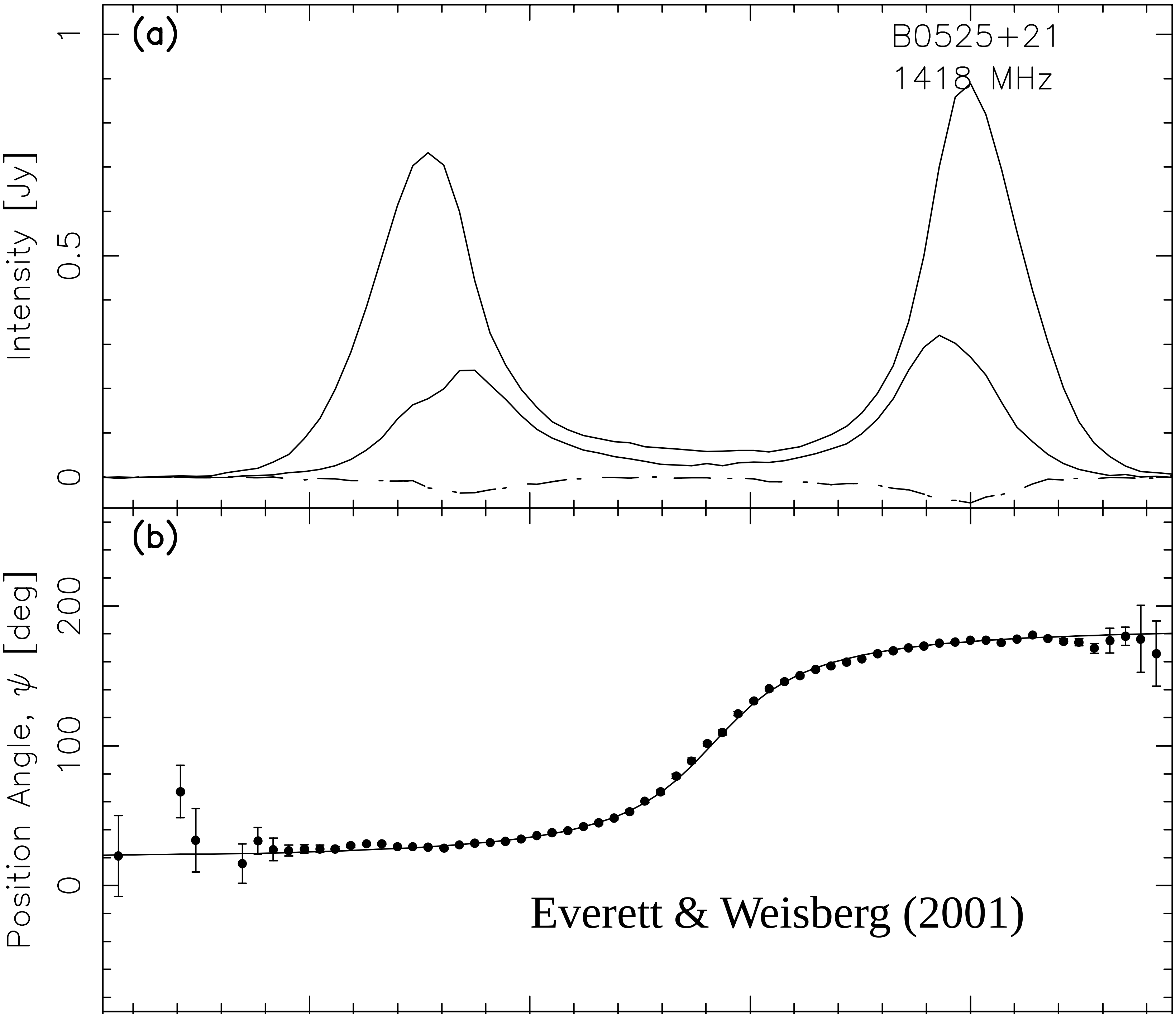
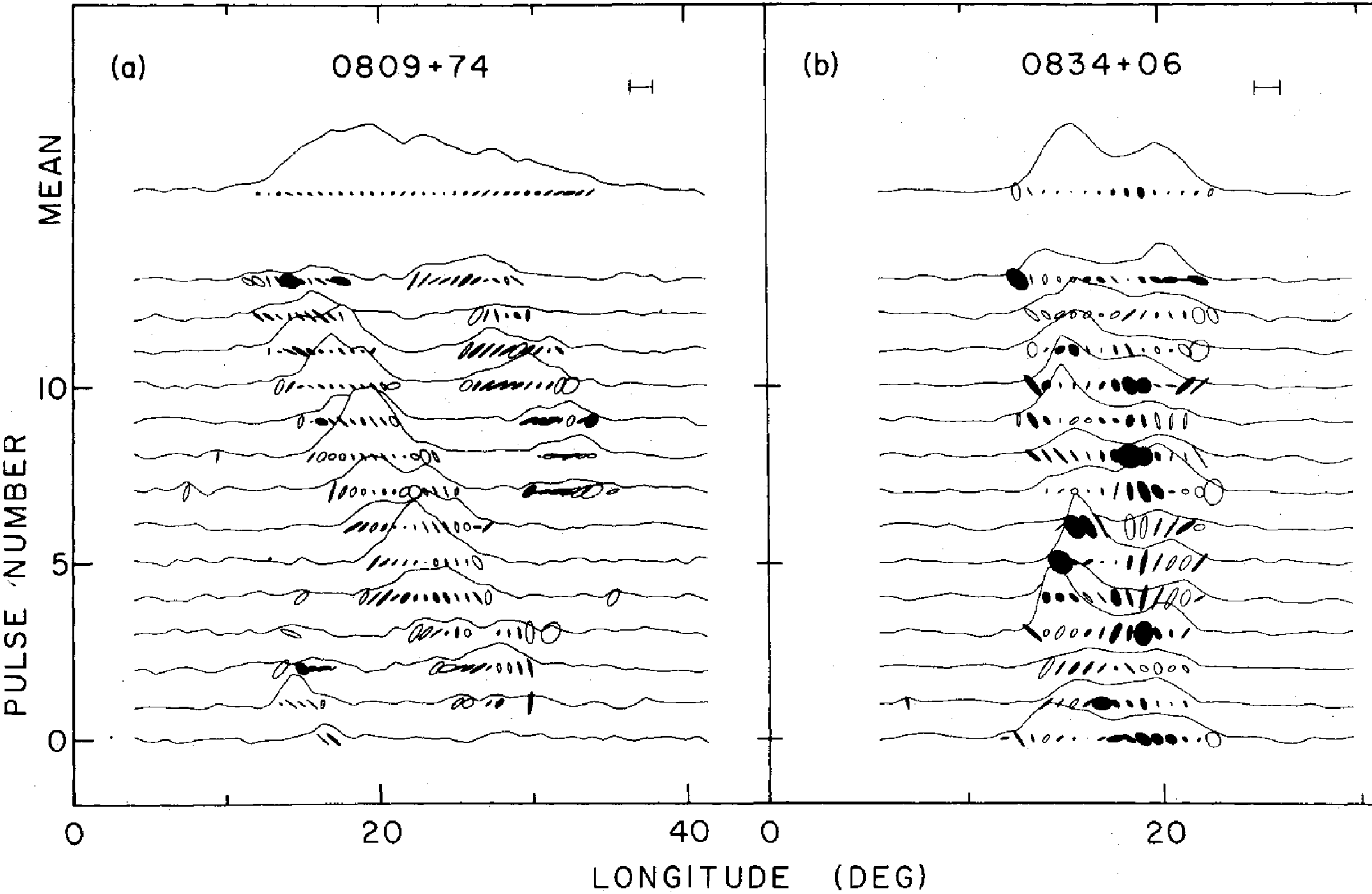
Pulsar electrodynamics is reviewed emphasizing the role of the inductive electric field in an oblique rotator and the incomplete screening of its parallel component by charges, leaving 'gaps' with $E_\parallel \neq 0$. The response of the plasma leads to a self-consistent electric field that complements the inductive electric field with a potential field leading to an electric drift and a polarization current associated with the total field. The electrodynamic models determine the charge density, ρ, and the current density, $\boldsymbol{J}$; charge starvation refers to situations where the plasma cannot supply ρ, resulting in a gap and associated particle acceleration and pair creation. It is pointed out that a form of current starvation also occurs implying a new class of gaps. The properties of gaps are discussed, emphasizing that static models are unstable, the



