Interferometric Polarimetry: A primer

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Polarimetry – Why Do It?

- EM radiation is a transverse wave, with two independent components.
- Polarimetry refers to the characteristics of these two components.
  - Their amplitudes, and the phase relation between them.
- Why do we care about polarization?
- Because various physical processes emit radiation which is partially polarized.
- Measuring the polarization gives us additional information into the physical processes at play.
- Examples:
  - Synchrotron radiation – orientation and strength of magnetic fields.
  - Zeeman splitting – strength of fields.
  - Electron scattering
  - Faraday rotation (of linear polarization due to magnetic fields)
  - Polarization of radiation from thermal bodies – measures the material refractive index.

Interferometric Polarimetry

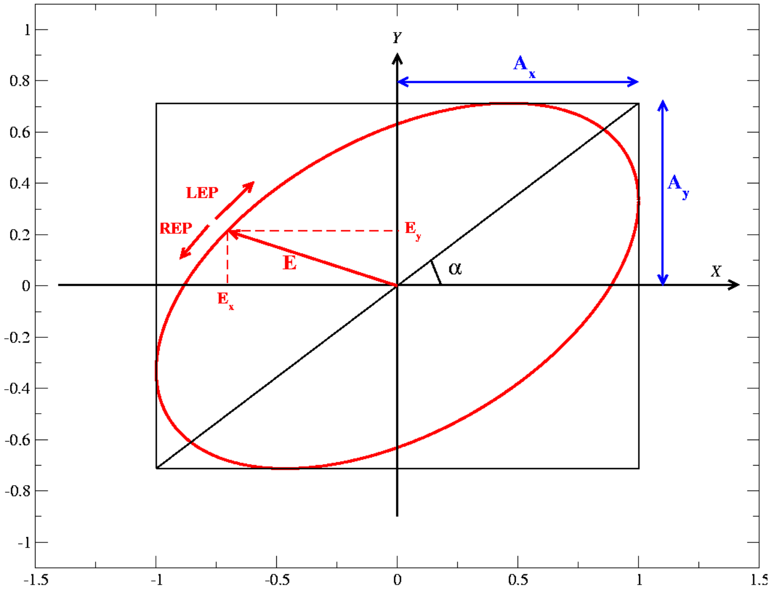
- The description of polarization usually begins with utilizing the 'quasi-monochromatic approximation'.
- Here we imagine analysis of radiation passed through a very narrow filter – say 1 Hz wide.
- The characteristics of the field are then quasi-stable for ~1 second.
- Maxwell's equations then tell us the electric field describes an ellipse.

In general, three parameters are needed to describe the ellipse.

- A_x – X-axis amplitude max
- A_y – Y-axis amplitude max
- $\alpha = \text{atan}(A_y/A_x)$ – an angle describing the orientation

If the E vector is rotating (as seen by the observer):

- Clockwise, the wave is Left Elliptically Polarized:
- Anti-clockwise, the wave is Right Elliptically Polarized.



Linear and Circular Bases

- It is easy to conceptualize an elliptically polarized propagating wave as the sum of two orthogonal linear components: E_x and E_y.
  - There are three factors: the two amplitudes, and the phase between them.
- But we can also describe the elliptical wave in terms of two oppositely rotating circular components.
- Again – three factors: E_r, E_l, and the phase between them.
- This is sufficient for the monochromatic case, but in general, radiation is broad-band, originating from an uncountably large number of electrons.
- This results in partial polarization, for which we need a fourth parameter.

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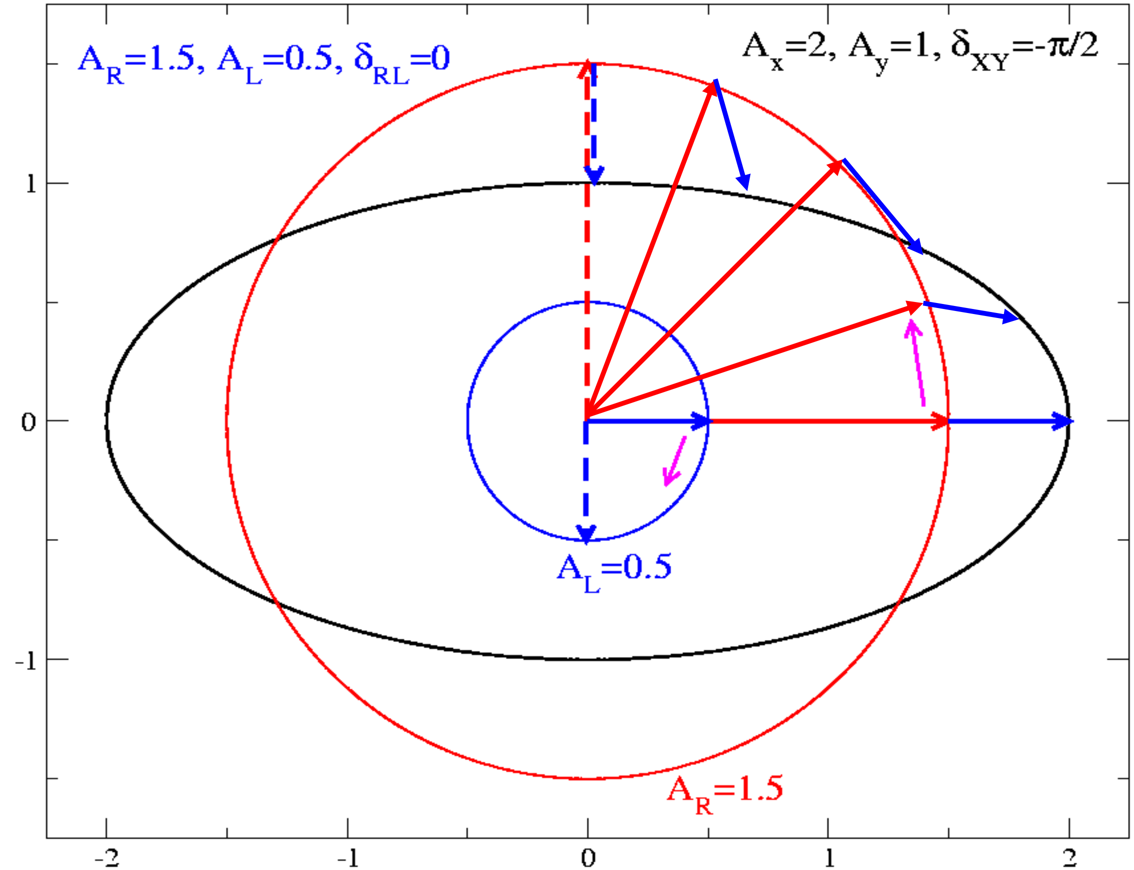
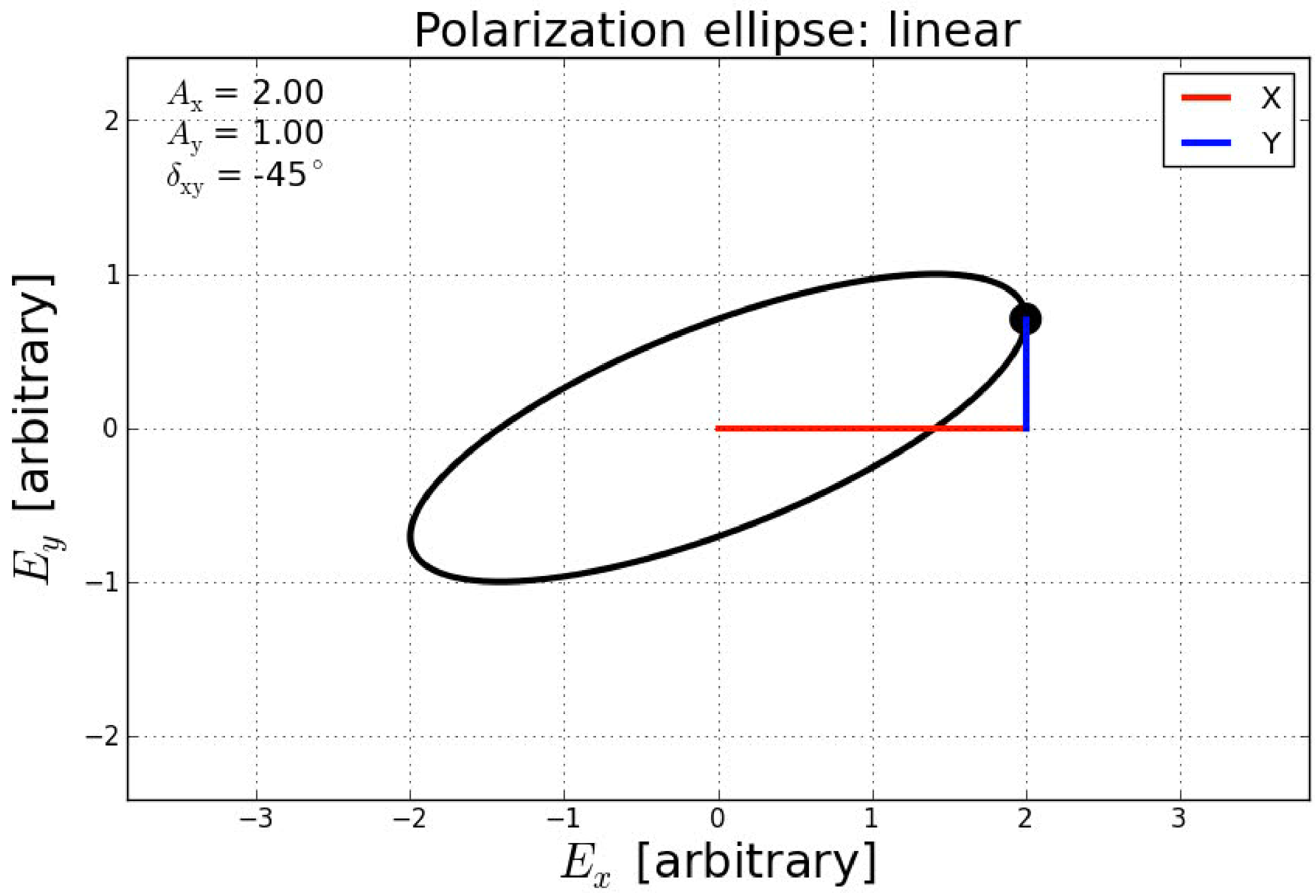
Circular Basis Example

