Wide field and wide band imaging

ATNF Radio Workshop

Rajan Chhetri | 26 Sep 2023

Outline – the 5W of (research) communication

- Who
- What
- Where
- When
- Why

The 5W of wide-field wide-band imaging

- Who should care?
- What
- Where
- When
- Why

The 5W of wide-field wide-band imaging

- Who should care?
- What are the effects to care about?
- Where
- When
- Why

The 5W of wide-field wide-band imaging

- Who should care?
- What are the effects to care about?
- Where do the effects arise from?
- When
- Why

The 5W of wide-field wide-band imaging

- Who should care?
- What are the effects to care about?
- Where do the effects arise from?
- When do we apply mitigating steps
- Why

The 5W of wide-field wide-band imaging

- Who should care?
- What are the effects to care about?
- Where do the effects arise from?
- When do we apply mitigating steps
- Why ?
  - "Because it's there"

**if you know why this expression is famous, come and talk to me later.*

Outline (following the “white book”)

- Effects of wide observing band
- Effects of time
- Using “normal” imaging to cover large areas – mosaic
- Non-coplanar baselines
- Direction dependent effects

** White book = Synthesis Imaging in Radio Astronomy II, (Taylor, Carrilli & Perley 1998)

Effects of wide observing band

- Why use wide band?
  - To increase sensitivity
    - Bandwidth is cheaper to increase than antennas

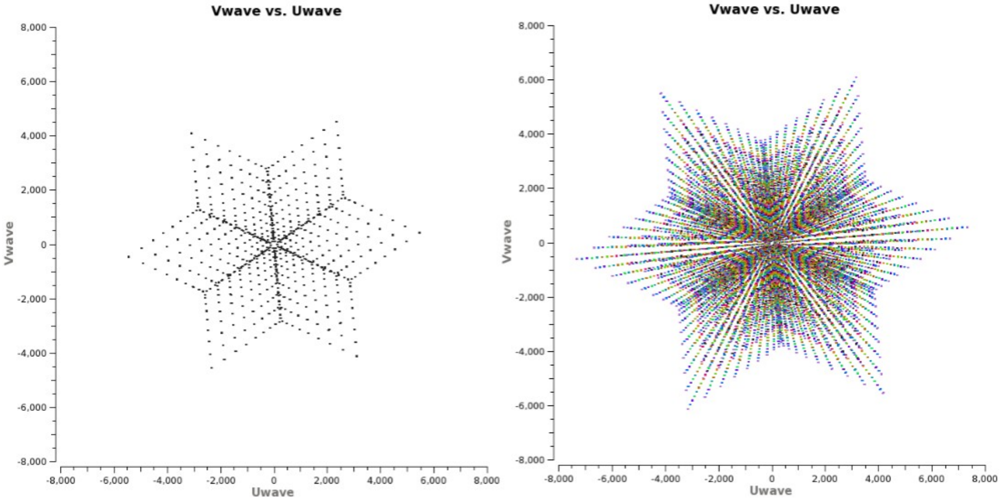
$$\sigma_S = \frac{2kT_s}{A_e[N(N-1)\,\Delta\nu\,\tau]^{1/2}}.$$

  - To improve u-v coverage (better images)

Improved UV coverage

- Different frequency fill different part of u-v plane (radially)

- u-v plane is filled quicker

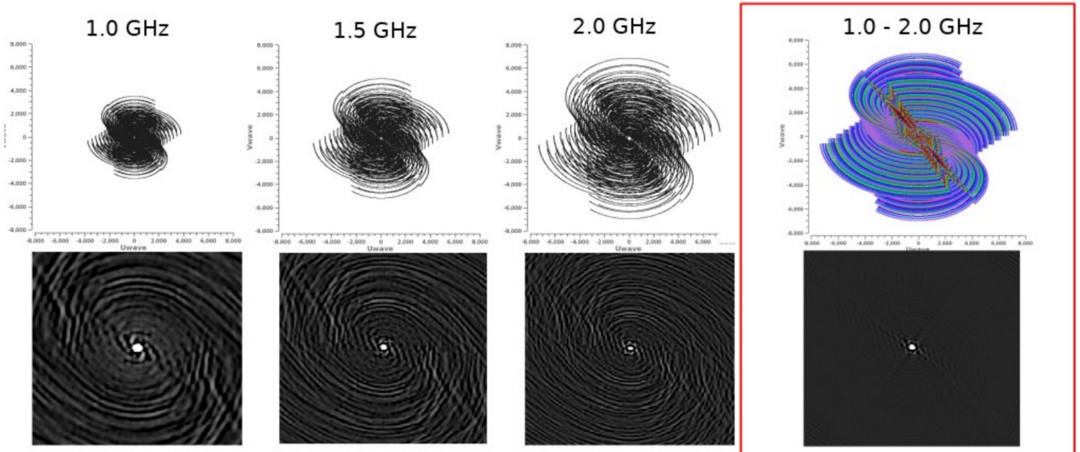


VLA snapshot u-v coverage: (lef)t single frequency (right) multiple frequencies

source: https://casaguides.nrao.edu/index.php?title=File:MultiFrequency_Synthesis_snapshot.png

Improved UV coverage

- Sampling function → F.T. → PSF
- Fewer gaps = better PSF

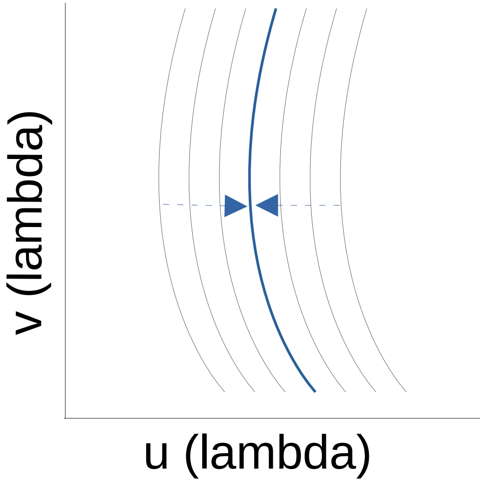


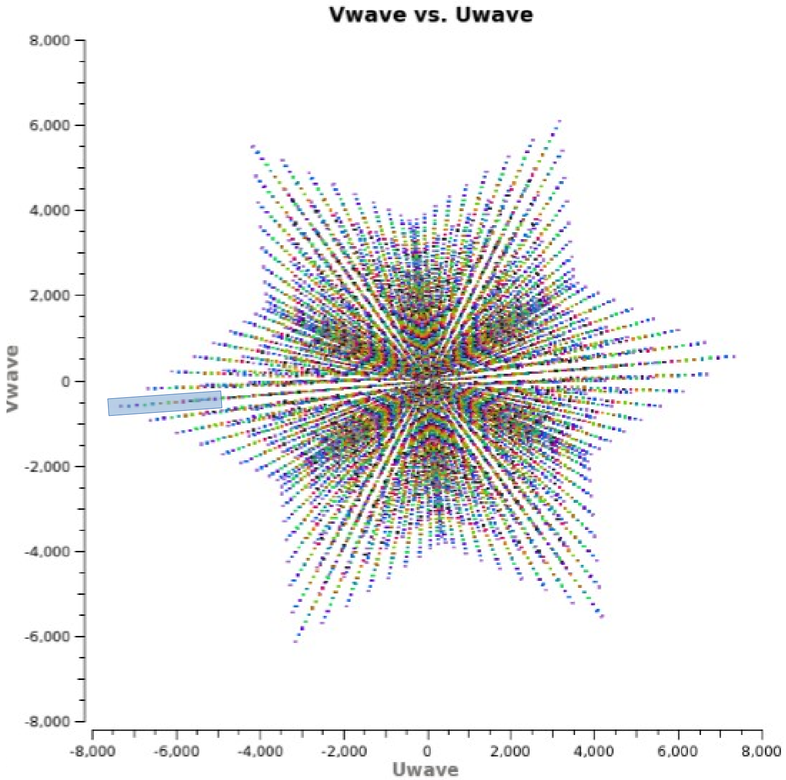
Observed image : $I_\nu^{obs} = I_\nu^{sky} * PSF_\nu$

$$I_{wb}^{obs} = \sum_\nu \left[I_\nu^{sky} * PSF_\nu \right]$$

source:

BUT averaging in u-v = smearing in image





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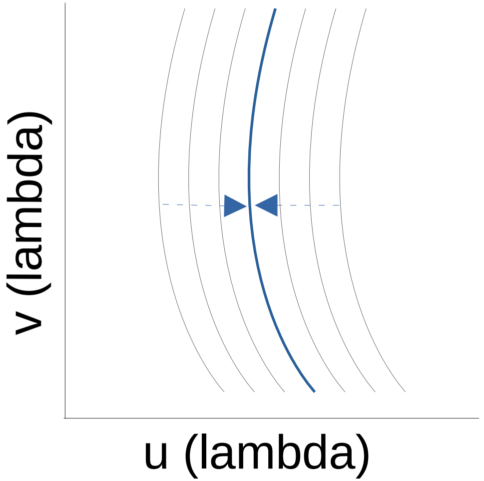


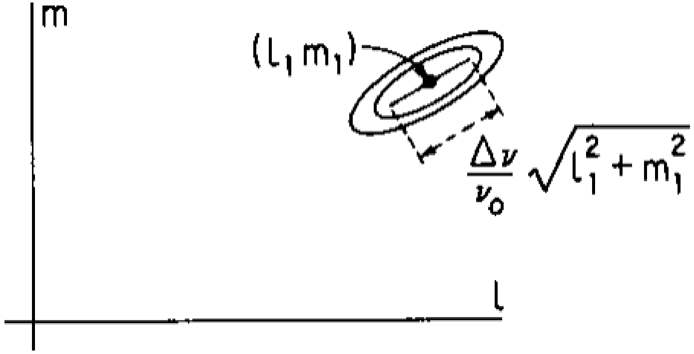


Fig. 2-17, SIRA-II

BUT averaging in u-v = smearing in image

- Bandwidth smearing (chromatic abberation) is:
  - Radial in nature (main lobe)
  - **Gets worse with distance from phase centre**
- Total flux is conserved
  - But peak flux decreases

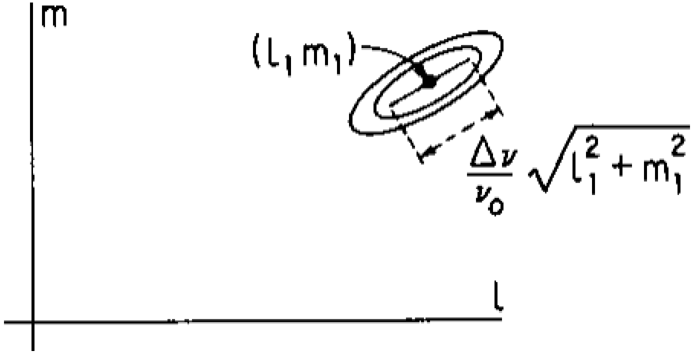
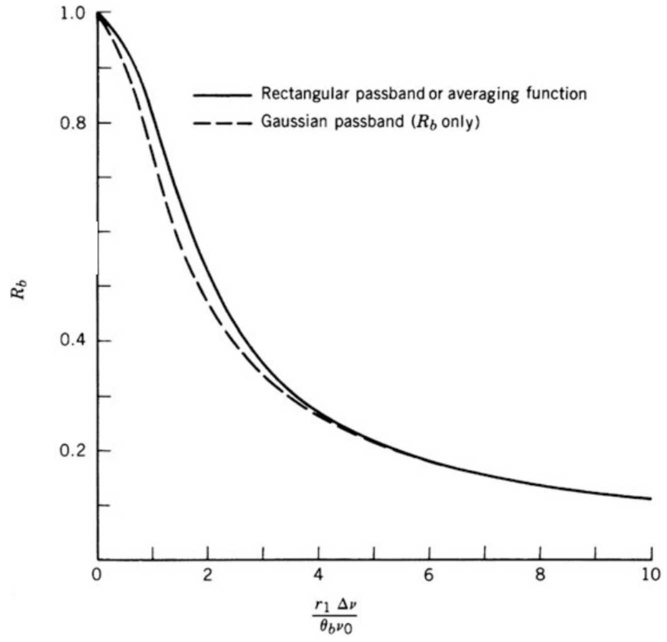


Fig. 2-17, SIRA-II

Effect of frequency averaging

- Peak amplitude drops with radial distance from the phase centre



Effect of frequency averaging

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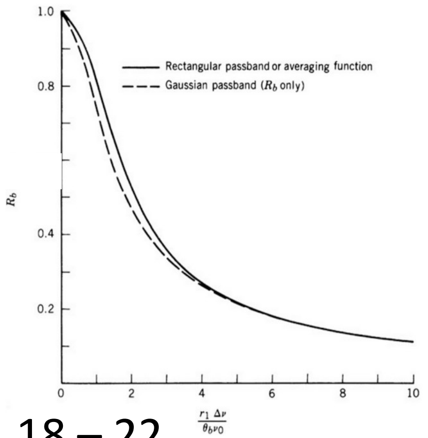


Observing freq (GHz):	1 – 2	4 – 8	18 – 22
Bandwidth (GHz):	1	4	4
Bandwidth ratio (nu_min:nu_max):	2:1	2:1	1.22:1
Fractional bandwidth (bandwidth/obs freq.):	0.67	0.67	0.20

BUT


WCS: (11:24:32.7, -54:16:39); Image: (7165, 5469); Polarization: Stokes I
Declination
-54:30:00
-55:00:00
30:00
-56:00:00
30:00
-57:00:00
Right ascension
55:00
50:00
45:00
11:40:00
35:00
30:00
25:00
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000

