Beam forming and phased array feeds

The technology behind ASKAP's wide field of view

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Outline

- Antenna optics review
- Introduction to phased arrays in radio astronomy
- Using a phased array as a feed for a dish antenna
- Beam formation methods
- Beam characterisation
- Beam maintenance
- Useful beam footprints for astronomy
- A footprint tiling scheme for large-area surveys
- Future possibilities

Dish antennas and the diffraction limit

- The spatial resolution that can be achieved by an imaging system is governed by the size of the light-gathering aperture
  - This limit is rarely approached at optical wavelengths, but in radio astronomy it is typically what defines the primary beam of a telescope

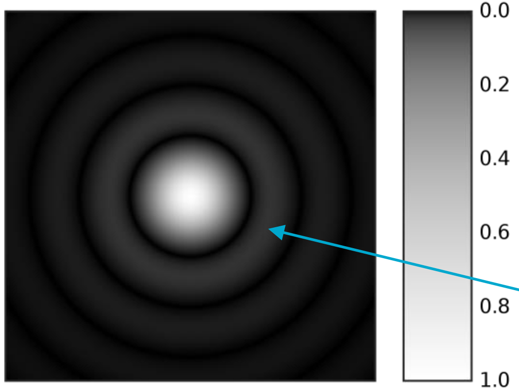
- The Airy pattern associated with diffraction due to a circular aperture is also the intensity profile (beam) of a uniformly-illuminated reflector
  - *Fun maths fact – the Airy pattern is a Bessel function of the 1st kind with order one*
  - The central peak of the Airy pattern is well approximated by a 2D Gaussian

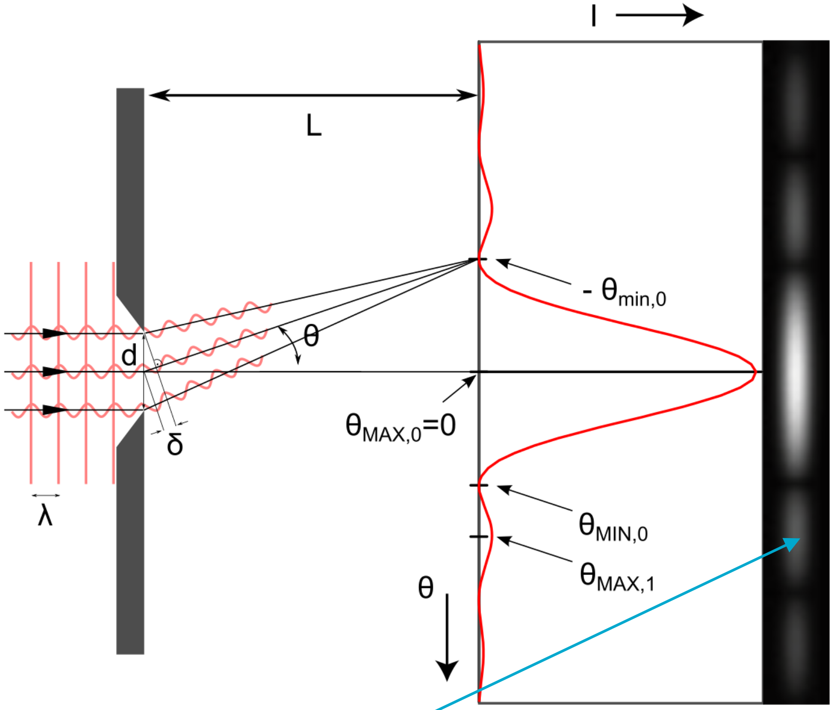
- There is an inverse relationship between aperture size and the extent of the diffraction pattern
  - Large antennas have lots of gain but also a small field of view

Directivity and amplification

Visualising the diffraction limit

- Diffraction calculations apply to a single wavelength
  - Beam size scales with frequency
- Diffraction causes significant side-lobes away from the main axis
  - These can be suppressed with tapered illumination (see later)





Side lobes

Image: Airy disk structure for a 2D circular aperture; Wikimedia Commons

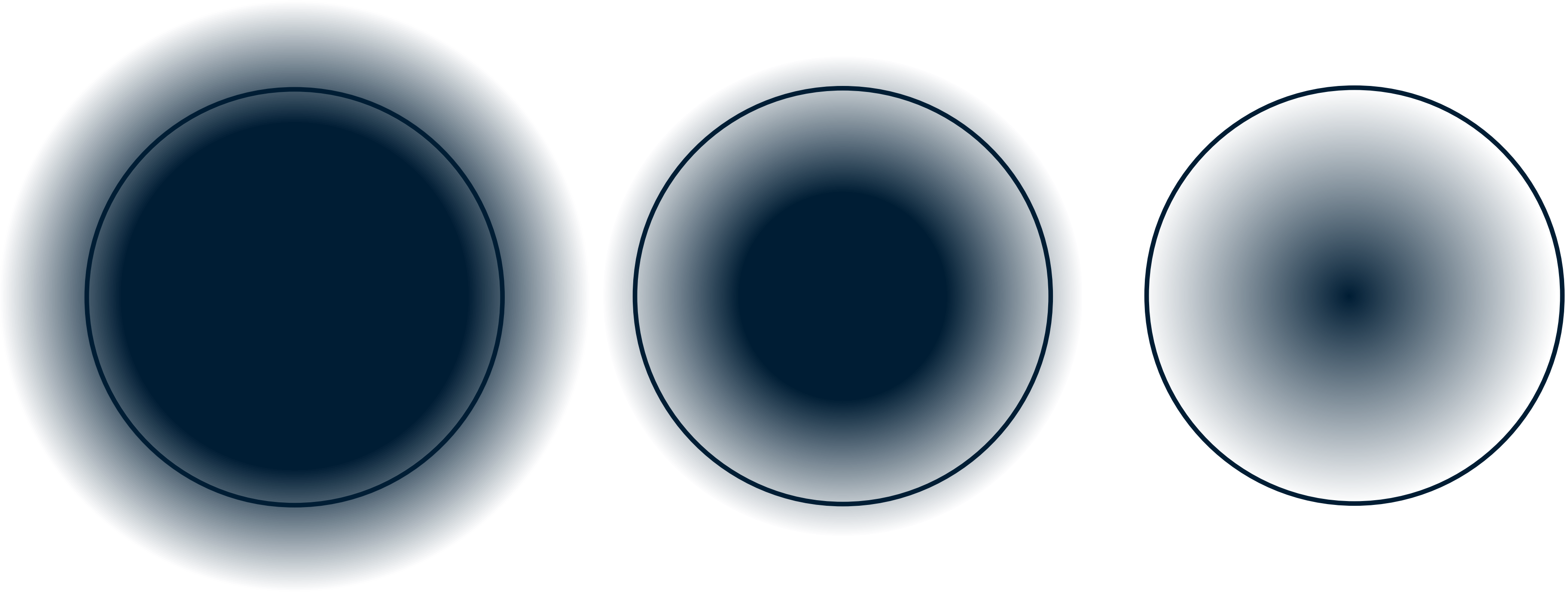
Image: Single slit diffraction – Jkrieger; Wikimedia Commons

Dish antenna feeds

- A feed is responsible for converting free-space waves into guided waves
  - The simplest configuration is a single feed placed at the focus on the optical axis
- A feed is itself an antenna!
  - It will impose its own radiation pattern on the reflector (known as illumination)
- Feeds are typically less sensitive to radiation coming from the edges of the dish, compared to the centre
  - Truly uniform illumination requires an immediate cut-off at the reflector boundary
  - In practice we must balance efficiency against spill-over and side-lobe levels

Image: Amateur radio telescope built by the Hydrogen Line Observing Group

Feed illumination & tapering



Near-uniform illumination

Tapered illumination

Under-illumination

The Gaussian primary beam approximation

- We often approximate the illumination pattern as a 2D Gaussian
  - Neglecting aperture blockage, reflections from support struts, etc.

