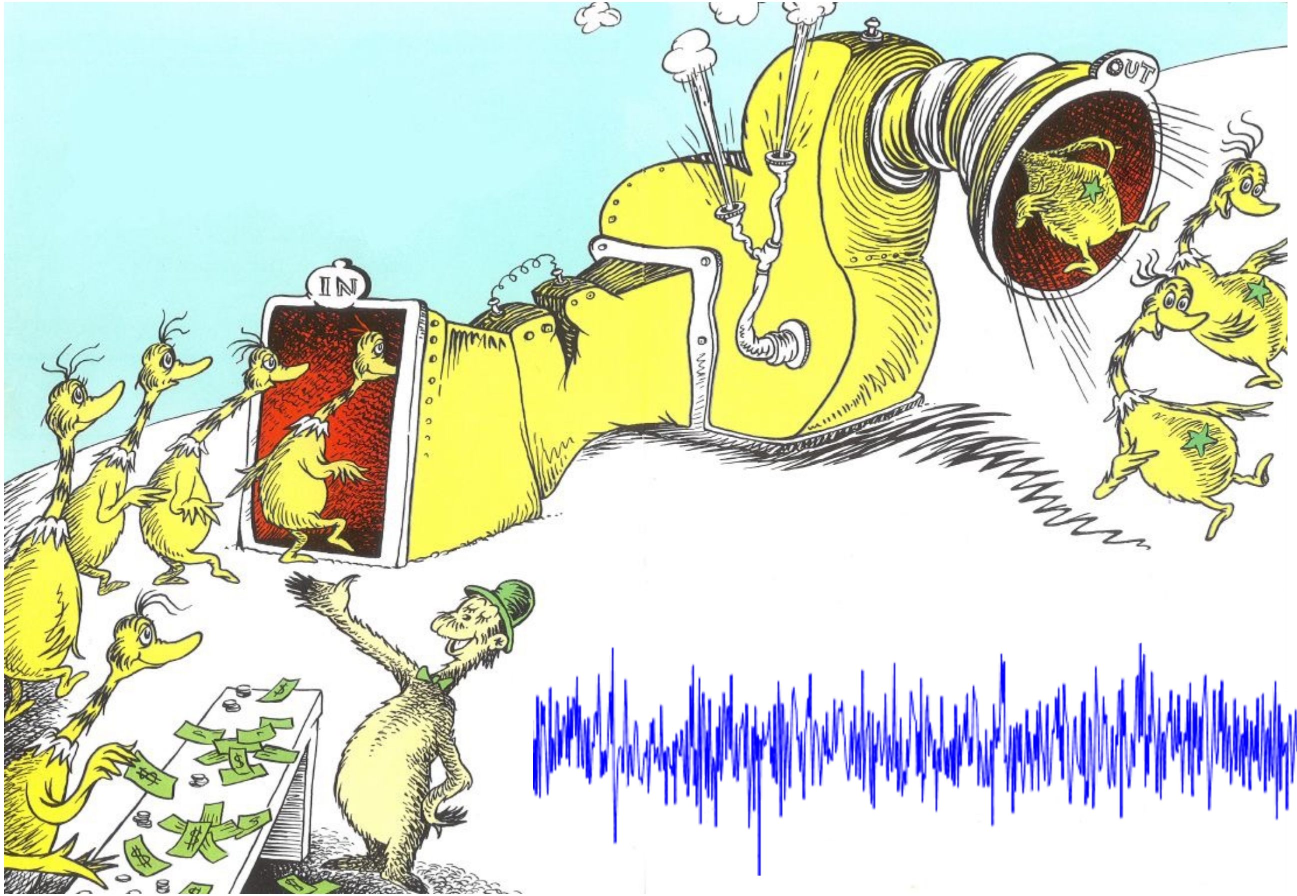
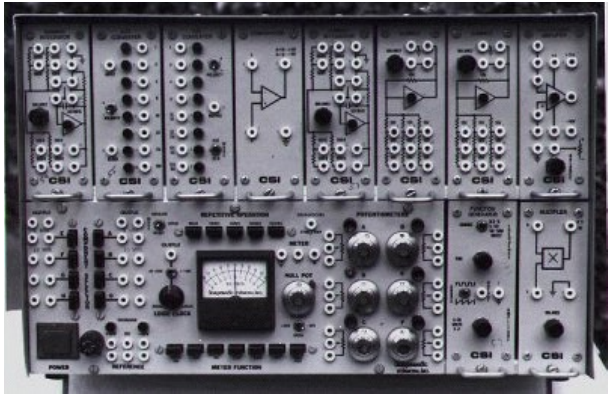
(Digital) Signal Processing

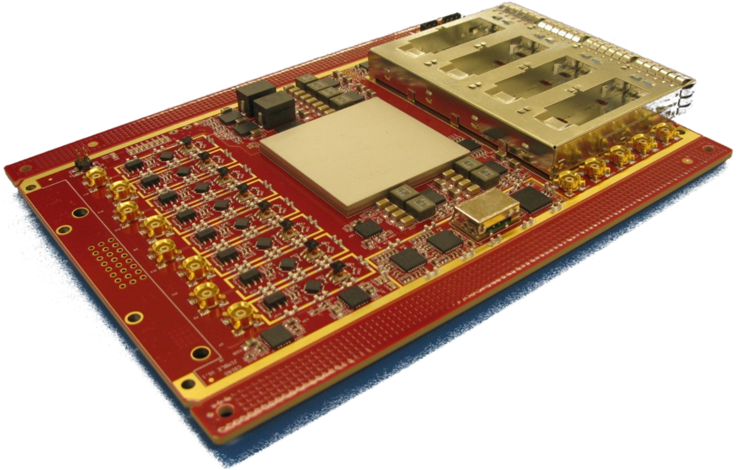
Chris Phillips| 25/9/2023



Digital Signal Processing - DSP

- Why digital:
  - Stability (time and environment)
  - Superior performance (e.g. filter shape)
  - Cost – can mass produce
- However:
  - Loss of dynamic range and SNR
  - Artifacts - aliasing & birdies
  - Cross talk and reliability
  - High power requirements

