Fundamentals of Radio Interferometry
1: The Basics

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Topics

- **Introduction:**
  - **Motivation: Mapping the Sky**
  - **Why Interferometry?**
- **The Basic Interferometer**
  - **Simplifying Assumptions**
  - **'Fringe' patterns**
  - **Sine and Cosine Fringes**
  - **Response to Extended Emission**
  - **The Complex Correlator and Complex Visibilities**
  - **Illustrations to aid visualization**
- **Important Note:**
  - **The concepts I'll be speaking about are not difficult – but they are unfamiliar to beginners.**

References

- Before launching into the material, here are five references I find most useful:

1. 'Principles of Optics': Max Born and Emil Wolf. My copy is the 6th edition (1980). Pergamon Press.
2. 'Interferometry and Synthesis in Radio Astronomy.' Thompson, Moran and Swenson. 2nd edition 2001. Wiley Interscience.
3. 'Synthesis Imaging in Radio Astronomy II'. Taylor, Carilli and Perley. ASP Conference Series Volume 180, 1999.
4. 'The Fourier Transform and its Applications'. Ron Bracewell. 2nd edition. McGraw-Hill. (Be careful of the 3rd edition, which has errors).

Our Goal: Mapping the Sky Brightness

- In astronomy, we wish to know the angular distribution of the emission of distant radiating sources.
  - This can be a function of frequency, polarization, and time.
- 'Angular Distribution' means we are interested in the **brightness distribution** of the emission.
- Measuring the brightness means **resolving the emission.**
- Because our targets are so far away, the emission is extremely weak, and of very small angular size.
  - Power from strongest radio source collected over the entire area of Australia in 1 GHz bandwidth = 0.1 watt!
  - 1 arcsecond = angular width of a coin at ~5 kilometres!
- Early (1950s) surveys of the radio sky employed single dishes.
- Nowadays, most (but not all!) observations are done with interferometers.

Why Interferometry?

- It's due to **Diffraction** – a consequence of the wave nature of light.
- Radio telescopes coherently sum electric fields over an aperture of size D. For this, diffraction theory applies – the angular resolution is:

  $\theta_{rad} \approx \lambda / D$ Or, in practical units $\theta_{\mathrm{arcsec}} \approx 2\,\lambda_{\mathrm{cm}} / D_{\mathrm{km}}$

- To obtain 1 arcsecond resolution at a wavelength of 21 cm, we require an aperture of ~42 km – not feasible.
- The (currently) largest single, fully-steerable apertures are the 100-m antennas in Bonn, and Green Bank. Nowhere big enough.
- Can we synthesize a larger aperture with separated antennas?
- The technique of synthesizing a larger aperture through combinations of separated pairs of antennas is called 'aperture synthesis'.

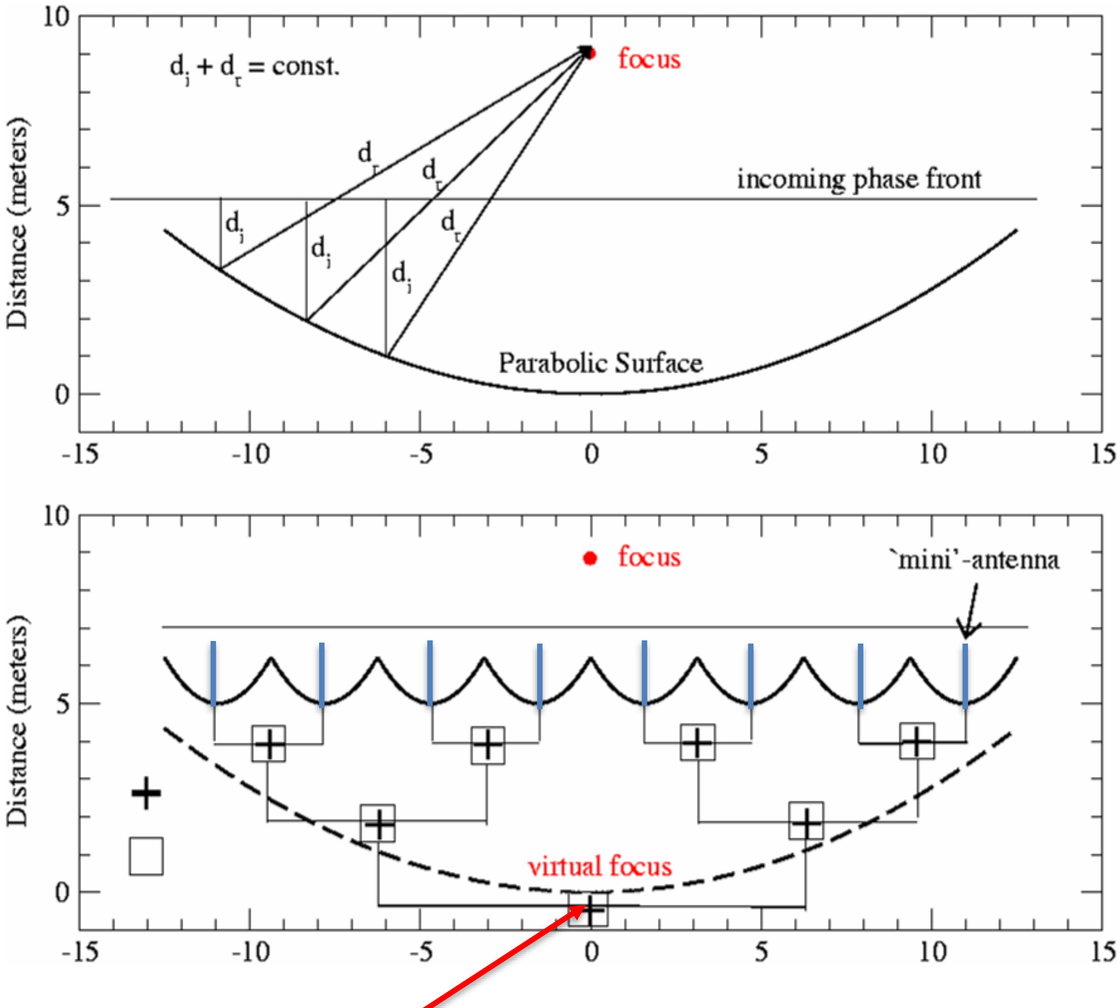
Interferometry – Basic Concept

- A parabolic dish coherently sums EM fields at the focus.

- The same result can be obtained by adding voltages from individual elements with a wired network.
  - Note – they need not be adjacent.

- This is the basic concept of interferometry – coherent summation of voltages from separated antennas.

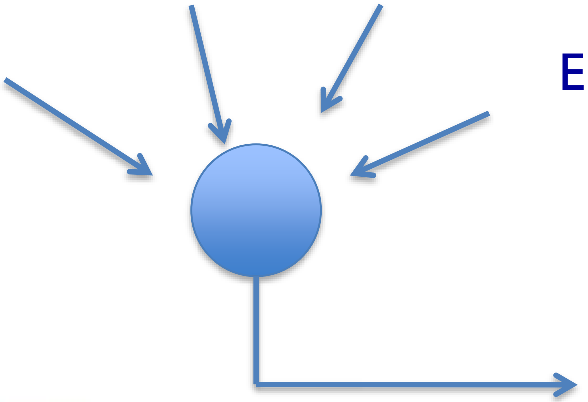
- Aperture Synthesis is an extension of this concept.



Signals summed here equivalent to those collected at prime focus

The Role of the Sensor (aka Antenna)

- Coherent interferometry is based on the ability to correlate the electric fields measured at spatially separated locations.
- To do this (without mirrors) requires conversion of the electric field E(**r**,ν,t) at some place (**r**) to a voltage V(ν,t) which can be conveyed to a central location for processing – without losing the phase information.
- For our purpose, the sensor (a.k.a. 'antenna') is simply a device which senses the electric field at some place and converts this to a voltage which faithfully retains the amplitudes and phases of the electric fields.

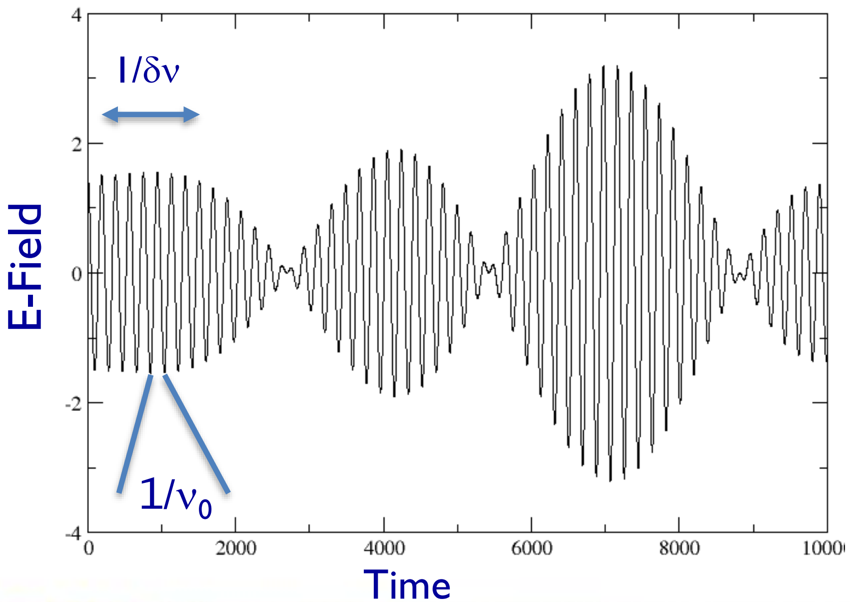


Quasi-Monochromatic Radiation

- Mathematical analysis of wideband noise is difficult, if not impossible.
- Analysis is simple if the fields are monochromatic.
  - Natural radiation is never monochromatic
- So we consider instead 'quasi-monochromatic' radiation, where the bandwidth $\delta\nu$ is very small, but not zero.
- Then, for a time dt $\sim 1/\delta\nu$, the electric fields will be sinusoidal, with unchanging amplitude E and phase ϕ, described by

$$E_{\upsilon}(t) = E\cos(2\pi\upsilon t + \phi)$$

The figure shows an 'oscilloscope' trace of a narrow bandwidth noise signal. The period of the wave is $T=1/\nu_0$, the duration over which the signal is closely sinusoidal is $T \sim 1/\delta\nu$. There are $N \sim \nu_0/\delta\nu$ oscillations in a 'wave packet'.