- The (voltage) beam of a reflecting antenna is the 2D Fourier transform of its aperture illumination pattern
  - The Fourier transform of a Gaussian is another Gaussian, so tapered illumination acts to suppress side-lobes
  - Using a Gaussian profile for primary beam correction is usually a good starting point
  - True beam patterns may differ and can be predicted using electromagnetic simulations or measured using techniques such as holography

- Knowing the primary beam is key to obtaining an accurate flux scale and polarisation characteristics
  - *For an interferometer, source position depends on visibility phases, not the primary beam*

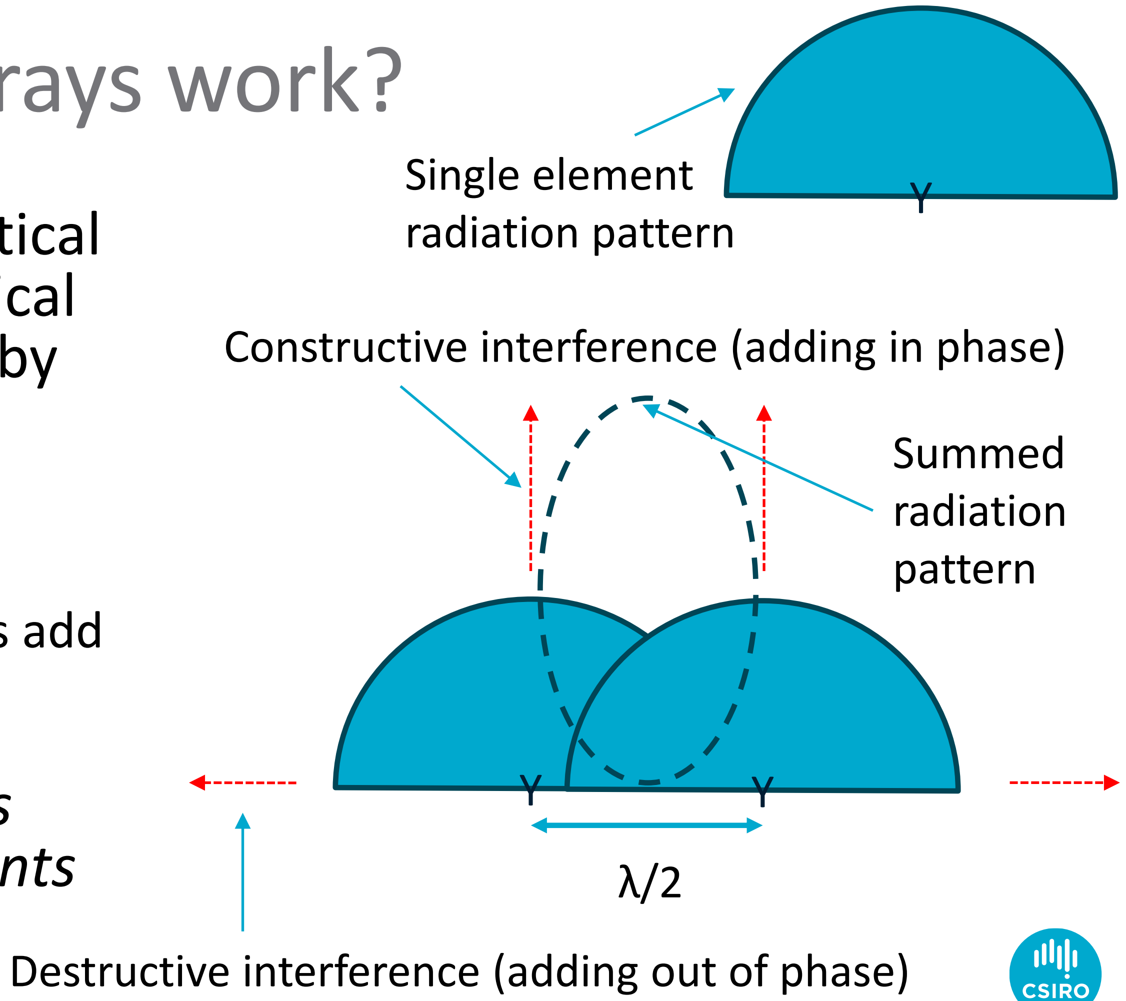
Do we need a dish?

- Reflecting antennas have some useful properties, but other technologies can be used, especially at low frequencies
  - Long wavelengths can be efficiently received using non-solid apertures
- Aperture arrays & phased arrays
  - Collecting area consisting of many low-gain elements combined

Image: Artist's impression of SKA-low; SKAO

How do phased arrays work?

- Consider a 1D line of identical antennas with hemispherical radiation pattern, spaced by half a wavelength
- In the far field, all signals overlap and add together
  - From overhead all wavefronts add in phase, but from the side, everything cancels
- *Directivity (gain) increases with the number of elements*



How do we steer a phased array?

- We can steer the beam away from vertical by applying a phase gradient across the elements before addition
  - We get the same directivity increase, in a different direction
- In this case we only need to adjust phases, but it's possible to apply scale factors too

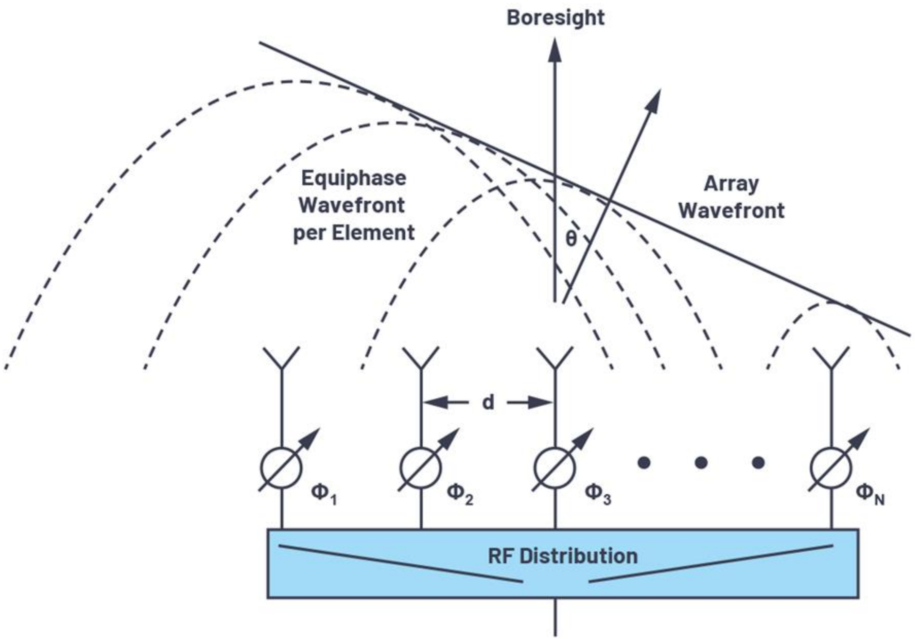


Image: From Phased Array Antenna Patterns - Part 1: Linear Array Beam Characteristics and Array Factor by Peter Delos, Bob Broughton, and Jon Kraft