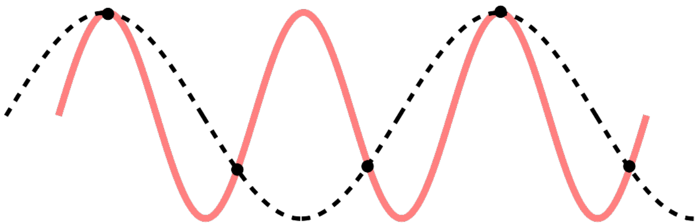


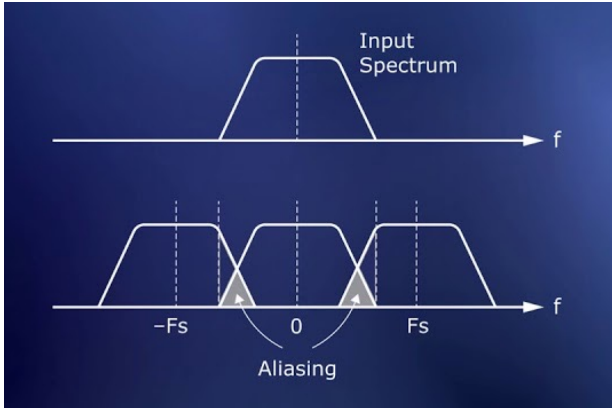


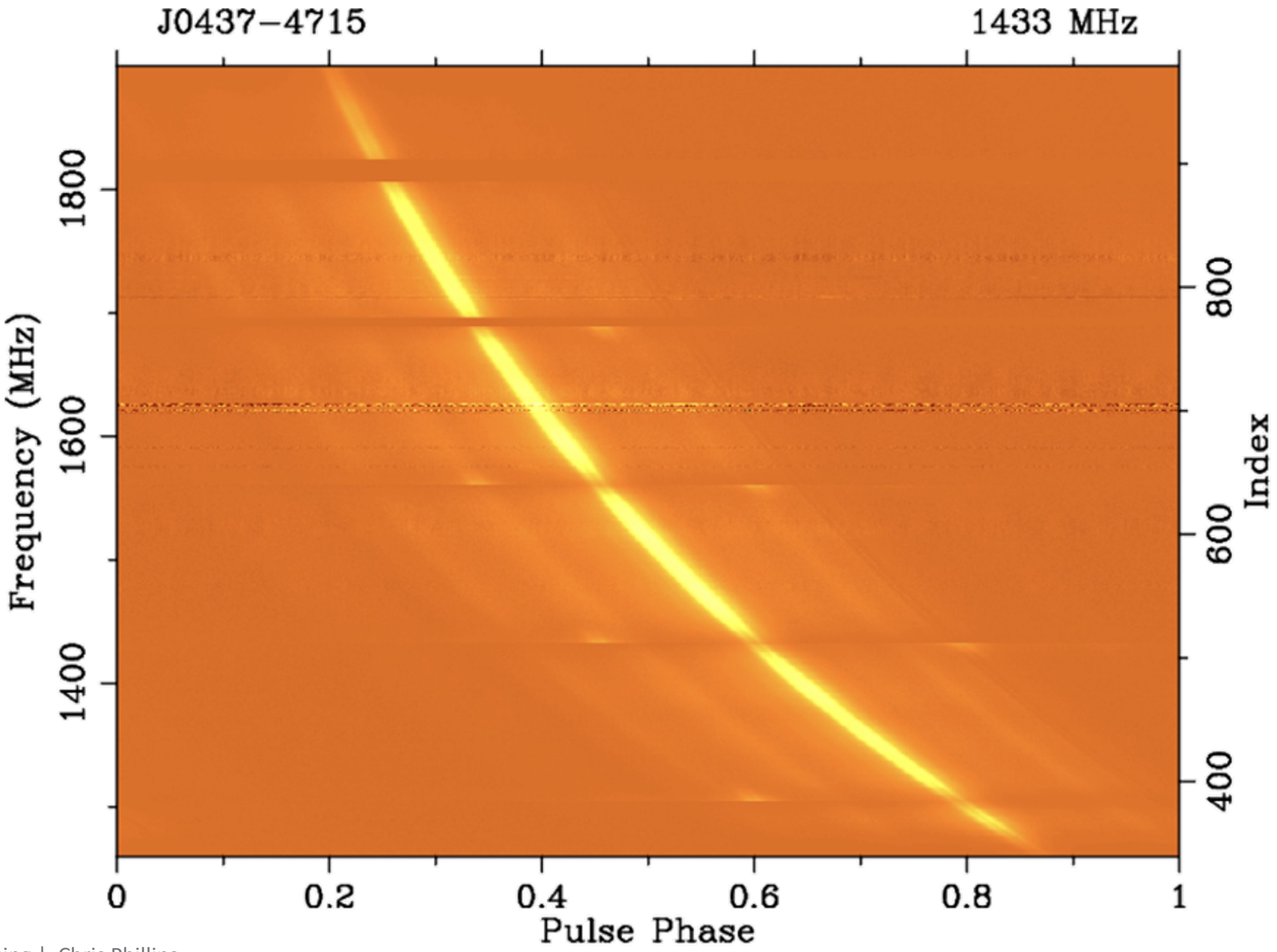
Nyquist/Shannon Sampling

Nyquist, H. (1928). "Certain topics in telegraph transmission theory."
Shannon, C. E. (1949). "Communication in the presence of noise."