Simplifying Assumptions

- We now consider the most basic interferometer, and seek a relation between the characteristics of the product of the voltages from two separated antennas and the distribution of the brightness of the originating source emission.
- To establish the basic relations, the following simplifications are introduced:
  - Isotropic sensors fixed in space – no rotation or motion
  - Quasi-monochromatic radiation
  - Source in the far-field ($D >> b^2/\lambda$)
  - No frequency conversions (an 'RF interferometer')
  - Single polarization
  - Propagation in vacuum, without distortions (no ionosphere, atmosphere …)
  - Idealized electronics (perfectly linear, perfectly uniform in frequency and direction, perfectly identical for both elements, no added noise.
  - I will later relax most of these restrictions.

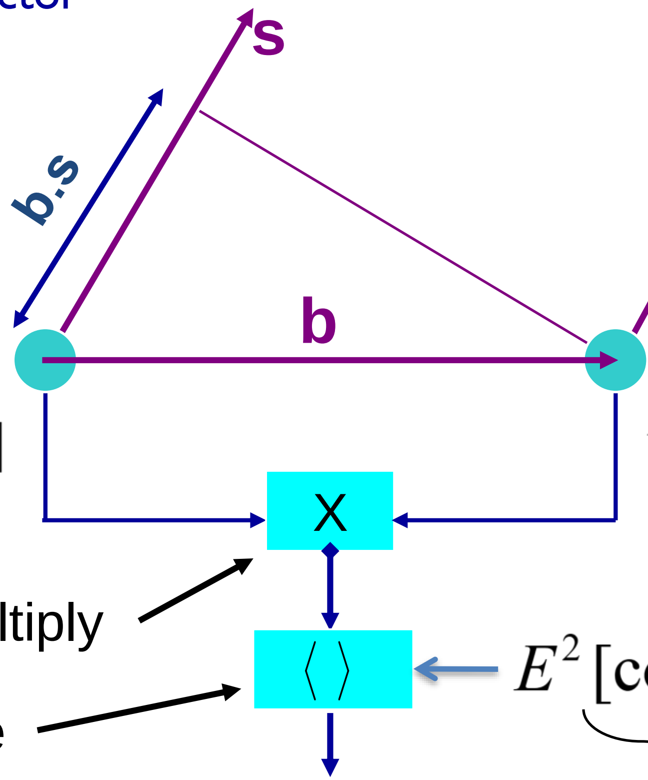
The Stationary, Quasi-Monochromatic Radio-Frequency Interferometer

s = Direction Unit Vector

b = Baseline Vector

Geometric Time Delay

$$\tau_g = \mathbf{b} \cdot \mathbf{s} / c$$



$$V_1 = E \cos[\omega(t - \tau_g)]$$

$$V_2 = E \cos(\omega t)$$

The path lengths from sensors to multiplier are assumed equal!

multiply

average

$$E^2 [\cos(\omega\tau_g) + \cos(2\omega t - \omega\tau_g)]$$

Unchanging

Rapidly varying

$$R_C = P \cos(\omega\tau_g)$$

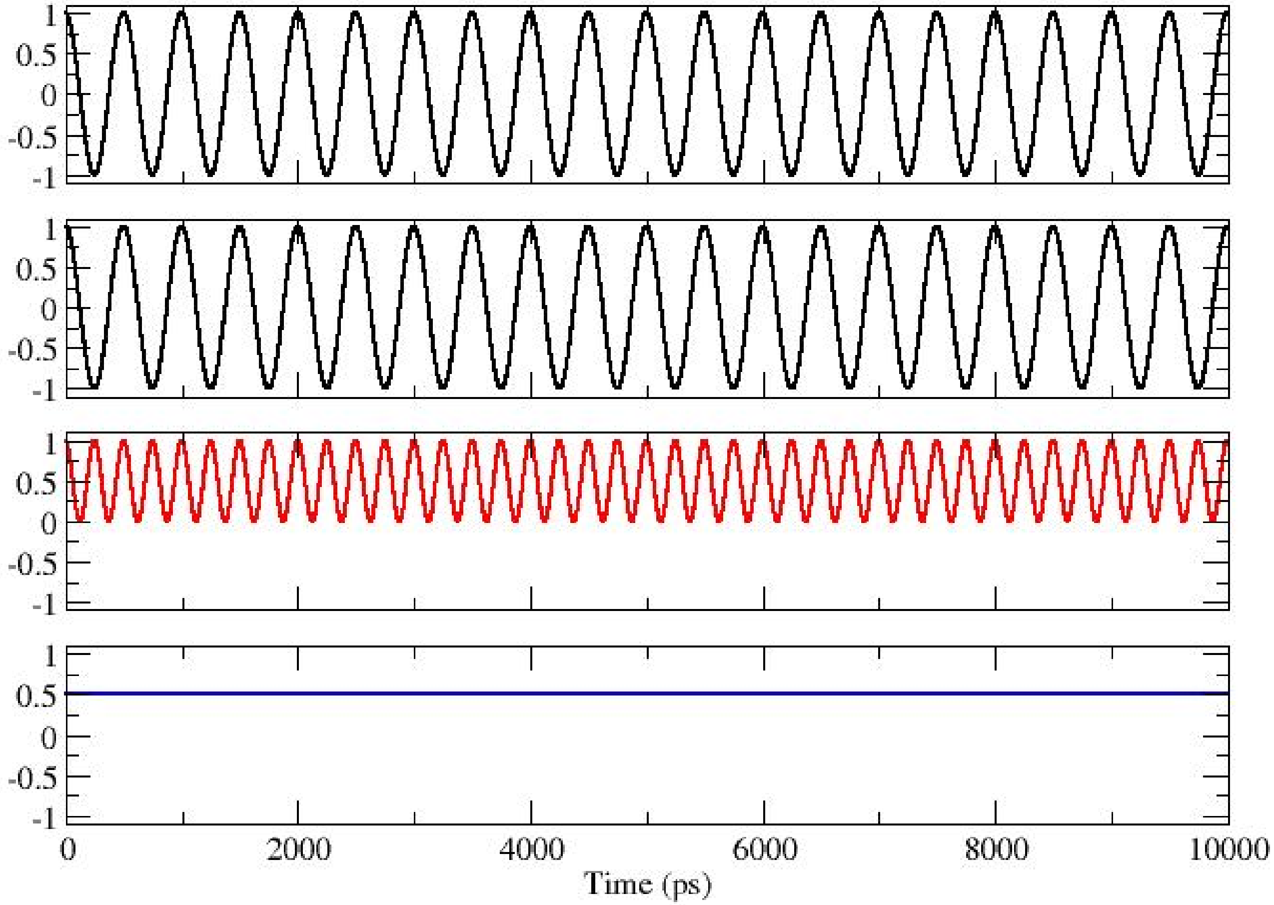
Where I replaced E^2 with P (power), and ignored factors of 2