BUT averaging in u-v = smearing in image

- Bandwidth smearing (chromatic abberation) is:
  - Radial in nature (main lobe)
  - **Gets worse with distance from phase centre**

- Total flux is conserved
  - But peak flux decreases

- **Solution**: multi-frequency synthesis

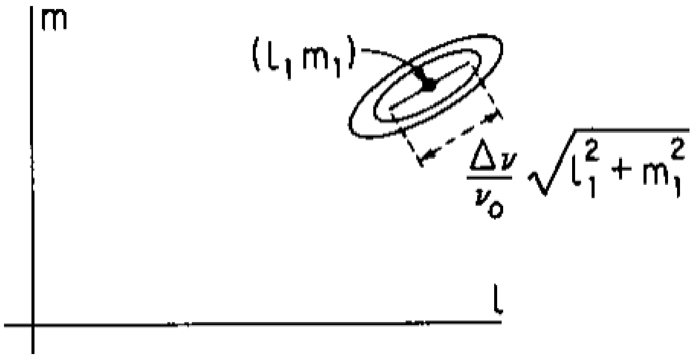


Fig. 2-17, SIRA-II

Multi-frequency synthesis (MFS)

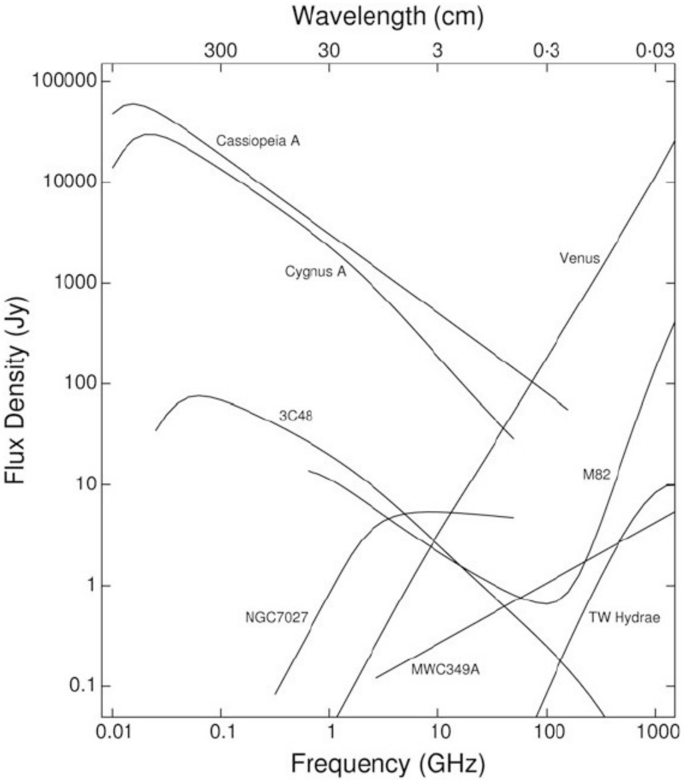
<u>Simple concept:</u>

- Divide the band into smaller channels
- Image each channel seprately
  - Removes smearing
- Add images at the end
  - Output image is smoothed to lowest resolution

<u>Challenges</u>

- Natural radio sources have different spectra
  - Flat, inverted, steep (or some mixture)
- Instrument response (e.g. primary beam) change with frequency
  - Can add artificial spectral index

Multi-frequency synthesis (MFS)



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<u>Challenges</u>

- Natural radio sources have different spectra
  - Flat, inverted, steep (or some mixture)
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  - Can add artificial spectral index

Multi-frequency synthesis (MFS)

<u>Challenges</u>

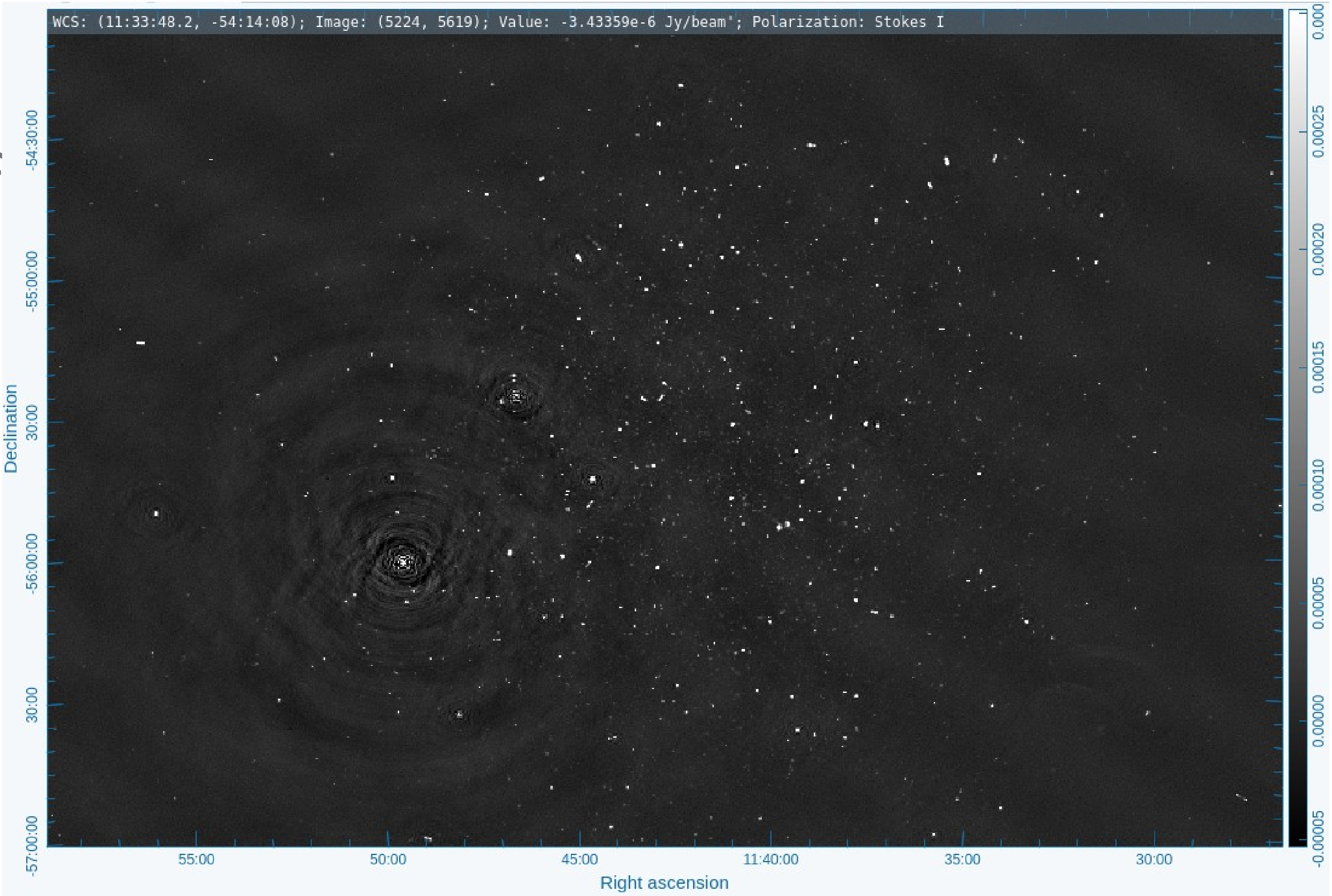
- Natural radio sources have different spectra
  - Flat, inverted, steep (or some mixture)
- Instrument response (e.g. primary beam) change with frequency
  - Can add artificial spectral index

<u>Solution:</u>

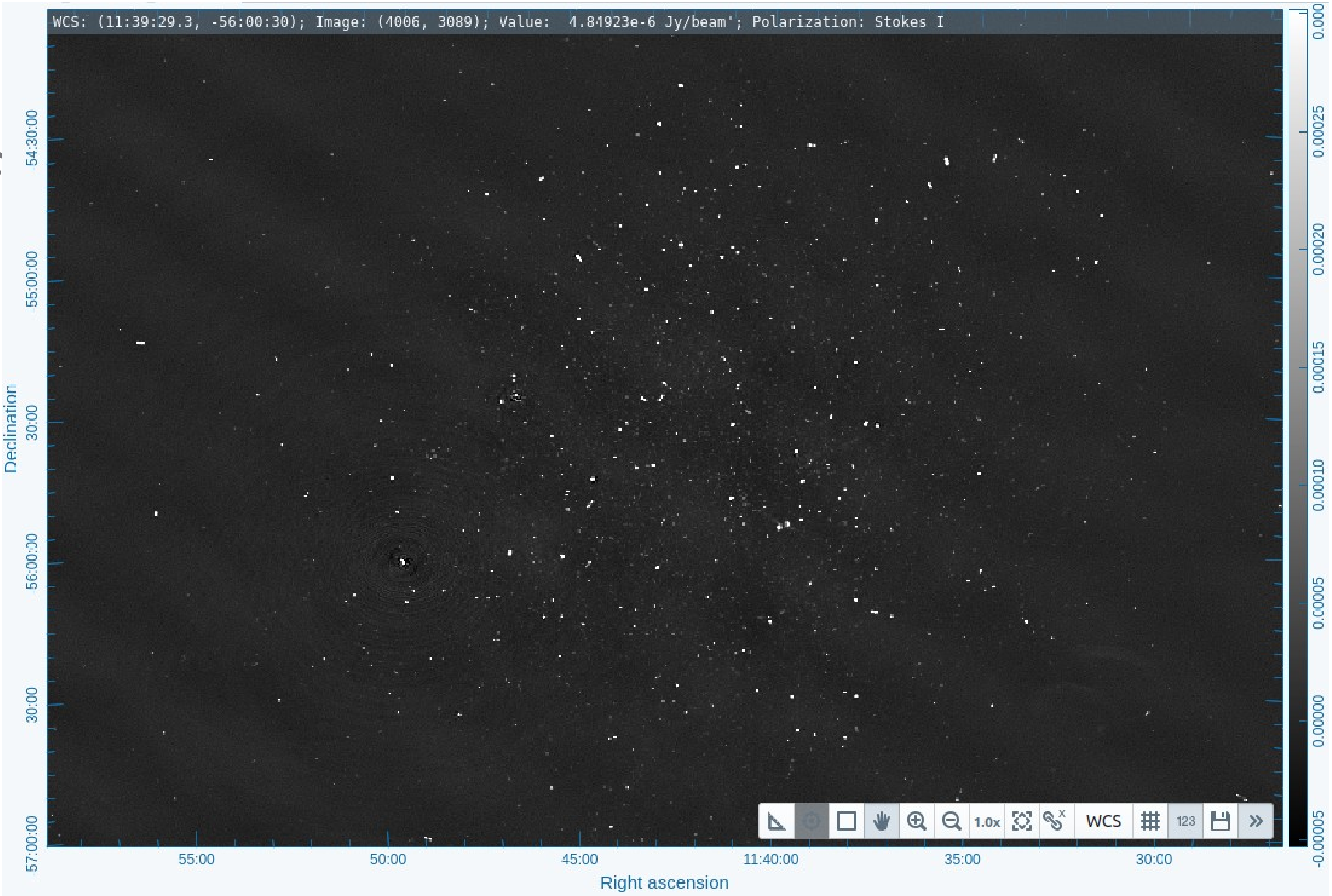
- Multi-term MFS
  - Fit for spectral index of each pixel using polynomial (Taylor term); (Rau & Cornwell 2011)
  - Image simultanteously