- A bandlimited signal can be perfectly reproduced if it is sampled at a frequency at least twice rate of maximum bandwidth
  - Critical sampling rate Nyquist
  - 100 MHz bandwidth signal needs to be sampled at 200 MHz
- *Aliasing* occurs if sampling at a lower rate
  - *Frequencies can be lost, folded and mixed*

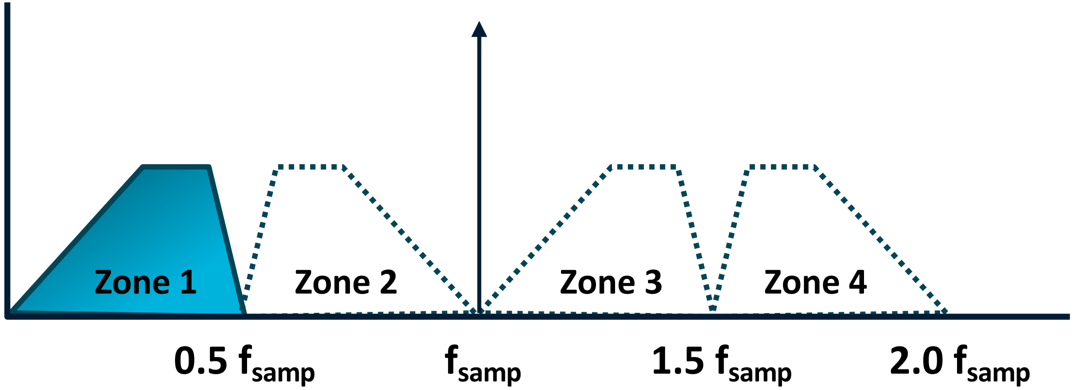





J0437−4715
1433 MHz
Frequency (MHz)
1800
1600
1400
Index
800
600
400
0
0.2
0.4
0.6
0.8
1
Pulse Phase

Aliasing and Nyquist zones

- Aliasing can be useful in combination with filtering
  - It acts to reflect everything above the critical frequency
  - This is effectively a form of down-conversion
    - Needs bandpass filter before digitizer to reduce range of frequencies
  - Digitisers often operate in these higher-order Nyquist zones
  - Virtually all ATNF digitizer systems filter in Zone 2 or 3

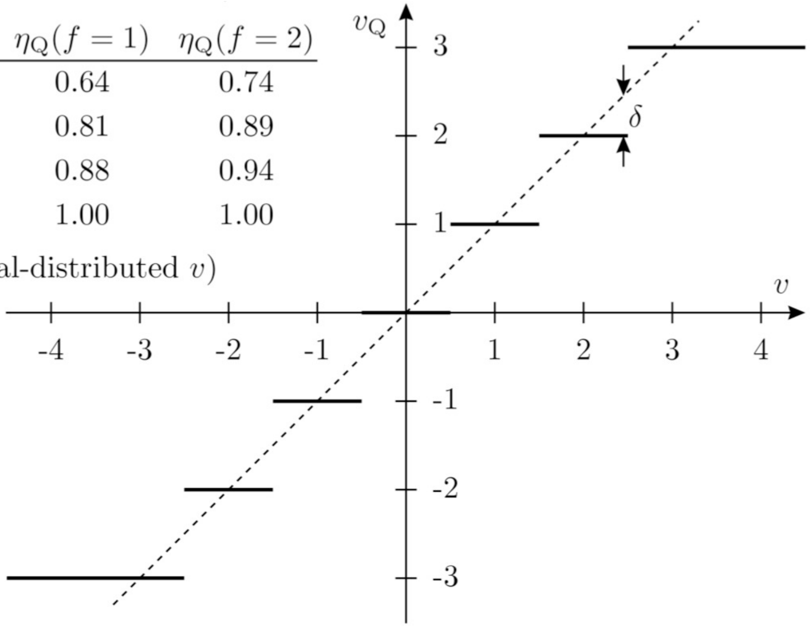


Quantisation

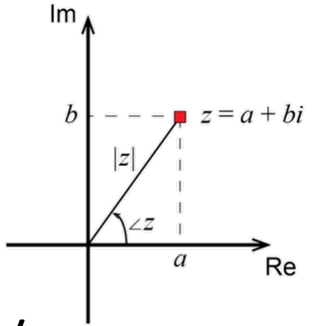
- DSP represents samples with limited number of bits
- 1 or 2 traditional
- 8 or 16 common now
- More bits give (slightly) more "efficiency" and more dynamic range (e.g. robustness against RFI)
  - 4 bits give 98.8% efficiency

N levels	$\eta_Q(f=1)$	$\eta_Q(f=2)$
2	0.64	0.74
3	0.81	0.89
4	0.88	0.94
∞	1.00	1.00

(For normal-distributed v)



Complex Voltages



- There are multiple ways of representing a signal
  - One representation is not more “right” than another
- Complex numbers are a convenient way to represent e/m waves or voltages
  - Nyquist rate is halved – data rate is unchanged
- Analogue to digital converters can produce real or complex output
- Digital filterbanks more naturally output complex values
- Complex numbers have a number of advantages
  - Slightly more efficient FFT implementation
  - “Natural” form for phase corrections

The Fourier transform

- The Fourier transform switches between the time and frequency domains
  - Or the image and spatial frequency (UV) domains, depending on context
- Often convenient to switch between representations to simplify the maths or interpretation
- FFT (Fast Fourier Transform is an algorithm to compute fourier transform efficiently