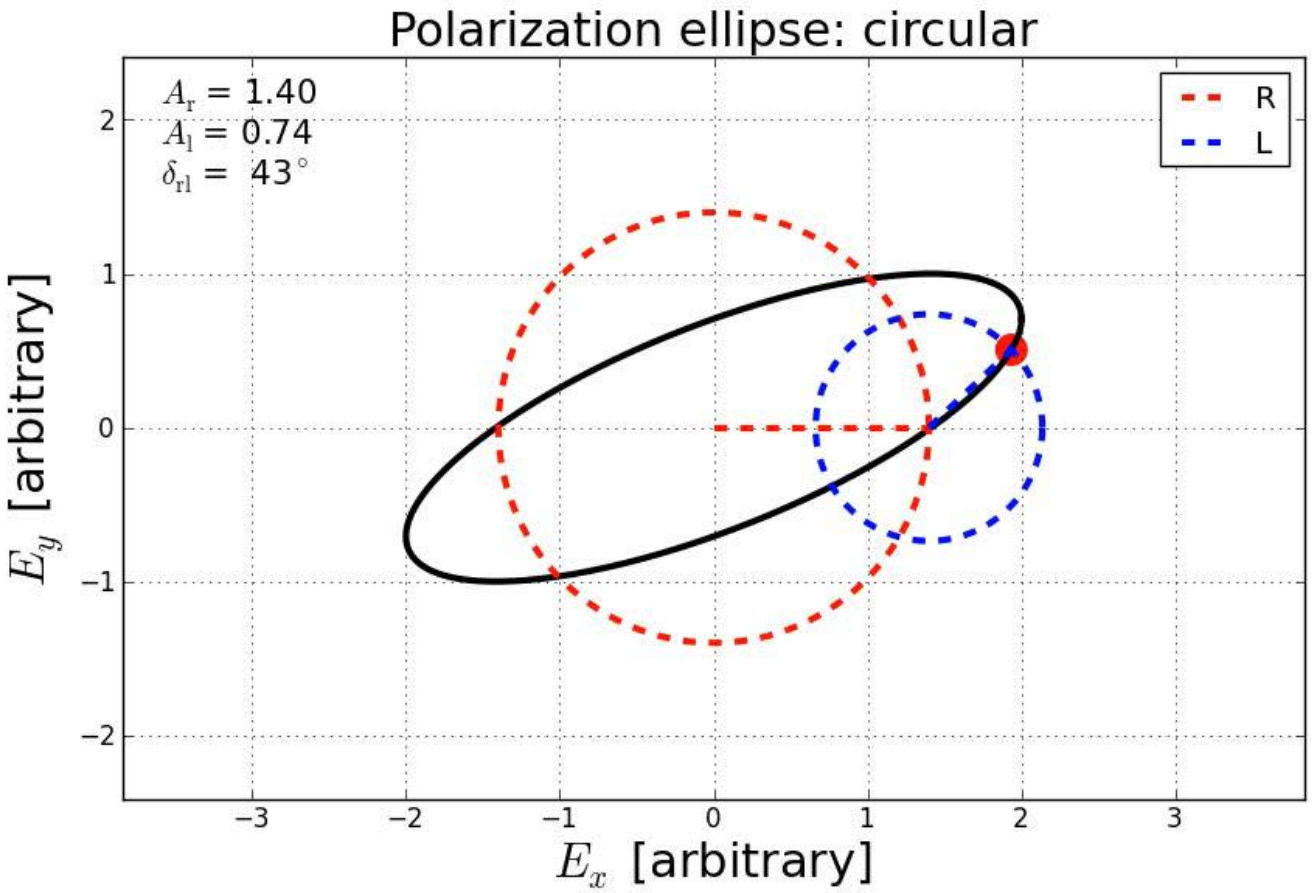
- The polarization ellipse (black) can be decomposed into an X-component of amplitude 2, and a Y-component of amplitude 1 which lags by ¼ turn.
- It can alternatively be decomposed into a counterclockwise (RCP) rotating vector of length 1.5 (red), and a clockwise rotating (LCP) vector of length 0.5 (blue).