Beam forming vs interferometry

- With an aperture array, each element has a direct view of the sky
  - The combined beam is defined by the element beam, number of antennas and their physical spacing
- Phased array beam formation involves a weighted sum of the elements
  - With unity weight, this will increase gain along the optical axis
  - Non-unity weights can be applied to steer and shape the beam

- Interferometry increases spatial resolution by correlating inputs
  - Correlation is essentially multiplication, with suitable delay tracking
  - *Correlation allows a telescope to image the region within its primary beam*
- Both beam formation and correlation are frequency-dependent

Beam forming in radio astronomy

- A beam former additively combines receiver elements (while applying complex weights or delays) to produce a new output
  - This can be done in hardware – e.g. cables with different lengths to introduce delays on the way to a signal combiner
  - Or digitally – by computing a numerical weighted sum post-digitisation

- Beam forming is done *before* correlation
  - Must occur without loss of information and maintain phase coherence
  - Correlation can later be done across beams that point in the same direction


Image: MWA delay lines; ICRAR/Curtin
Delay Line Rev 7
CSIRO
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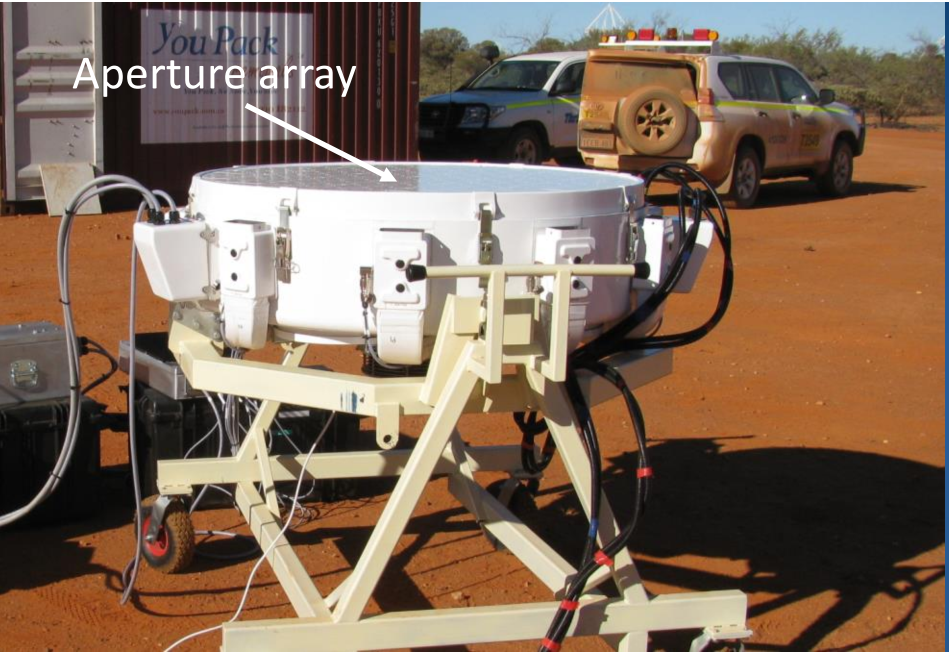
Aperture arrays past and future

- Phased arrays were used at the very beginning of radio astronomy
- Jansky's famous "merry go round" is an example of a dipole array and was used to discover radio emission from the Milky Way
- SKA low will be an aperture array consisting of tiles
  - Several beams are formed using the elements within a tile
  - Beams from different tiles that point in the same direction are correlated

Image: Artist's impression of SKA-low; SKAO

Using a phased array as a dish feed

- The same hardware can fill two roles with different beam weights
  - Illumination of the dish is determined by the beam weights used



The ASKAP chequerboard PAF

- 2D (dual polarisation) array of “bow tie” dipoles on a square grid
- Elements are spaced by 90 mm across roughly 1 m diameter

- Broad frequency coverage, from 700 MHz to 1800 MHz
- Complete sampling of the wavefront in the focal plane
- 188 elements, fed from the corners of the diamonds

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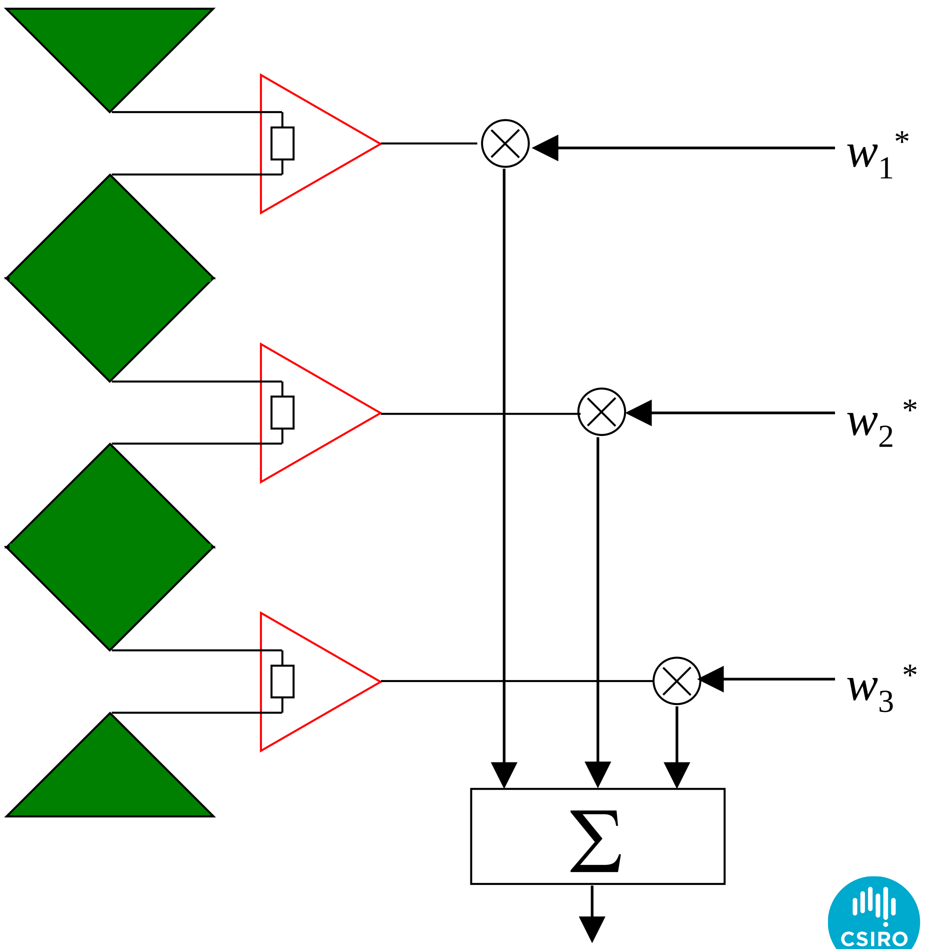
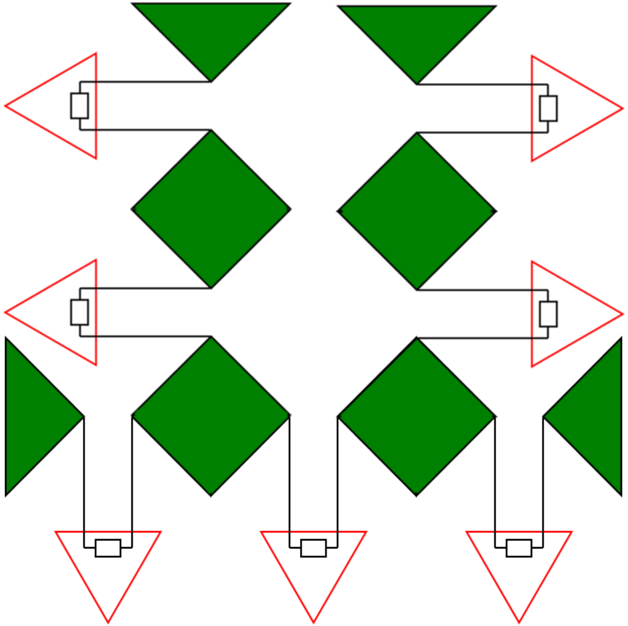
Forming beams