Pictorial Example: Signals In Phase

2 GHz Frequency, with voltages in phase:

b.s = $n\lambda$, or $\tau_g = n/\nu$

- Antenna 1 Voltage
- Antenna 2 Voltage
- Product Voltage
- Average



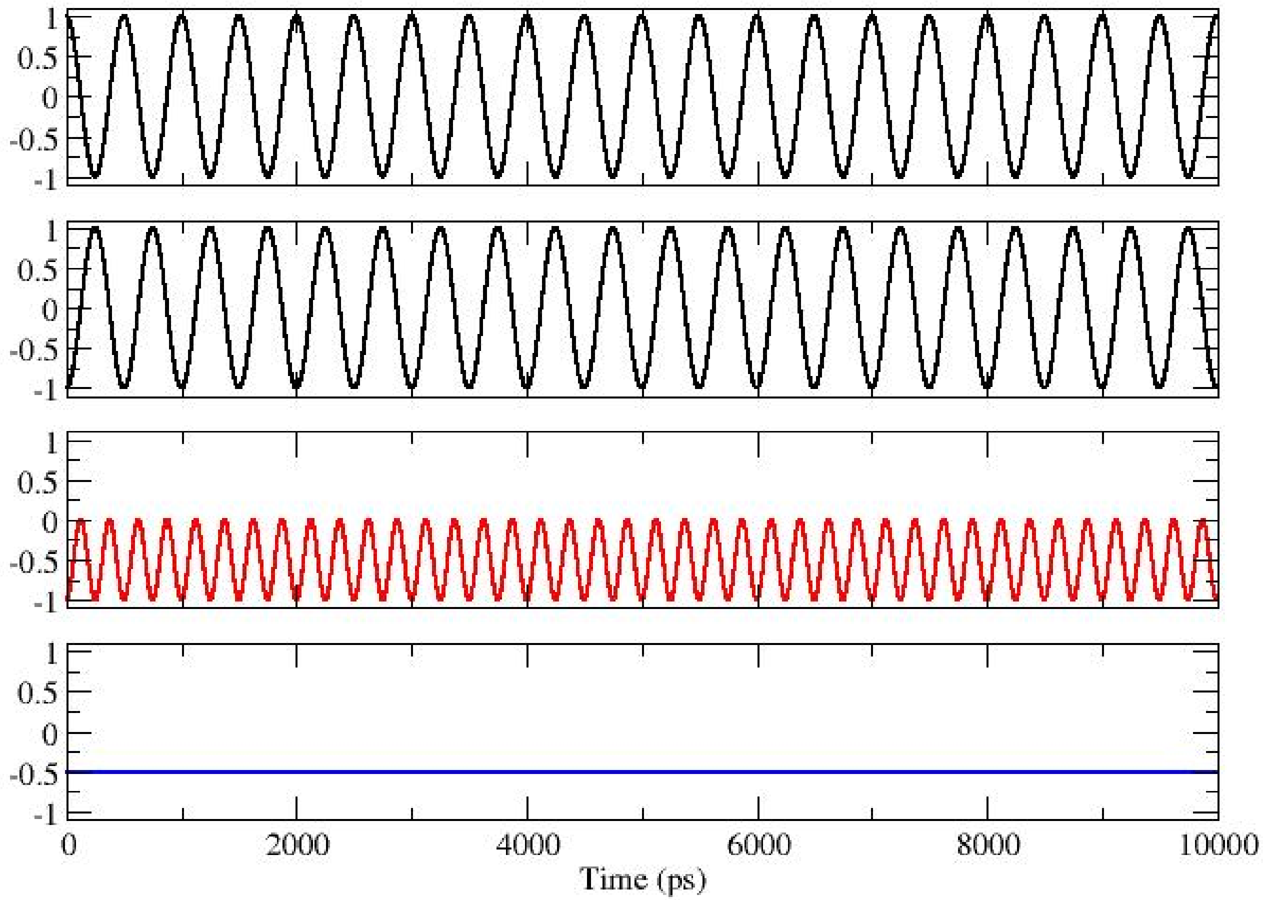
Positive constant

Pictorial Example: Signals out of Phase

2 GHz Frequency, with voltages out of phase:

b.s=(n +/- ½)λ τ_g = (n +/- ½)/ν

- Antenna 1 Voltage
- Antenna 2 Voltage
- Product Voltage
- Average



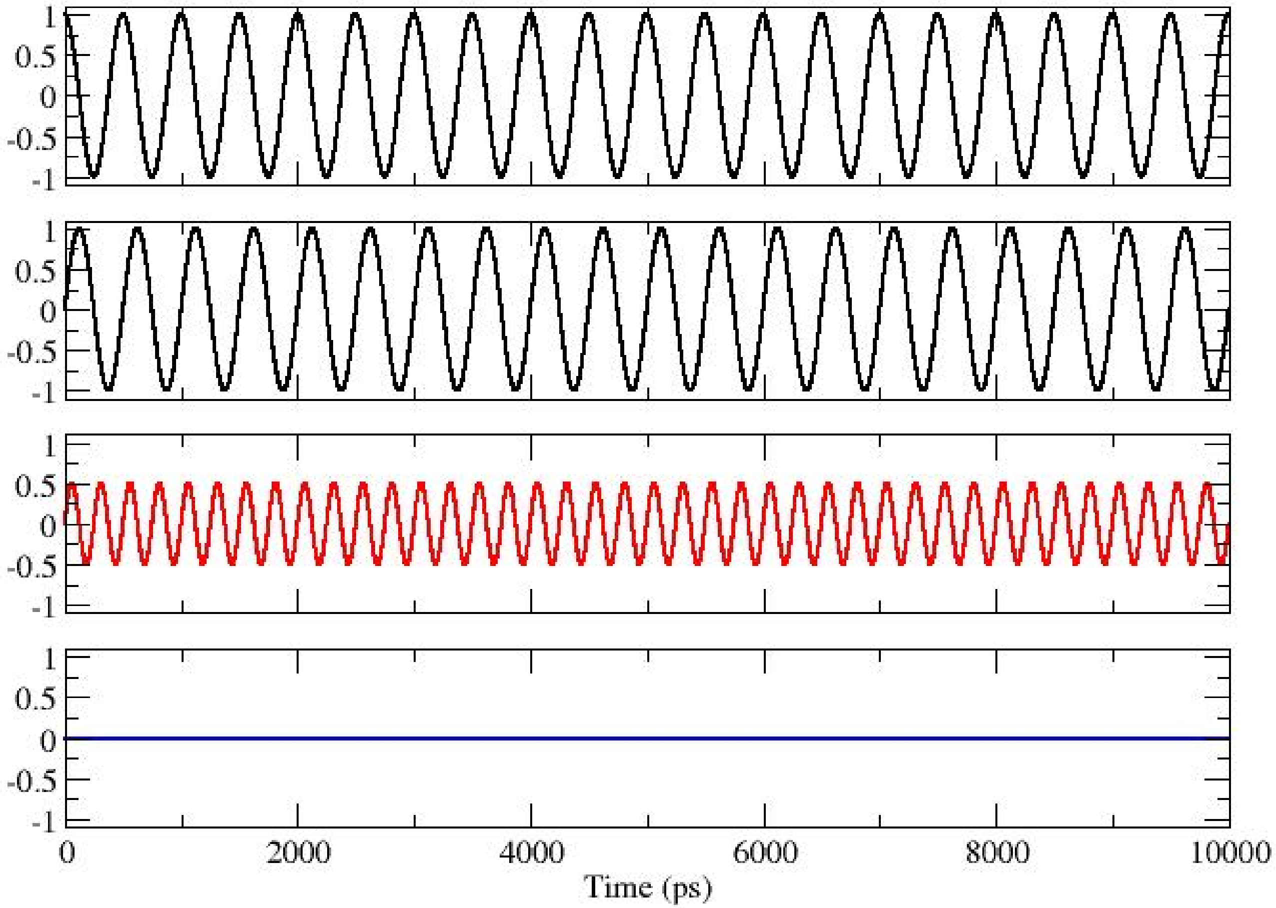
Negative constant

Pictorial Example: Signals in Quad Phase

2 GHz Frequency, with voltages in quadrature phase:

$b.s=(n \pm 1/4)\lambda$, $\tau_g = (n \pm 1/4)/\nu$

- Antenna 1 Voltage
- Antenna 2 Voltage
- Product Voltage
- Average



Zero constant

General Comments on this Product

- The averaged product R_C is dependent on the received power, $P = E^2/2$ and geometric delay, τ_g, and hence on the baseline orientation and source direction. From slide 10, we have:

$$R_C = P\cos(\omega\tau_g) = P\cos\left(2\pi\frac{\mathbf{b}\cdot\mathbf{s}}{\lambda}\right)$$

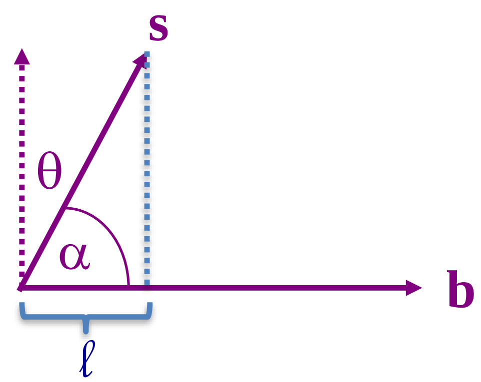
- Note that R_C is not a function of:
  - The time of the observation -- provided the source itself is not variable!
  - The location of the baseline -- provided the emission is in the far-field.
  - The actual phase of the incoming signal – the distance to the source -- provided the source is in the far-field.

- The strength of the product is also dependent on practical factors such as the antenna sizes, electronic gains, bandwidth and time averaging – but these factors or ignored here (and can be calibrated for). .

Define the Direction Cosine

- Consider a single baseline, and define the x-axis to extend along this baseline.
- Write $\mathbf{b} = u\hat{\mathbf{x}}$ where u $=|\mathbf{b}|/\lambda$ is the baseline length in wavelengths, and $\hat{\mathbf{x}}$ = the unit direction vector.
- Define the `direction cosine' as: $l = \hat{\mathbf{x}} \bullet \mathbf{s} = \cos\alpha = \sin\theta$
- Then:

$$\frac{\mathbf{b} \bullet \mathbf{s}}{\lambda} = u\cos\alpha = u\sin\theta = ul$$



- So the interferometer response is:

$$R_C = P\cos(\omega\tau_g) = P\cos\left(2\pi\frac{\mathbf{b}\cdot\mathbf{s}}{\lambda}\right) = P\cos(2\pi ul)$$

Two Illustrative Examples

- Consider the response R_c, as a function of angle, for two different baselines with u = b/λ = 10, and u = 25 wavelengths.
- Since

$$R_C = \cos(2\pi u l)$$

- We have, for u = 10: $R_C = \cos(20\pi\, l)$
- And, for u = 25: $R_C = \cos(50\pi\, l)$
- These are simple functions of angle on the sky.

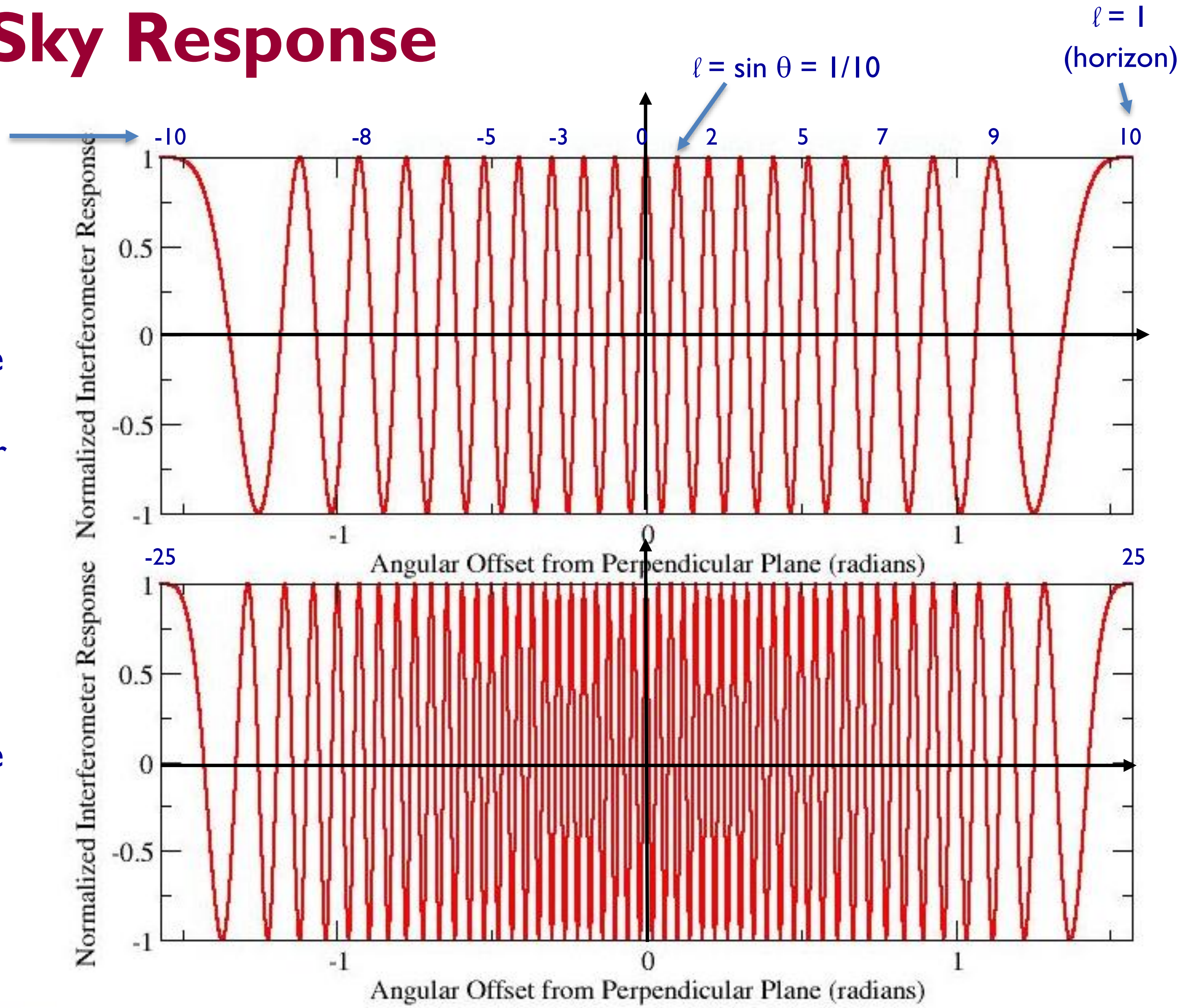
Remember:

u = baseline length in wavelengths

ℓ = sin θ,

θ = angular offset from perpendicular plane

Whole-Sky Response



- Top:

  u = 10

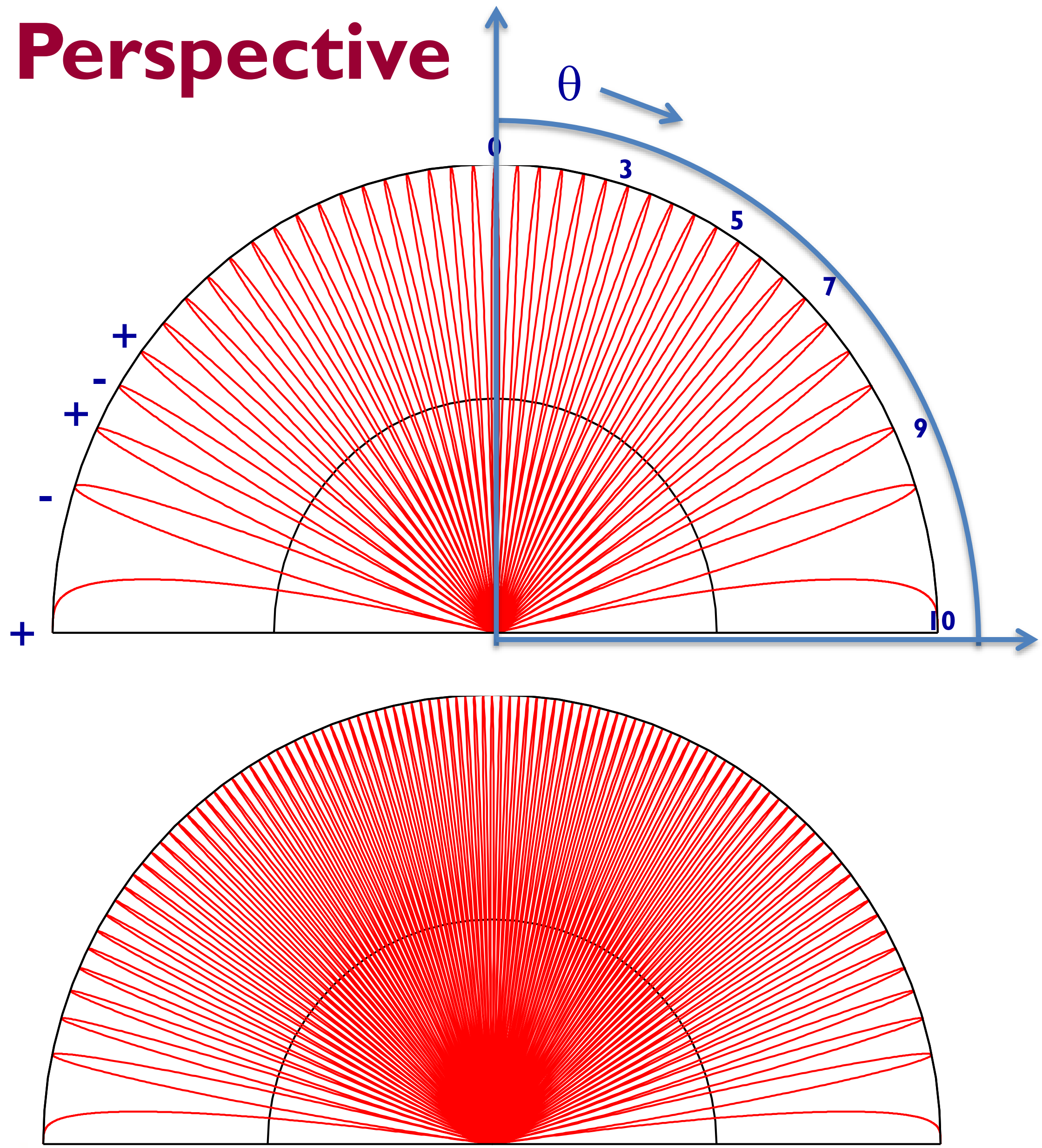
  There are 21 fringe maxima, and 20 fringe minima over the sphere.

- Bottom:

  u = 25

  There are 51 fringe maxima over the sphere

From an Angular Perspective



Top Panel:

The absolute value of the response for u = 10, as a function of angle.

The 'lobes' of the response pattern alternate in sign.

Bottom Panel:

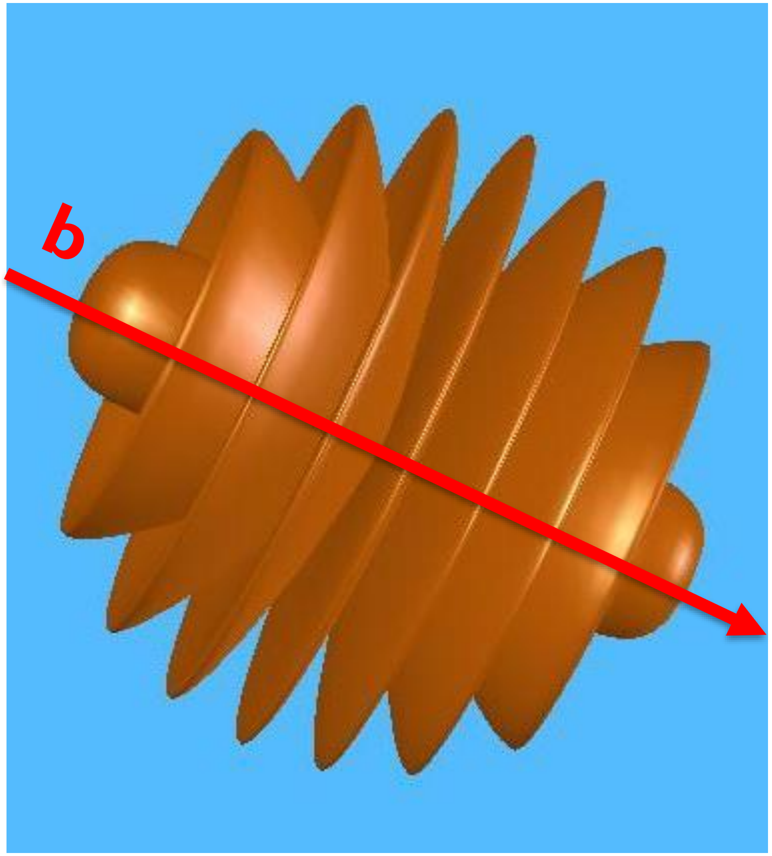
The same, but for u = 25.

Angular separation between lobes (of the same sign) is

$\delta\theta \sim 1/u = \lambda/b$ radians.

Hemispheric Pattern

- The preceding plot is a meridional cut through the hemisphere, oriented along the baseline vector **b**.
- In the three-dimensional space, the fringe pattern consists of a series of coaxial cones, oriented along the baseline vector.
- The figure is a two-dimensional representation when u = 4.
- As viewed along the baseline vector, the fringes show a 'bulls-eye' pattern – concentric circles.

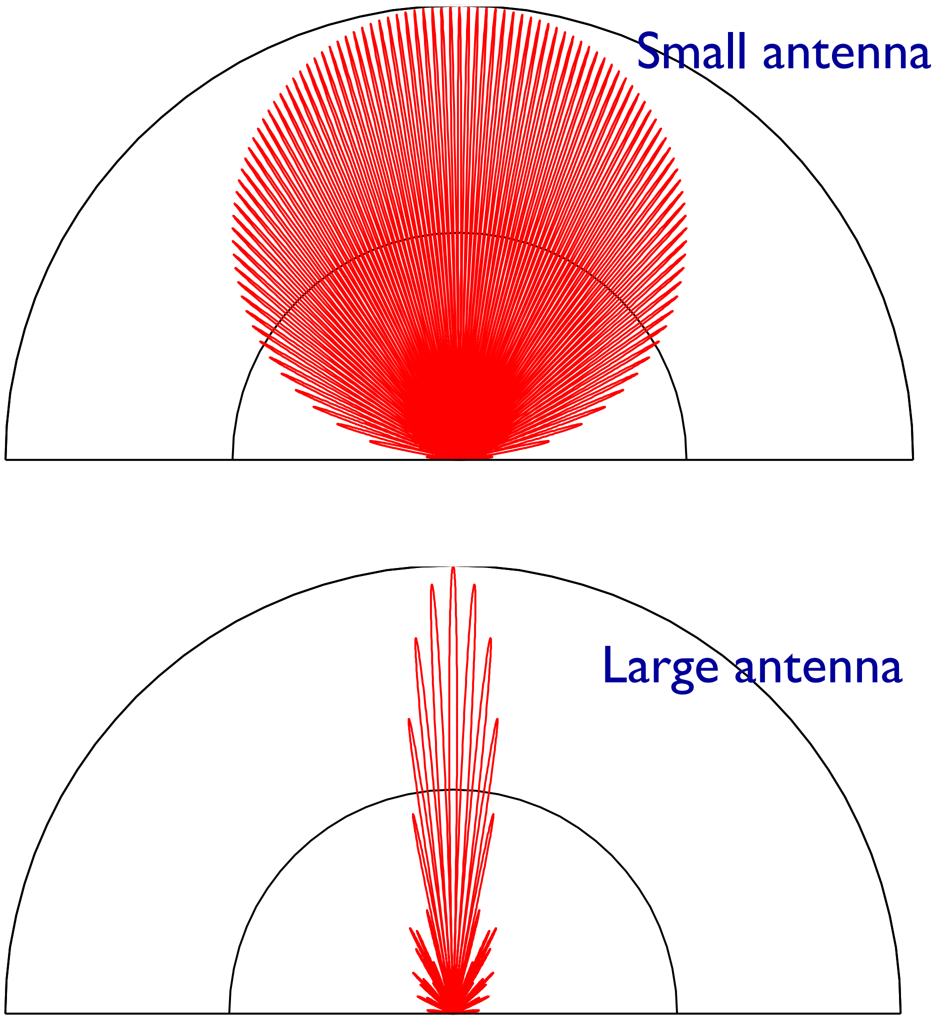


The Effect of the Sensor (aka Antenna)

- The patterns shown presume the sensor has isotropic response.
- This is a convenient assumption, but doesn't represent reality.
- Real sensors impose their own patterns, which modulate the amplitude and phase of the interferometer response.
- Large sensors have good angular resolution and high gain
  - very useful for some applications like imaging individual objects.
- Small sensors have poor angular resolution and low gain
  - useful for wide-angle surveys.
- Emphasis: The fringe pattern is a function of the baseline length (in wavelengths) and orientation, and is not affected by the antenna patterns.