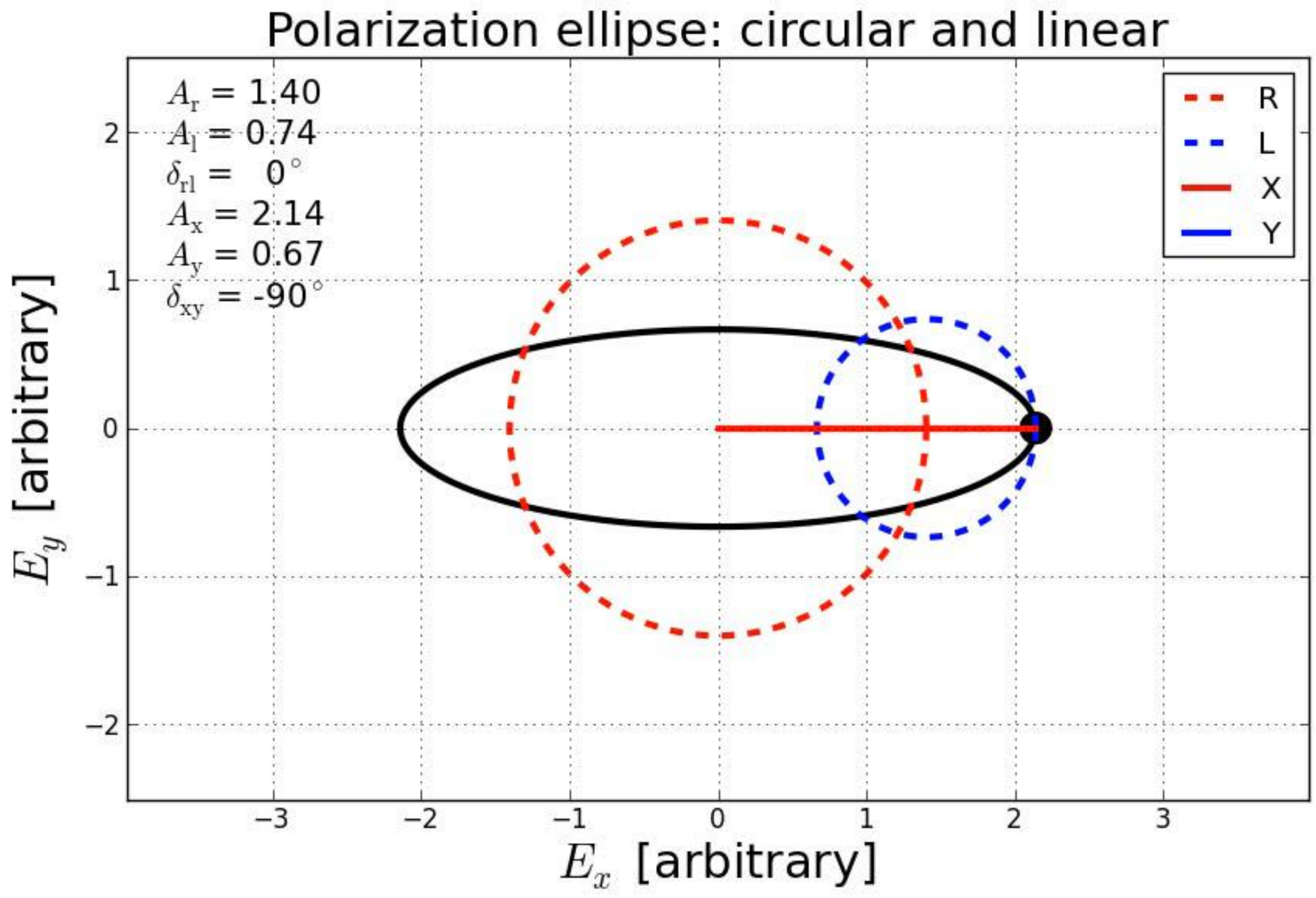
- Elliptical Wave, decomposed into orthogonal linear components.



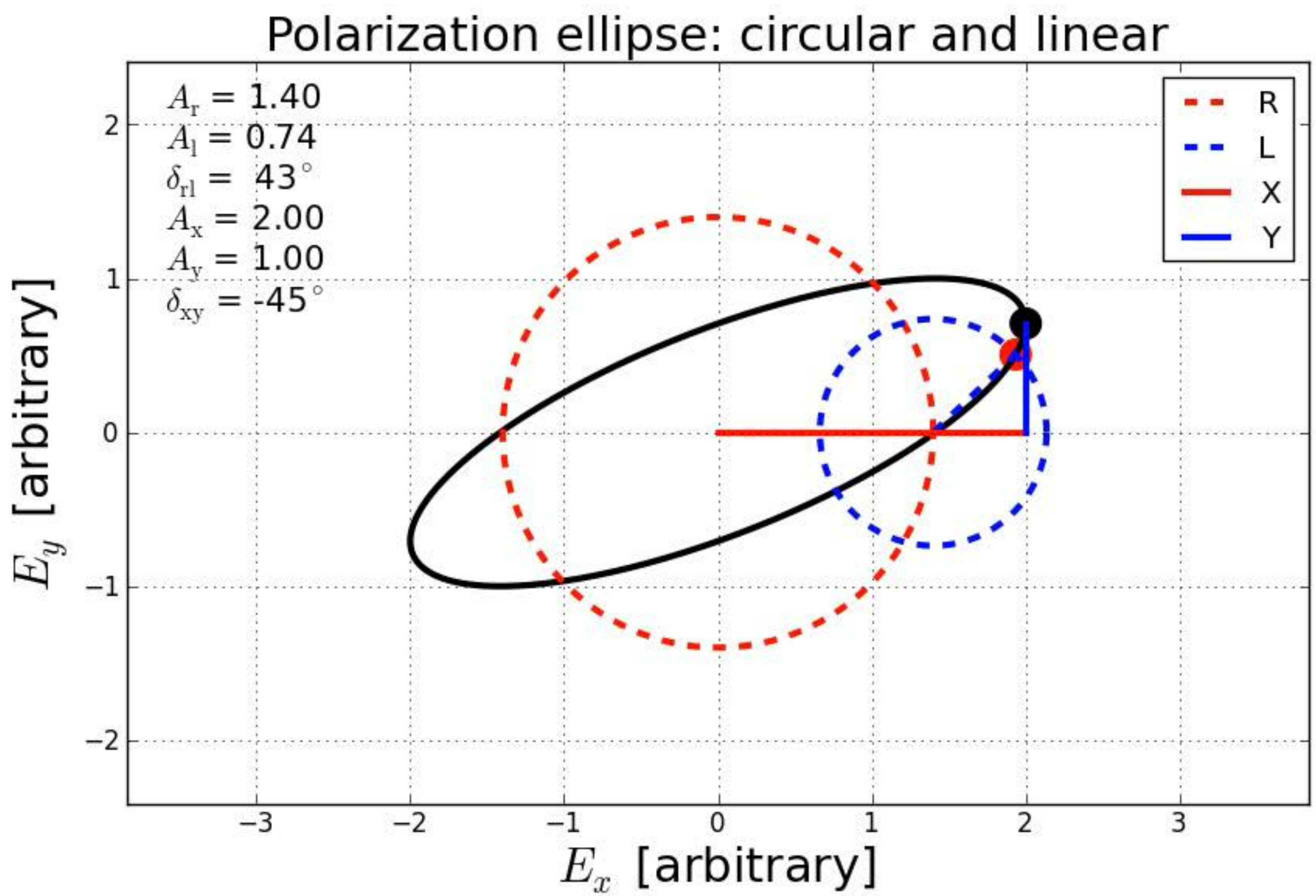
- The same wave, decomposed into orthogonal circular components



- Both decompositions, for a horizontal ellipse

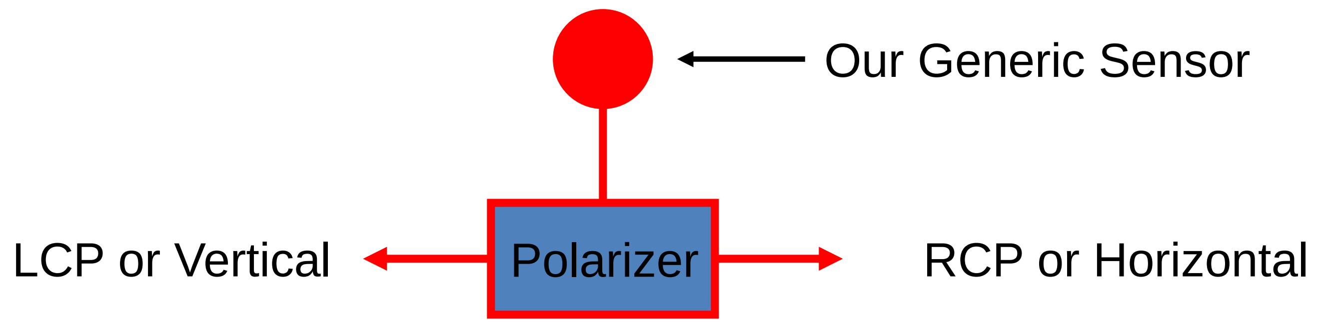


- Both decompositions, for a tilted ellipse.



Antennas are Polarized!