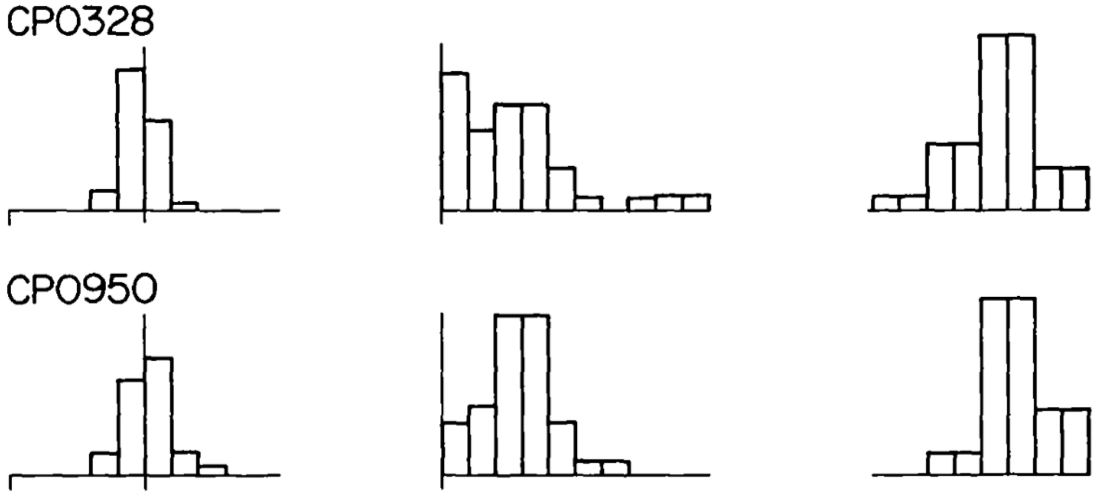
Radhakrishnan & Cooke (1969)

$$\tan(\psi' - \psi'_0) = \frac{\sin\alpha \sin(\phi - \phi_0)}{\sin\zeta\cos\alpha - \cos\zeta\sin\alpha\cos(\phi - \phi_0)}$$

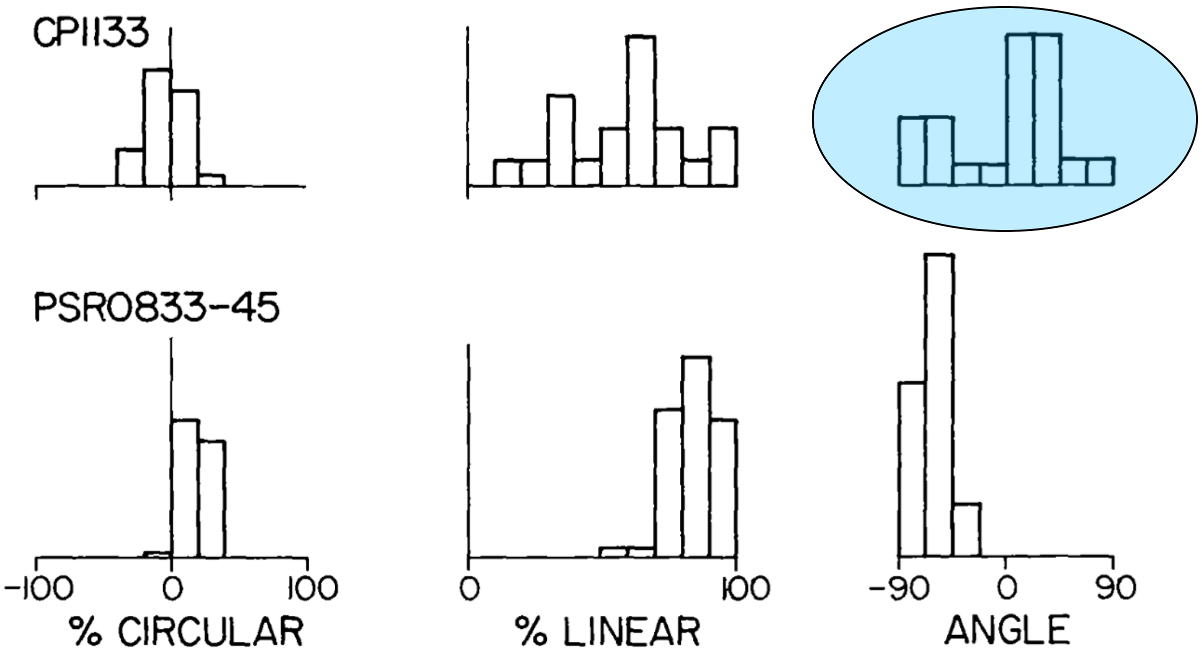

(a)
B0525+21
1418 MHz
Intensity [Jy]
1
0.5
0
(b)
Position Angle, ψ [deg]
200
100
0
Everett & Weisberg (2001)



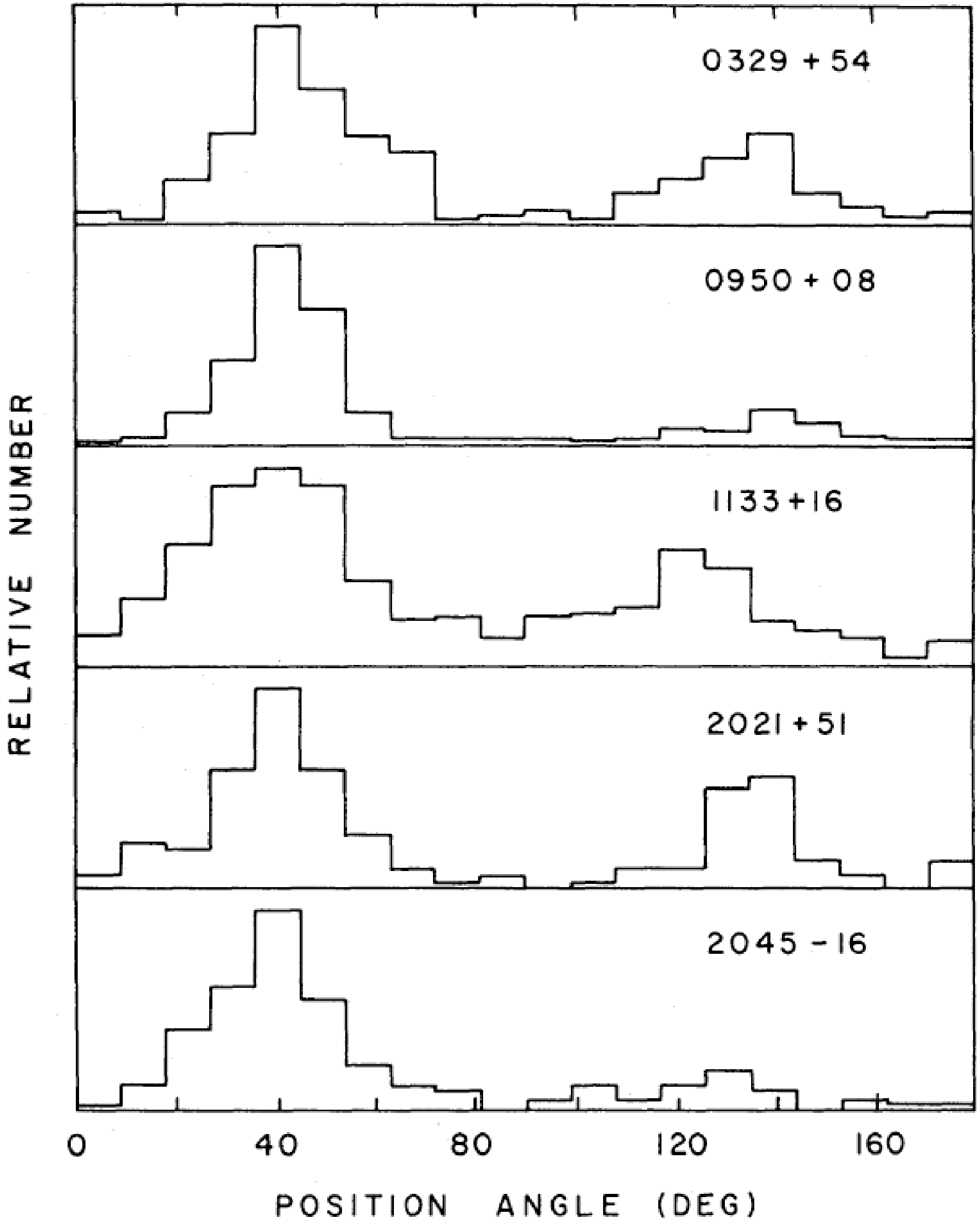
Manchester, Taylor, and Huegenin (1975)