The Effect of Sensor Patterns

- Sensors (or antennas) are not isotropic, and have their own responses – both amplitude and phase.
- **Top Panel:** The interferometer pattern with a cos(θ)-like sensor response.
- **Bottom Panel:** A multiple-wavelength aperture antenna has a narrow beam, but also sidelobes.
- **Emphasis:** The fringe pattern is a function of the array geometry. The angular modulation is from the sensor.



From Point Sources to Extended Emission

- By construction, everything I stated earlier holds for 'point sources'. What are these?
- Point Source: Emission from an object whose angular extent is much, much smaller than the interferometer fringe scale.
- That is: $$\theta_{rad} \ll \lambda / B$$

  where θ = angular size, λ = wavelength, B = baseline
- What happens when the source angular size is comparable to, or larger than, the fringe scale?
- Short answer: Under nearly all conditions, you just 'add up' the individual responses from the components of the extended source.

The Response from an Extended Source

- For an extended source, the voltage output from a single antenna is the spatial integration of the E-field emission over the primary beam:

$$V = \iint E(\mathbf{s})\, d\Omega$$

- So the correlator response becomes

$$R_C = \left\langle \iint E_1(\mathbf{s})\, d\Omega_1 \times \iint E_2(\mathbf{s})\, d\Omega_2 \right\rangle$$

- The averaging and integrals can be interchanged and, **providing the emission is spatially incoherent**, we get

$$R_C = \iint I_\nu(\mathbf{s}) \cos(2\pi\nu \mathbf{b}\cdot\mathbf{s}/c)\, d\Omega$$

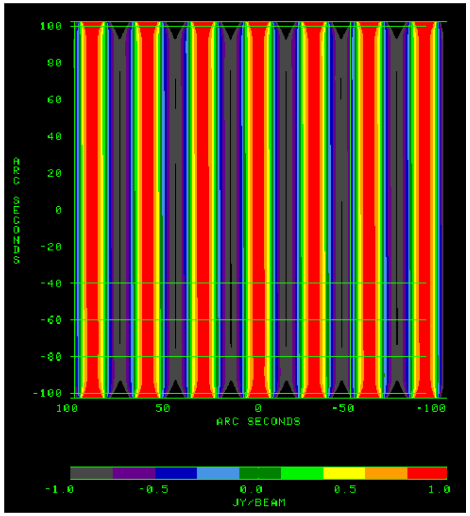
- The response is the sum of the spatial integration of the brightness modulated by the cosinusoidal interferometer pattern.
- This expression links what we want: the brightness on the sky, $I_\nu(\mathbf{s})$, to something we can measure - R_C, the interferometer response.
- Can we recover $I_\nu(\mathbf{s})$ from observations of R_C?

A picture is worth 1000 words …

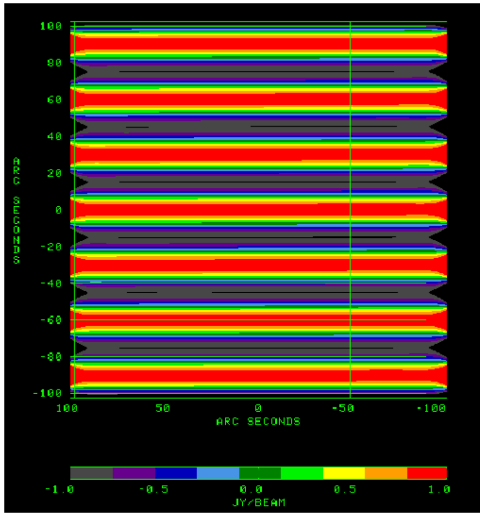
- As stated earlier, these concepts are not difficult, but are unfamiliar. We need to think in new ways, to get a deeper understanding of how all this works.
- To aid, I have generated images of interferometer fringes, of various baseline lengths and orientations.
- I then 'observe' a real source (Cygnus A, of course), to show what the interferometer actually measures.
- For all these, the 'observations' are made at 2052 MHz. The Cygnus A image is take from real VLA data.
- To keep things simple, all simulations are done at meridian transit.

'Real' Fringes … 1Km Baseline at 2052 MHz

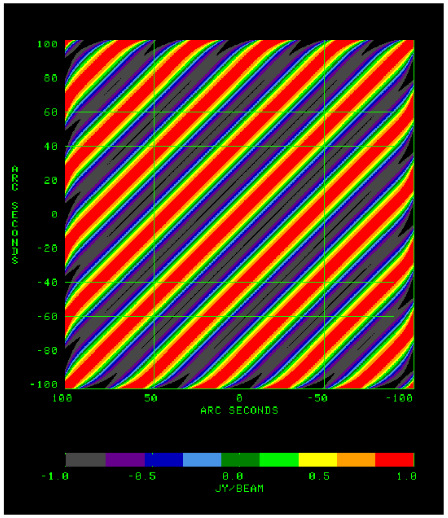
- The fringe separation given by baseline length in wavelengths, the orientation given by the orientation of the baseline.



East-West baseline makes vertical fringes



North-South baseline makes horizontal fringes



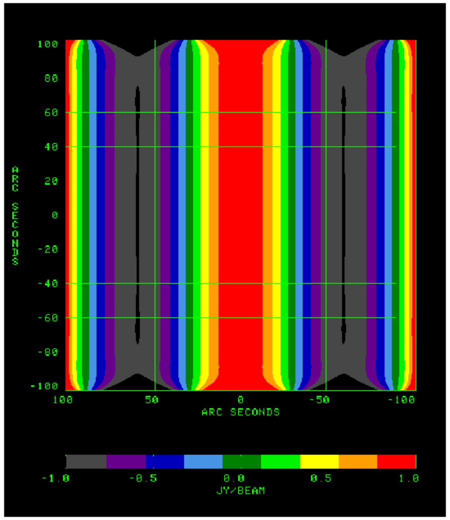
Rotated baseline makes rotated fringes

- Red = positive maximum. Black = negative maximum. Green = zero
- Fringe angular spacing given by baseline length in wavelengths:

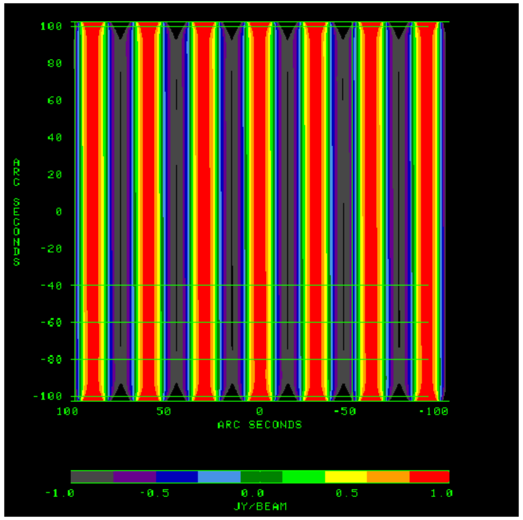
$$\Delta\theta = \lambda / B = 30.2''$$

Longer Baselines => Smaller Fringe Spacing

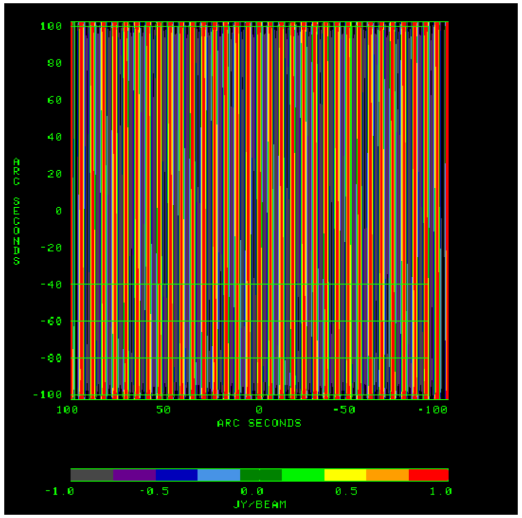
- With longer baselines (**in wavelengths!**) come finer fringes:



250 meter baseline
120 arcsecond fringe



1000 meter baseline
30 arcsecond fringe

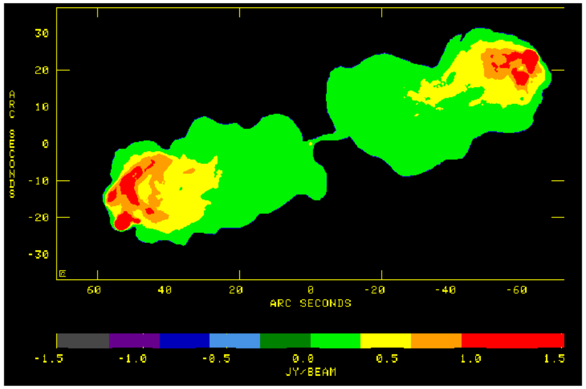


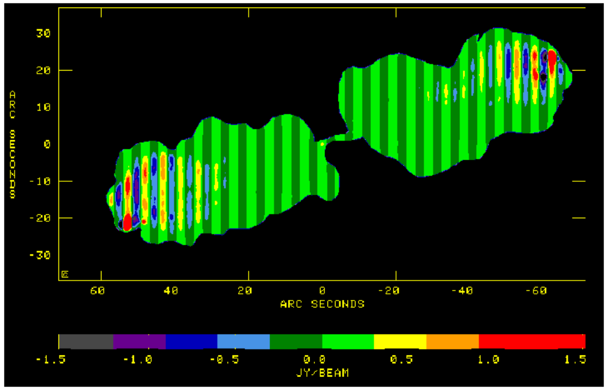
5000 meter baseline
6 arcsecond fringe

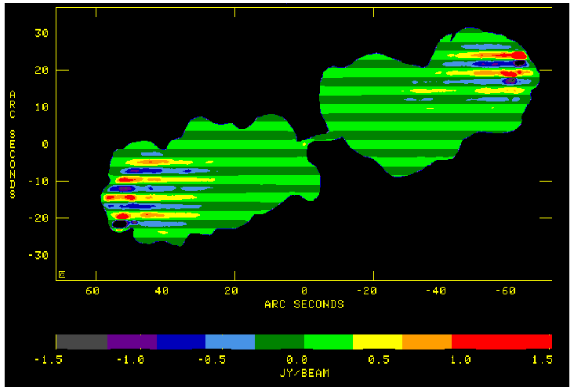
- What the interferometer measures is the integral (sum) of the product of this pattern with the actual brightness.