$$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi i k x} dk$$

$$e^{ix} = \cos x + i \sin x$$ Euler's formula

Standard Transforms

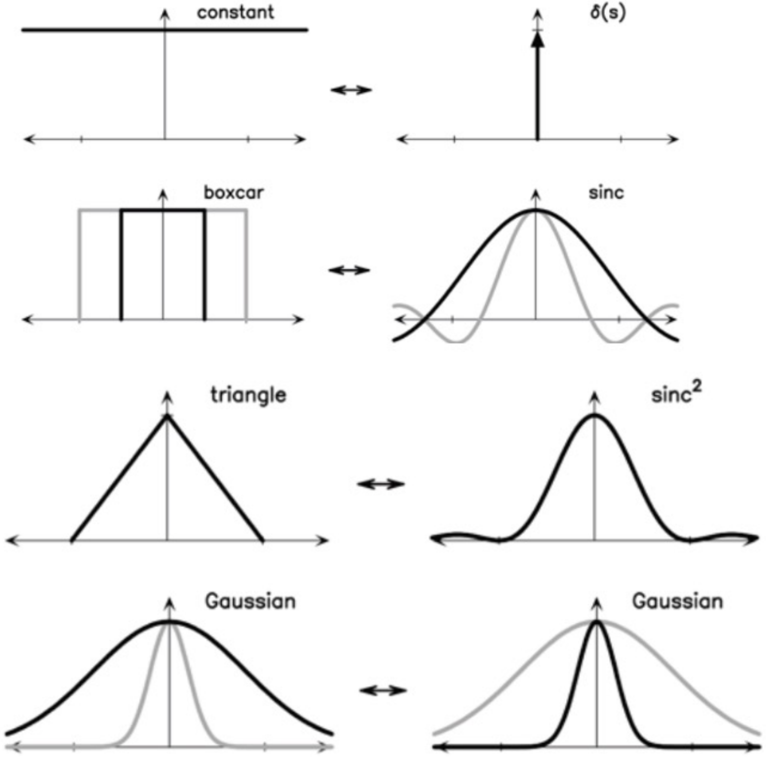


Image Credit: Josh Marvil

Fourier Theorems

Fourier shift theorem

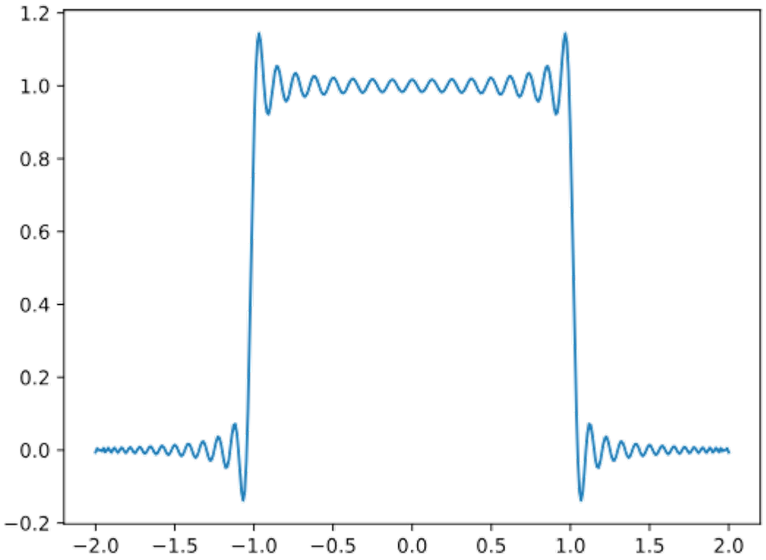
- Applying a phase gradient in Fourier space shifts a function

Convolution theorem

- Convolution of two functions is equivalent to multiplication of their Fourier transforms

Spectral Ringing (Gibbs Phenomenon)

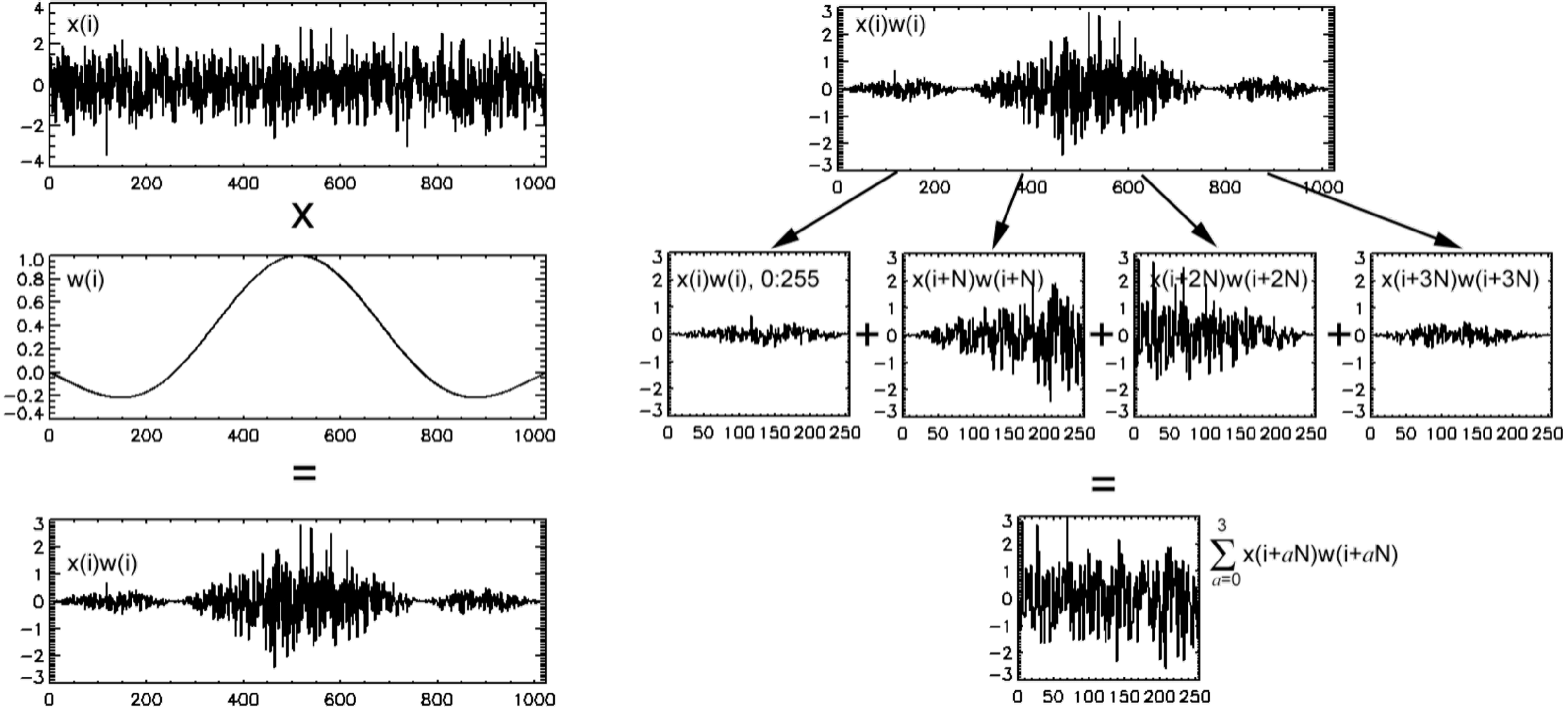
- Fourier theory assumes an infinite length function
- Realistic transforms (FFTs) limited size
- Equivalent of multiplying with a box car
- FT of boxcar sinc function
- “Sharp” features will convolved with sinc…