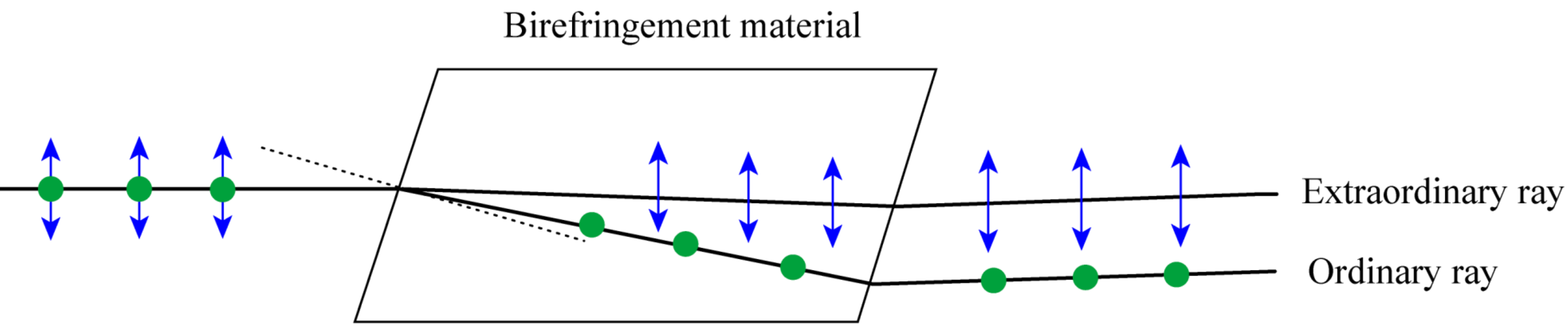
In CP 1133 the polarization structure in the preceding subpulse seems to be the result of the addition of two linearly polarized components with angle differing by about 90°. One component at approximately 10° lasts throughout the subpulse, but near the center of the subpulse it is dominated by a stronger component at 90°. A similar phenomenon



Ekers & Moffet (1969)



Manchester, Taylor, & Huegenin (1975)



Melrose & Stoneham (1977); Melrose (1979); Allen & Melrose (1982) Barnard & Arons (1986); McKinnon (1997); Kennett & Melrose (1998); Petrova & Lyubarskii (2000); Wang, Lai & Han (2010); Beskin & Philippov (2012)

Birefringence in Pulsar Magnetosphere

- Natural modes are linearly polarized
  - X-mode travels along straight ray path
  - O-mode "ducted" along magnetic field lines

- separated by many beam widths
  - modes are mutually exclusive (disjoint)

- beams overlap (superposed)
  - "generalized Faraday rotation"
  - "Cotton–Mouton birefringence"

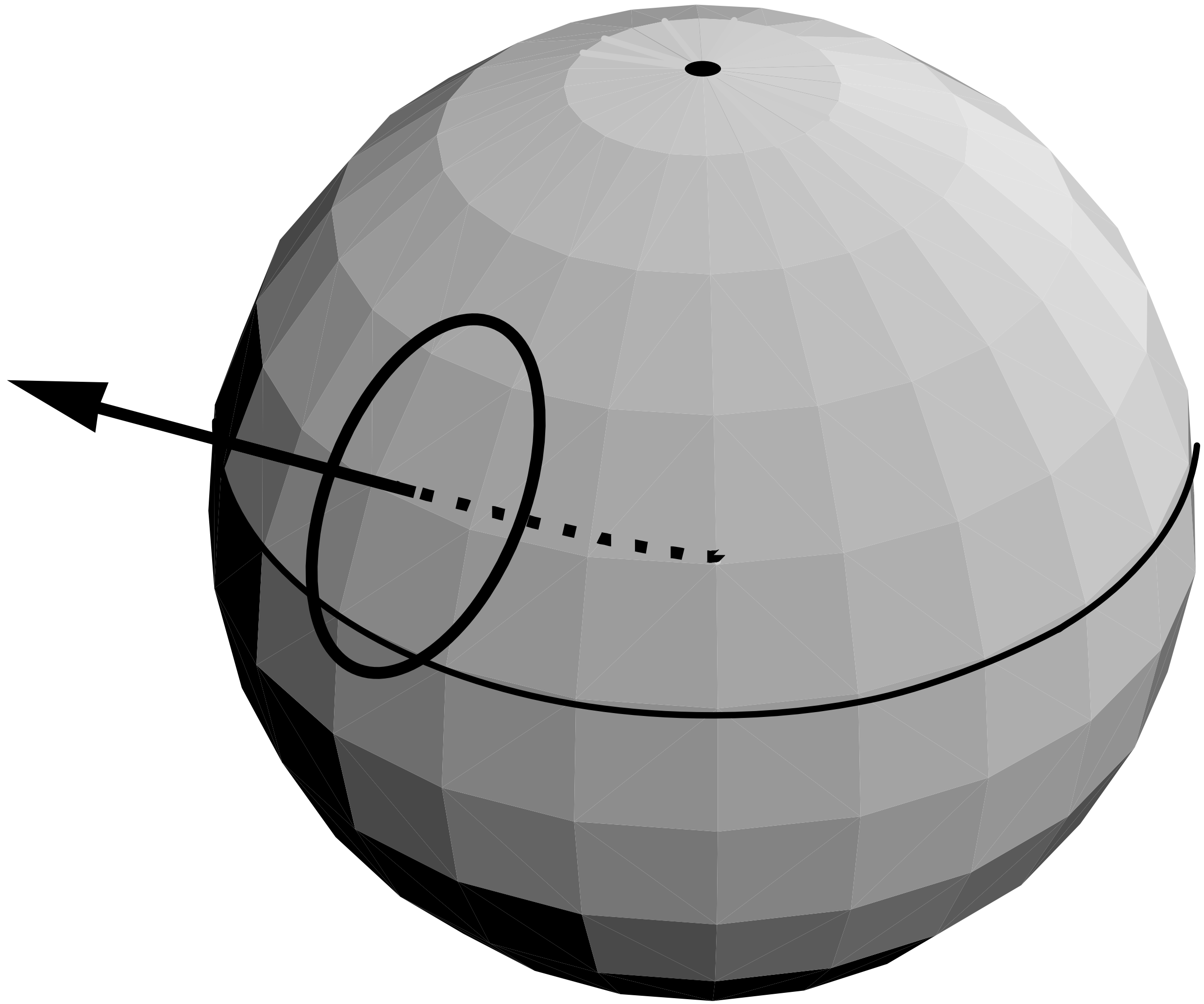
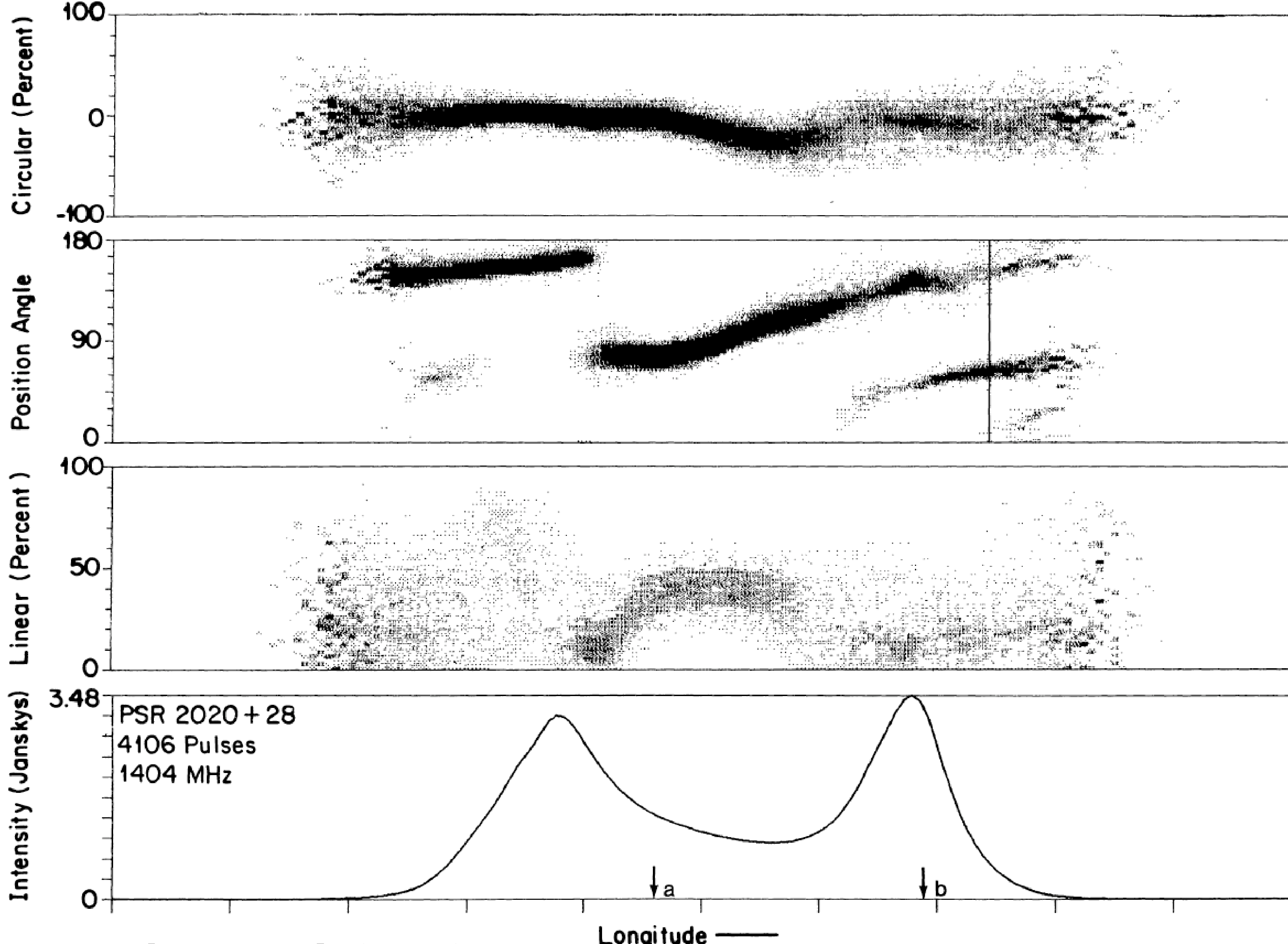