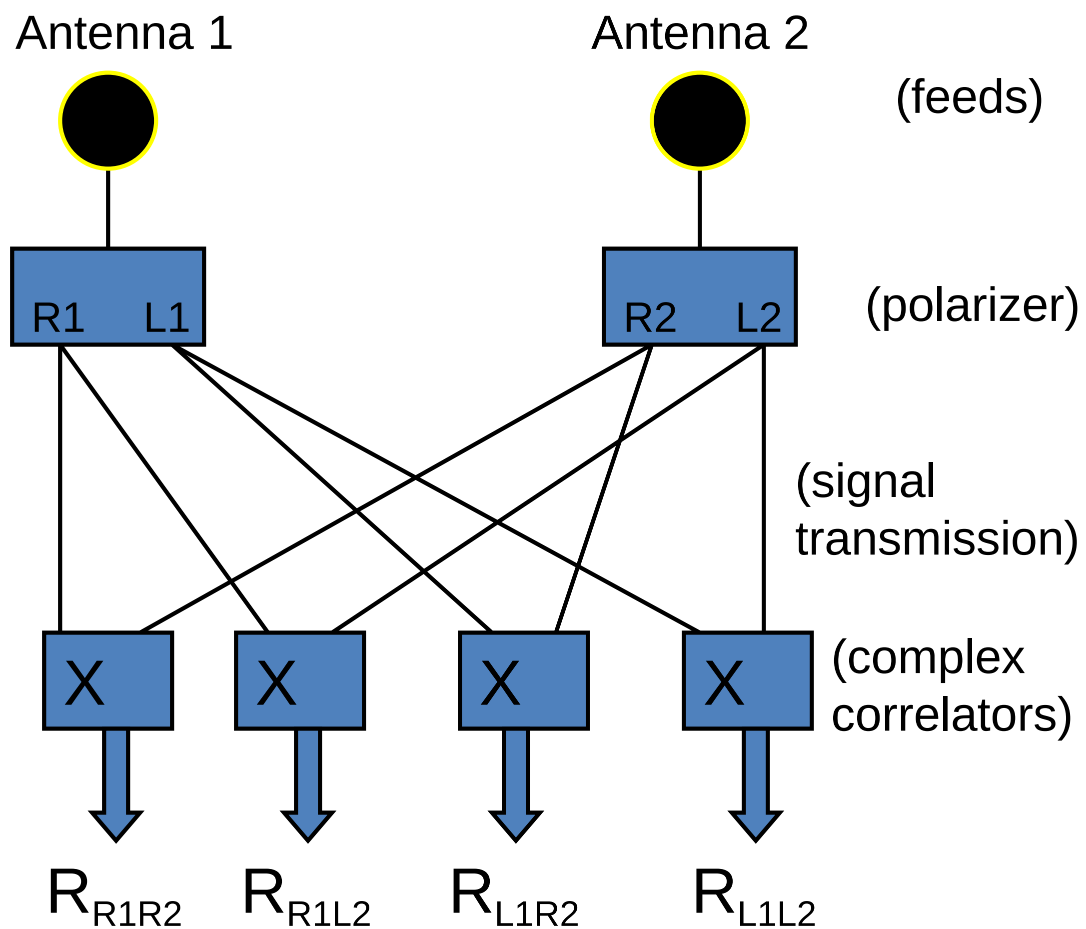
- The choice of basis for the description is useful, since antenna/receiver systems are themselves naturally polarized.
- They are designed to output signals (voltages) proportional to the amplitude and phase of either the linear, or circular, components.
- They provide two simultaneous voltage signals whose values are (ideally) representations of the electric field components – either in a circular or linear basis.



- We have two antennas, each with two polarized outputs.
- We can then form four complex correlations.

Four Complex Correlations per Pair of Antennas

- Two antennas, each with two differently polarized outputs, produce four complex correlations.
- The 'RR' and 'LL' (or VV and HH) correlations are called the 'parallel hands'.
- The 'RL' and 'LR'(or VH and HV) correlations are called 'cross-hands'.



What is the relation between these correlations and polarimetry?

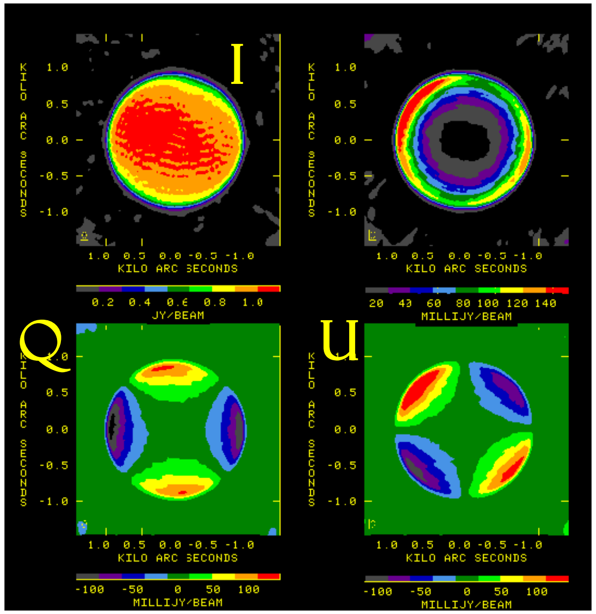
Stokes Parameters -- Definition

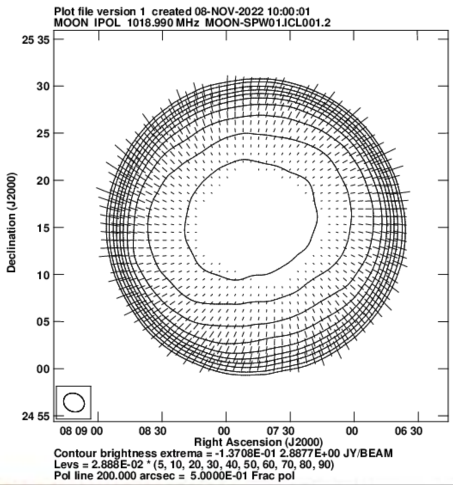
- The Stokes Parameters (named for George Stokes, 1842) are now commonly used to describe astronomical signal polarization.
- They have units of spectral power, or brightness.
- I describes the brightness ('total power').
- Q and U describe the linear polarization:

  +Q => vertical EVPA, -Q => horizontal EVPA

  +U => EVPA at 45deg, -U => EVPA at -45 deg

  EVPA: $\chi = 0.5 \arctan(U/Q)$
- V describes circular polarization:
  - +V => Right CP, -V => Left CP
- In general, the signal is a mixture of Q, U, and V.
- Always, $I^2 > Q^2 + U^2 + V^2$





Stokes Parameters

- Three parameters are sufficient to describe the monochromatic EM wave properties.
- It is most convenient to have the three parameters share the same units, and have easily grasped physical meanings.
- It is standard in radio astronomy to utilize the parameters defined by George Stokes (1852), and introduced to astronomy by Chandrasekhar (1946):

Linear Basis **Circular Basis**

$$I = A_X^2 + A_Y^2 \qquad = A_R^2 + A_L^2$$

$$Q = A_V^2 - A_H^2 \qquad = 2A_R A_L \cos\delta_{RL}$$

$$U = 2A_V A_H \cos\delta_{VH} = 2A_R A_L \sin\delta_{RL}$$

$$V = 2A_V A_H \sin\delta_{VH} = A_R^2 - A_L^2$$

I describes the total flux

Q and U describe linear poln

V describes circular poln

- Note that $I^2 = Q^2 + U^2 + V^2$
- But, wideband signals are partially polarized:

$$I^2 > Q^2 + U^2 + V^2$$

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Stokes Visibilities for Interferometry

- You will all know that the Visibility Function, V(u,v), is related to the sky brightness by Fourier Transform:

  $V(u,v) \Leftrightarrow I(l,m)$ (a Fourier Transform Pair)

- In basic derivations, 'I' referred to a single polarization (like 'H' of 'V').
- We will now be more formal, and consider the Stokes brightness distributions for I, Q, U, and V.
- Define the **Stokes Visibilities** $\mathcal{I}$, $\mathcal{Q}$, $\mathcal{U}$, and $\mathcal{V}$, to be the Fourier Transforms of these brightness distributions.
- Then, the relations between these are:
- $\mathcal{I} \Leftrightarrow I$, $\mathcal{Q} \Leftrightarrow Q$, $\mathcal{U} \Leftrightarrow U$, $\mathcal{V} \Leftrightarrow V$
- Stokes Visibilities are complex functions of (u,v), while the Stokes Images are real functions of (l,m).
- All Stokes visibilities are Hermitian (V(u,v) = V*(-u,-v))
- Our task is now to measure these Stokes visibilities.