Multi-f

1 channel

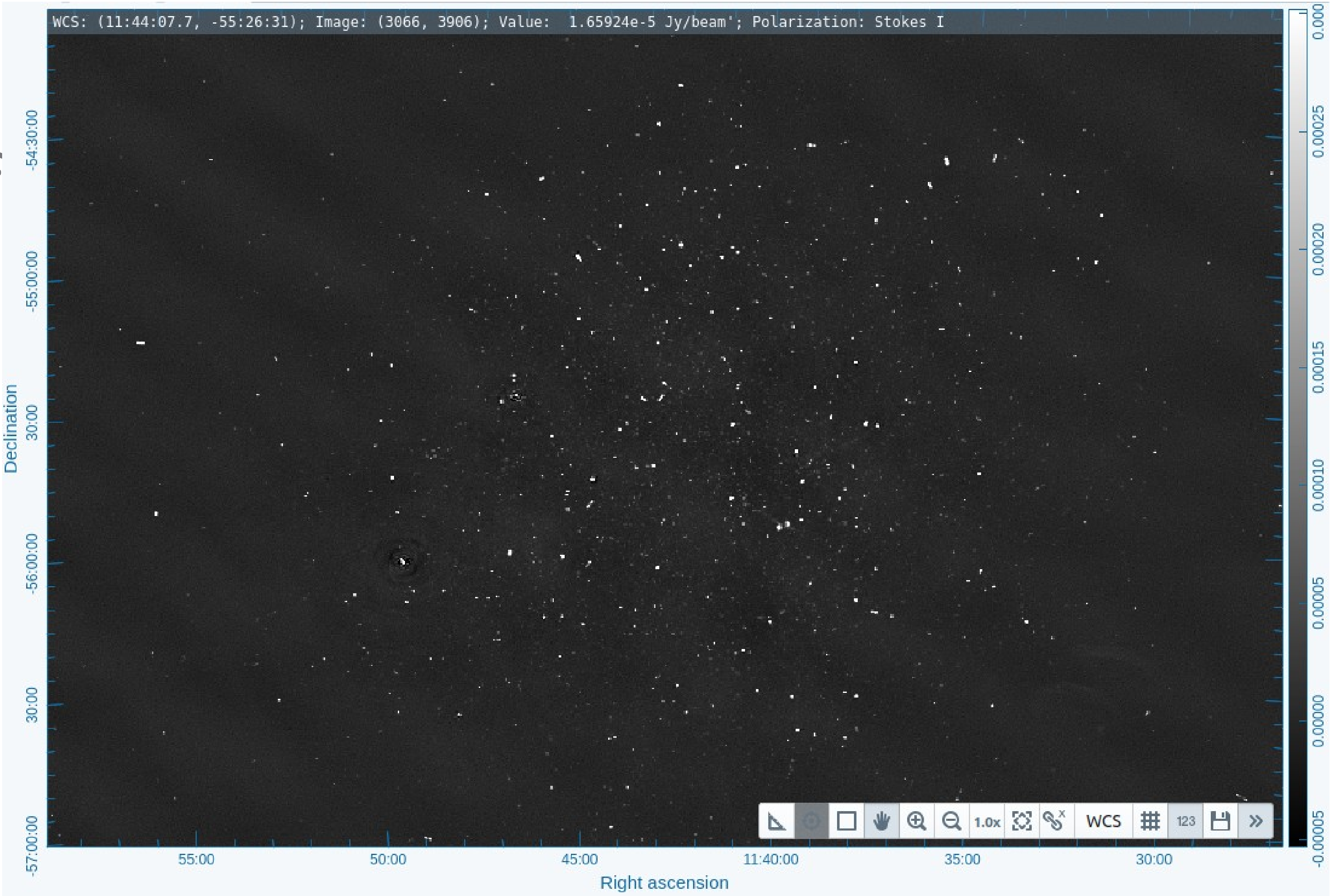


Multi-f



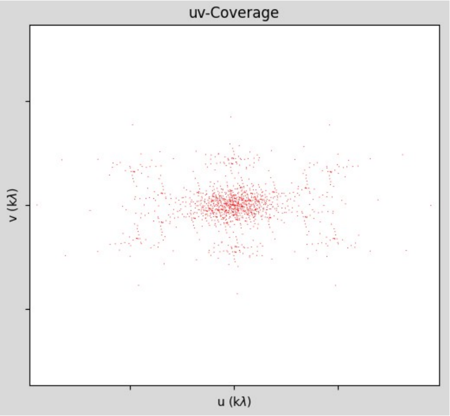
4 channels

Multi-f



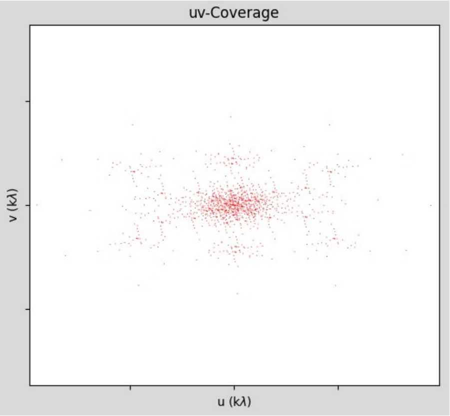
12 channels

Averaging in time

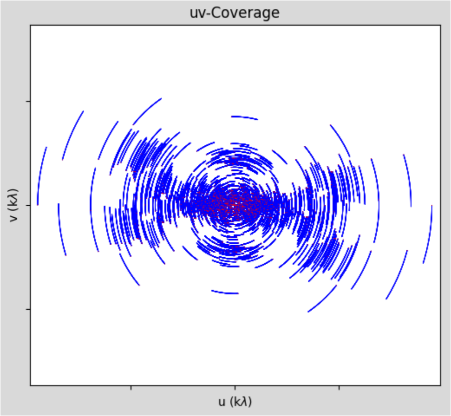


ASKAP 10s u-v coverage

Averaging in time

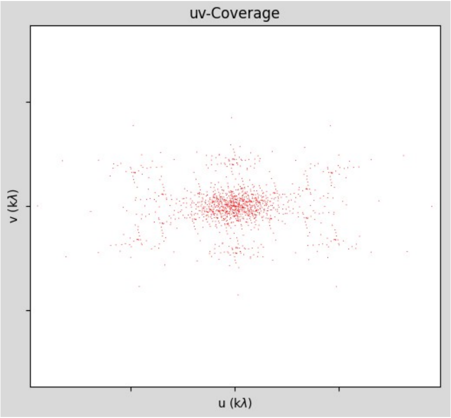


ASKAP 10s u-v coverage

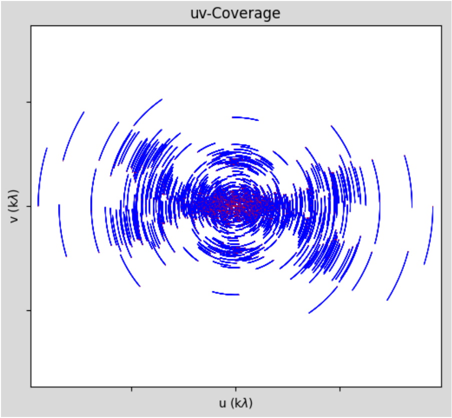


ASKAP 1hr u-v coverage

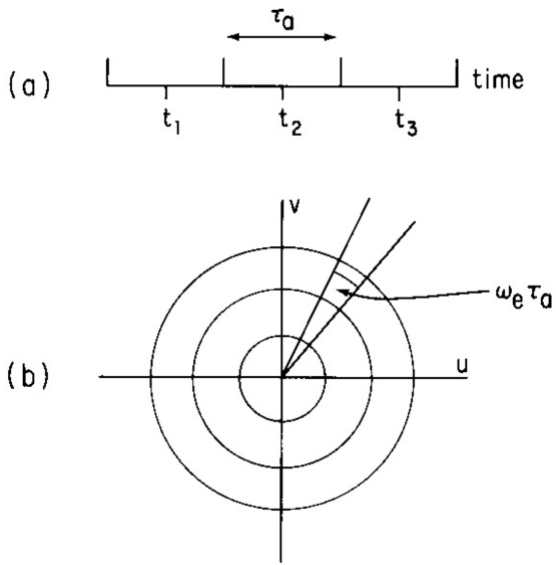
Averaging in time



ASKAP 10s u-v coverage



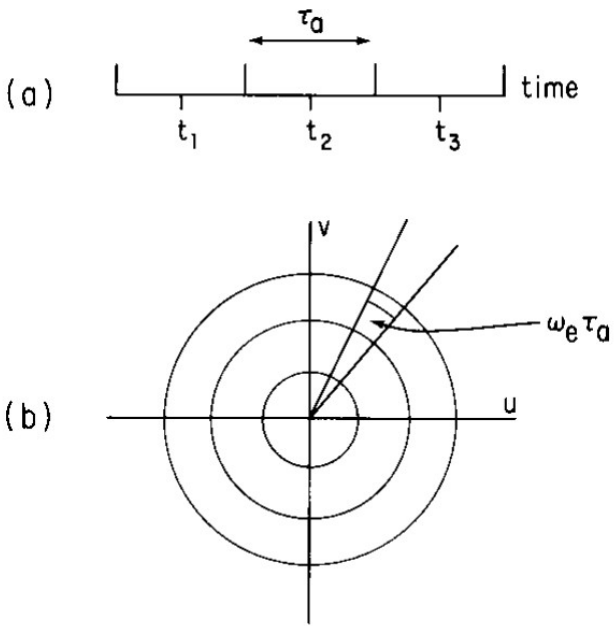
ASKAP 1hr u-v coverage (pole)



Averaging in time (Fig 2-18, SIRA-II)

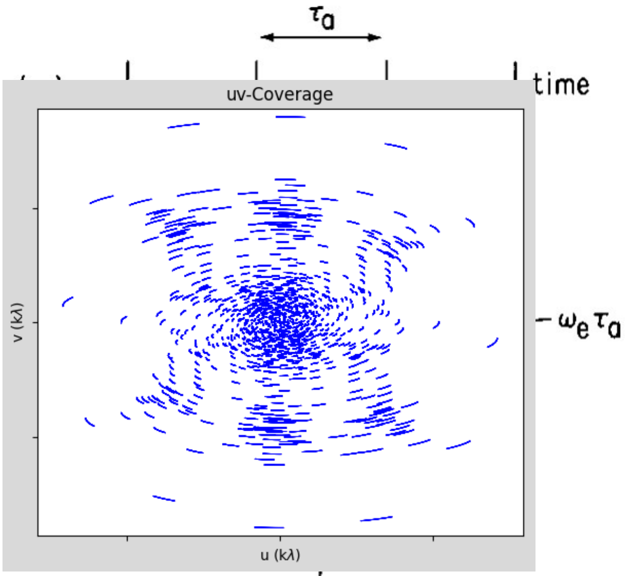
Averaging in time

- Circular u-v plane only when observing the poles
  - Effect is azimuthal



Averaging in time

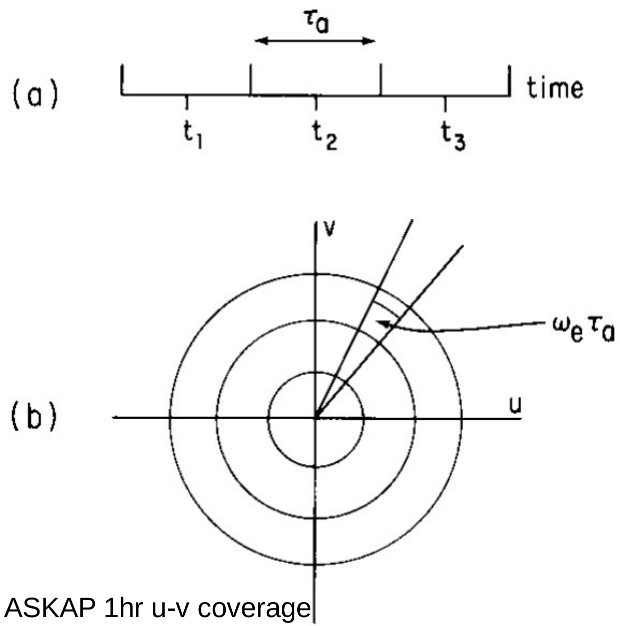
- Circular u-v plane only when observing the poles
  - Effect is azimuthal
- But It’s not practical to always point at the poles
  - Effect becomes complicated



ASKAP 1hr u-v coverage
(Dec -8 deg)

Averaging in time - mitigation

- Use short intervals to observe/image
- Baseline dependent averaging
  - Effect is a function of uv distance, so average as a function of u-v distance
- Optimising time series filtering
-
  -



ASKAP 1hr u-v coverage (Dec -8 deg)