- Amplifiers are connected to the inside corners of each dipole pair
- A single conductive patch contributes to several elements
- The signal from each element is digitally sampled and sliced into frequency channels
- Samples from each element are multiplied by their own complex weight
- Weighted voltages are summed to a single number per time sample and frequency channel



Dual orthogonal polarisations

- Polarisation is determined by the orientation of the pair that combines to form an element
- ASKAP PAF elements are linearly polarised
  - Half of the 188 elements are aligned in X, the other half in Y (94 each)
- Beams are formed exclusively from similarly-oriented elements
  - Physical orthogonality is very good by construction
  - In theory it is possible to optimise further using cross-polarisation elements in the beam solutions, with suitable constraints



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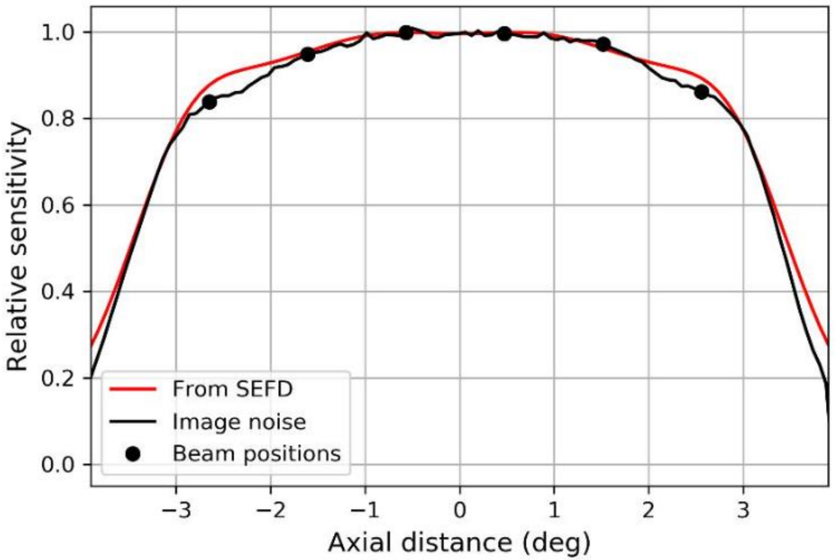
Multiple simultaneous beams

- We receive ample photons at radio wavelengths
  - Many copies of element signals can be made without loss of quality
- Each set of copies can be combined using different beam weights
  - We can steer several beams in different directions simultaneously!
- PAF beams can be closely packed on the sky
  - Once beams start to overlap, they exhibit correlated noise due to contributions from the same physical elements

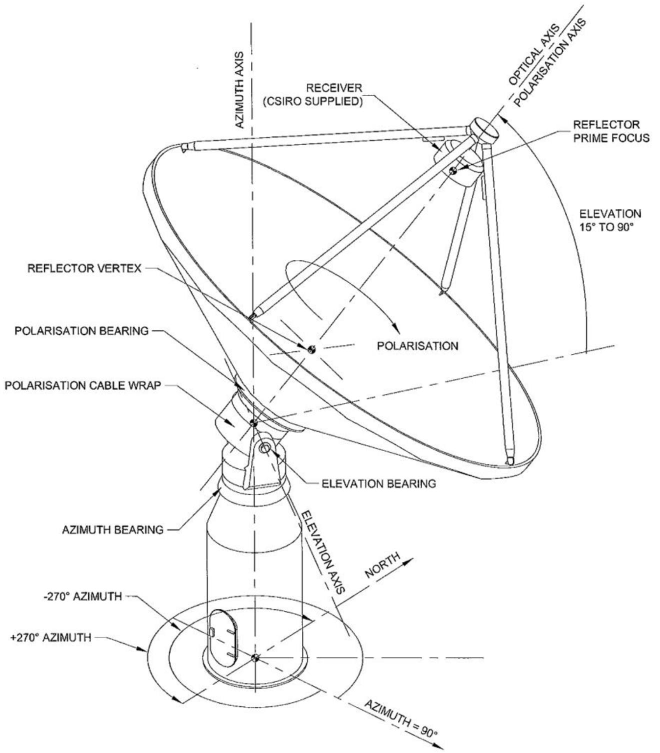
Off-axis feeds and the focal plane

- A parabolic reflector will focus plane waves incident along the optical axis to a unique point
- At small angles offset from the optical axis there is still a strong focussing effect, though with some coma distortion
- Off-axis beams are viable, within a decreasing sensitivity envelope
- We can consider the antenna to have a focal plane