Stokes Visibilities – Special Case

- For simplicity, I omit (for this slide) the orientation of the dipoles, and presume they are aligned with the (α,δ) sky coordinates.
- This applies to equatorial-mounted antennas.

Perfect Circular	Perfect Linear
$\mathcal{I} = R_{R1R2} + R_{L1L2}$	$\mathcal{I} = R_{V1V2} + R_{H1H2}$
$\mathcal{V} = R_{R1R2} - R_{L1L2}$	$\mathcal{V} = i\left(R_{H1V2} - R_{V1H2}\right)$
$\mathcal{Q} = R_{R1L2} + R_{L1R2}$	$\mathcal{Q} = \left(R_{V1V2} - R_{H1H2}\right)$
$\mathcal{U} = i\left(R_{L1R2} - R_{R1L2}\right)$	$\mathcal{U} = \left(R_{V1H2} + R_{H1V2}\right)$

- All quantities here are complex valued.
- For both systems, Stokes 'I' is the sum of the parallel-hands.
- Stokes 'V' is the difference of the crossed hand responses for linear, (good) and is the difference of the parallel-hand responses for circular (bad).
- Stokes 'Q' involves only cross-hand correlations in the circular system (good), but involves all four correlations in the linear (bad).

Stokes Visibilities – General Case

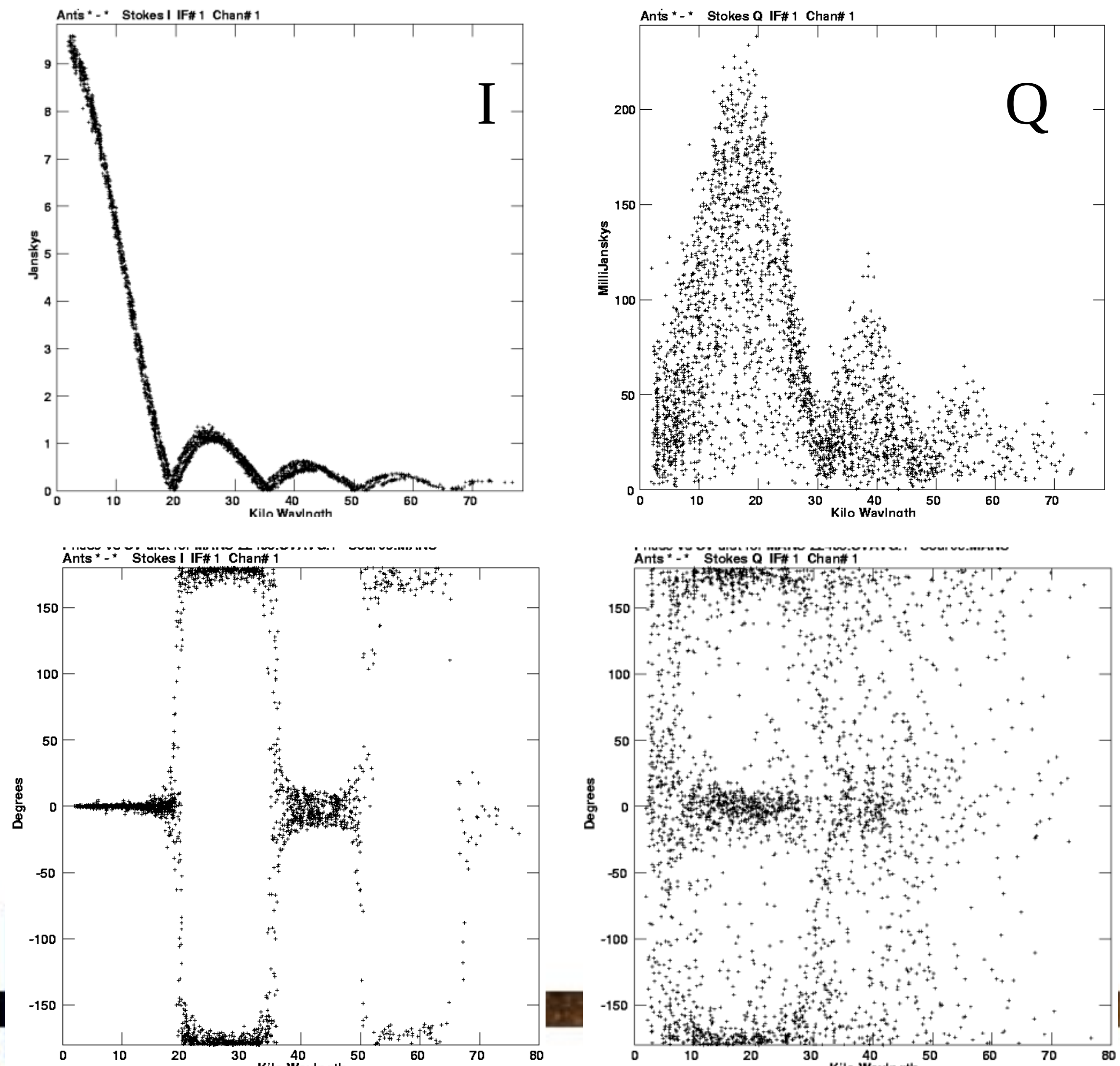
- The more general form, which includes the orientation of the antenna w.r.t. the celestial coordinate frame (described by the 'parallactic angle' looks like these:

Circular	Linear
$\mathcal{I} = R_{R1R2} + R_{L1L2}$	$\mathcal{I} = R_{V1V2} + R_{H1H2}$
$\mathcal{V} = R_{R1R2} - R_{L1L2}$	$\mathcal{V} = i\left(R_{H1V2} - R_{V1H2}\right)$
$\mathcal{Q} = e^{i2\Psi_P} R_{R1L2} + e^{-i2\Psi_P} R_{L1R2}$	$\mathcal{Q} = \left(R_{V1V2} - R_{H1H2}\right)\cos 2\Psi_P - \left(R_{V1H2} + R_{H1V2}\right)\sin 2\Psi_P$
$\mathcal{U} = i\left(e^{-i2\Psi_P} R_{L1R2} - e^{i2\Psi_P} R_{R1L2}\right)$	$\mathcal{U} = \left(R_{V1V2} - R_{H1H2}\right)\sin 2\Psi_P + \left(R_{V1H2} + R_{H1V2}\right)\cos 2\Psi_P$

- Note that in the circular system, the linear components (Q and U) are uniquely found in the cross-hand components, while in the linear system, they require all four correlations.
- This is a major advantage to circular systems (if linear polarization is what you're interested in).