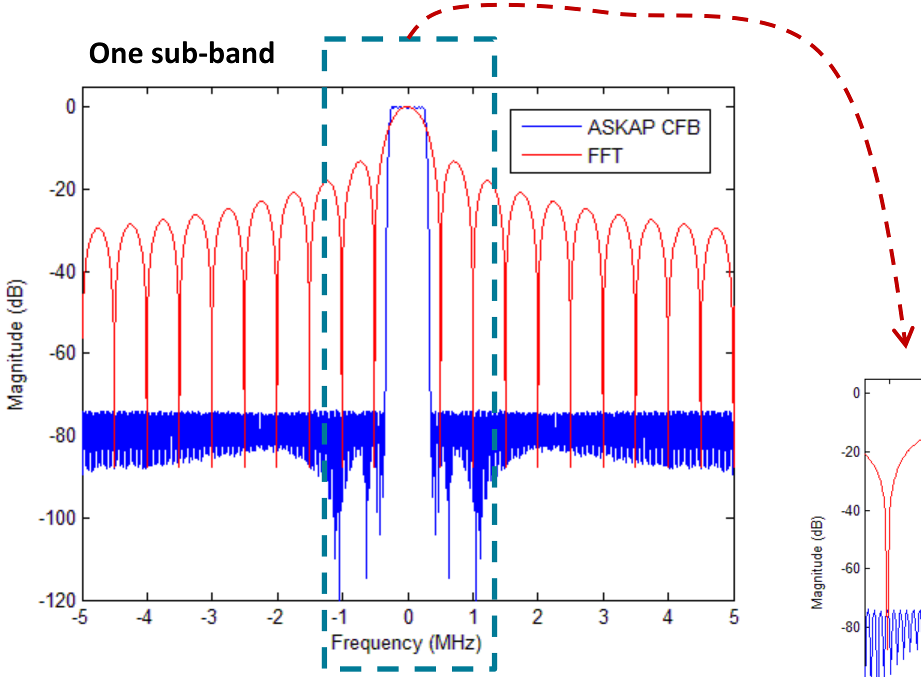
Polyphase Filterbanks (PFB)

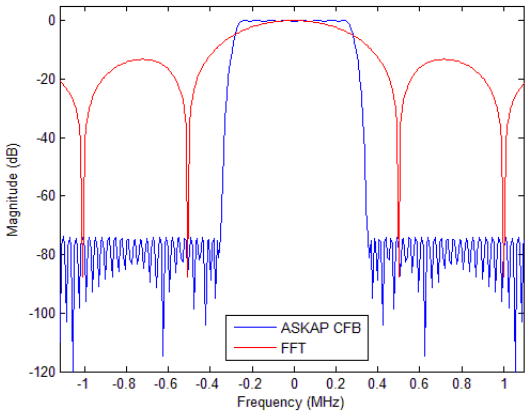
- FFTs have poor spectral response
  - ”Leakage” of signal from one channel to another
- Tapering of the signal in the time domain of smoothing (convolving) in frequency domain can reduce ringing
- PFB applies ”pre-filter” to *multiple* FFT windows then averages data before performing FFT
- Less spectral leakage and “squarer” filter shape
- Similar results can be achieved with larger FFTs and averaging output channels in frequency

PFB Implementation



FFT Compared to PFB





768-point FFT

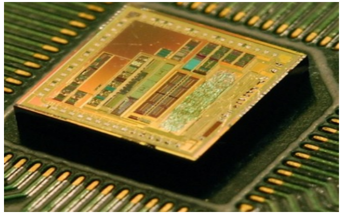
12,288-tap polyphase filter + 768-point FFT

Image Credit: John Tuthill

Technologies

Hard-wired logic	Stored (programmed) logic

ASIC's
Application-**S**pecific **I**ntegrated **C**ircuit

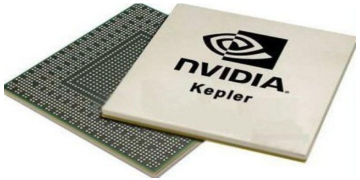


EVLA, ALMA

FPGA's
Field **P**rogrammable **G**ate **A**rray

CABB, ASKAP

GPU's
Graphics **P**rocessing **U**nit



MWA,BIGCAT

CPU's/DSP's
Central **P**rocessing **U**nit/ **D**igital **S**ignal **P**rocessor