Time smearing - example

- VLBA image
  - Longest baselin 8500 km
  - 2 sec averaging

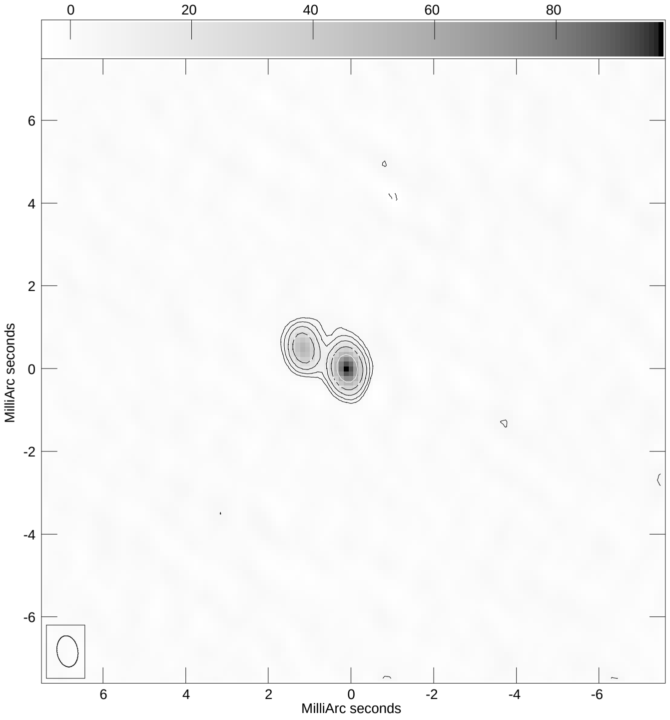


Image credit: Cormac Reynolds

Time smearing - example

- VLBA image
  - Longest baselin 8500 km
  - 20 sec averaging

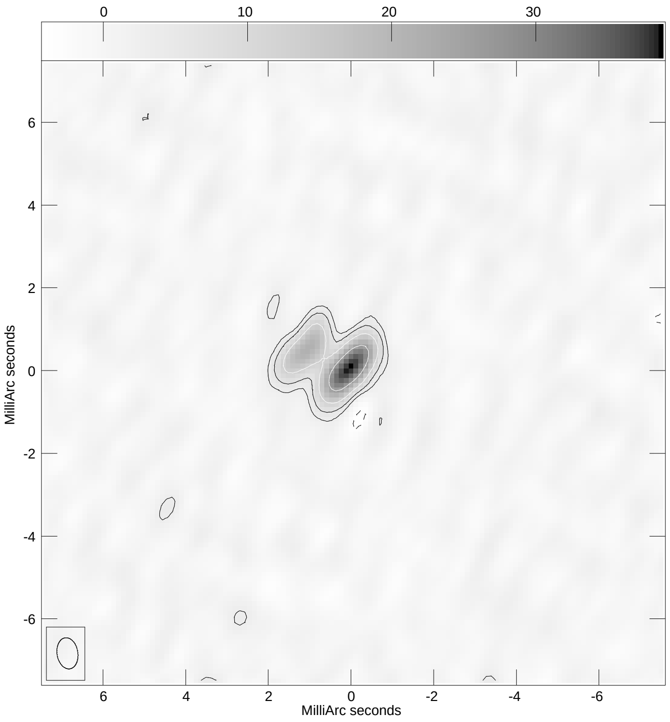


Image credit: Cormac Reynolds

Time smearing - example

- VLBA image
  - Longest baselin 8500 km
  - 40 sec averaging

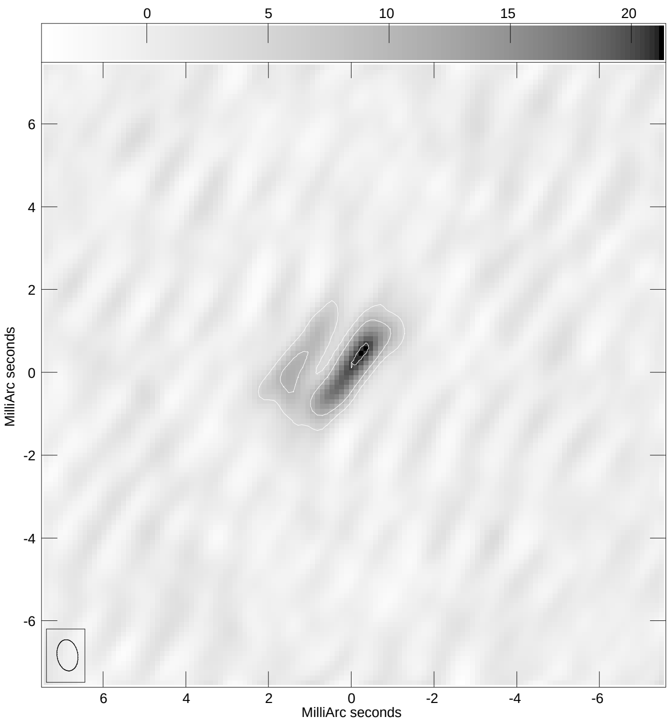
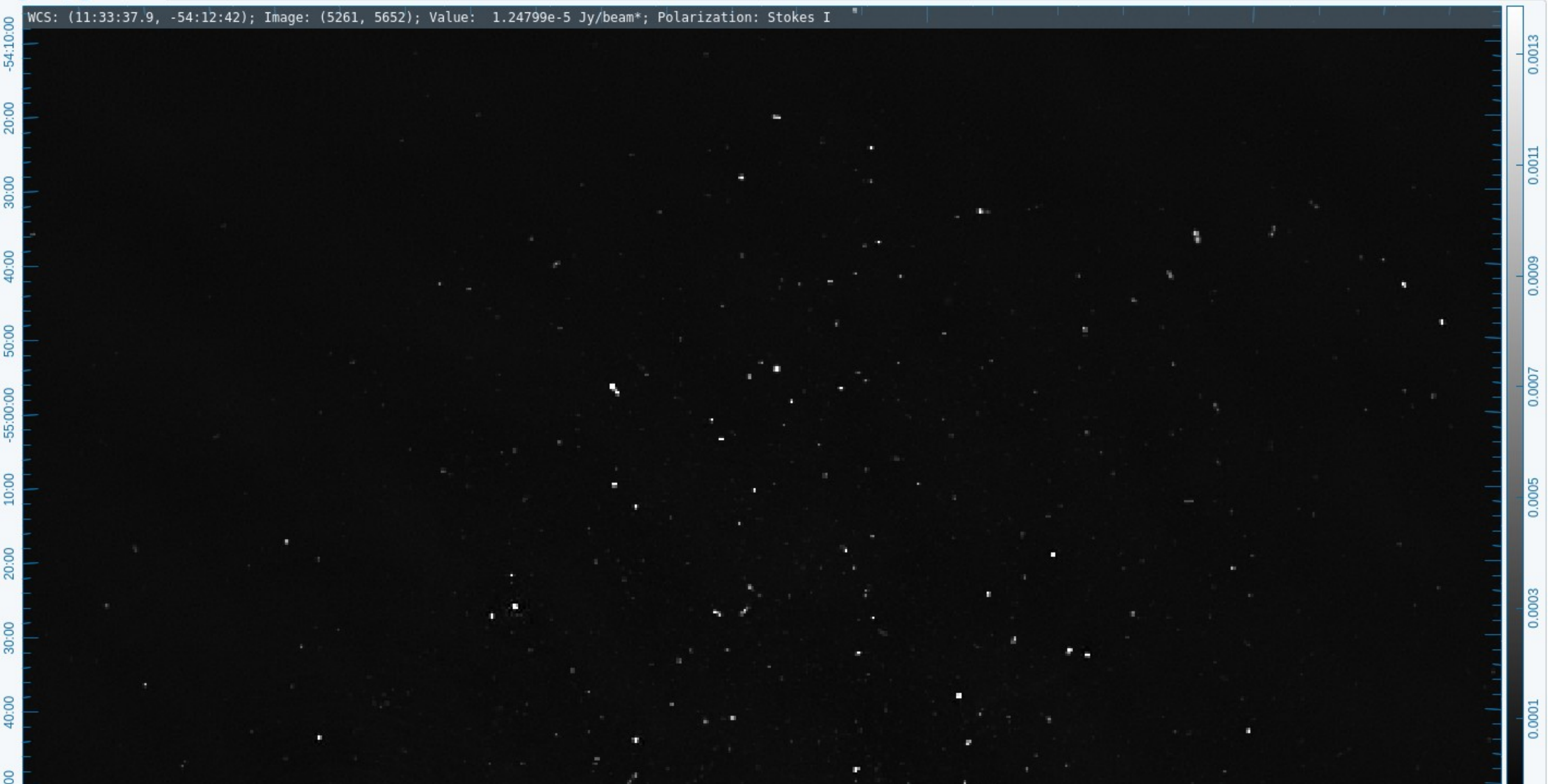
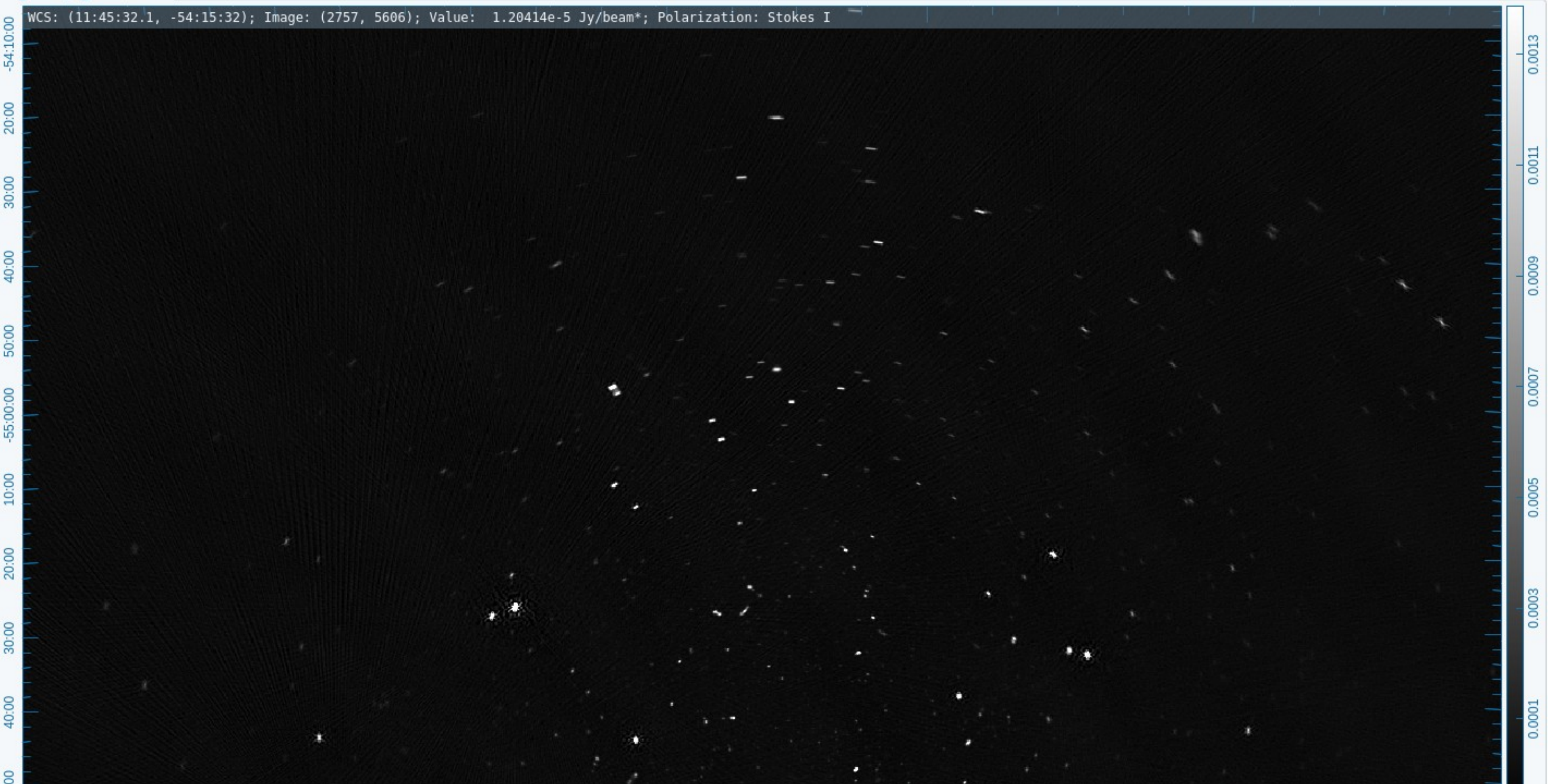


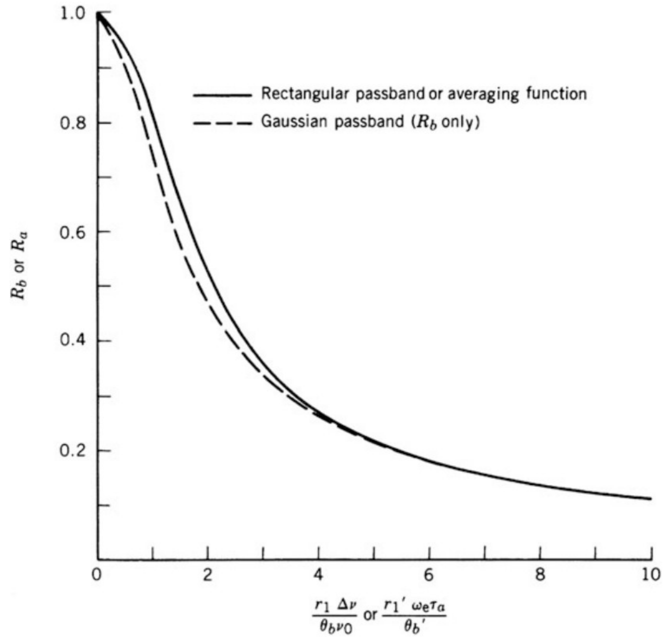
Image credit: Cormac Reynolds


WCS: (11:33:37.9, -54:12:42); Image: (5261, 5652); Value: 1.24799e-5 Jy/beam*; Polarization: Stokes I
-54:10:00
20:00
30:00
40:00
50:00
-55:00:00
10:00
20:00
30:00
40:00
0.0013
0.0011
0.0009
0.0007
0.0005
0.0003
0.0001


WCS: (11:45:32.1, -54:15:32); Image: (2757, 5606); Value: 1.20414e-5 Jy/beam*; Polarization: Stokes I
-54:10:00
20:00
30:00
40:00
50:00
-55:00:00
10:00
20:00
30:00
40:00
0.0013
0.0011
0.0009
0.0007
0.0005
0.0003
0.0001

Effect of frequency/time averaging

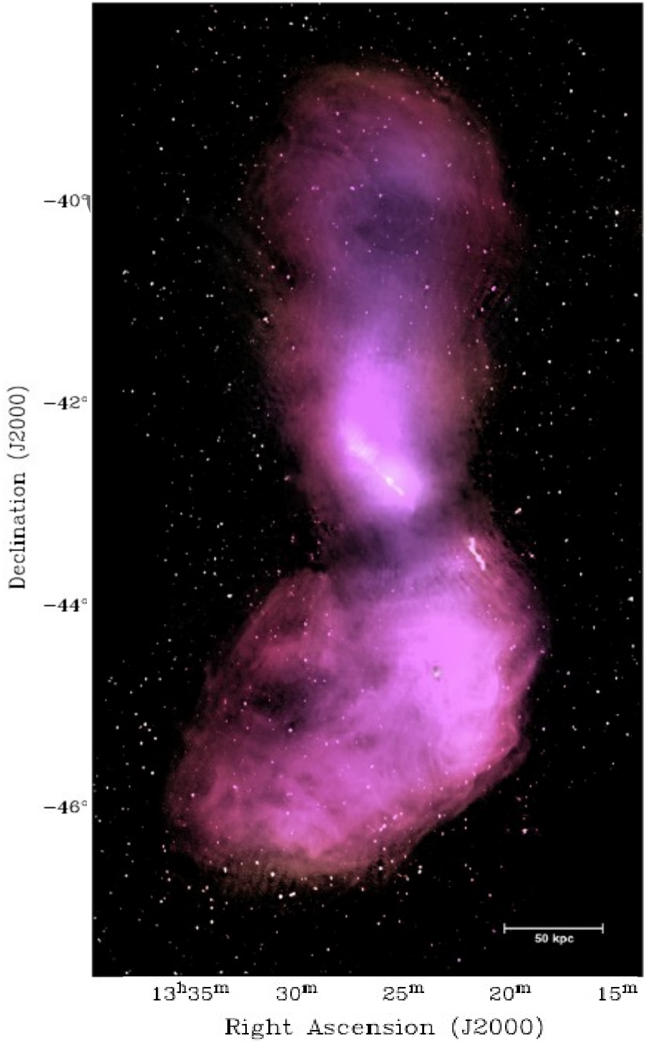
- Peak amplitude drops in a similar manner, as a function of distance