$\mathcal{I}$ and $\mathcal{Q}$ Visibilities for Mars at 23 GHz

VLA, 23 GHz, 'D' Configuration, January 2006



Amplitude

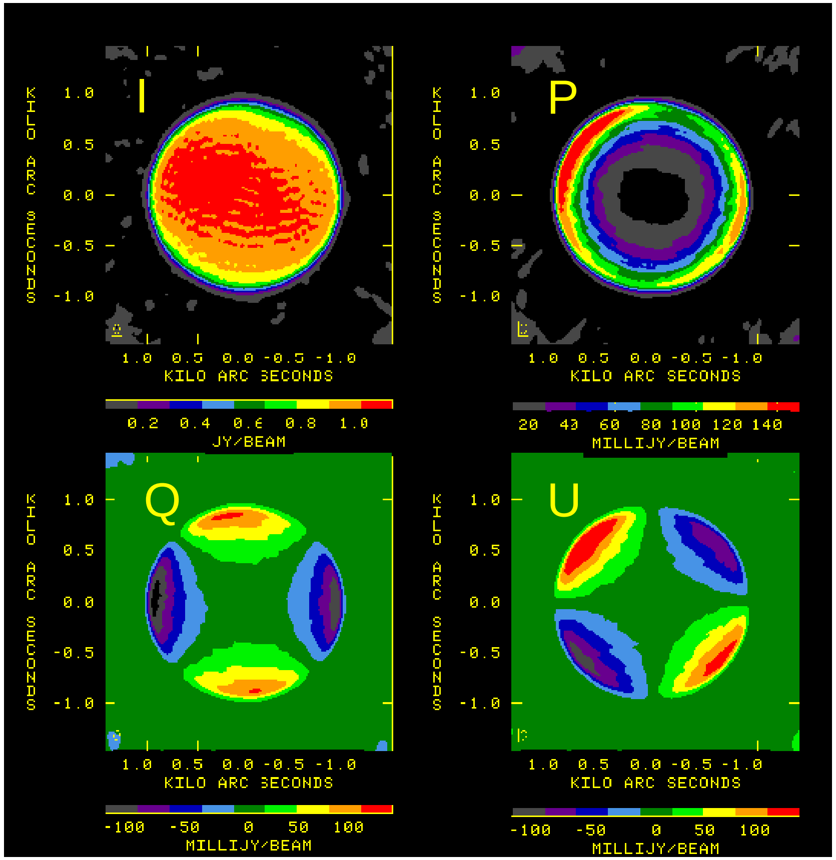
- $|\mathcal{I}|$ is close to a J_0 Bessel function.
- Zero crossing at 20kλ tells us Mars diameter ~ 10 arcsec.
- $|\mathcal{Q}|$ amplitude ~0 at zero baseline.
- $|\mathcal{Q}|$ zero at 30 kλ means polarization structures ~ 8 arcsec scale.

Phase

- $\mathcal{I}$ phase alternates between 0 and π.
- $\mathcal{Q}$ phase = both 0 and π in the 'main lobe' – this tells us there are both positive and negative structures, at different PA.

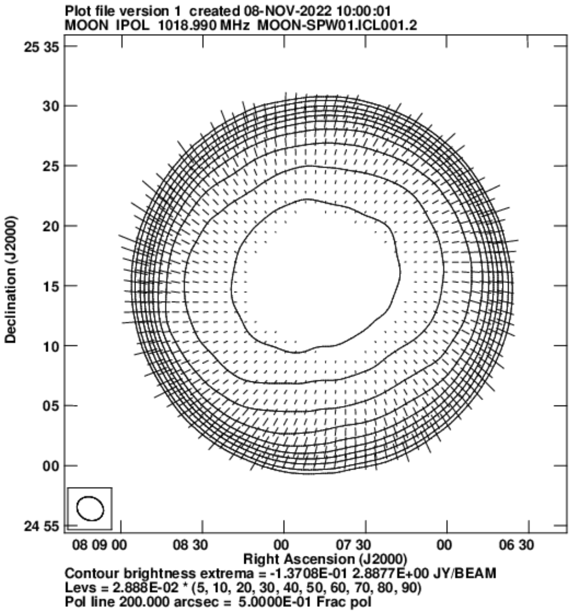
Imaging – Polarization of the Moon

- Shown here are the total intensity (I), polarized intensity (P), and Q and U images at 1040 MHz.
- The apparent elliptical brightness shape (in both I and P) is real – observations were taken in June (sun at high dec, moon was low)
- The edge-brightened polarization maximum is exactly as expected. See Perley & Butler, ApJSupl, **206**, 16 (2013) for details.
- The Q and U images tell us right away that the EVPAs are very nearly radial (as expected).

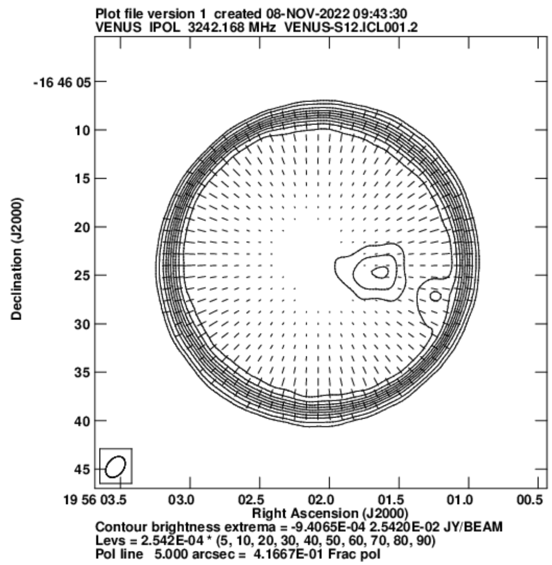


Example Images: Moon, Venus, Mars

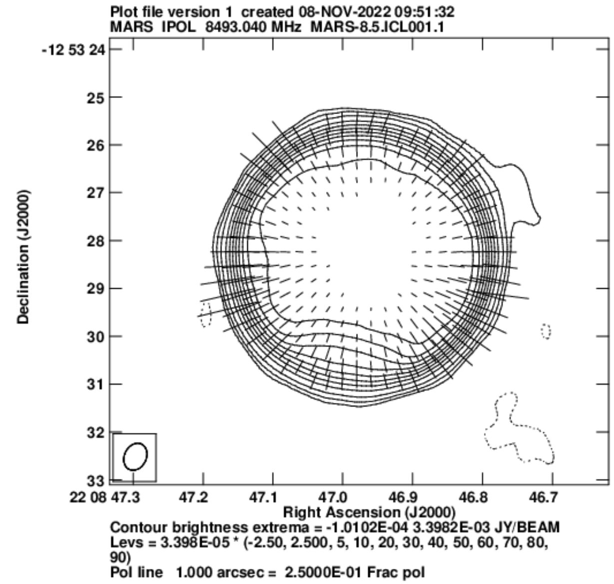
- Theory tells us that thermal radiation emitted from underneath the surface of a solid planet must be radially polarized, reaching about 30% near the limb.
- The maximum polarization depends on the dielectric constant of the material.
- We can use the observed position angle to calibrate our instruments.



Moon at 1.02 GHz
30 arcmin diameter
Limb darkening due to primary beam attenuation.



Venus at 3.24 GHz
30 arcsec
Cold regions are elevated terrain (Ovda and Thetis Regio)



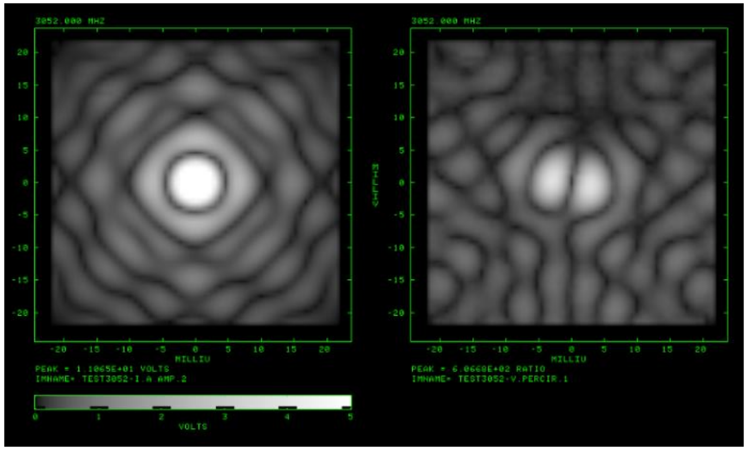
Mars at 8.49 GHz
5 arcsec

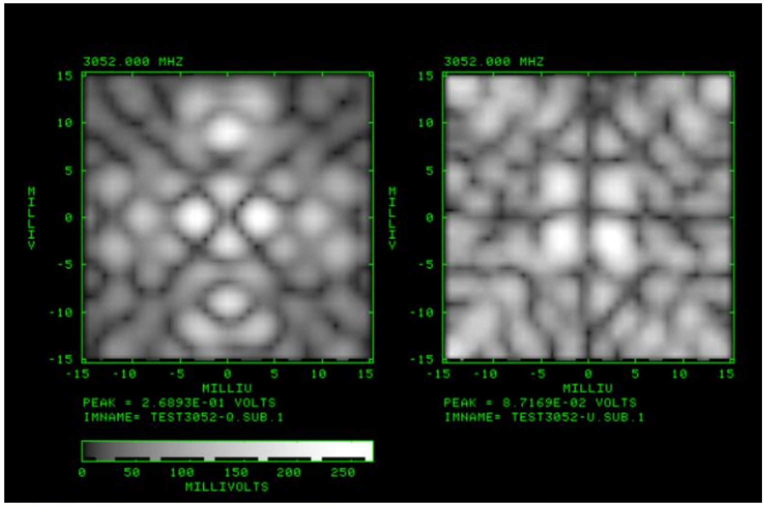
Not as Simple as it Seems …

- From this, you may be led to think this is easy.
  - Add polarizers, cross-multiply, calibrate, image, and done!
- Sadly, the reality is a bit more complex.
  - The polarizers are not perfect.
  - Real electronics 'leak' signals from one polarization to the other.
- And – to heap insult upon insult
  - Real antennas are differentially spatially polarized – their polarization is a function of angle on the sky.
- Bottom line here is that the antenna output labelled (say) 'R' is not wholly 'R', but contains a little bit of 'L'.
- This is an issue of design, and of the software needed to correct for the contamination.