For a Real Source (Cygnus A = 3C405)

- Cygnus A is a powerful, nearby radio galaxy.
- The left panel shows the color-coded brightness at 2052 MHz. (2" res^{n})
- The other two panels show how 6.9 km EW and NS interferometers 'see' the source.







Zero-Spacing Image
Sum = 1029 Jy

6.3 km EW spacing
Sum = 70 Jy

6.3 km NS spacing
Sum = -19 Jy

- Dark green, blue, and purples => Negative correlation
- Light green, yellow, and reds => Positive correlation
- Don't be alarmed by the negative flux in the third panel.

So … What Next?

- The interferometer casts a cosinusoidal pattern on the sky, with the result that we obtain a response which is some function of the source brightness and the fringe separation and orientation.
- By definition, 'Cosine' fringes have a peak passing through the central position of the source ('phase center').
- But in fact, something is missing. 'Cosine' fringes are not sufficient to allow recovery of the sky brightness.
- To answer why …
- Time for some mathematics…. Starting with a seeming digression about odd and even functions.
- (All will be clear shortly…)

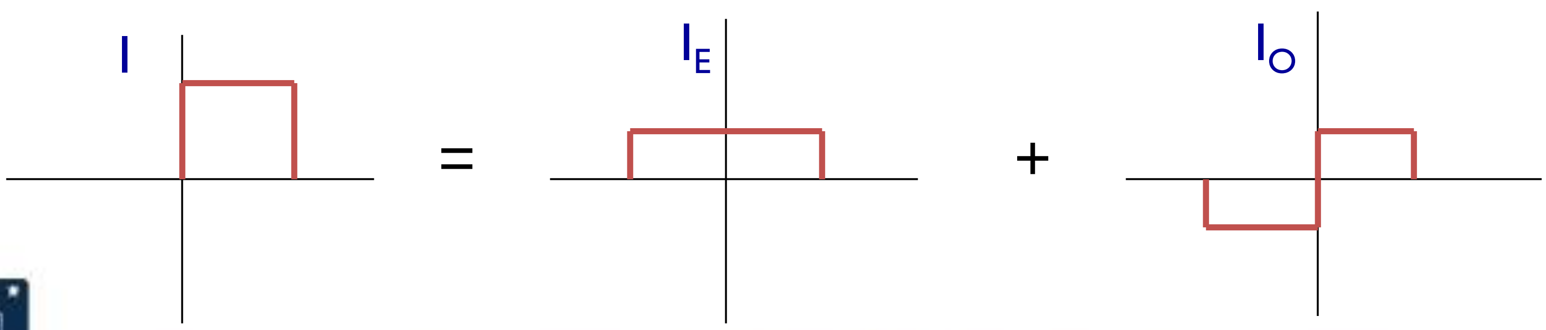
A Short Mathematics Digression – Odd and Even Functions

- Any real function, I(x,y), can be expressed as the sum of two real functions which have specific symmetries:

$$I(x,y) = I_E(x,y) + I_O(x,y)$$

An even part: $$I_E(x,y) = \frac{I(x,y) + I(-x,-y)}{2} = I_E(-x,-y)$$

An odd part: $$I_O(x,y) = \frac{I(x,y) - I(-x,-y)}{2} = -I_O(-x,-y)$$



The Cosine Correlator is Blind to Odd Structure

- Suppose that the source of emission has a component with odd symmetry, for which

$$I_o(x) = -I_o(-x)$$

- Since the cosine fringe pattern is even, the response of our interferometer to the odd brightness distribution is 0!

$$R_c = \iint I_o(\mathbf{s})\cos(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)\,d\Omega = 0$$

- Thus, the cosine correlator response R_c:

$$R_C = \iint \big(I(\mathbf{s}) \big) \cos(2\pi\nu\,\mathbf{b}\cdot\mathbf{s}/c)\,d\Omega = \iint \big(I_E(\mathbf{s}) \big) \cos(2\pi\nu\,\mathbf{b}\cdot\mathbf{s}/c)\,d\Omega$$

is sensitive to the even component of the source structure.

Hence, we need more information if we are to completely recover the source brightness.

Thus: Two Correlations are Needed !!!

- The integration of the cosine response, R_c, over the source brightness is sensitive to only the even part of the brightness:

$$R_C = \iint I(\mathbf{s})\cos(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega = \iint I_E(\mathbf{s})\cos(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega$$

  since the integral of an odd function (I_O) with an even function (cos x) is zero.
- To recover the 'odd' part of the brightness, I_O, we need an 'odd' fringe pattern.
- Let us replace the 'cos' with 'sin' in the integral, to get

$$R_S = \iint I(\mathbf{s})\sin(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega = \iint I_O(\mathrm{s})\sin(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega$$

  since the integral of an even times an odd function is zero.

- To obtain this necessary component, we must make a 'sine' pattern.

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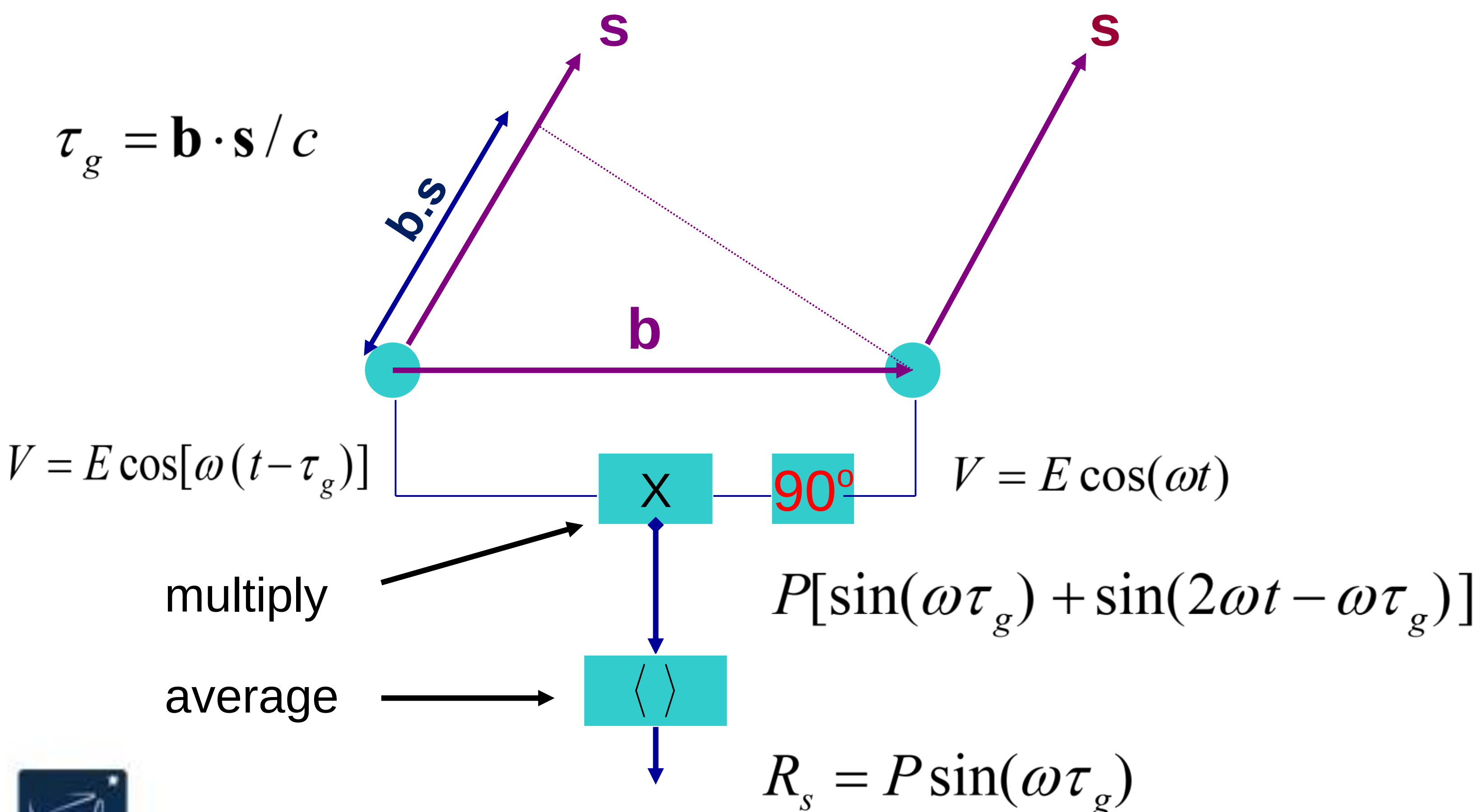
Making a SIN Correlator

- We generate the 'sine' pattern by inserting a 90 degree phase shift in one of the signal paths.



Define the Complex Visibility

- We now DEFINE a complex function, the complex visibility, V, from the two independent (real) correlator outputs R_C and R_S:

$$V = R_C - iR_S = Ae^{-i\phi}$$

where

$$A = \sqrt{R_C^2 + R_S^2}$$

$$\phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

- This gives us a beautiful and useful relationship between the source brightness, and the response of an interferometer:

$$V_\upsilon(\mathbf{b}) = R_C - iR_S = \iint I_\nu(\mathbf{s})\, e^{-2\pi i\nu \mathbf{b}\cdot\mathbf{s}/c} d\Omega$$

- Under some circumstances, this is a 2-D Fourier transform, giving us a well established way to recover $I(\mathbf{s})$ from $V(\mathbf{b})$.