Combine small images to large - Mosaicing

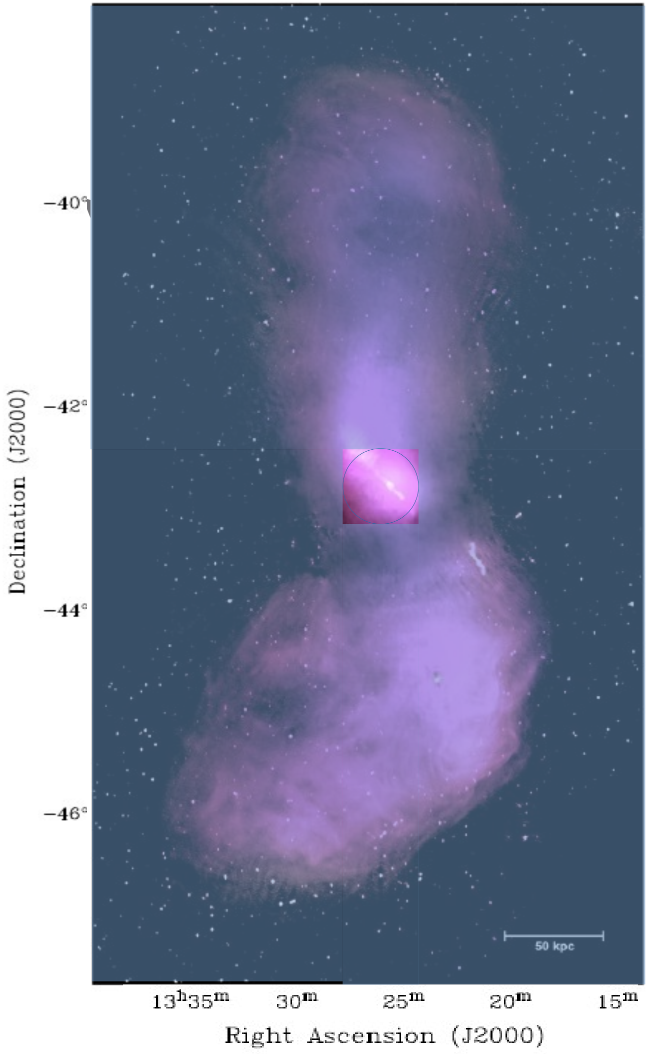
Combine small images to large

- Want to image large area



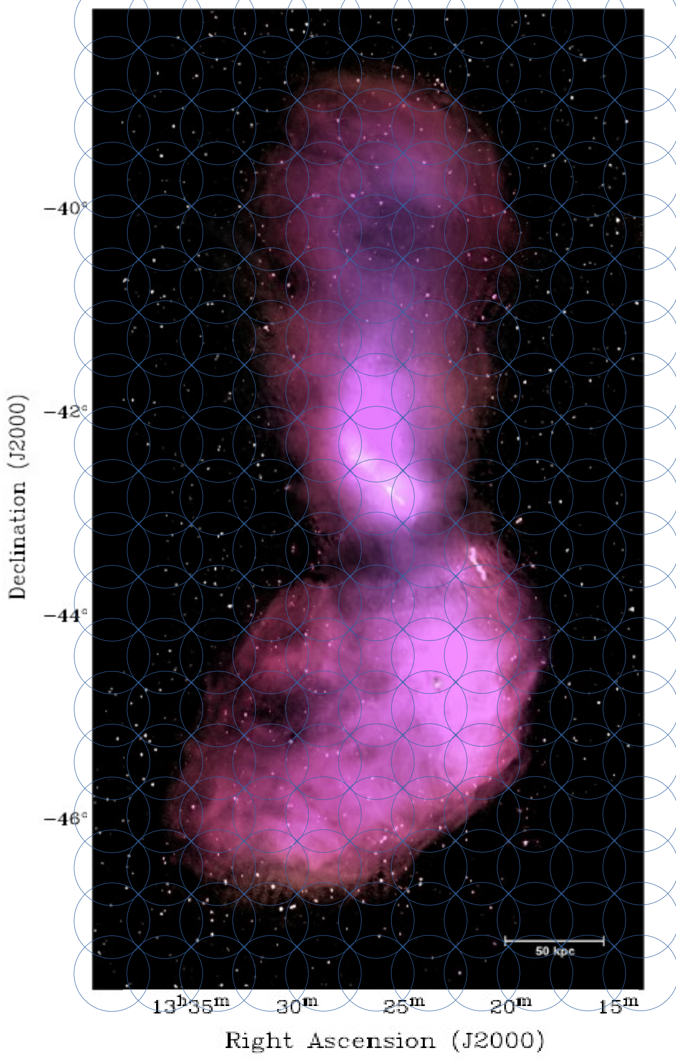
Combine small images to large

- Want to image large area
  - But your PB has limited size



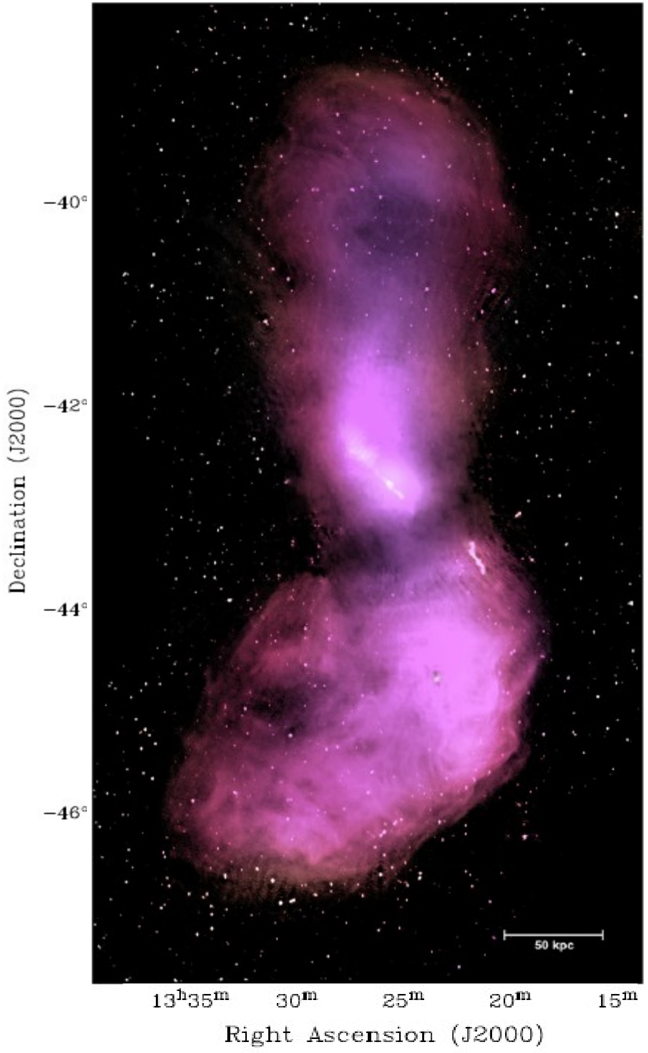
Mosaicking

- Want to image large area
  - But your PB has limited size
- Adequately sample multiple locations over the region of interest in the sky
- Combine images
  - deconvolved separately, or
  - jointly
  - Using appropriate weights e.g. antenna primary beam



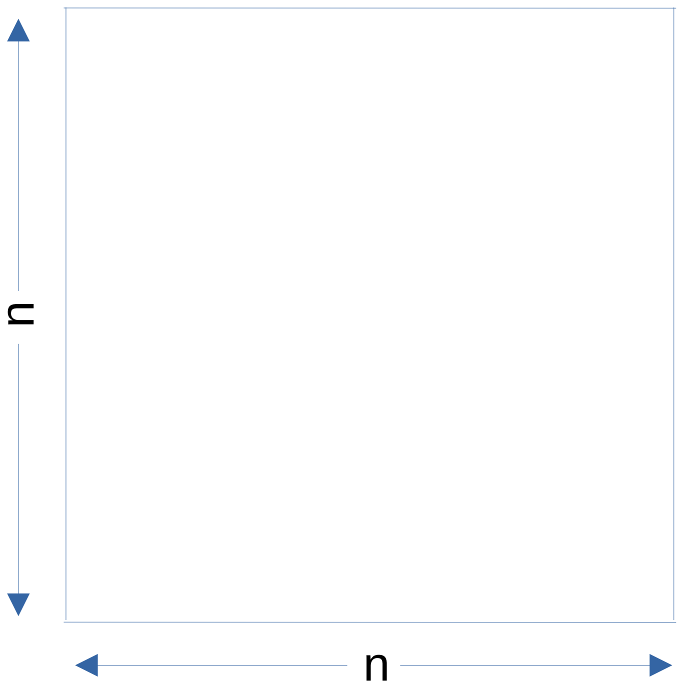
Mosaicking

- Mosaiked image with 406 pointings of ATCA + Parkes data
- Some newer instruments (e.g. ASKAP) form multiple primary beams simultaneously using phased array feed.



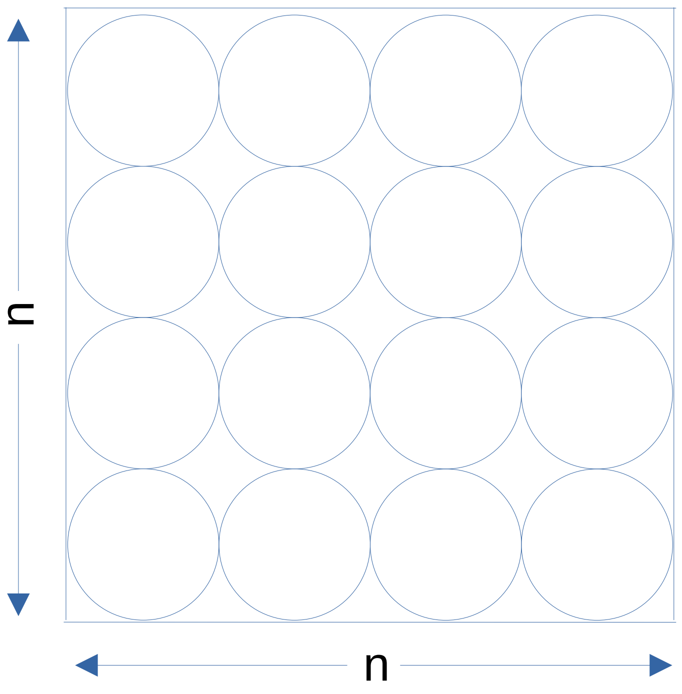
Mosaicking

- To image area with side n,



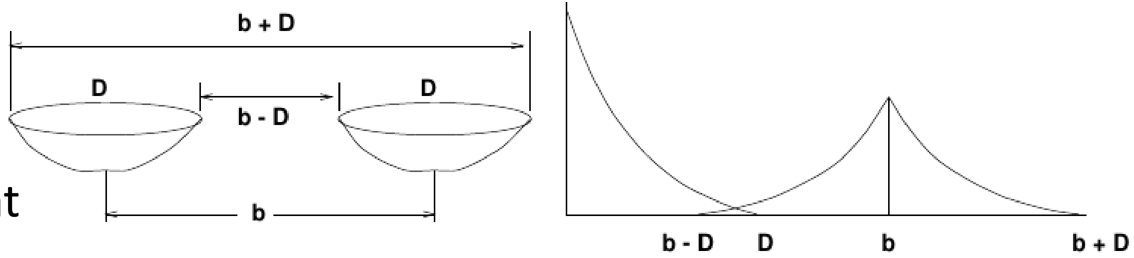
Mosaicking

- To image area with side n,
  - need sampling of n^2 x that required for each beam
  - Sampling < the diameter of antenna needed



Mosaicking

- To image area with side n,
  - need sampling of n^2 x that required for each beam
  - Sampling < the diameter of antenna needed
  - The act of scanning across the field provides extra information (Ekers & Rots 1979)
  - Add single dish



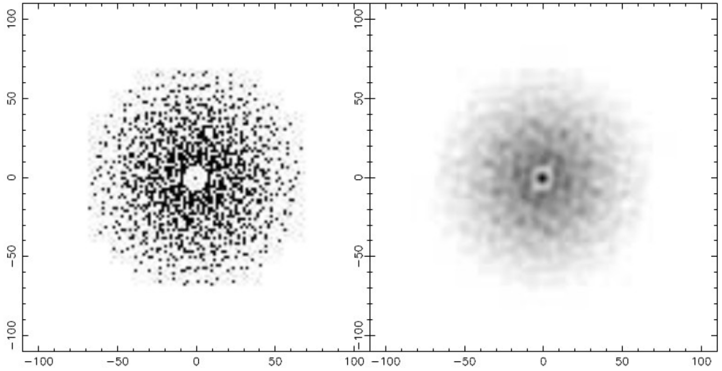
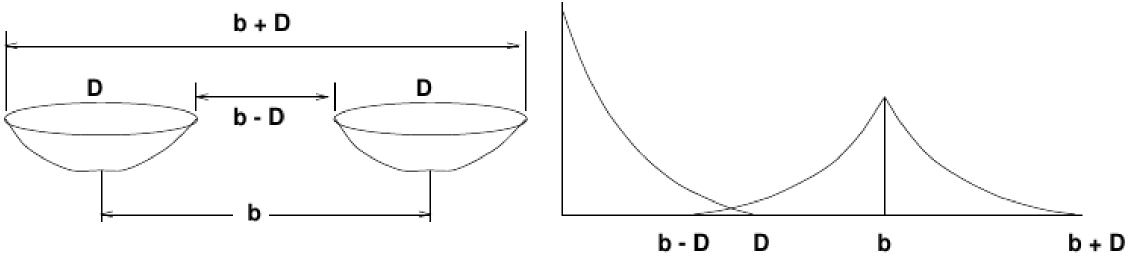


Figure 20–6. The effective (u, v) coverage of a sample compact MMA configuration.