DiFX

- Less flexible
- Lower power/computation
- Higher initial development

↔

- More flexible
- Higher power/computation
- Lower initial development

Image Credit: John Tuthill

Correlators

Complex Visibilities

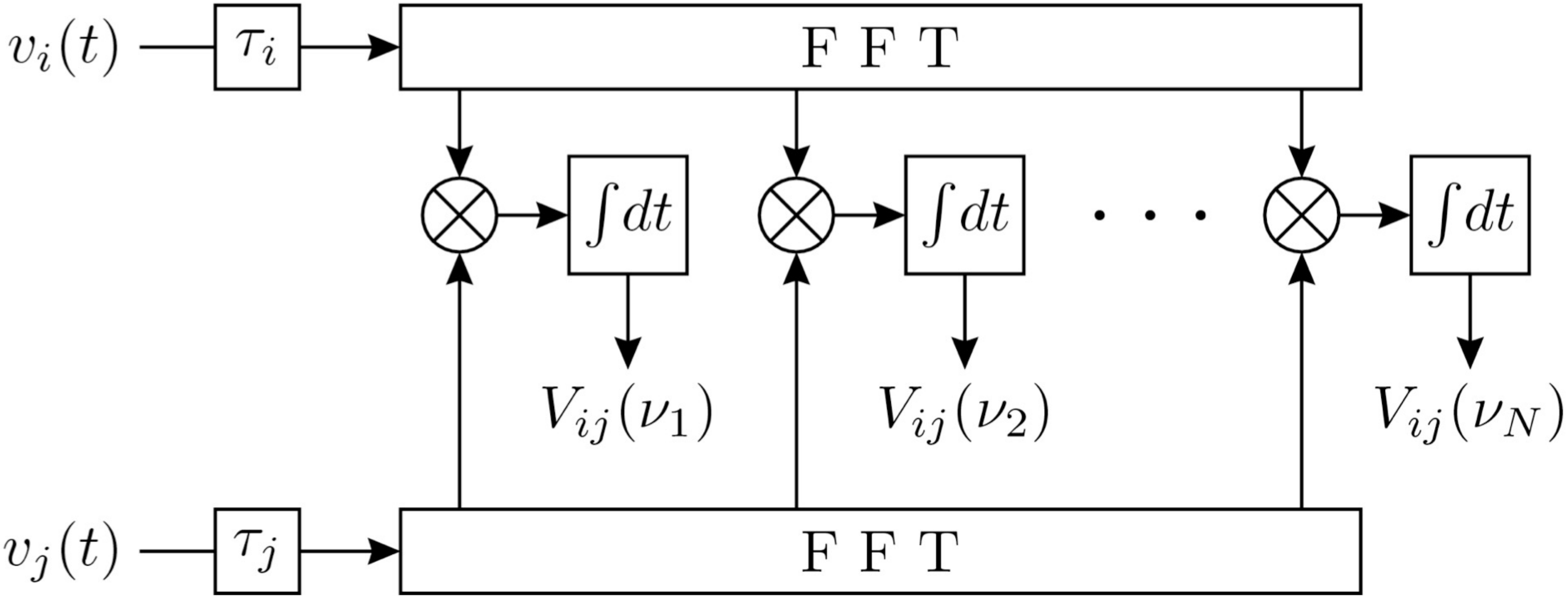
$$V_\upsilon(\mathbf{b}) = R_C - iR_S = \iint I_\nu(s) e^{-2\pi i\nu \mathbf{b}\cdot\mathbf{s}/c} d\Omega$$

$$R_C = \iint I_E(\mathbf{s}) \cos(2\pi\nu\, \mathbf{b}\cdot\mathbf{s}/c) d\Omega$$

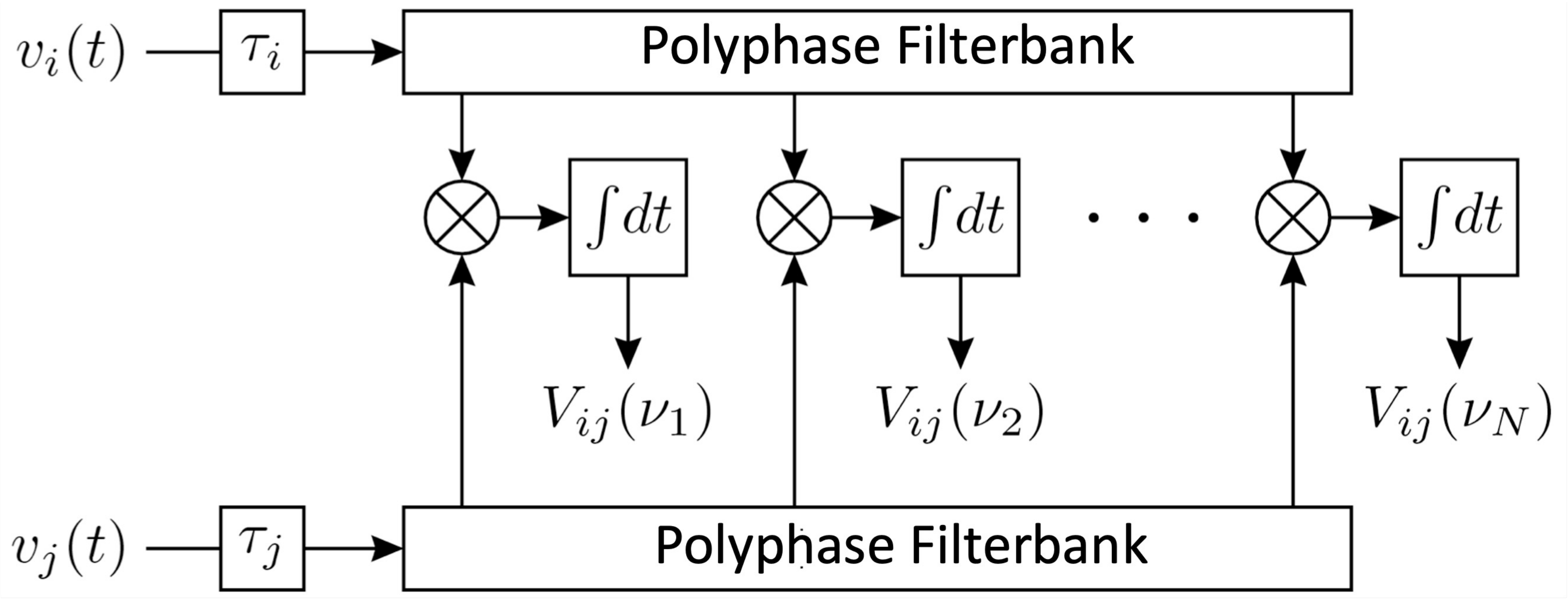
$$R_S = \iint I_O(\mathrm{s}) \sin(2\pi\nu\, \mathbf{b}\cdot\mathbf{s}/c) d\Omega$$

$$V_f(\mathbf{r}_1, \mathbf{r}_2) = E_f(\mathbf{r}_1) \times E_f^*(\mathbf{r}_2)$$