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The Complex Correlator and Complex Notation

- A correlator which produces both 'Real' and 'Imaginary' parts – or the Cosine and Sine fringes, is called a 'Complex Correlator'
  - For a complex correlator, think of two independent sets of projected sinusoids, separated by 1/4 fringe spacing.
  - In our scenario, both components are necessary, because we have assumed there is no motion – the 'fringes' are fixed on the source emission, which is itself stationary.
- The complex output of the complex correlator also means we can use complex analysis throughout: Let:

$$V_1 = A\cos(\omega t) \rightarrow Ae^{-i\omega t}$$

$$V_2 = A\cos[\omega(t - \mathbf{b}\bullet\mathbf{s}/c)] \rightarrow Ae^{-i\omega(t-\mathbf{b}\bullet\mathbf{s}/c)}$$

- Then:

$$P_{corr} = \langle V_1 V_2^* \rangle = A^2 e^{-i\omega \mathbf{b}\bullet\mathbf{s}/c}$$

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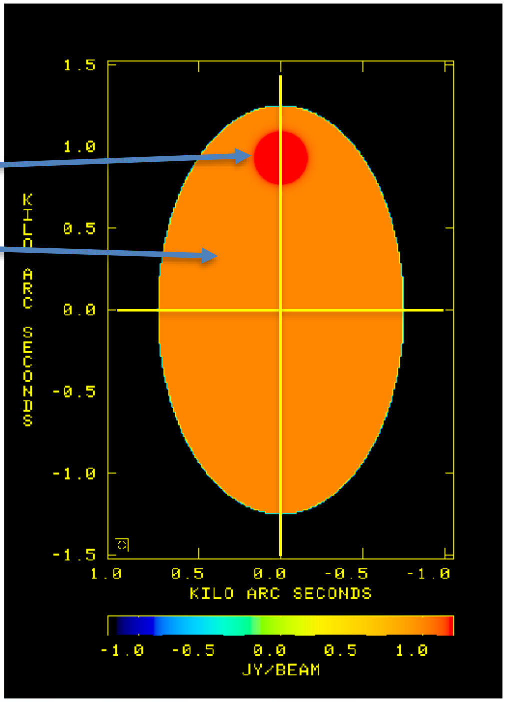
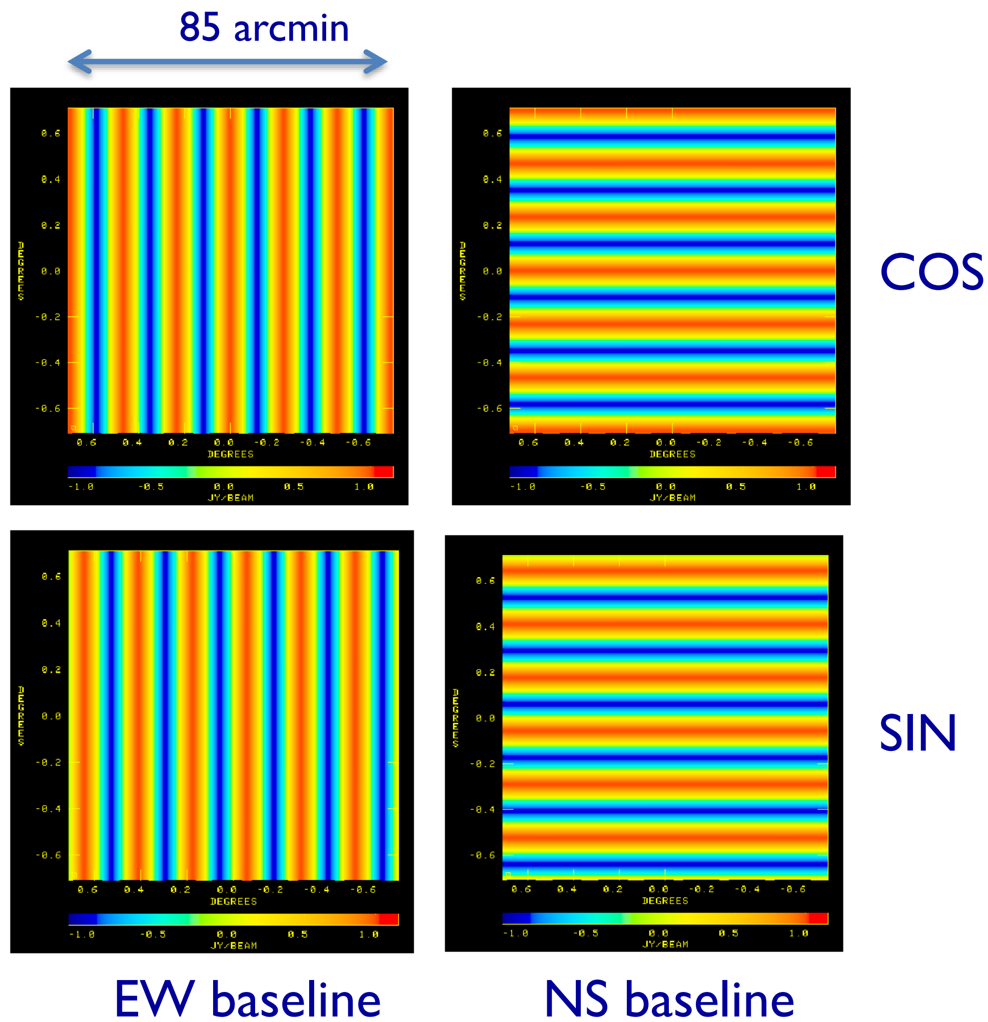
Visualizing Visibilities

- An intimate familiarity with interferometer visibilities is essential in understanding how interferometers work.
- Fortunately, the concepts can be easily grasped through pictures.
- To illustrate, I have generated a mock source, consisting of a flat elliptical disk, (1500" x 2500") and a bright, circular gaussian 'spot' of width 150" near one end.
    - Flux of disk: 1283 Jy.
    - Peak brightness of hotspot: 6 Jy/beam
    - Brightness of disk: 1 Jy/beam
    - Flux of hotspot: 56 Jy
    - Beamsize = 45 arcseconds
    - The odd color wedge is chosen to illustrate the main points
        - Color wedge runs from -1.25 Jy/beam (black) to +1.25 Jy/beam (red).



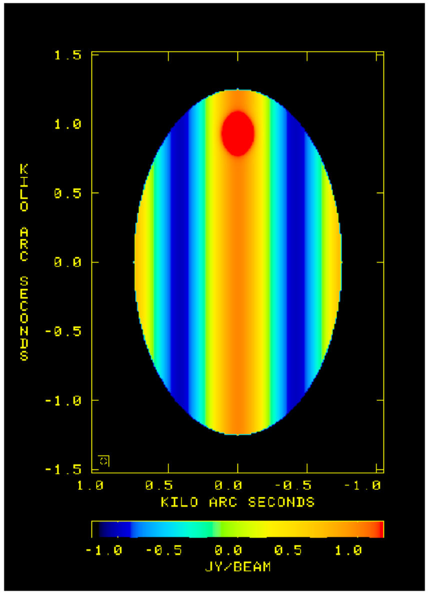
Our Mock Interferometer Response

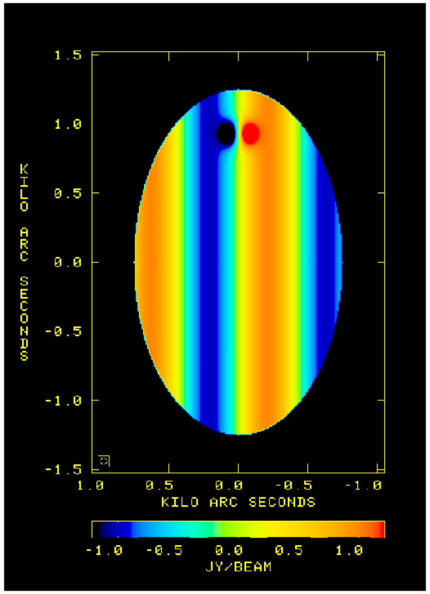
- We now `observe' this source with an interferometer, with seven baselines running from 12 through 525 meters, at a frequency of 1471 MHz.
  - These numbers are appropriate for the MeerKAT interferometer, but can be scaled to any other.
- The mock interferometer has only EW and NW spacings.
- Shown here are the two fringe patterns (Cos and Sin) for the two 50-meter baselines.
- Fringe spacing = 841”
- =300*206265/(1471*50)



What does the interferometer 'do'?

- Recall that the interferometer multiplies the actual brightness by the fringe pattern (both COS and SIN), and integrates (adds) over the field of view.
- The complex visibility is made from these products as:
  - COS => Real Part
  - SIN => Imaginary Part
- Shown are the COS and SIN products for the 50-m EW baseline.





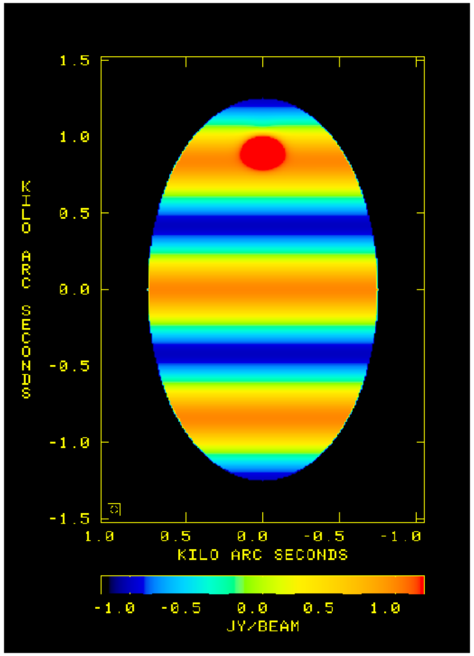
Cos Σ = -104 Jy   Sin Σ = -0.14 Jy

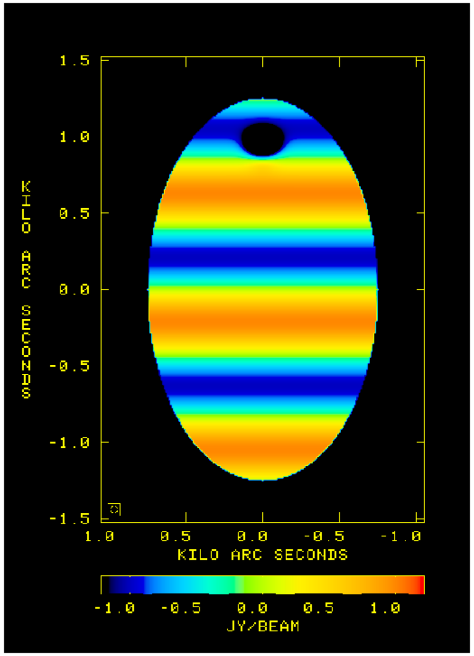
Thus: A = 104 Jy, ϕ = 180 degrees.