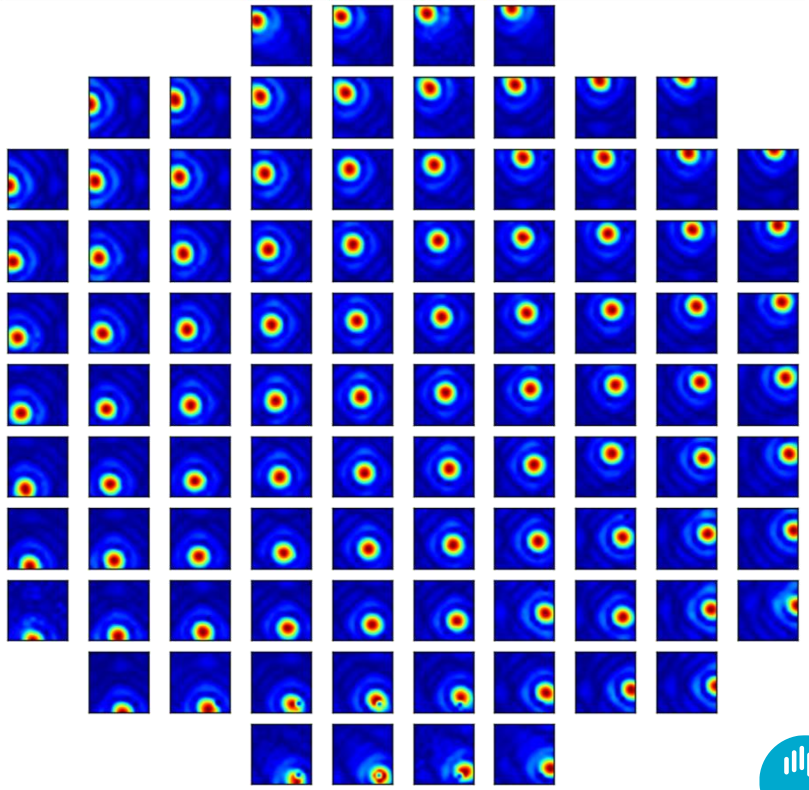
Tracking with offset beams

- Alt-Az telescope drives can track a sky position, but not with constant orientation
  - When using on-axis beams, this causes the field to rotate as the antenna tracks
  - When using off-axis beams, it causes the field centre to move!
- ASKAP has a third driving axis that spins the reflector to keep offset beams fixed on the sky



An element-level perspective

- Each PAF element has its own view of the sky via the dish
  - Off-axis elements all see slightly different parts of the sky
- Any composite beam is a linear combination of single element beams
- Element beams are primarily governed by the dish
  - They have a lot of spill-over!



Nuances of beam forming

- PAFs typically use an adaptive beam forming approach
  - Beams are formed in response to measured parameters
- Each element's amplifier has its own complex gain
  - In particular, the phase of each amplifier is independent (though fixed)
  - This means that beam weights are unique to an individual PAF

- Each element emits thermal radiation that is received by its neighbours
  - Adjacent elements do not receive completely independent signals

- Beam forming algorithms can be constrained in many ways, but for ASKAP we use a simple method that optimises sensitivity

Generalised beam forming

- We can express beam forming as a mathematical operation:

PAF element outputs at time *i*

$$y_k[i] = \mathbf{w}_k^T \mathbf{x}[i]$$

Beam *k* output at time *i*

Weight vector for beam *k*

Maximum sensitivity beam forming

- Applebaum (1976) derived a simple expression for the weights that maximise sensitivity to an unresolved source:

$$\mathbf{w}_k = \widehat{\mathbf{R}}_n^{-1} \hat{\mathbf{v}}_k$$

Noise covariance matrix for the *n* elements

Steering vector (response of individual elements to a point source in the direction of beam *k*)

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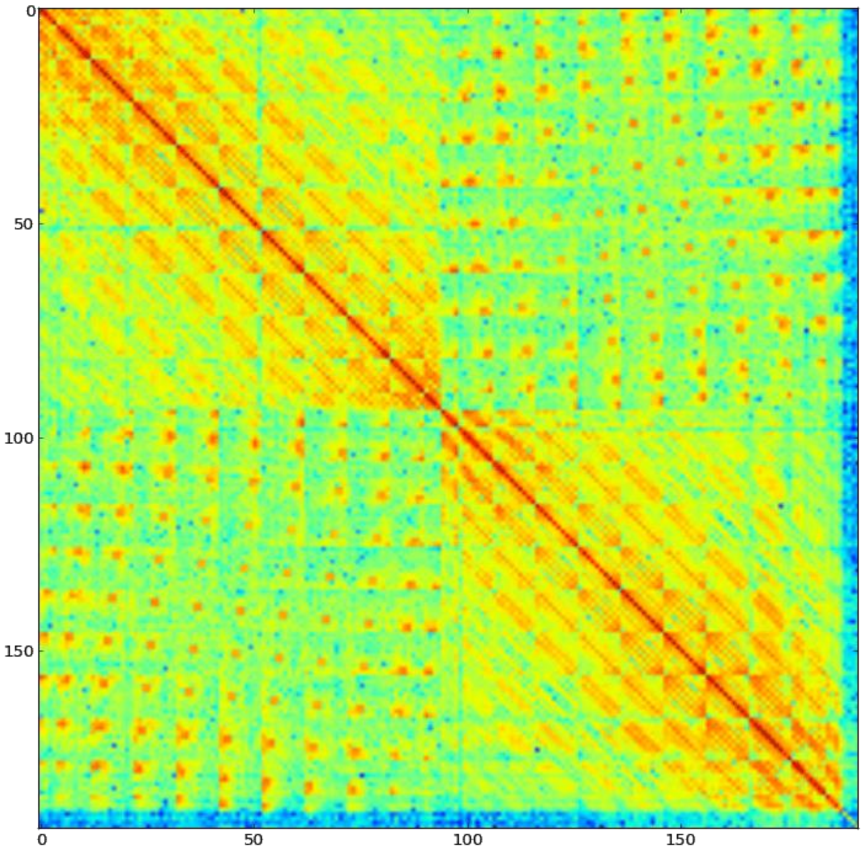
Noise covariance

- The beam former can compute the Array Covariance Matrix
  - ACM = the correlation of each element with every other element
- There is always some correlation between nearby ports (even between different polarisations)
- The ACM's structure is mostly determined by the PAF's physical geometry