VLA's Polarized Beams at 3 GHz.

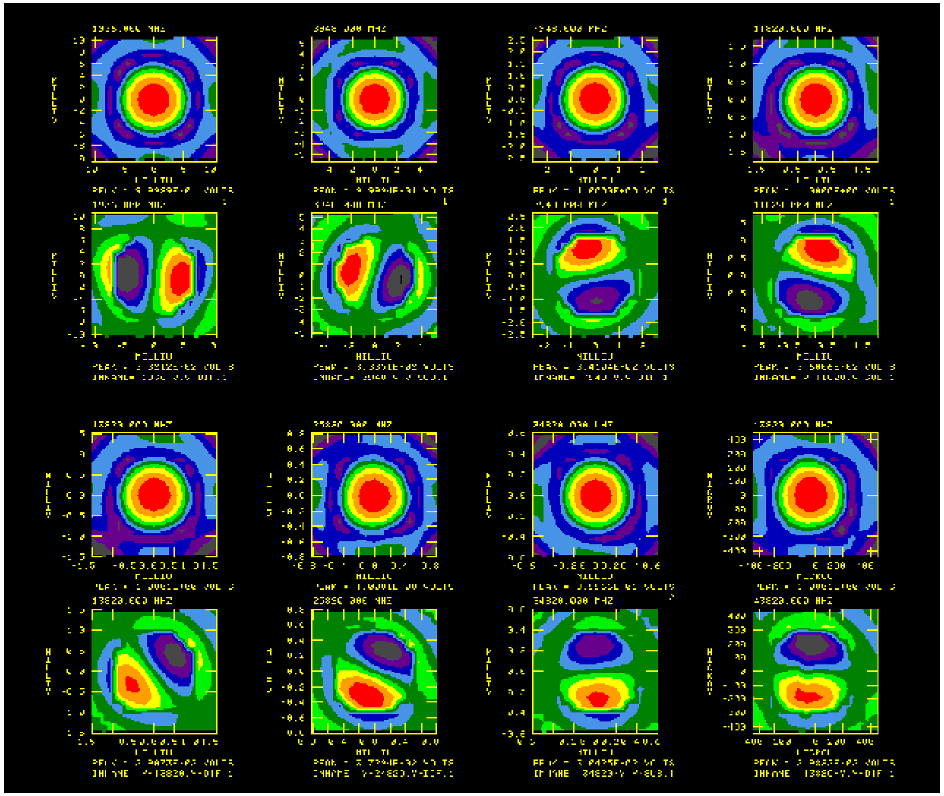
- The VLA's primary antenna response is significantly polarized.
- This is due primarily to asymmetries in the optical design.
- V polarization due to offset of the feed from axis of symmetry.
- Q, U polarizations due to parabolic reflector.
- These antenna-imposed signals must be removed from data to enable wide-field astronomical polarimetry.





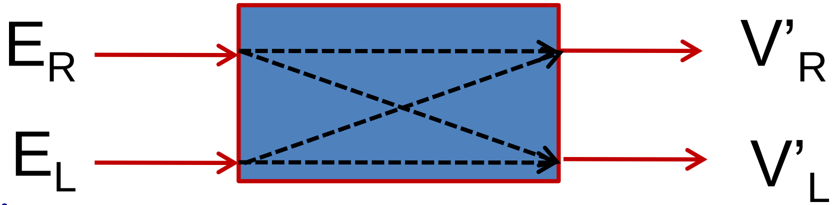
VLA's I and V beams – all 8 bands

- I and V beam patterns for all eight JVLA bands.
- I beams (scaled) are all very similar.
- V beams rotate according to the position angle of the offset feed.
- V>0 => Red = RCP
- V< 0 => Purple = LCP



Jones Matrix Algebra

- The analysis of how a real interferometer, comprising real antennas and real electronics, is greatly facilitated through use of Jones matrices.
- In this, we break up our general system into a series of 4-port components, each of which is presumed to be linear.
- Chain them all together, and represent the telescope as:



- And write:

$$\begin{pmatrix} V'_{R'} \\ V'_{L} \end{pmatrix} = \begin{pmatrix} G_{RR} & G_{LR} \\ G_{RL} & G_{LL} \end{pmatrix} \begin{pmatrix} E_R \\ E_L \end{pmatrix}$$

- Or, in shorthand **V' = JV**
- The four G components of the Jones matrix describe the linkages within the 'grey box'.

The Generalized Formulation (circular basis)

- For an array with the same parallactic angle for each element, ignoring the gains, an alternate form can be written:

$$\begin{pmatrix} R_{R1R2} \\ R_{R1L2} \\ R_{L1R2} \\ R_{L1L2} \end{pmatrix} = \begin{pmatrix} 1 & D^*_{LR2} & D_{LR1} & D_{LR1}D^*_{LR2} \\ D^*_{RL2} & 1 & D_{LR1}D^*_{RL2} & D_{LR1} \\ D_{RL1} & D_{RL1}D^*_{LR2} & 1 & D^*_{LR2} \\ D_{RL1}D^*_{LR2} & D_{RL1} & D^*_{RL2} & 1 \end{pmatrix} \begin{pmatrix} (\mathcal{I}+\mathcal{V})/2 \\ e^{-2i\Psi_P}(\mathcal{Q}+i\mathcal{U})/2 \\ e^{2i\Psi_P}(\mathcal{Q}-i\mathcal{U})/2 \\ (\mathcal{I}-\mathcal{V})/2 \end{pmatrix}$$

- The D's are (unimaginatively) called the 'D-terms', and describe the amplitude and phase of the cross-over signals from R to L, and L to R.
- **Main Point:** The effect of an impure polarizer is to couple all four of the Stokes visibilities to all four cross-products.
- If the 'D' terms are known in advance, this matrix equation can be easily inverted, to solve for the Stokes visibilities in terms of the measured Rs, and the known Ds.

Calibration of Polarimetric Data

- While it's easy to write down these equations, it's not so simple to determine the necessary calibration constants.
- For 'perfect' polarizers, we have a number of calibration parameters to determine:
  - The parallel-hand gain amplitudes
  - The parallel-hand phases (w.r.t. a reference antenna).
  - The parallel-hand delays (w.r.t. a reference antenna).
  - The cross-hand phases.