Figure 3—Representative point for radiation in an anisotropic medium rotates about the diagonal joining the points for the two natural modes.



Stinebring et al. (1984)

Disjoint or Superposed?

$$\langle d_L{}^2 \rangle \equiv \frac{\langle |L|^2 \rangle}{\langle I^2 \rangle}$$

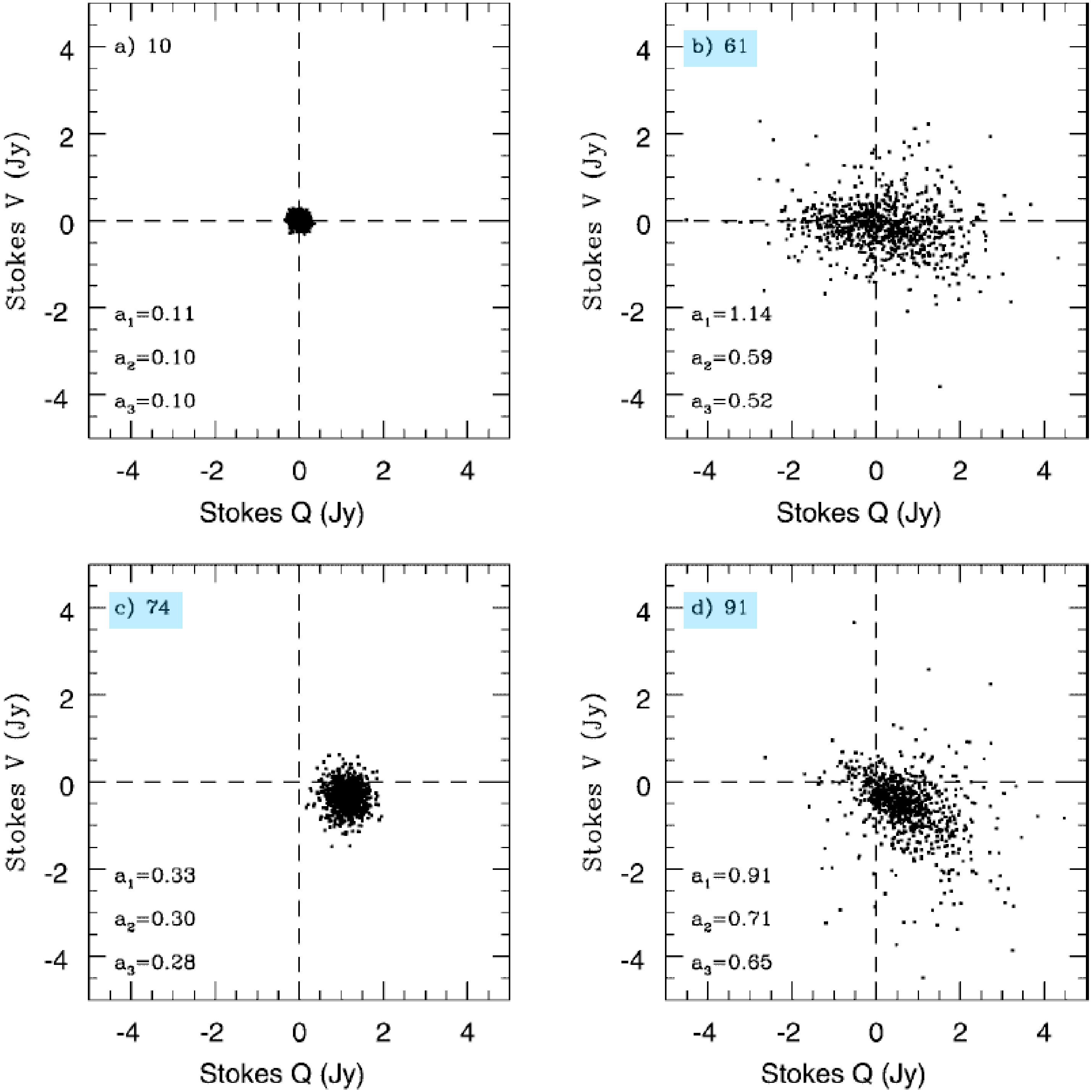
$$= \frac{\langle L_1{}^2 \rangle + \langle L_2{}^2 \rangle}{\langle (I_1 + I_2)^2 \rangle} \quad \text{(disjoint)}$$

$$= \frac{\langle L_1{}^2 \rangle + \langle L_2{}^2 \rangle - 2|\langle L_1 \rangle|\,|\langle L_2 \rangle|}{\langle (I_1 + I_2)^2 \rangle} \,, \quad \text{(superposition)} \quad (5)$$

Cordes, Rankin & Backer (1978)