NB: The EW even symmetry requires the SIN integral = 0, so the phase must be either 0 or 180.

While for the NS baseline…

- The N-S asymmetry of the model means the NS baselines ‘see’ a wider range in the SIN (imaginary) component:





Cos Σ = 91.9 Jy

Sin Σ = -30.6 Jy

Thus: A = 96.9 Jy, ϕ = -18.4 degrees.

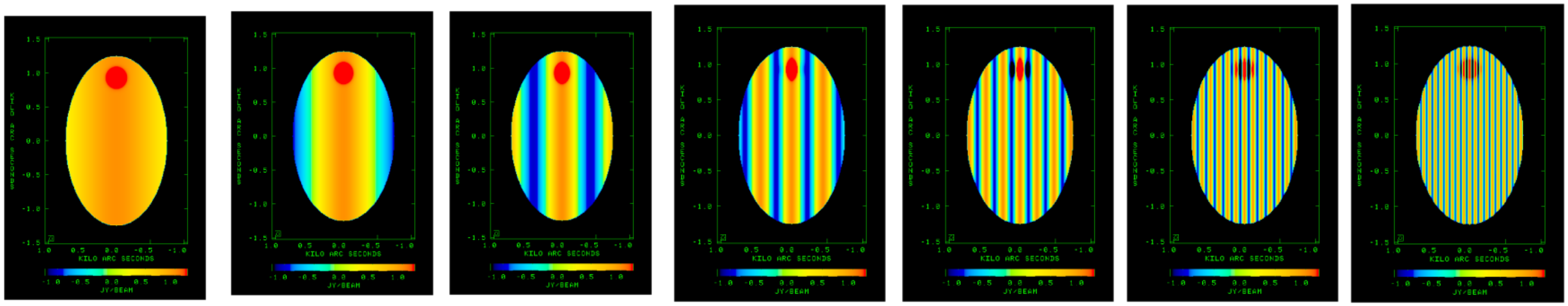
NB: There are no symmetries here, so phases can be any value.

Visibilities as a function of baseline length

- Showing a sequence as the E-W baselines gets longer:

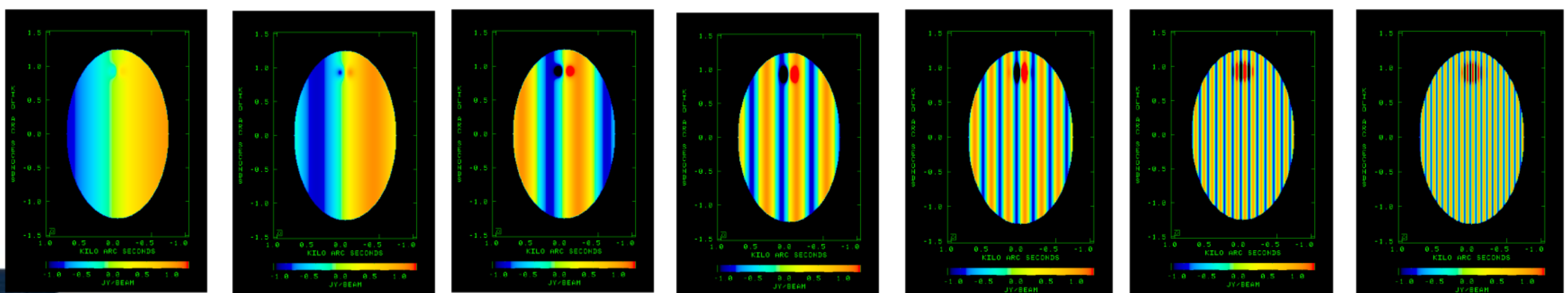
Baseline:	12.5m	25m	50m	95m	175m	350m	525m

COS



Σ	1069	429	-104	8.6	13.3	6.0	5.15
A	1069	429	104	8.6	13.3	6.0	5.15
ϕ	-0.3	0.1	180	0.0	0.0	0.0	0.0
Σ	-6.3	0.6	-0.1	0.0	0.1	0.0	0.1

SIN



- Note that the 'SIN' fluxes are all very low, so the visibility phases are ~zero or 180 degrees.

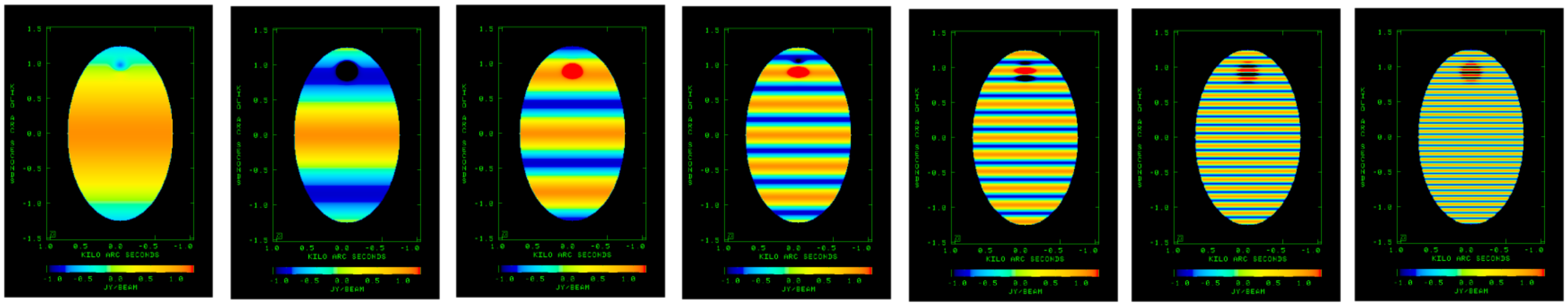
NRAO

As a function of N-S baseline …

- Showing a sequence as the baseline gets longer:

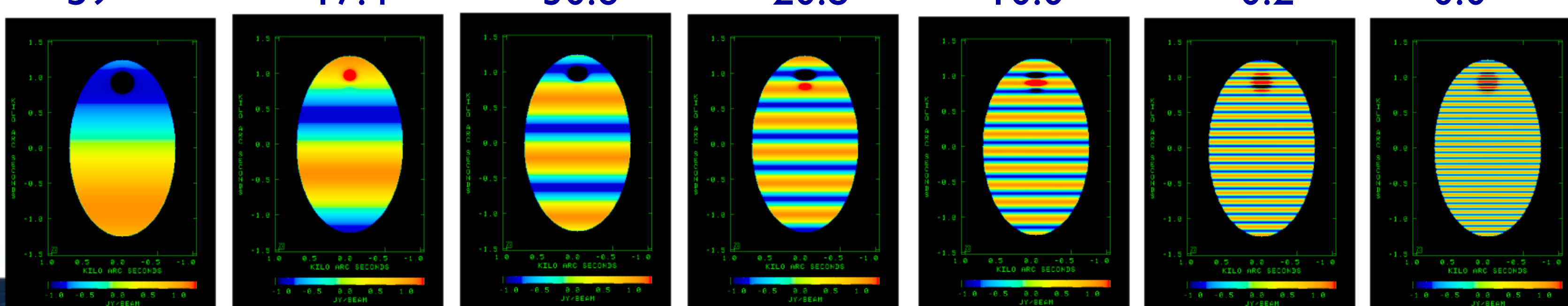
Baseline:	12.5m	25m	50m	95m	175m	350m	525m

COS



Σ	624	-201	91.9	4.2	14.9	3.5	6.8
A	627	202	96.9	21.2	17.7	3.5	6.8
ϕ	-5.4	175	-18	-78	34	3.6	0
Σ	-59	17.4	-30.6	-20.8	10.0	0.2	0.0

SIN



- Note that the 'SIN' fluxes are now much larger, and the visibility phases far from zero or 180.

Some Take-Aways…

- In general, as the baseline gets longer, the visibility amplitude declines.
- The decline need not be smooth – complicated emission will often have an oscillating, decaying, visibility function.
- The visibility from a 'zero-spacing' interferometer (aka 'a single dish') equals the total flux density of the source.
  - For Stokes 'I', this cannot be less than all visibilities at longer spacings.
- The visibilities for a source with even symmetry about some axis must be real (phase 0 or 180) for fringes parallel to that axis.
  - An extreme case of this is a smooth circular source: The emission is even about all axes (through the center), so all visibilities are real.
- The visibilities for a source with odd symmetry about some axis must be imaginary (phase 90 or -90) for fringes parallel to that axis.
  - This is not possible for Stokes 'I', but is possible for Q, U, or V.

Some Thoughts to Ponder

- The complex visibility **amplitude** is independent of the source location*, and linearly related to source flux density.
- The complex visibility **phase** is a function of source location, and independent of source flux density.
- These two statements, restated, are:
  - 1) Doubling the source brightness doubles the visibility amplitude, but doesn't change the visibility phase
  - 2) Shifting the source position changes the phase, but does not change the visibility amplitude.*
- Reversing the elements of an interferometer negates the phase of the complex visibility, and leaves the amplitude unchanged.
- For those of you familiar with Fourier transforms, the equivalent statement is that:
  - 'As the source brightness is a real function, its Fourier transform is Hermitian'.

* Ignoring any attenuation due to the primary beam…

Visibilities Lead to Images …

- The Visibility is a unique function of the source brightness.
- The two functions are related through a Fourier transform. $V_{\upsilon}(u,v) \Leftrightarrow I(l,m)$
- How we go from visibilities to images is the subject of a later lecture.
- An interferometer, at any one time, makes one measure of the visibility, at baseline coordinate (u,v).
- `A sufficient number of measures' of the visibility function (as derived from an interferometer) will provide us a `reasonable estimate' of the source brightness.
- How many is 'sufficient', and how good is 'reasonable'?
- These simple questions do not have easy answers…

Final Comments …

- The formalism presented here presumes much … including that there is no motion between source and interferometer.
- Real interferometers:
  - Are on a rotating platform
  - Use wide bandwidths
  - Average data over time
  - Employ frequency downconversion
  - Have to deal with corruptions due to many causes…
- How we manage these issues are the subjects of my next lecture, and by following lectures.