Mosaicking

- Images can be mosaicked:
  - Linearly
    - Image individually
    - Deconvolve individually
    - Combine
  - Non linearly:
    - Image individually
    - Combine
    - Jointly deconvolve (using MEM)
  - Other variations e.g. Sault et al. (1996)



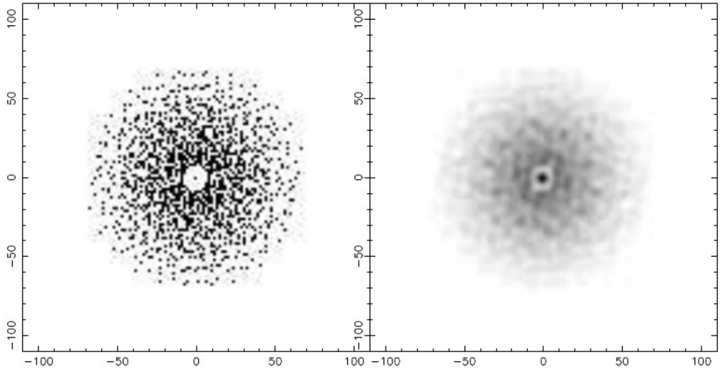
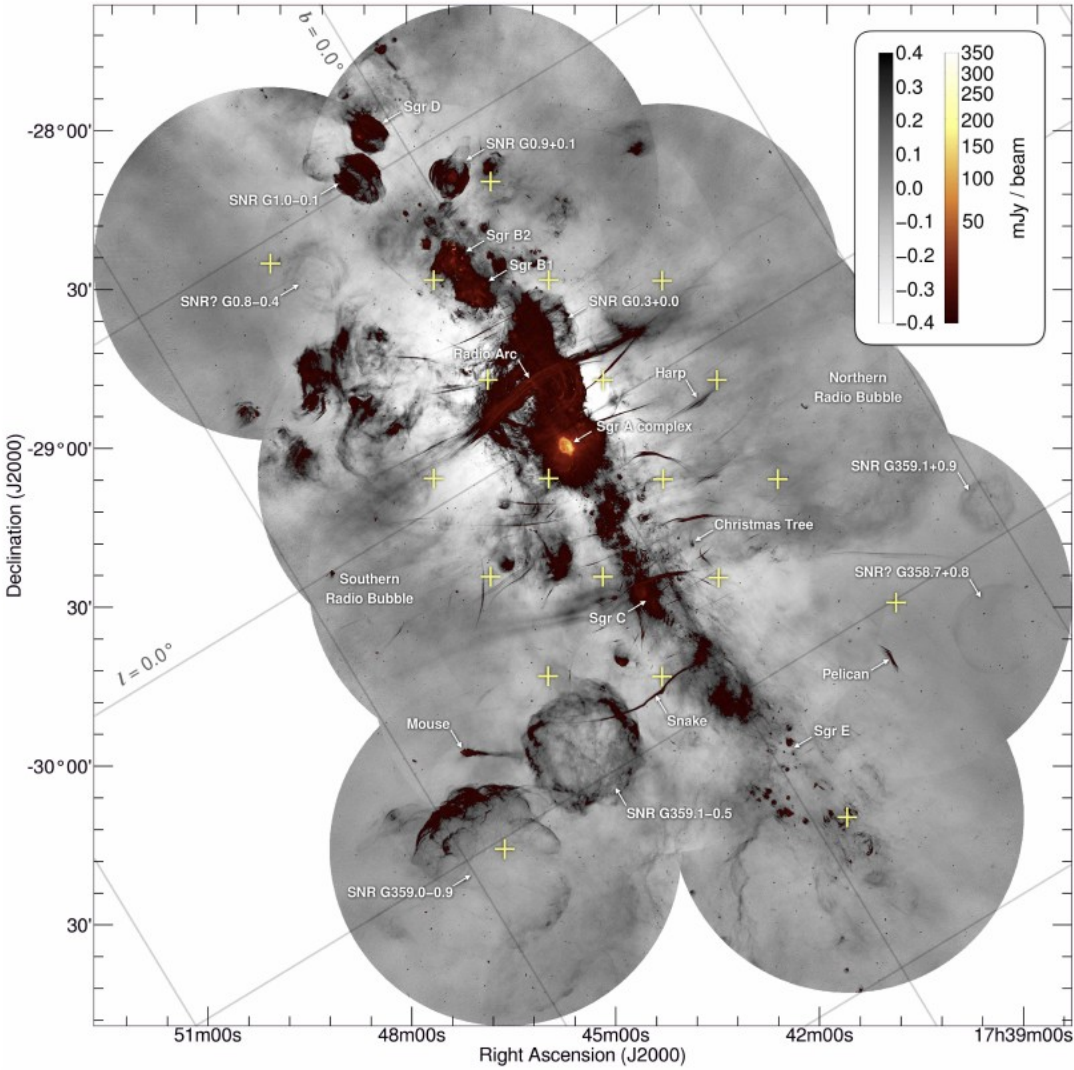


Figure 20–6. The effective (u, v) coverage of a sample compact MMA configuration.

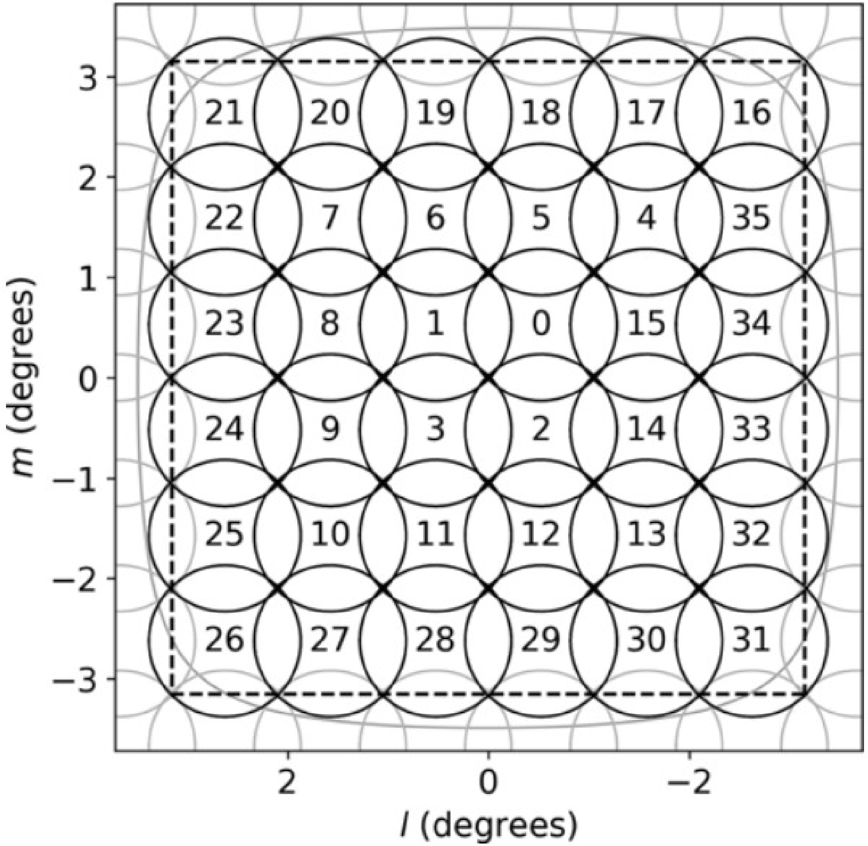
Mosaicking example

- Mosaicked Meerkat image of Galactic centre
  - Variance weighting
  - square of PB attenuation as weighting function
  - Total image area: 6.5 sq deg



Mosaicing

- Newer instruments form multiple primary beams simultaneously using phased array feed (e.g. ASKAP).
  - ASKAP's mosaiced images produce ~30 sq deg. sky coverage simultaneously



Breaking the 2-D FT – the *w*-term

The measurement equation in 2-D

$$V_{jk}(t,\nu) = g_j(t,\nu)g_k^*(t,\nu)S_{jk}(t,\nu) \iint I(l,m)\, e^{-i2\pi(ul+vm)}dldm$$

Gains are antenna-based and independent of direction

Sky is fixed over the course of an observation

2D Fourier transform between sky and gridded visibilities

The measurement equation – general

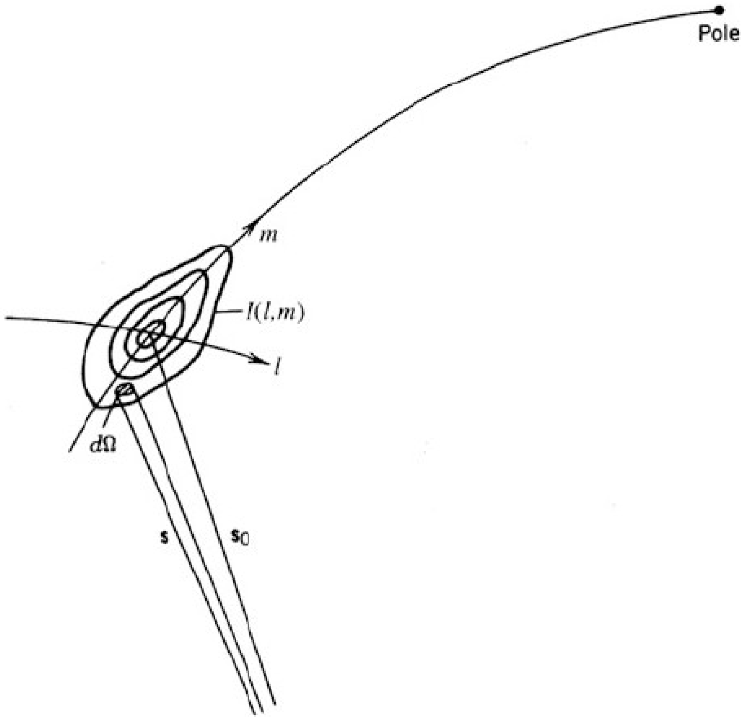
$$V(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(l,m)e^{-2\pi i\left[ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right]}\frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

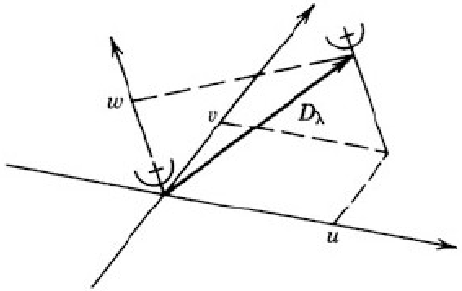
The measurement equation – general

$$V(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(l,m) e^{-2\pi i\left[ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right]} \frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

- Reduces to 2-D familiar expression when $w\left(\sqrt{1-l^2-m^2}-1\right)$ is close to 0 or <<1
  - Co-planar baselines
    - w=0 when E-W baselines with pointing parallel to Earth's rotation axis
  - Imaging region close to phase centre
    - ($l^2+m^2 \sim 0$) i.e. $\sqrt{1-l^2-m^2} \sim 1$