Statistical Models

- McKinnon & Stinebring (1998)
  - Disjoint or superposed?
- McKinnon & Stinebring (2000)
  - Covariant? Mode-separated profiles
- McKinnon (2002)
  - Model circular polarization of modes
- McKinnon (2003)
  - directional statistics (spherical)
- McKinnon (2004)
  - 3D covariance analysis (Cartesian)



a) 10
a₁=0.11
a₂=0.10
a₃=0.10
b) 61
a₁=1.14
a₂=0.59
a₃=0.52
c) 74
a₁=0.33
a₂=0.30
a₃=0.28
d) 91
a₁=0.91
a₂=0.71
a₃=0.65
Stokes V (Jy)
Stokes Q (Jy)

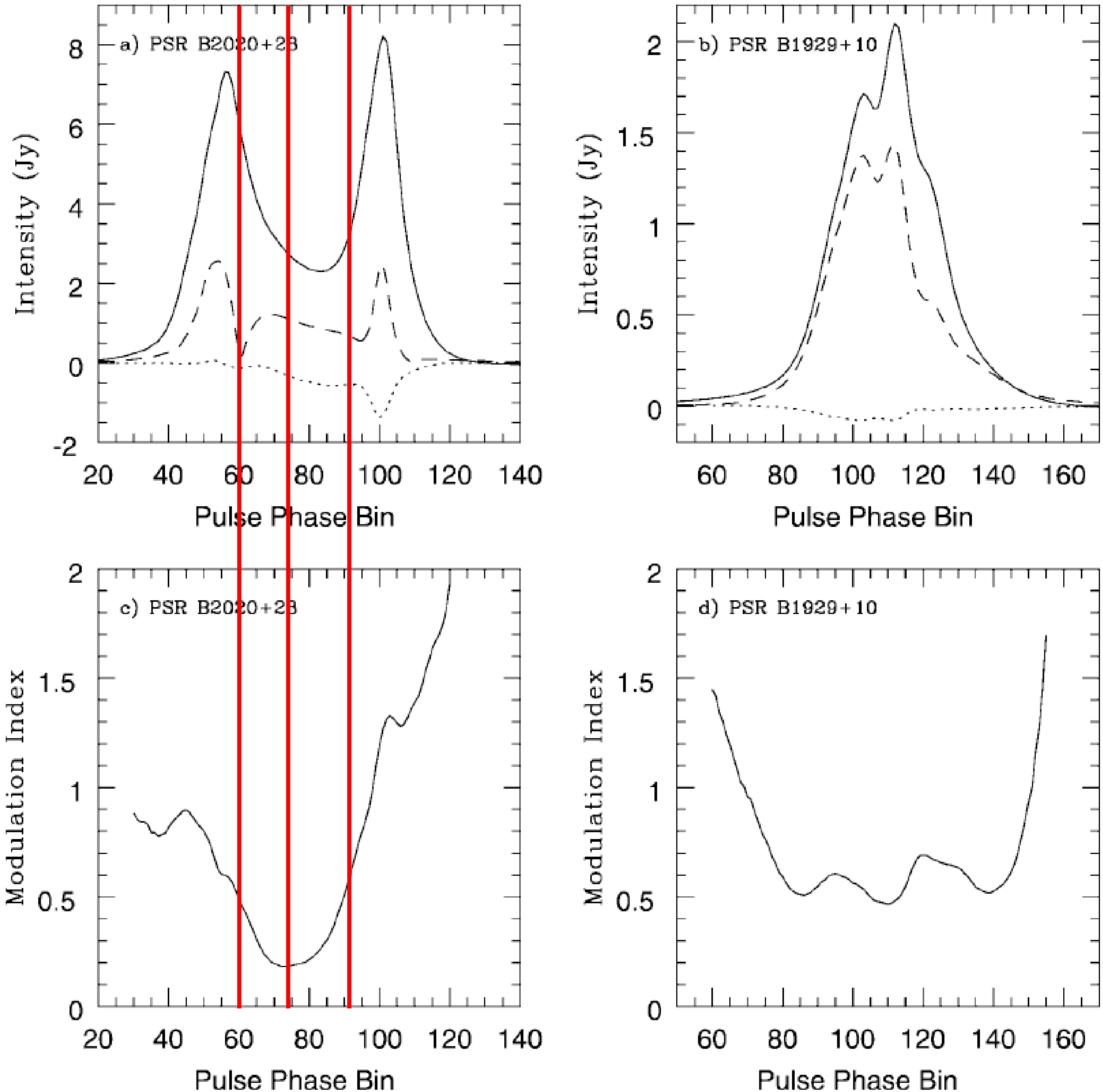

a) PSR B2020+28
b) PSR B1929+10
c) PSR B2020+28
d) PSR B1929+10
Intensity (Jy)
Modulation Index
Pulse Phase Bin
8
6
4
2
0
-2
1.5
1
0.5
20
40
60
80
100
120
140
160

Table 1
Eigenvalue Analysis from McKinnon (2004)

β	S_0 (Jy)	σ_0	σ_1	$\sigma_\perp$
PSR B2020+28				
0.45	5.5	2.48	1.14	0.59
0.19	2.8	0.54	0.33	0.30
0.56	3.1	1.74	0.91	0.71
PSR B1929+10				
0.55	0.68	0.38	0.32	0.09
0.68	1.35	0.92	0.63	0.13
0.65	1.00	0.65	0.46	0.10

$$\sigma_0 = \beta S_0 > \sigma_1$$

No excess polarization dispersion

Covariances between Stokes Parameters

$$C_{ij} = n^{-1} (S_i S_j - ½\, \eta_{ij} S^2)$$

$$S^2 = S_0{}^2 - |\mathbf{S}|^2$$

Brosseau & Barakat (1992)
van Straten (2009)

$$\mathrm{S} = [S_0, |\boldsymbol{S}|, 0, 0]$$

$$\mathbf{C}' = \varsigma^2 \begin{pmatrix} \|S\|^2 & 2S_0|\boldsymbol{S}| & 0 & 0 \\ 2S_0|\boldsymbol{S}| & \|S\|^2 & 0 & 0 \\ 0 & 0 & S^2 & 0 \\ 0 & 0 & 0 & S^2 \end{pmatrix}$$

$$\|S\|^2 = S_0{}^2 + |\boldsymbol{S}|^2$$

$$S^2 = S_0{}^2 - |\boldsymbol{S}|^2$$

$$\mathrm{S} = [S_0, |\boldsymbol{S}|, 0, 0]$$

$$\mathbf{C}' = \varsigma^2 \begin{pmatrix} \|S\|^2 & 2S_0|\boldsymbol{S}| & 0 & 0 \\ 2S_0|\boldsymbol{S}| & \|S\|^2 & 0 & 0 \\ 0 & 0 & S^2 & 0 \\ 0 & 0 & 0 & S^2 \end{pmatrix}$$

$$||S||^2 = S_0{}^2 + |\boldsymbol{S}|^2$$

$$S^2 = S_0{}^2 - |\boldsymbol{S}|^2$$

Covariant Mode Intensities

$$C_{00} = \varsigma_A^2 \|A\|^2 + \varsigma_B^2 \|B\|^2 + 2\varrho\, \varsigma_A \varsigma_B A_0 B_0$$

$$C_{11} = \varsigma_A^2 \|A\|^2 + \varsigma_B^2 \|B\|^2 - 2\varrho\, \varsigma_A \varsigma_B |\boldsymbol{A}||\boldsymbol{B}|$$

McKinnon & Stinebring (1998)
van Straten (2009)

Two Problems

1. $\sigma_0 > \sigma_1$ even when no OPM

2. Incomplete statistical model

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Disjoint or Superposed?

$$\langle {d_L}^2 \rangle \equiv \frac{\langle |L|^2 \rangle}{\langle I^2 \rangle}$$