$\hat{\mathbf{R}}_n$

Steering vectors

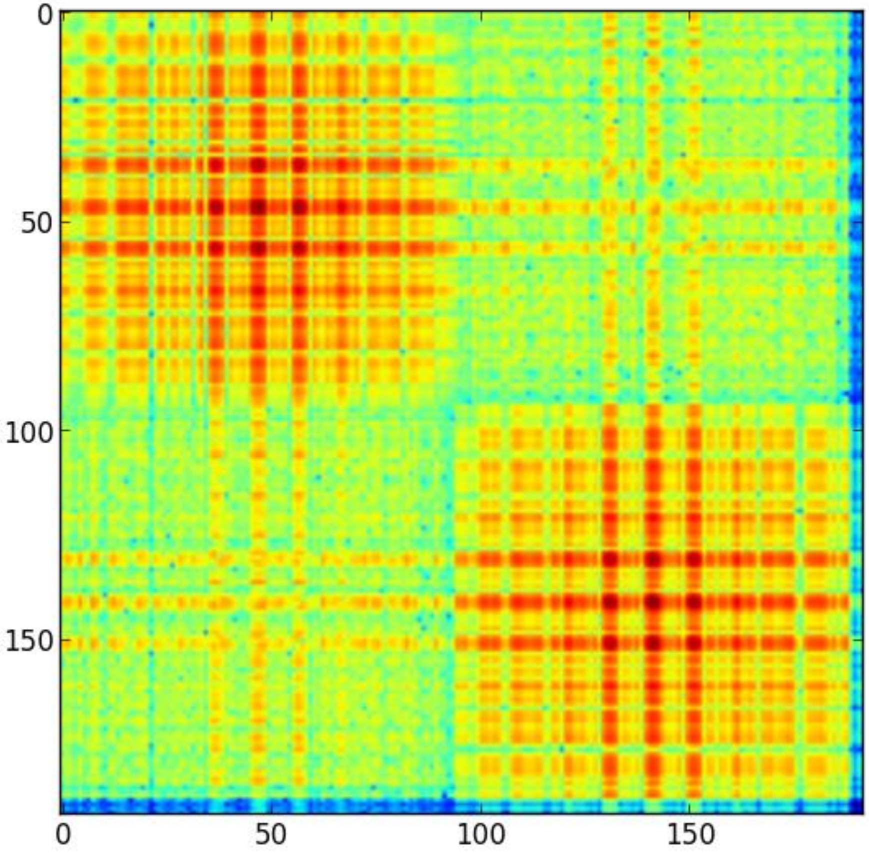
- Can be measured using single-dish ACM observations
- Recording ACMs while pointed at a strong source yields: $\hat{\mathbf{R}}_{s+n}$
- The required steering vector is the Eigenvector of the difference corresponding to the dominant eigenvalue λ (see Landon et al. 2010):

$$[\hat{\mathbf{R}}_{s+n} - \hat{\mathbf{R}}_{n}]\mathbf{v} = \lambda\mathbf{v}$$

- With an interferometer, we can also measure the steering vector directly by pointing a reference antenna at a strong source
  - This requires sending signals from individual elements to the correlator

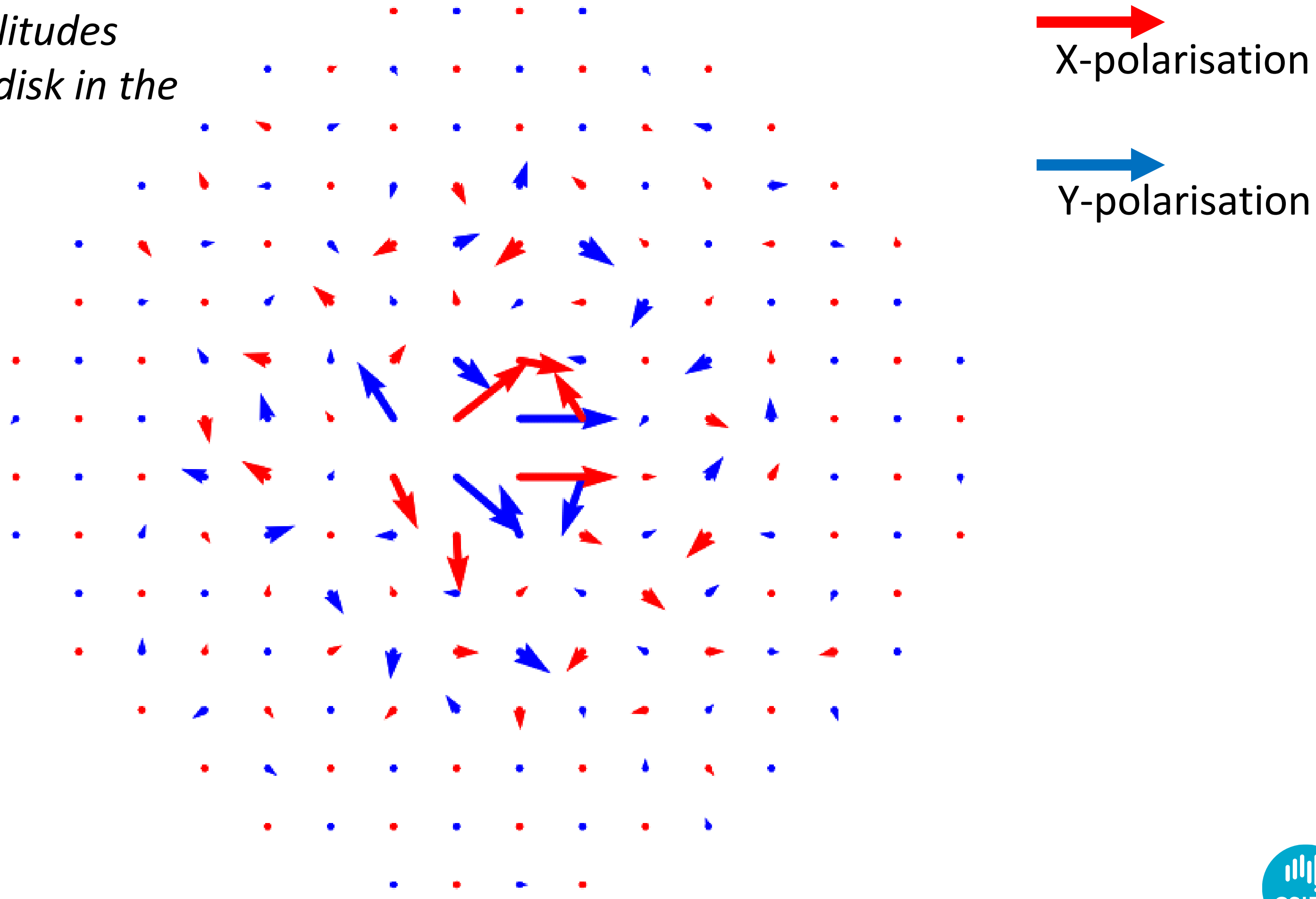
Signal covariance

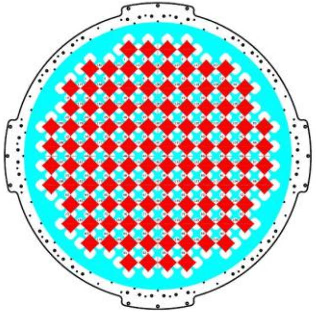
- The quality of the weights is determined by the strength of the reference source relative to the noise matrix
  - In practical terms, we need a very strong source to form beams with ASKAP's 12m dish antennas



Signal + noise covariance matrix $\hat{\mathbf{R}}_{s+n}$

Beam weight amplitudes trace out the Airy disk in the focal plane!