FFT Correlator - FX

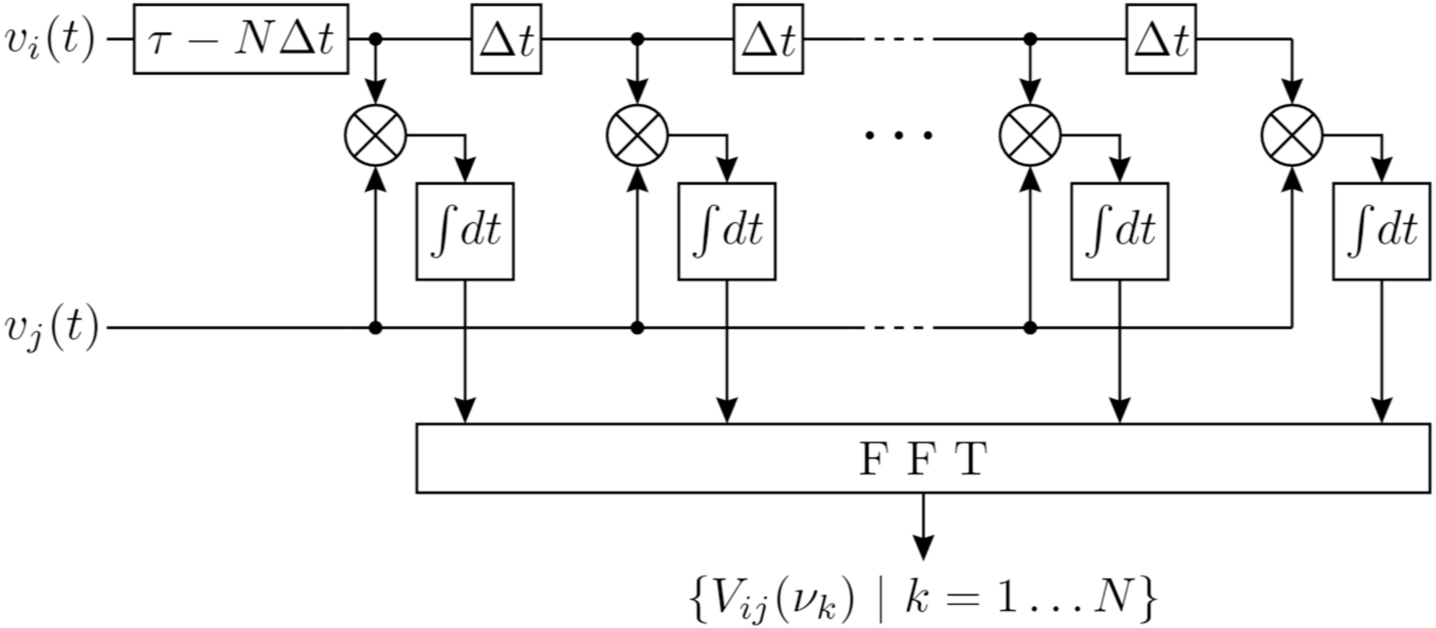


PFB Correlator - FX



Lag Correlator - XF

$$\mathcal{F}\{f\}.\mathcal{F}\{g\} = \mathcal{F}\{f*g\}$$



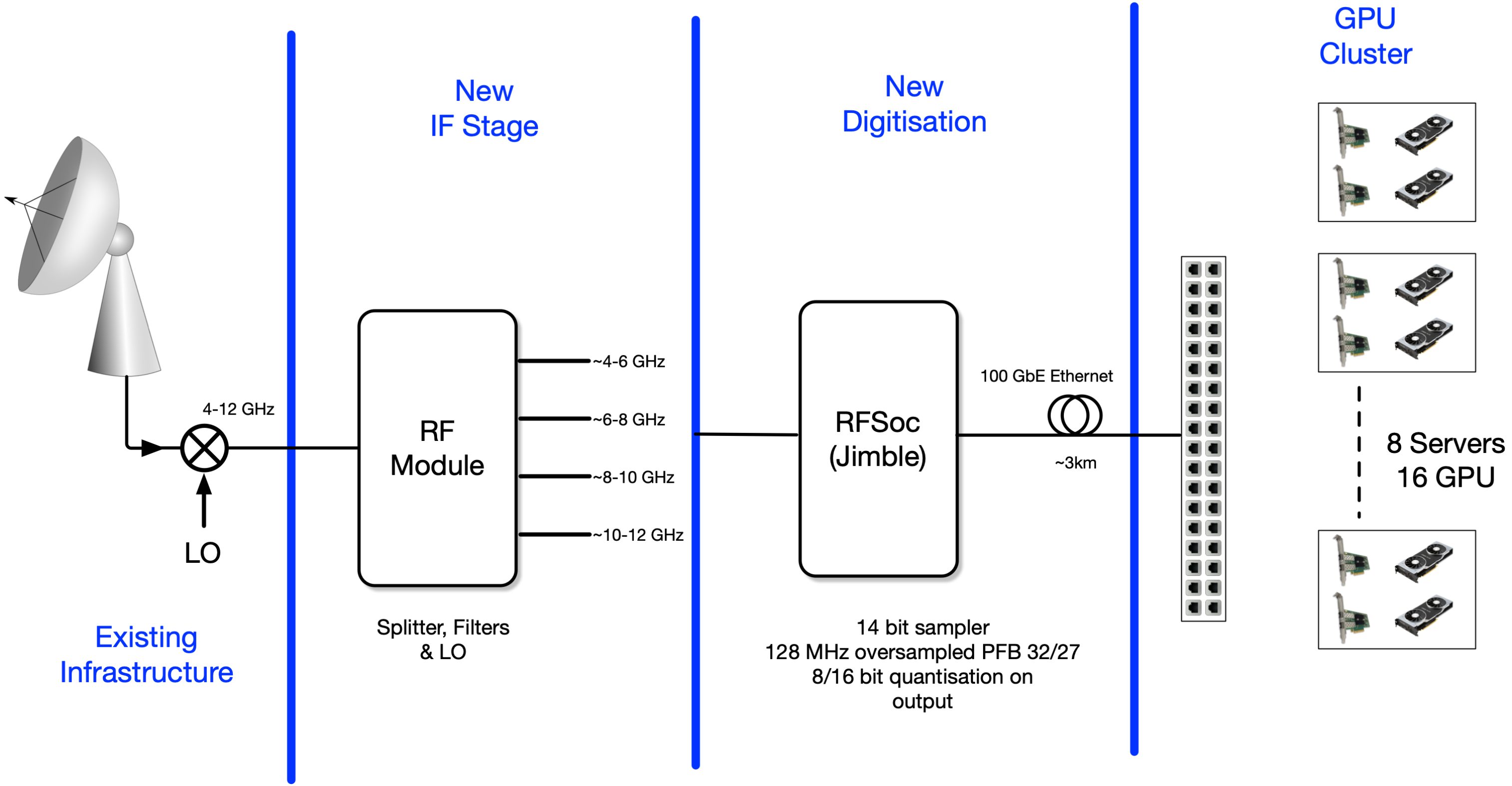
Practical Implementation of Correlator

- Receive time tagged digitized voltages
- Select appropriate sample, allowing for geometric delay
  - Delay often represented as N^{th} order polynomial
- Apply time based phased correction
  - "Fringe Rotation" – equivalent to Doppler correction
- FFT or PFB
- Fine delay correction
  - Linear phase ramp in frequency
- Cross multiple all antenna and polarisation pairs