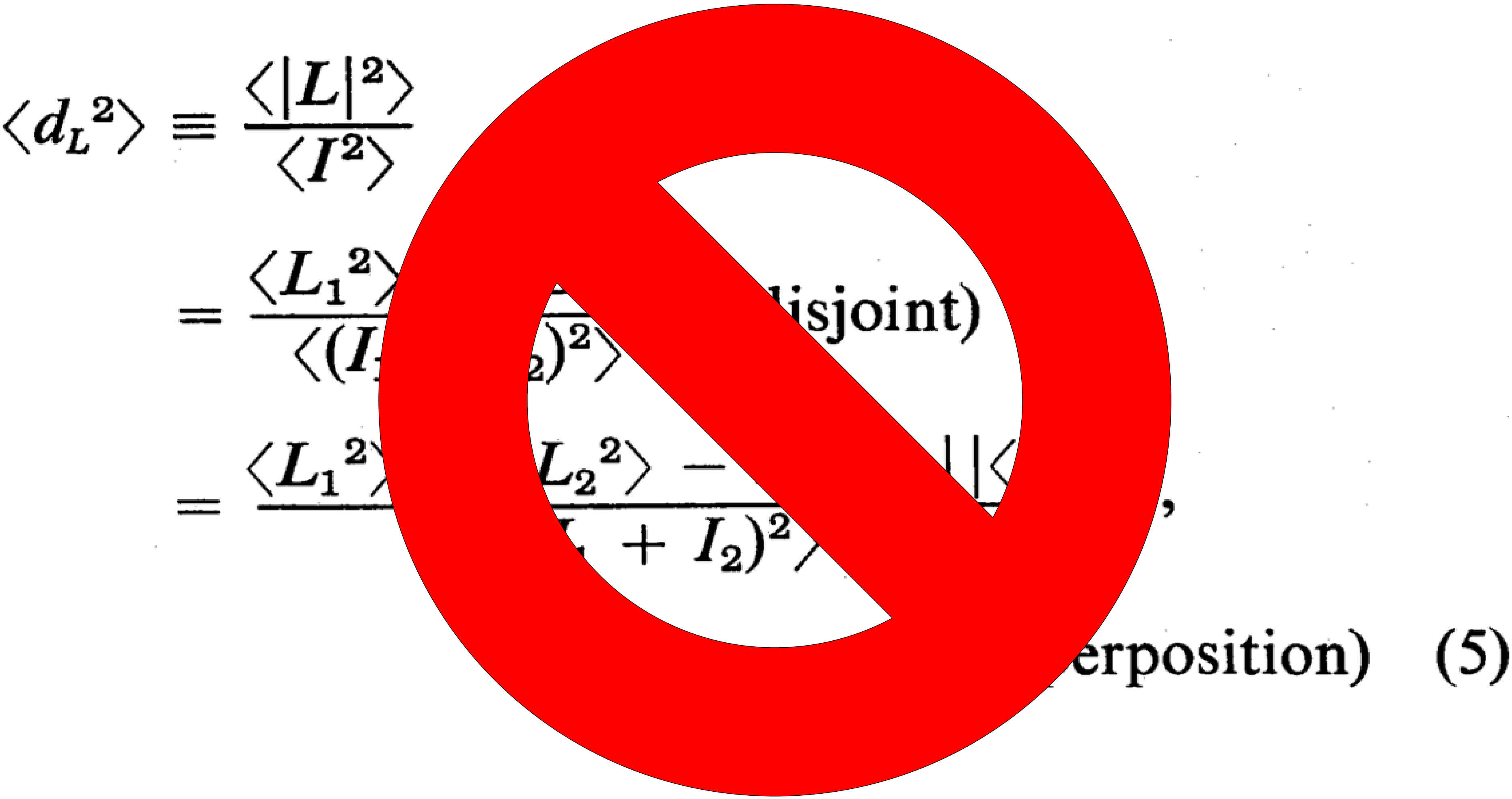
$$= \frac{\langle {L_1}^2 \rangle + \langle {L_2}^2 \rangle}{\langle (I_1 + I_2)^2 \rangle} \quad \text{(disjoint)}$$

$$= \frac{\langle {L_1}^2 \rangle + \langle {L_2}^2 \rangle - 2|\langle L_1 \rangle| \, |\langle L_2 \rangle|}{\langle (I_1 + I_2)^2 \rangle} \,,$$

$$\text{(superposition)} \quad (5)$$

Cordes, Rankin & Backer (1978)

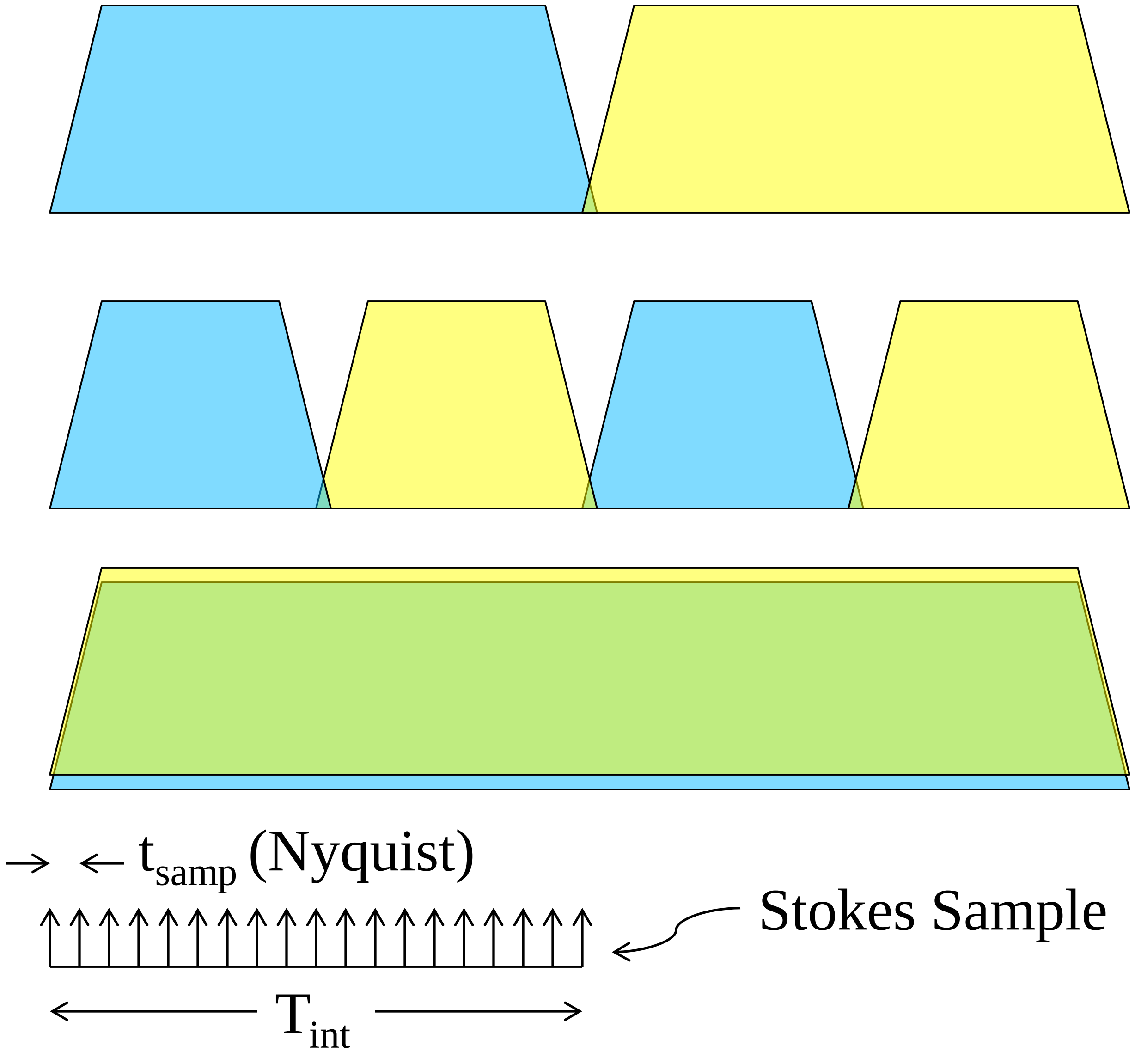
Valid only for instantaneous Stokes parameters