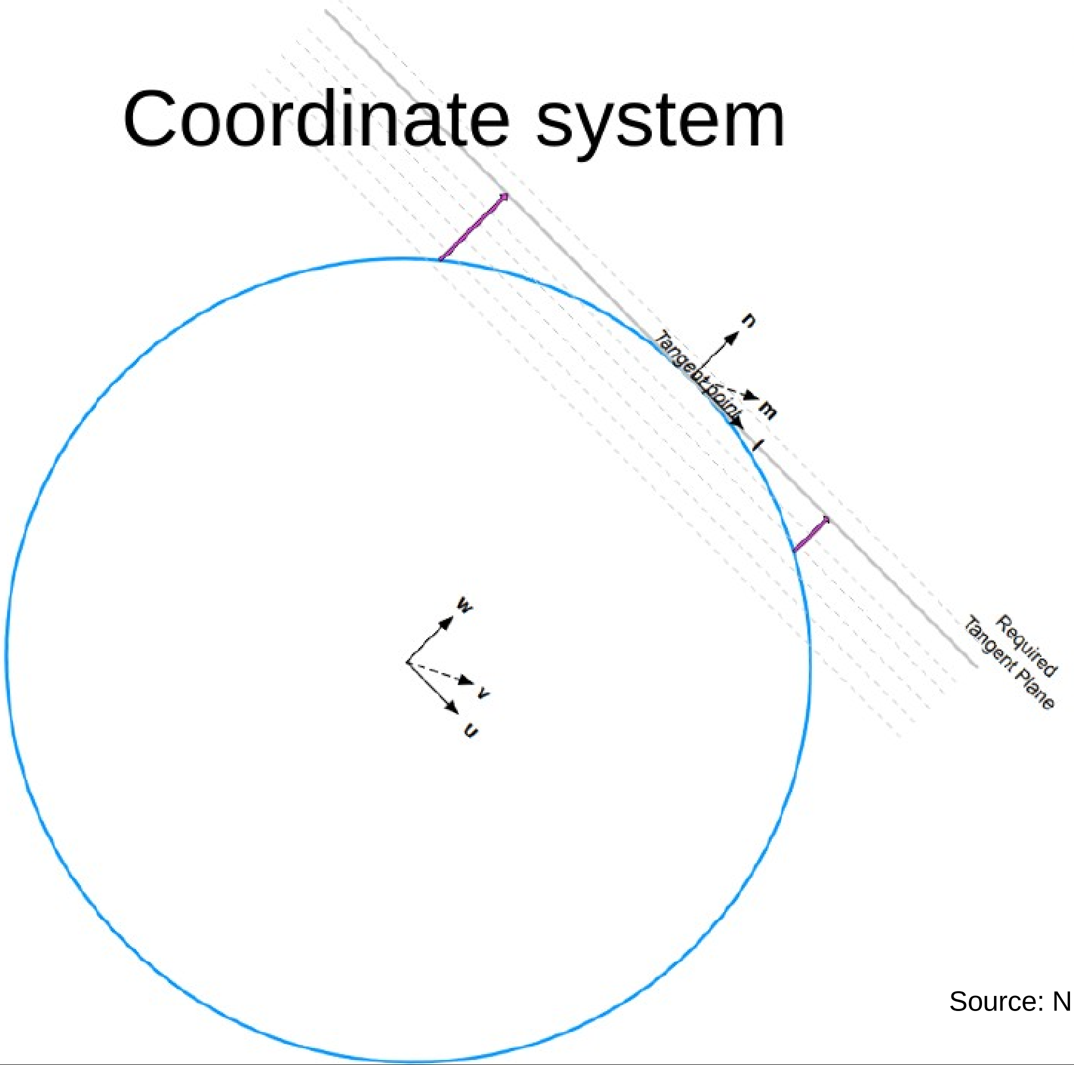
The origin of w





The origin of n

$n = \sqrt{1-l^2-m^2}$

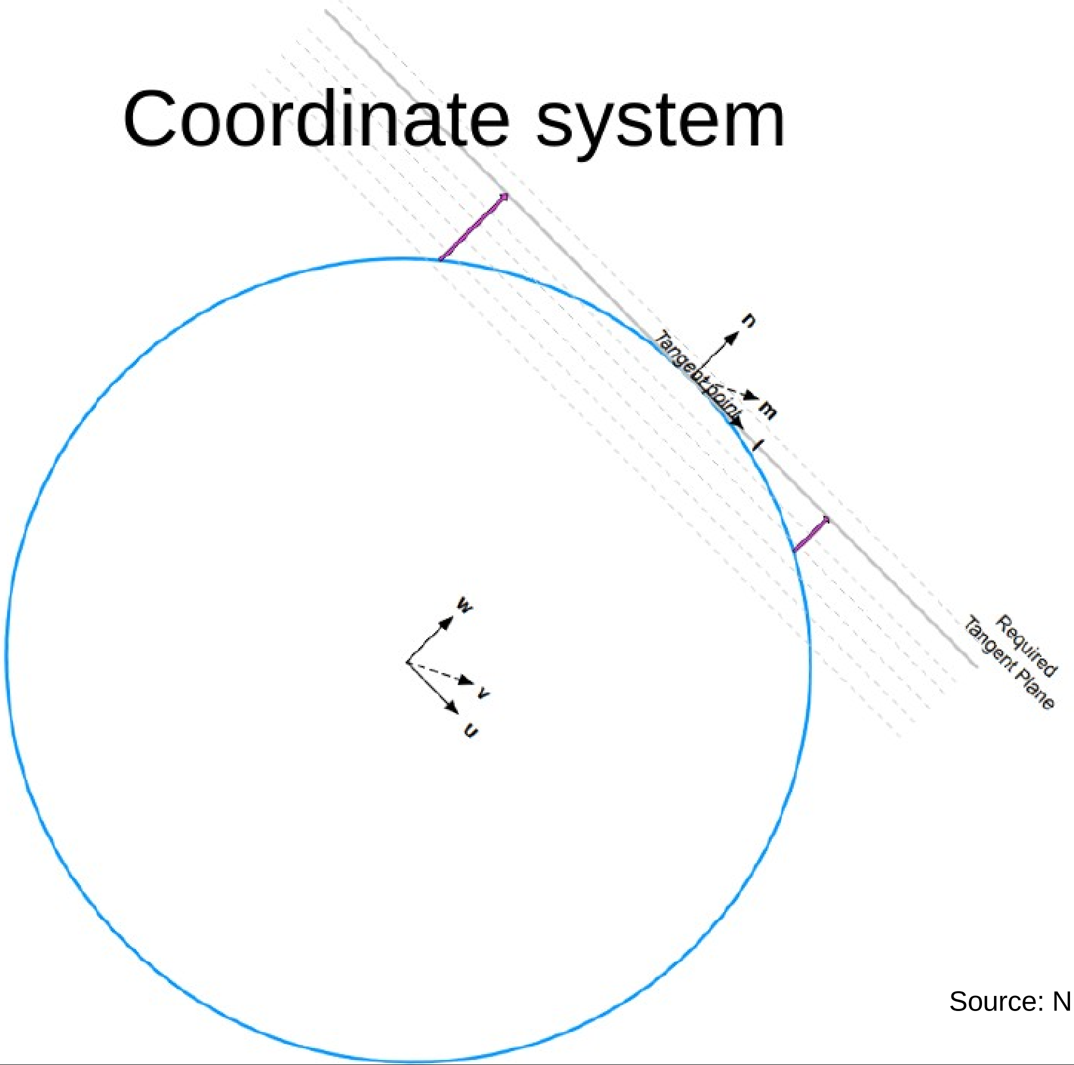


Source: NH-W

Coordinate system

The origin of n

$n = \sqrt{1-l^2-m^2}$



In both cases,
phase error:

$$\text{error} \approx \pi w \theta^2$$

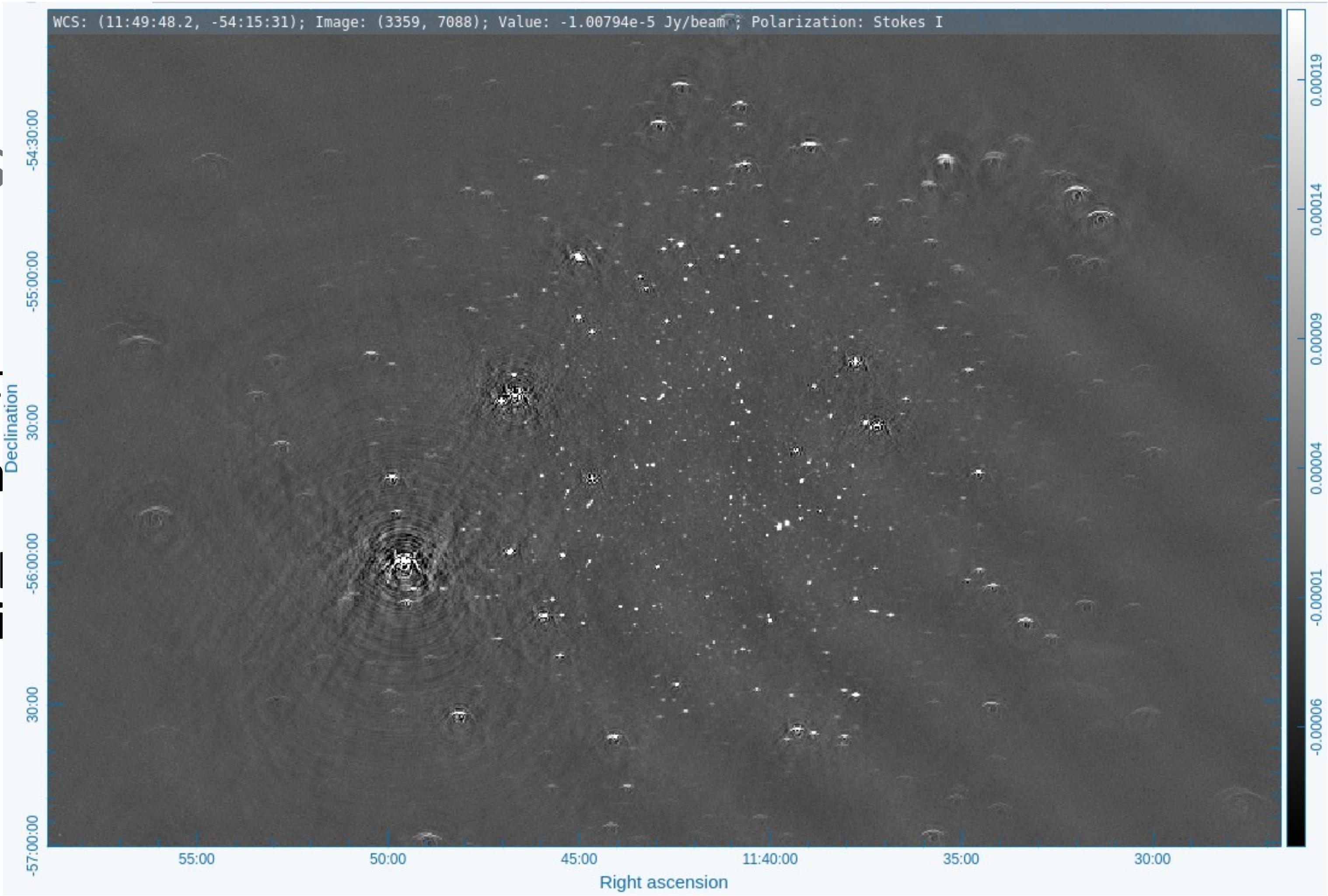
Source: NH-W

Menifestation

- Co-planar arrays:
  - Positional shift
- Non-coplanar baselines show
  - Complicated menifestation

Menifes

- Co-planar
  - Posit
- Non-copla
  - Com
    meni



Mitigation

- Snapshot images
  - Some instruments (e.g. VLA) baselines are almost coplanar
  - Note: fails for very disparate geo locations (e.g. hills-valleys), VLBI
- 1 D E-W instruments (e.g. ATCA, Westerbrok) are coplanar even for long integrations.

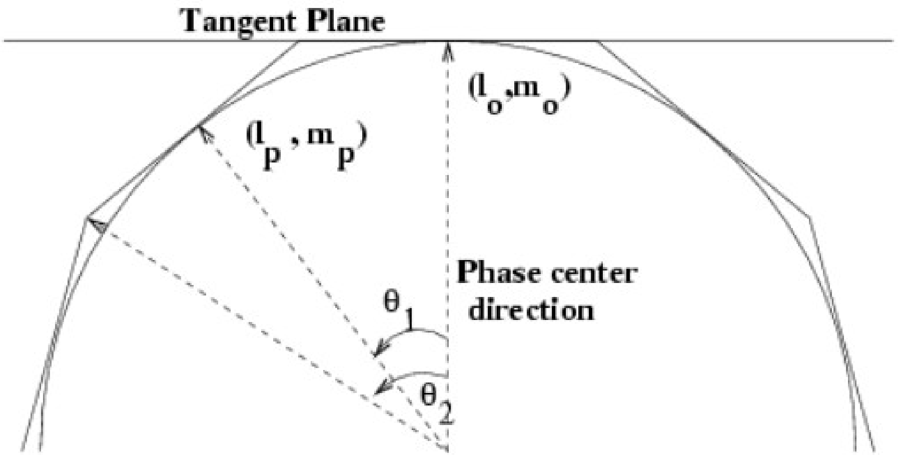
- 3 D fourier transform
  - Computationally intensive
  - Some work being done.

Mitigation – more common

- Facets
- W-projection (e.g. CASA)
- W-stacking (e.g. WSclean)
- Hybrids of the above (e.g. ASKAPsoft)

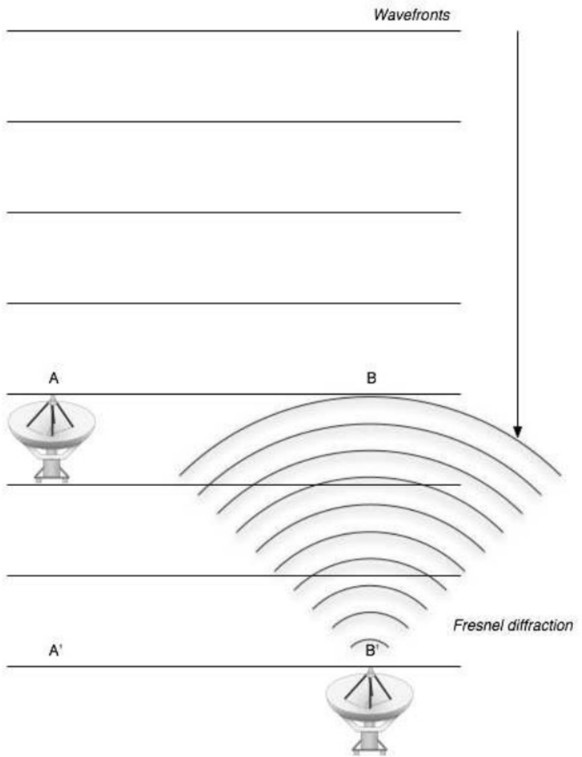
Mitigation: facets

- Approximate the celestial sphere with multiple smaller tangents (facets)
  - 2-D FT is now valid
- Phase rotate per facet
- Image each facet with it's phase reference centre
- Reproject to the tangent plane
- Widely used but slow, can cause artifacts at joins



Mitigation: w-projection

- Different interpretation:
  - E field travelling from B to B’ will have diffracted
  - B and B’ are related by Fresnel diffraction kernel
  - Frater & Docherty (1980): reprojection to [and from] any position in (u,v,w) space [to and] from w=0 plane *using convolution with known kernel*