- Time average

BIGCAT

- New backend for ATCA
- Doubles total bandwidth to 8 GHz
- Hybrid FPGA/GPU design
  - Very flexible configurations
- FPGA ADCs and coarse filterbank
  - Streamed over commodity 100 Gbps Ethernet
- To be installed first half 2024
- 1.75 Tbps total data rate from telescopes!!!
  - 117 Gbps per GPU

BIGCAT Design



Thank you

Space and Astronomy
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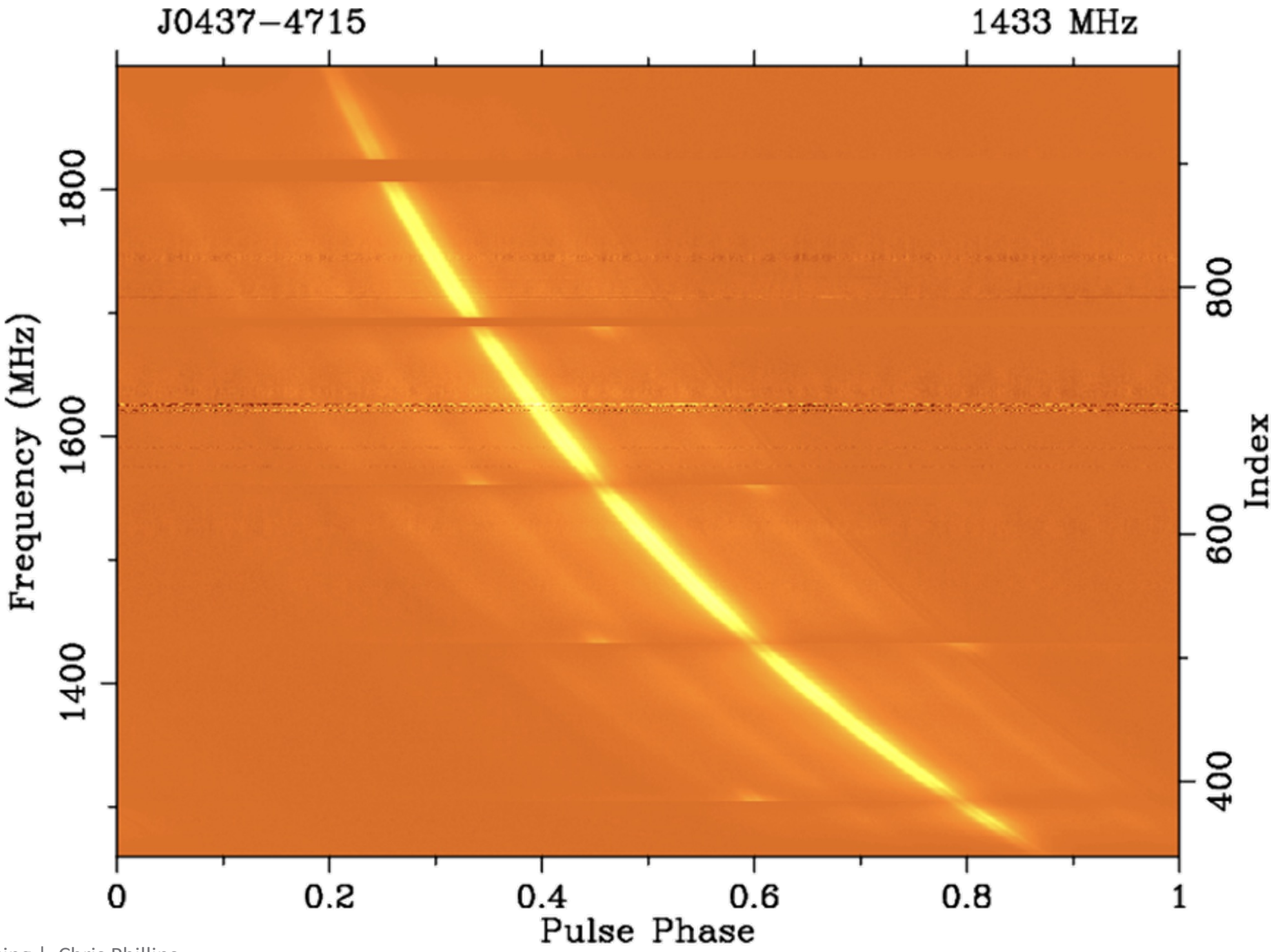
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1433 MHz
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