Cordes, Rankin & Backer (1978)

Three Regimes

- Disjoint
- Composite
- Superposed



$$\bar{\mathbf{C}}_{\rm disjoint} = F\bar{\mathbf{C}}_A + (1-F)\bar{\mathbf{C}}_B + F(1-F)(A-B)^{\otimes 2}$$

$$\bar{\mathbf{C}}_{\rm composite} = f\bar{\mathbf{C}}_A + (1-f)\bar{\mathbf{C}}_B$$

$$\bar{\mathbf{C}}_{\rm superposed} = \bar{\mathbf{C}}_A + \bar{\mathbf{C}}_B + n^{-1}\left(A \otimes B + B \otimes A - \boldsymbol{\eta} A \cdot B\right)$$

van Straten & Tiburzi (submitted)

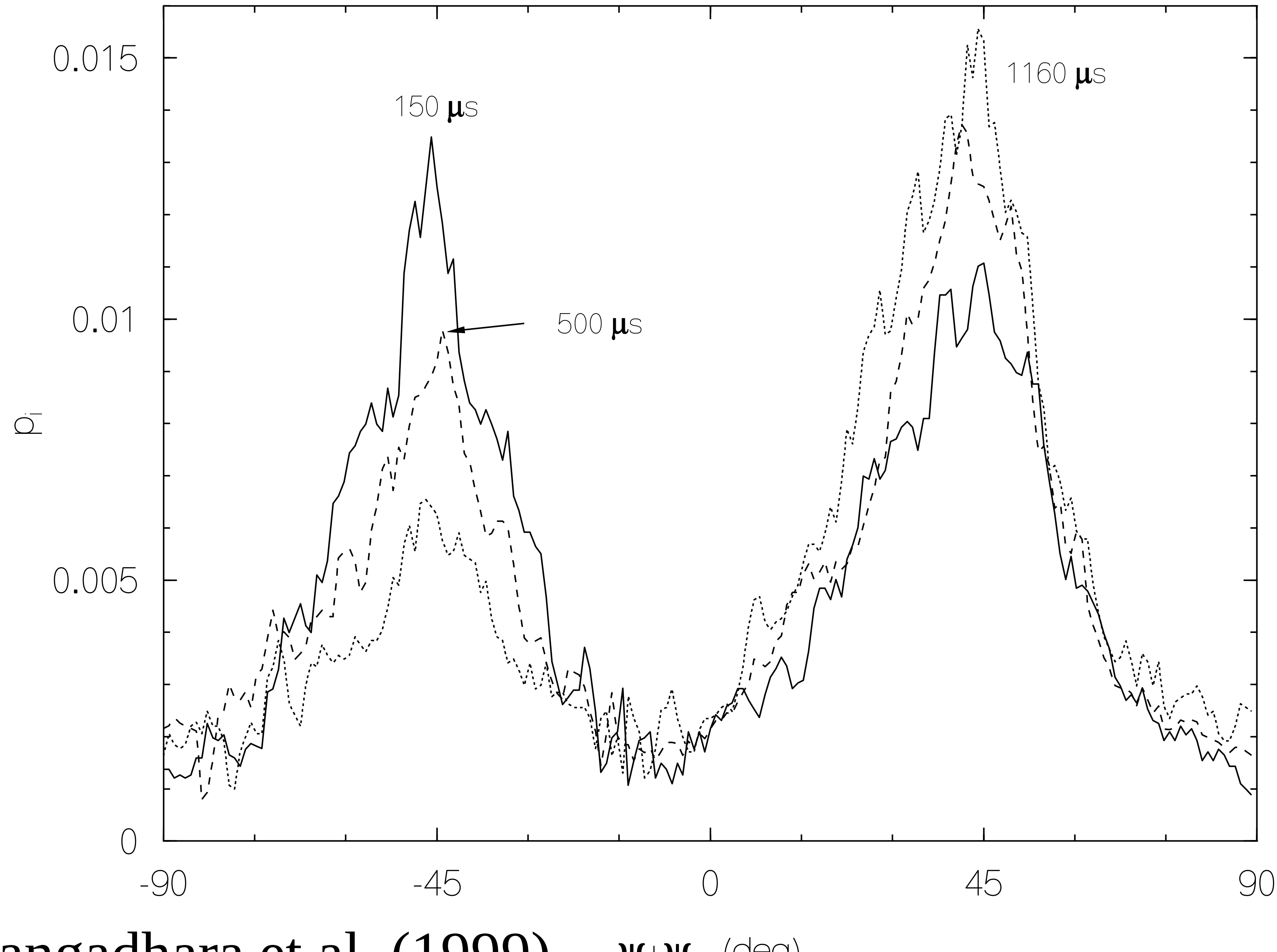
Composite samples

A. Satisfy previously proposed arguments in favour of superposed modes:

- Depolarization (Stinebring et al. 1984)
- Eq. 5 of Cordes, Rankin & Backer (1978)

B. Depend on instrumental resolution

i.e. distinction between “superposed” and disjoint modes is blurred by resolution



Gangadhara et al. (1999)

Two Problems

1. $\sigma_0 > \sigma_1$ even when no OPM

2. Incomplete statistical model

Therefore, cannot be explained by covariant mode intensities

$$\mathbf{C}' = \varsigma^2 \begin{pmatrix} \|S\|^2 & 2S_0|\boldsymbol{S}| & 0 & 0 \\ 2S_0|\boldsymbol{S}| & \|S\|^2 & 0 & 0 \\ 0 & 0 & S^2 & 0 \\ 0 & 0 & 0 & S^2 \end{pmatrix}$$

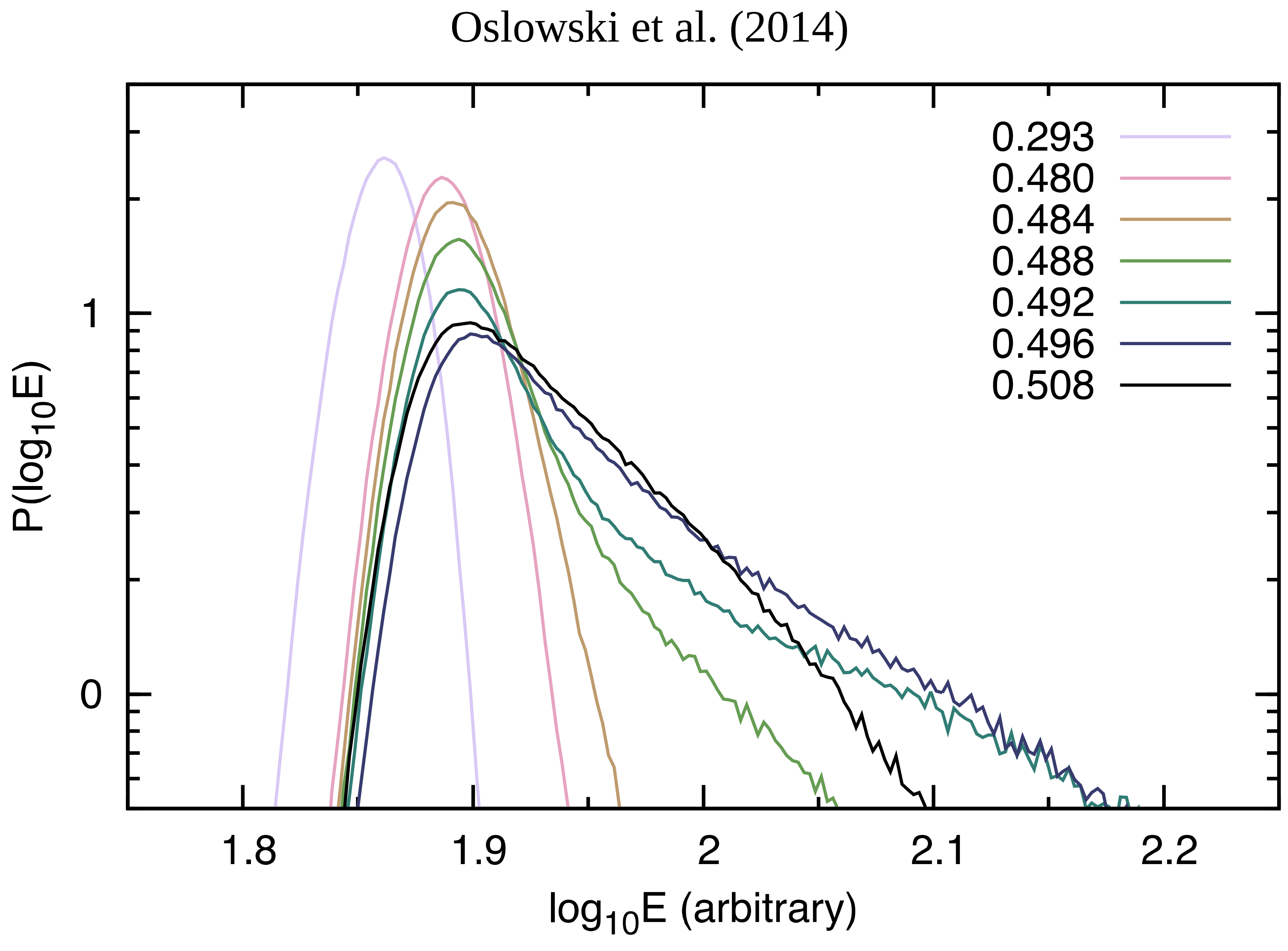
Brosseau & Barakat (1992)
van Straten (2009)