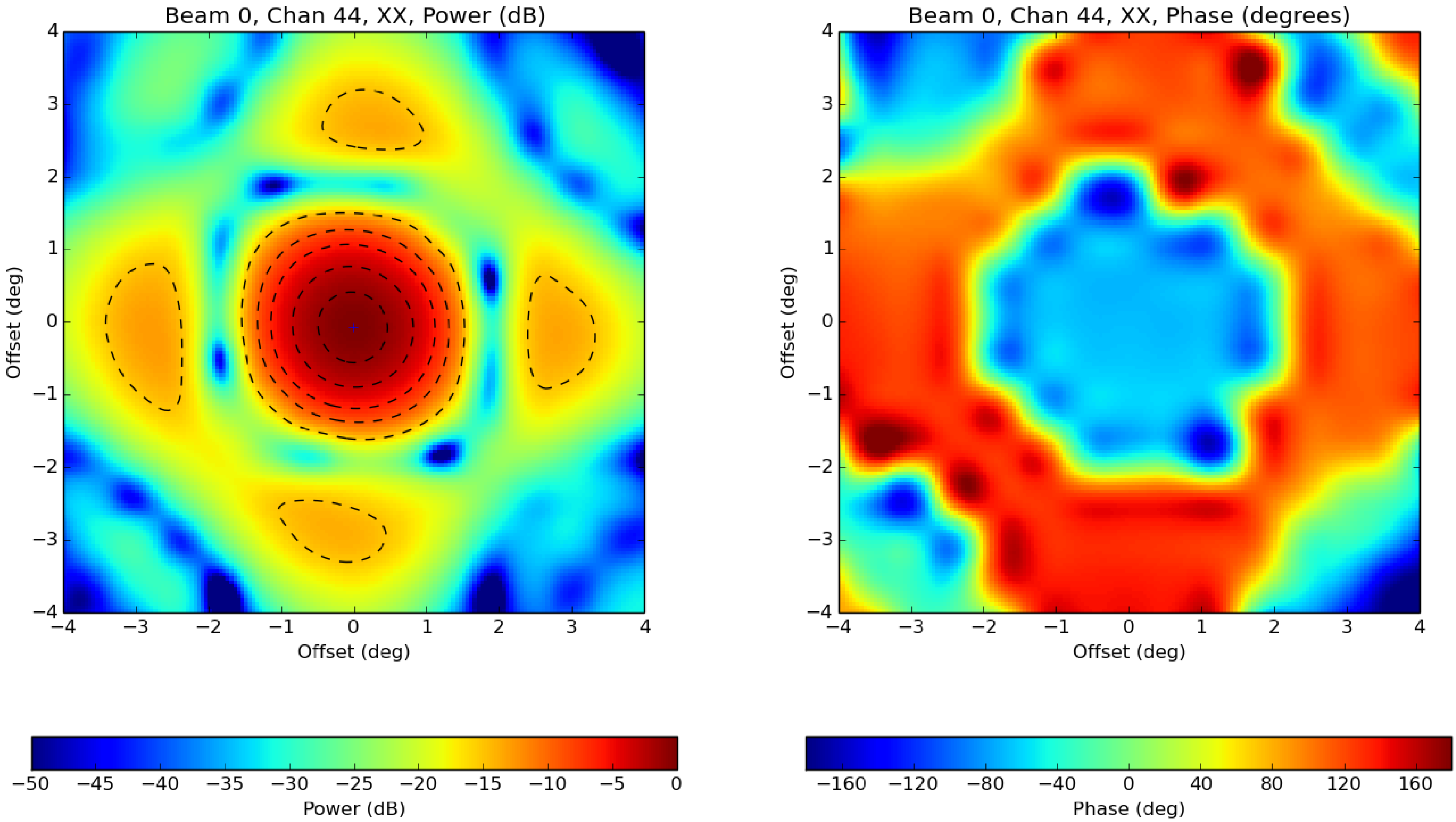
Beam forming and frequency dependence

- Antenna optics are naturally frequency-dependent
  - The diffraction properties of a fixed-size aperture depend on wavelength
- Beam forming for ASKAP is done independently on 1 MHz intervals
  - There can be phase and amplitude discontinuities at the interval boundaries
- Calibration should remove these features, but residuals can remain if the bandpass changes slightly
  - We offer wide-interval beam former modes to reduce the chance of a spectral line falling on a boundary

PAF antenna illumination

- A feed horn uses roughly 60% of the dish surface area
  - The illumination pattern of a horn is fixed by its design, but the illumination pattern of a PAF is determined by beam weights
- The maximum sensitivity algorithm tends to favour increased efficiency (~75%) at the cost of higher side-lobes
- The PAF and its supports present a significant aperture blockage
  - 4-fold symmetry can be seen in beam shapes as a result

- Beam weight solutions should be reproducible, but variations can occur
- Optimising for sensitivity leaves other parameters free
  - Side-lobe level, symmetry, etc.



Beam 0, Chan 44, XX, Power (dB)
Beam 0, Chan 44, XX, Phase (degrees)
Offset (deg)
Offset (deg)
Offset (deg)
Offset (deg)
4
3
2
1
0
−1
−2
−3
−4
−50
−45
−40
−35
−30
−25
−20
−15
−10
−5
0
Power (dB)
−160
−120
−80
−40
0
40
80
120
160
Phase (deg)

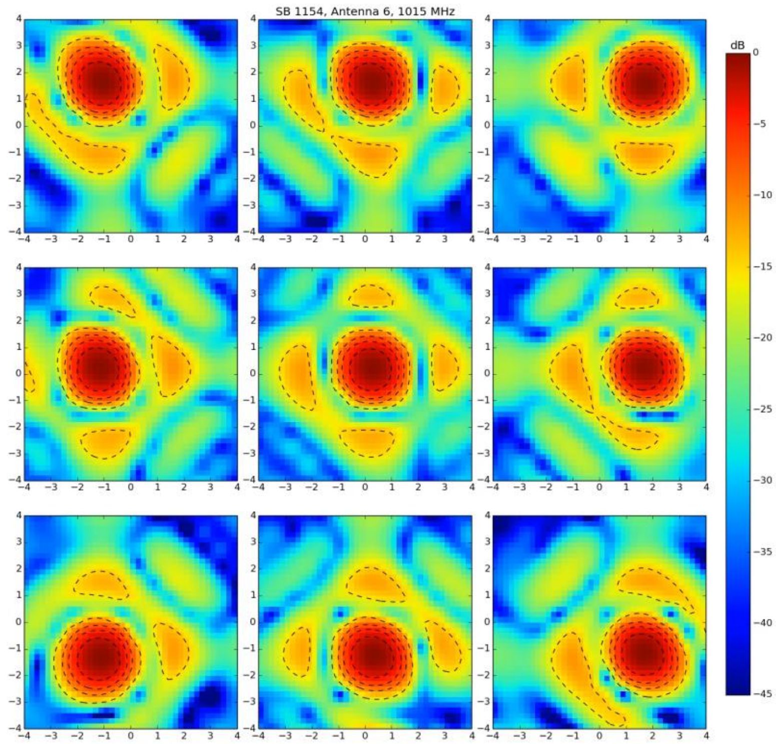
Holography for primary beam mapping

- When new beams are formed, we need to directly measure their shape to ensure accurate primary beam correction
  - This includes all polarisation products for off-axis leakage correction

