The Most Distant Observable Massive Objects

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Outline

- Detection statistics
- Redshift bias
- Horizon distances
- Conclusions
Do **GW** exist in a certain **universe**?

Frequentist assumptions:
- There are *universes* with no **GWs**.
- There are *universes* with **GWs**.
- The best way to distinguish them is with the $F$–statistic.
Do **coffee pods** exist in a certain **university**?

Frequentist assumptions:

- There are **universities** with no **coffee pods**.
- There are **universities** with a certain type of **coffee pods**.
- The best way to distinguish them is with the **number of published papers**.
Do coffee pods exist in a certain university?

Distribution of the number of papers in different universities:
Do **coffee pods** exist in a certain **university**?

We choose a threshold $T$ such that the probability of a university without any **coffee pods** having number of papers $> T$ is smaller than 0.1%.
Do **coffee pods** exist in a certain university?

However, if the **coffee pods** are weak, the probability that the number of papers $> T$ is also small among universities with coffee pods.

![Graph showing probability density of number of papers with and without coffee pods](image)
Do coffee pods exist in a certain university?

The coffee pods need to be strong enough, so that at least 95% of universities with coffee pods will produce a number of papers $> T$. 

![Bar graph showing the distribution of number of papers for different coffee pods: No coffee, Dolce, and Intenso.](image)
Do coffee pods exist in a certain university?

Summarising:

- We choose a threshold $T$ such that the probability of a university without any coffee pods having number of papers $> T$ is smaller than 0.1%.

- However, if the coffee pods are weak, the probability that the number of papers $> T$ is also small among universities with coffee pods.

- Therefore, coffee pods need to be strong enough, so that at least 95% of universities with coffee pods will produce a number of papers $> T$. 

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Horizon Distances
Do **GWs** exist in a certain **universe**?

Summarising:

- We choose a threshold $T$ such that the probability of a universe without any **GWs** having $\mathcal{F}$–statistic $> T$ is smaller than 0.1%.

- However, if the **GWs** are weak, the probability that the $\mathcal{F}$–statistic $> T$ is also small among universes with **GWs**.

- Therefore, **GWs** need to be strong enough, so that at least 95% of universes with **GWs** will produce a $\mathcal{F}$–statistic $> T$. 
Do GWs exist in our universe?

In other words:

- We choose a threshold $T$ such that the False Alarm Probability is $< 0.1\%$.
- To be detectable, GWs need to be strong enough in order to ensure a Detection Probability $> 95\%$.

How strong is strong enough?
Do GWs exist in our universe?

We assume that GWs are detectable if their *signal-to-noise ratio* is $S/N > 8$. This ensures (for the $\mathcal{F}$-statistic) a DP $> 95\%$ for a FAP $< 0.1\%$. 

![Detection Probability vs S/N](image)
Horizon distance

≡ maximum distance at which a GW source can be detected.
≡ maximum distance at which a GW source produces $S/N > 8$. 

Horizon distance
Redshift bias

Distant binaries can be brighter than closer ones!
Redshift bias

Explained in words:

*By observing a binary in a specific frequency band (fixed by the detector), there is a bias in which a later (brighter) stage of the inspiral is selected.*
Redshift bias

Explained in math:

\[ h(t) = \frac{4\pi^{2/3} [GM_z]^{5/3}}{c^4 D_L(z)} [f(t)]^{2/3}, \]

where

\[ M_z = M_c (1 + z). \]

But it is wrong to assume \( M_c \approx M_z \)!!
Redshift bias

S/N for a fixed chirp mass and observed GW frequency:
Redshift bias

There is a ‘special’ redshift...

\[(1 + z_{\text{min}}) \frac{d \ln[D_L(z)]}{dz} \bigg|_{z_{\text{min}}} \geq \frac{5}{3},\]

which, under Λ-CDM:

\[z_{\text{min}} \approx 2.63.\]

If you can see a binary inspiralling there, you can see it everywhere!
Horizon distances

How bright is a $10^{9.8} M_\odot$ binary at different redshifts?
How bright is a $10^{10} M_\odot$ binary at different redshifts?
Horizon distances

How bright is a $10^{10.2} M_\odot$ binary at different redshifts?

![Graph showing the brightness of a binary as a function of redshift and observed frequency.](graph.png)
Horizon distances

Detectable $10^{9.8} M_\odot$ binaries:

![Graph showing detectable horizon distances for $10^{9.8} M_\odot$ binaries. The graph plots log10(Initial GW observed frequency / Hz) on the x-axis and log10(Redshift) on the y-axis. The color scale on the right indicates log10(S/N).]
Horizon distances

Detectable $10^{10} M_\odot$ binaries:
Detectable $10^{10.2} M_\odot$ binaries:

![Graph showing detectable horizon distances](image)
Horizon distances for Advanced LIGO, LISA, and the PPTA:
Conclusions

- Redshift effects have to be considered. \((M_z \neq M_c)\!\)
- Current detectors are sensitive to high-\(z\) binaries!
- PTAs can detect \(M_c > 10^{10} M_\odot\) binaries \textit{at any distance}.
- Future PTAs will detect lighter binaries \textit{at any distance}.
- Single sources vs. background dilemma should be revisited.
- Redshift bias affects other types of GW sources.
- ...what about astrophysical/cosmological implications?

Thank you!