Valid only for normally distributed electric field

$$\mathbf{C}' = \varsigma^2 \begin{pmatrix} \text{[illegible]} & \text{[illegible]} & 0 & 0 \\ \text{[illegible]} & \|S\| \text{[illegible]} & 0 & 0 \\ 0 & 0 & S^2 & 0 \\ 0 & 0 & 0 & S^2 \end{pmatrix}$$

Brosseau & Barakat (1992)

van Straten (2009)

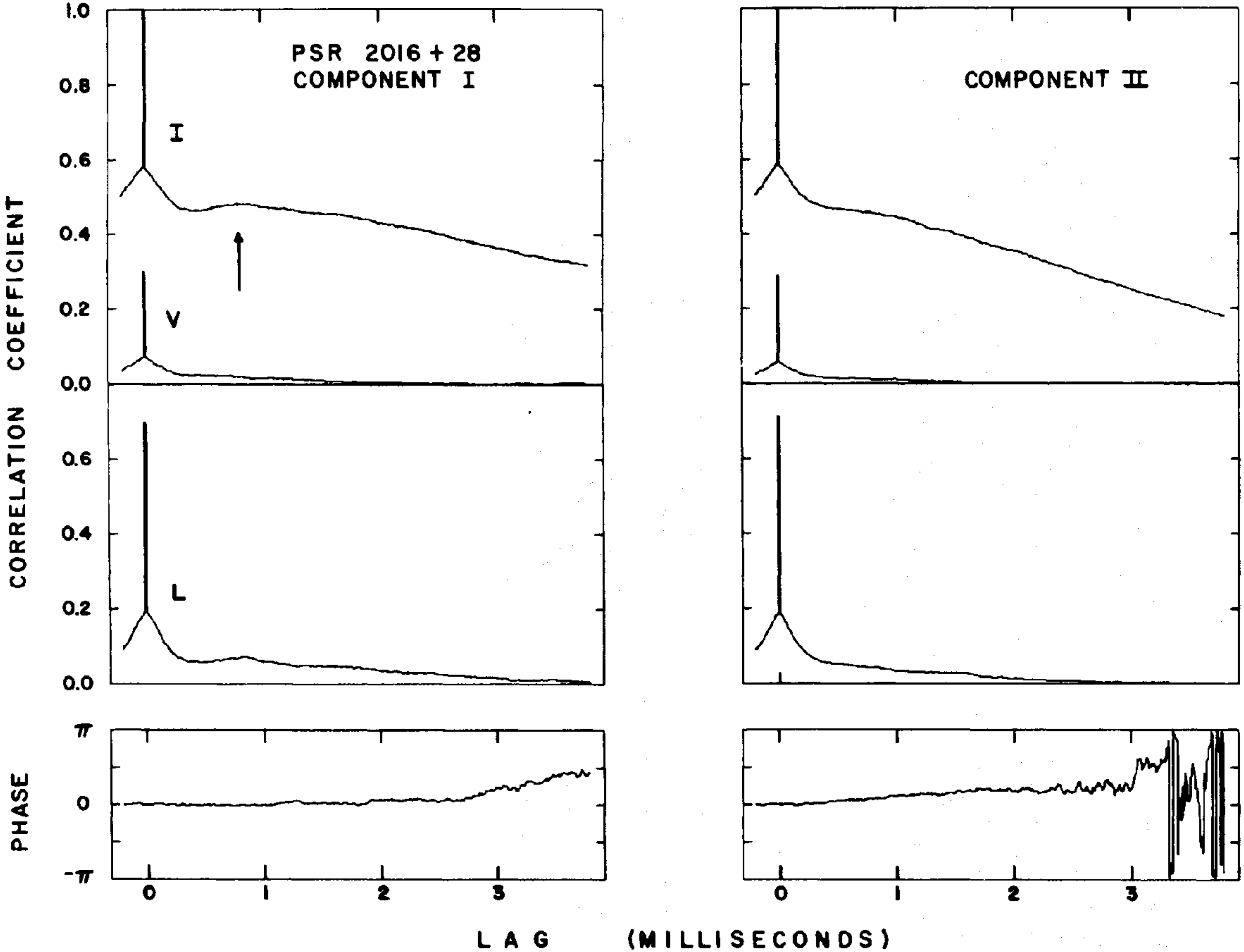

Oslowski et al. (2014)
0.293
0.480
0.484
0.488
0.492
0.496
0.508
1
0
P(log10E)
1.8
1.9
2
2.1
2.2
log10E (arbitrary)

Amplitude Modulation

$$\mathbf{C}' = \left(\varsigma_u^2 + 1\right)\mathbf{C} + \varsigma_u^2\, S \otimes S$$

Valid only for instantaneous Stokes parameters.

Cordes & Hankins (1977)



Conclusions

- The sample mean Stokes parameters admit three idealized regimes:
  - disjoint (mutually exclusive and resolved)
  - composite (unresolved mutual exclusivity)
  - superposed (classical wave superposition)

Conclusions

- The 4x4 matrix of covariances between the Stokes parameters provides new insight
  - “modal broadening” explained by self noise
  - no need for additional randomly polarized component
  - potential to differentiate between regimes

Conclusions

- Amplitude modulation must be considered
  - including temporal correlations / structure of amplitude modulating function

Conclusions

- Ekers & Moffet (1969) discussion
  - observed emission may originate in different regions of pulsar magnetosphere
  - observed polarisation variations could be due to emission mechanism or propagation

Conclusions

- Statistical approach
  - e.g. McKinnon (2004); Edwards (2004)
  - can be applied to weaker pulsars
  - may enable broader studies of population
  - more than one process involved

THE STATISTICS OF RADIO ASTRONOMICAL POLARIMETRY: DISJOINT, SUPERPOSED, AND COMPOSITE SAMPLES

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(Received)

ABSTRACT

A statistical model is presented for the study of the orthogonally polarized modes of radio pulsar emission via the covariances between the Stokes parameters. Particular consideration is given to the effects of integration over finite intervals appropriate to the analysis of single-pulse observations. If the interval over which the polarization state is estimated is longer than the timescale for switching between disjoint modes, then the resulting composition of modes exhibits properties that have been attributed to mode superposition; in fact, composite modes satisfy all of the statistical tests of superposed modes that have been proposed to date. Because the distinction between composite and disjoint modes depends on the temporal resolution of the observing instrumentation, the evidence in favour of superposed modes of pulsar emission is revisited. First, to accommodate the typically heavy-tailed distributions of single-pulse radio flux density, the fourth-order joint cumulants of the electric field are used to derive the covariances between the Stokes parameters when two sources with