Parallel-Hand Gains

- For perfect circular or linear systems, the four correlations are related to the Stokes visibilities by:

$$R_{R1R2} = G_{R1}G_{R2}^{*}(\mathcal{I} + \mathcal{V})/2$$
$$R_{L1L2} = G_{L1}G_{L2}^{*}(\mathcal{I} - \mathcal{V})/2$$
$$R_{R1L2} = G_{R1}G_{L2}^{*}(\mathcal{Q} + i\mathcal{U})e^{i2\psi_P}/2$$
$$R_{L1R2} = G_{L1}G_{R2}^{*}(\mathcal{Q} - i\mathcal{U})e^{-i2\psi_P}/2$$

$$R_{V1V2} = G_{V1}G_{V2}^{*}(\mathcal{I} + \mathcal{Q}\cos 2\Psi_P + \mathcal{U}\sin 2\Psi_P)/2$$
$$R_{H1H2} = G_{H1}G_{H2}^{*}(\mathcal{I} - \mathcal{Q}\cos 2\Psi_P - \mathcal{U}\sin 2\Psi_P)/2$$
$$R_{V1H2} = G_{V1}G_{H2}^{*}(-\mathcal{Q}\sin 2\Psi_P + \mathcal{U}\cos 2\Psi_P + i\mathcal{V})/2$$
$$R_{H1V2} = G_{H1}G_{V2}^{*}(-\mathcal{Q}\sin 2\Psi_P + \mathcal{U}\cos 2\Psi_P - i\mathcal{V})/2$$

- By far the simplest approach is to utilize unpolarized calibrators!
- But, most calibrators are polarized, so we must deal …
- In fact, circular polarization is very low for most calibrators, (<< 1%), so circular systems have a decided edge!
- For linearly polarized systems, must know, or be able to derive, the linear polarization of the calibrators as part of the calibration regimen.

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Crossed-Hand Phase

- Parallel-hand calibration treats each polarization independently.
- For amplitudes, this is appropriate.
- But for phases, there remains one unknown variable – the cross-hand phase of the reference antenna.
  - This is because phases are not absolute – interferometers measure the phase difference between antenna signals, so an arbitrary phase offset between the parallel hand channels will remain.
- This offset has interesting implications in polarimetry:
- For Circular systems, the effect is to rotate the EVPA of the observed linearly polarization by twice the phase offset – the rotation is about the V axis in the Poincare sphere.
- For Linear systems, the effect is rotate Q, U, and V about an axis in the (Q,U) plane. For an equatorial antenna, the rotation is about the Q axis in the Poincare sphere.

Calibrating the cross-hand phase

- The best way is to design the electronics so that the phase differential between the signal channels (R-L, or V-H) is continuously measured.
- These values can then be fed to the software, which makes the necessary adjustments.
- Sadly, the VLA has no such on-board calibration system. So, for VLA polarimetry, one must solve for the residual cross-hand phase by observation of a polarized calibrator source with known EVPA.
- For linear systems, without functioning monitoring, the procedure is a bit more difficult. One can show that, (see EVLA Memo 219) in the presence of a cross-hand phase, the apparent Stokes' visibilities are:

$$I' = I$$

$$Q' = Q\cos 2\Psi_P + U\sin 2\Psi_P$$

$$U' = (-Q\sin 2\Psi_P + U\cos 2\Psi_P)\cos\phi + V\sin\phi$$

$$V' = (Q\sin 2\Psi_P - U\cos 2\Psi_P)\sin\phi + V\cos\phi$$

Cross-hand phase for Linears

- The apparent U and V visibilities then give the required phase:

$$\tan\phi = V^{'}/U^{'}$$

  - Provided that the calibrator has no actual V polarization.
- This works well if you have enough polarized signal.
- Also note that this method fails when the actual Q and U signals meet the following criterion:

$$U\cos 2\Psi_P - Q\sin 2\Psi_P = 0$$

- For equatorial mounts, this becomes U = 0.
- For alt-az mounts, and long observations, this condition will only occur twice per day.

Feed Handedness Issues -- Circulars

- I finish up with an amusing side topic.
- What happens if the engineers/technicians incorrectly connect the feeds.
- For circulars, only one error can be made: R⇔ L.
  - If only one antenna is 'wired backwards', the diagnosis is simple – the high parallel-hand power will show up in the cross-hand channels for the baseline connecting the backwards wired antenna to a correct one.
- More interesting is when ALL the antennas are reversed-wired.
- Examination of the fundamental equations show that:

$$I \Rightarrow I$$

$$V \Rightarrow -V$$

$$Q \Rightarrow Q$$

$$U \Rightarrow -U$$

Linear Mis-Assignments

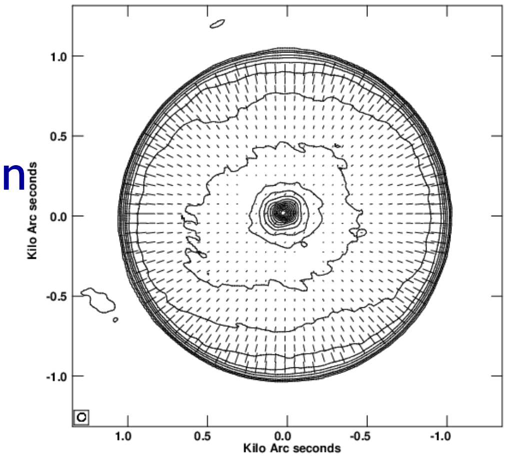
- For Linear systems, the situation is more interesting … (and confusing).
- There are two errors possible:
  - The 'H' and 'V' dipoles are interchanged, or/and
  - One of the two is 'backwards' phased (rotated by 180 degrees).
- The effect of connecting a dipole 'backwards' is to invert the phase by 180 degrees. A 'truth' table is useful:

H,V Correct	H,V Reversed	H,V Correct	H,V Reversed
Phase Correct	Phase Correct	Phase Reversed	Phase Reversed
I	I	I	I
V	-V	-V	V
Q	-Q	Q	-Q
U	U	-U	-U

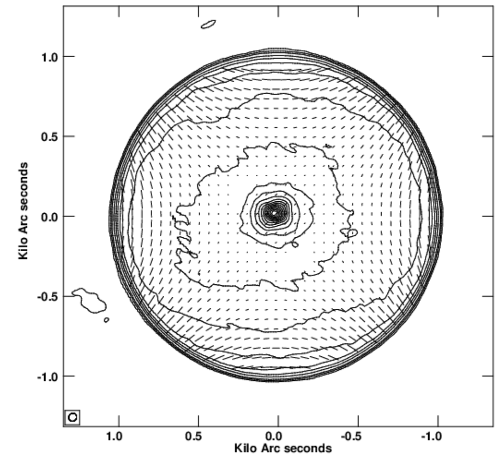
A picture is worth 1000 words …

- So, what does these errors do to a polarimetric image?
- Use the Moon as an example. (MeerKAT data at 867 MHz)

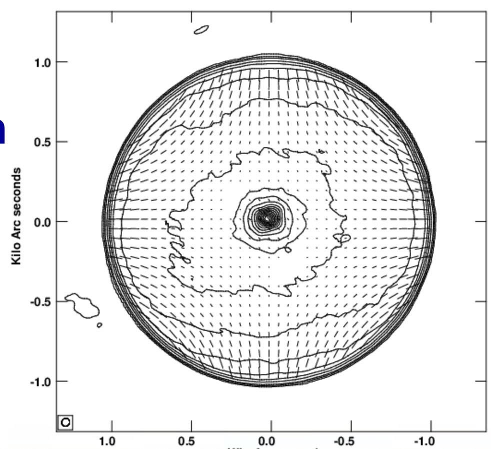
Correct Orientation and Phasing



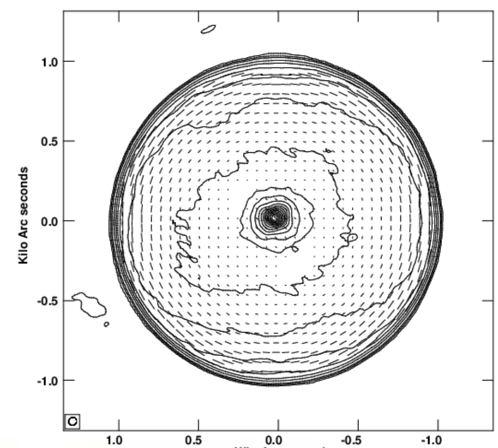
Correct Phasing Reversed Orientation



Correct Orientation Reversed Phasing



Reversed Phasing Reversed Orientation



Why is this important?

- Because mistakes happen!
- On the VLA, nobody knew which dipole fed which channel.
- To further complicate matters, new software reversed the assignments.
- On MeerKAT, nobody told the engineers that the IAU/IEEE standard was for 'X' to be vertical, and 'Y' horizontal.
- Investigations showed that indeed, the signal channel assignments were indeed reversed.
- (And in addition, it seems the S-band receiver has the phase inverted as well).