- A dedicated holography observing mode is used to map beams
  - Point one reference antenna at an astronomical source
  - The others observe a grid of closely spaces positions covering the field of view
  - Extract only the visibilities from the reference antenna to each test antenna
  - Grid and interpolate to obtain the beam map as a function of angular distance

Example beam shapes from holography

- Offset beams are elongated due to coma distortion
- Side lobes have asymmetric structure and high level
- Beam positions can vary from one antenna to the next
    - Beam pointing will absorb fixed mechanical pointing offsets



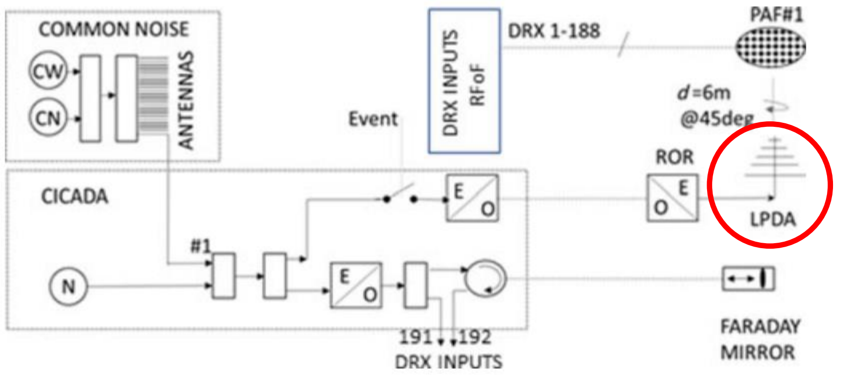
The lifetime of beam weights

- If nothing changes, a weight solution should last forever
- However, many things can change over time
  - Slow gain drifts due to temperature variation and hardware aging
  - Abrupt changes due to digital delay jumps upon hardware reset
  - Dramatic changes like LNA failure due to lightning strikes

- New beam solutions for ASKAP are derived every few months
- These solutions are also updated before every observation
  - Updates can be done much more quickly than a full solution

On-dish calibration

- The ODC system provides a reference against which we can measure element gain changes
  - Low-power broad-band noise that uniformly illuminates the PAF
- This reference can't determine what the weights should be (since it's in the near field), but it can tell us what has changed



Designing beam footprints for surveys

- Up to 36 beams can be placed anywhere within ASKAP's field of view
- For practical reasons, regularly spaced packing schemes are used
  - Square (larger field of view)
  - Closepack (more uniform sensitivity)
- Adjacent footprints should tesselate neatly

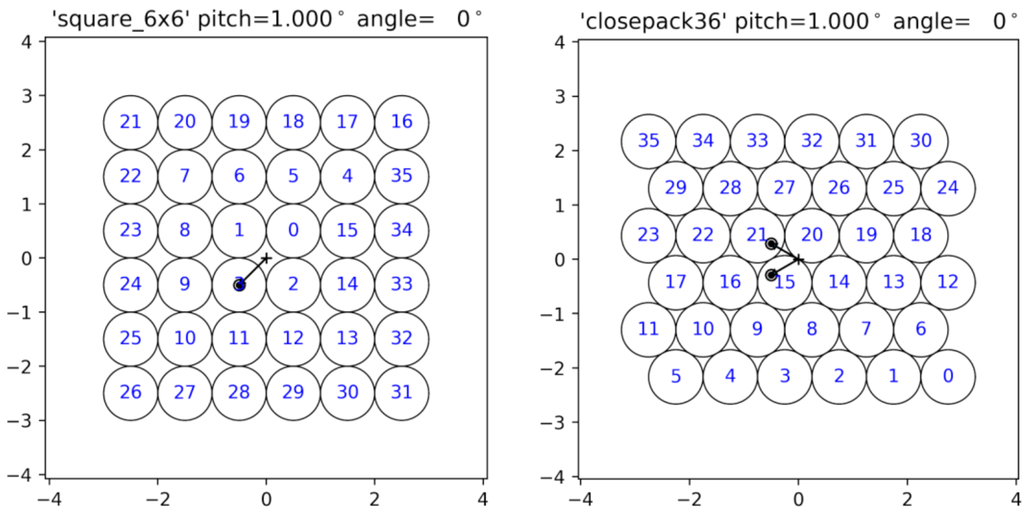


Image: Footprint configurations; David McConnell, ASKAP observation guide

Tiling multiple fields for surveys

- Rectangular footprints stack horizontally and vertically
- We slice the sky into rings of constant declination
- Towards the poles, footprint corners overlap more and more
  - At high latitudes we use a polar cap to maintain survey efficiency
- Roughly 1000-2000 tiles are needed to cover the observable sky (depending on frequency and beam spacing)

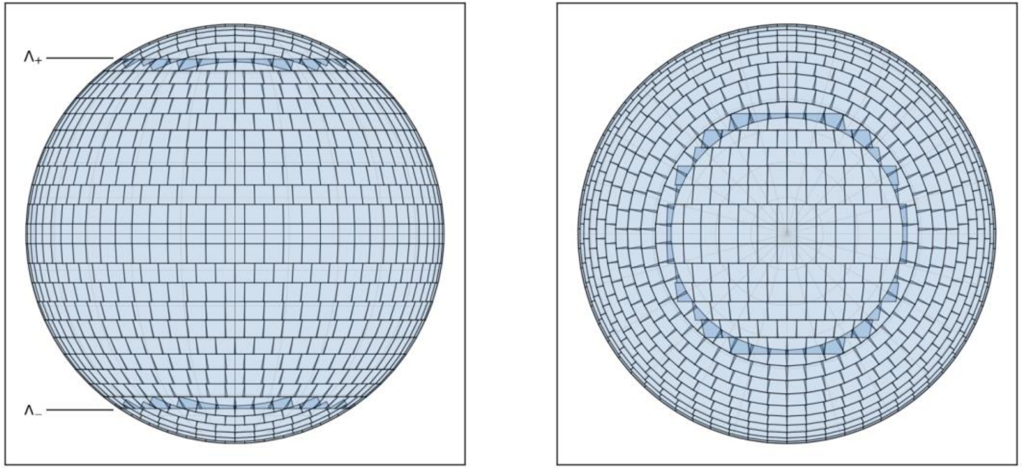


Image: Survey tiling; David McConnell, ASKAP observation guide

Future possibilities

- Digital beam forming provides great flexibility (and complexity)
- It may be possible to exert more control over beam properties
  - Additional constraints should be well matched to calibration requirements
  - There would likely be a trade-off with raw sensitivity
- Adaptive nulling could improve interference mitigation
  - Place beam nulls on known sources of RFI (satellites, the Sun, etc.)
  - Requires updating beam weights during an observation

In conclusion

- Phased arrays increase gain using a weighted sum of their elements
- We can feed a dish antenna using a phased array in the focal plane
  - The Airy disk of the dish is imprinted on the feed elements
- Phased array feeds allow control of beam properties, at the cost of complexity in processing and potentially calibration
  - More processing required to form beams and then correlate them
  - Beam weight maintenance and beam shape measurement
- ASKAP's large field of view is made possible by forming 36 simultaneous beams packed regularly into a rectangular area