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$$V(u,v,w) = \int \frac{I(\ell,m)}{\sqrt{1-\ell^2-m^2}} G(\ell,m,w)\, e^{-2\pi i[u\ell+vm]} d\ell dm \tag{10}$$

$$G(\ell,m,w) = e^{-2\pi i[w(\sqrt{1-\ell^2-m^2}-1)]} \tag{11}$$

Applying the Fourier convolution theorem, we find that:

$$V(u,v,w) = \tilde{G}(u,v,w) * V(u,v,w=0) \tag{12}$$

Mitigation: w-projection

- Correction:
  - Correct this with a convolution with inverse kernel during gridding of visibilities
  - Different kernels for different w values

$$V(u,v,w) = \int \frac{I(\ell,m)}{\sqrt{1-\ell^2-m^2}} G(\ell,m,w)\, e^{-2\pi i[u\ell+vm]} d\ell dm \qquad (10)$$

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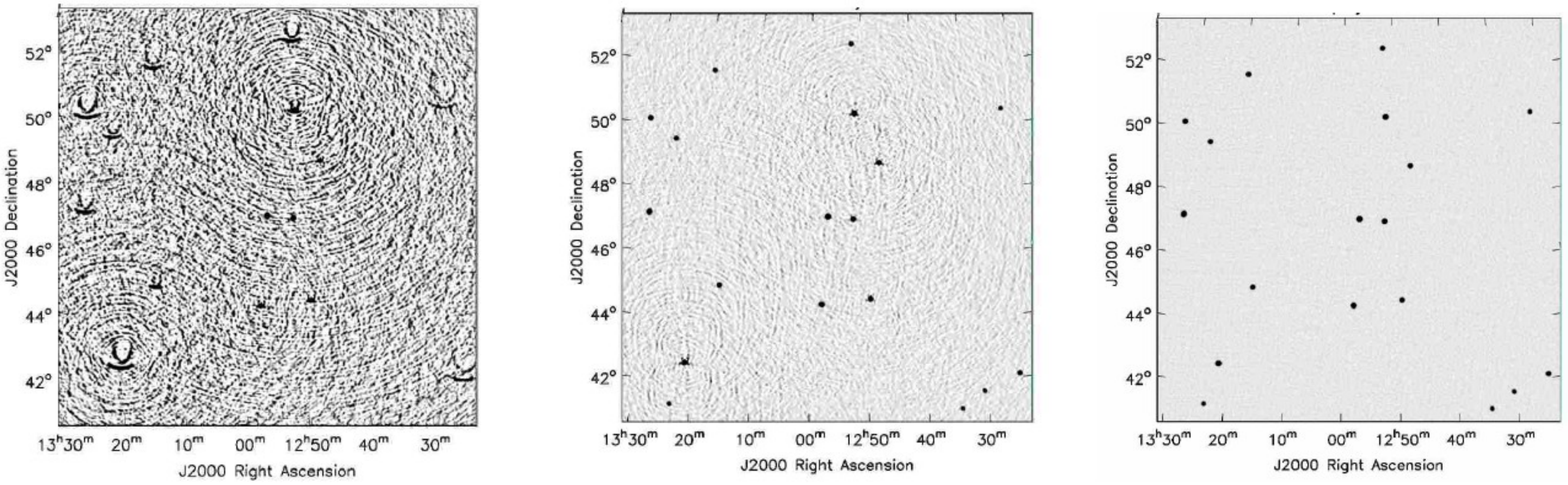
w-projection performance



(a) standard Fourier transform

(b) uvw-space facets (9 x 9)

(c) W-projection (128 $\tilde{G}_T$ planes)

Mitigation: w-stacking (e.g. WSclean)

- Somewhat similar mathematical approach to w-projection,

$$V(u,v,w) = \int\int \frac{I'(l,m)\mathrm{e}^{-2\pi \mathrm{i} w(\sqrt{1-l^2-m^2}-1)}}{\sqrt{1-l^2-m^2}} \times \mathrm{e}^{-2\pi i(ul+vm)}\mathrm{d}l\mathrm{d}m.$$

Mitigation: w-stacking (e.g. WSclean)

- Somewhat similar mathematical approach to w-projection,
  BUT:
  - Grid equal w-value samples on a uniform grid
  - Calculate inverse FFT
  - Apply phase shift to each layer
  - Add all layers

$$\frac{I'(l,m)\,(w_{\max}-w_{\min})}{\sqrt{1-l^2-m^2}} = \int_{w_{\min}}^{w_{\max}} e^{2\pi i w\left(\sqrt{1-l^2-m^2}-1\right)} \times \int\int V(u,v,w)e^{2\pi i(ul+vm)}\,du\,dv\,dw.$$

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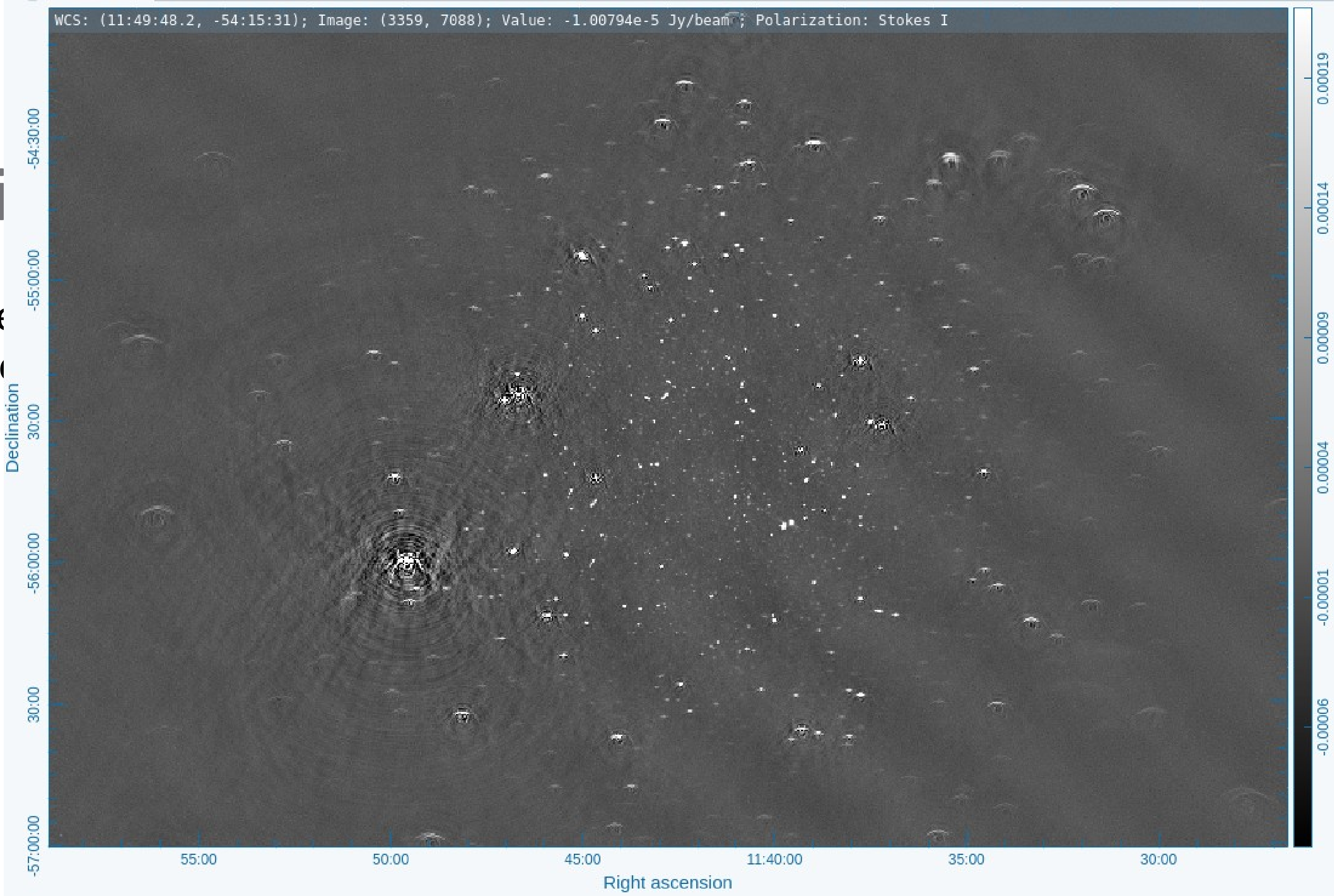

Miti

- Some appr BUT:
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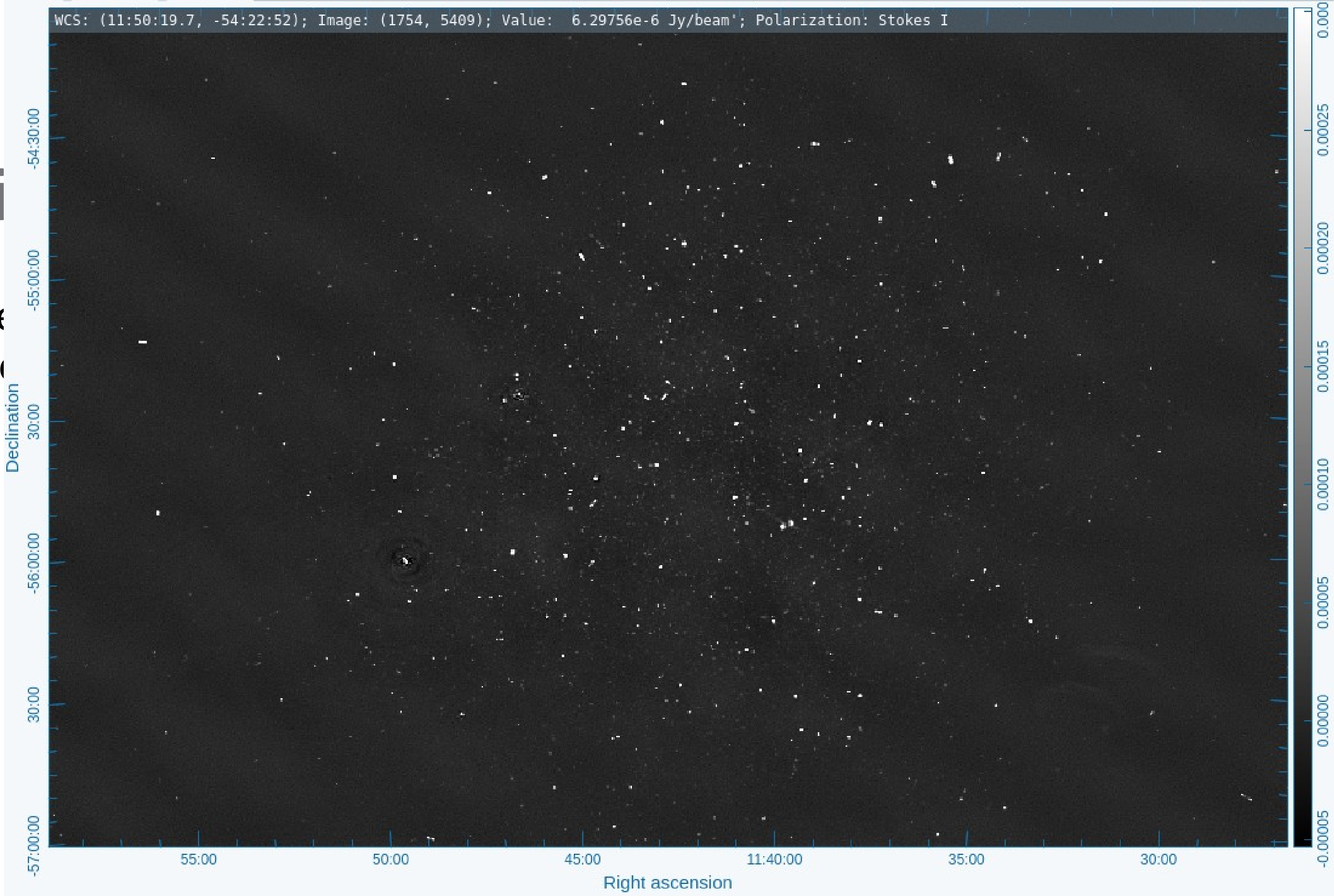

Some take-aways:

W-projection (e.g. CASA):

- Correction in visibility plane
- Computationally challenged for large images >1000s pixels
- Suited for imaging wide fields with instruments e.g. VLA

W-stacking (WSclean):

- Correction in image plane
- Efficient for large images (up to an order of mag. faster for MWA images)
- Suited for large FoV instruments e.g. MWA

Other direction dependent effects

Direction dependent effects

Calibration is applied independent of direction

Direction dependent effects

Calibration is applied independent of direction doesn’t always hold true in wide-field observations

Direction dependent effects

Calibration is independent of direction
doesn't always hold true in wide-field observations

- **Due to isoplanatic patches**
- Troposphere
  - Small effect except for VLBI/mm
- Ionosphere –
  - Frequency dependent
  - Patches smaller than the field of view
  - Corrections must be made to patches

Instrumental:

- Different response in different direction (primary beam)
- Polarisation changes across reception pattern

Direction dependent effects - mitigation

Calibration is independent of direction
doesn’t always hold true in wide-field observations

Direction dependent calibration

- Obtain complex gains at different grid points in the sky (facets) calibration with a source in the middle of the facet
- Approached in a similar manner to w-projection, implement a correcting convolution kernel (A projection)

Summary

- Need to be mindful of a number of different effects

- A combination of techniques needed to obtain desired results.

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