

**Analysis of Phase Cal Extraction Algorithms
and
a Proposal for a Parallel Multitone Phase Cal
Extraction Software Development**

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0. Introduction

Phase cal signals are currently used in VLBI observations for a monitoring of a phase properties of receivers and data acquisition racks. It was also proposed [1] to use these signals for a system temperature calibration.

The phase cal signals are injected into VLBI data stream by the means of a picosecond pulse generator driven by 1 MHz reference clock. When downconverted to a videoband, the signals look like harmonics of 1 MHz frequency shifted by some frequency offset. It means that when a broad band (up to 16 MHz) is used, it contains up to 16 harmonics simultaneously.

An attitude to get complete information about phases and amplitudes of all harmonics in the analysed band may give us an additional increase of the calibration accuracy.

Hardware approach to the parallel multitone phase cal extraction was proposed in [2]. Here algebraic analysis of the parallel multitone extraction algorithm is presented.

1. Algorithms analysis

To determine phases and amplitudes of the calibration tones we need to measure, basically, accumulated voltages of the complex harmonics of interest:

$$(1) \quad V_{\text{tone}} = \sum_k S(\text{dt} \cdot k) * \exp\{-2\pi i \cdot (F_{\text{tone}} + F_{\text{offset}}) \cdot \text{dt} \cdot k\},$$

where $S(\text{dt} \cdot k)$ - input signal sampled at a time $t = \text{dt} \cdot k$,
 dt - sampling interval, nominally $\text{dt} = 1 \mu\text{sec} / N_s$,
 $N_s = 1, 2, \dots, 32$ - whole number,
 k - sample number with respect to the pre-determined reference time,
 F_{tone} - tone frequency, nominally $F_{\text{tone}} = 1 \text{MHz} \cdot N_{\text{tone}}$,
 $N_{\text{tone}} = 1, 2, \dots, 16$ - whole number,
 F_{offset} - offset frequency, frequently used as 10 KHz.

Expression (1) is the basic one to be used and it is directly implemented (with some approach of complex exponent presentation) in the current hardware and software phase cal extractors.

To clarify the idea of the parallel multitone extraction some algebraic transforms of (1) need to be done.

Lets choose a whole number K as

$$(2) \quad K = M / (1\text{MHz} \cdot dt) = N_s \cdot M, \quad M=1,2,\dots - \text{whole number.}$$

Actually K is a several microsecond period, expressed in terms of a sampling intervals dt.

Than it is possible to re-write (1) as follows:

$$(3) \quad V_{\text{tone}} = \sum_k S(dt \cdot k) \cdot \exp\{-2\pi i \cdot F_{\text{offset}} \cdot dt \cdot k\} \cdot \exp\{-2\pi i \cdot F_{\text{tone}} \cdot dt \cdot \text{mod}_K(k)\},$$

Substitution of k by $\text{mod}_K(k)$ in the second exponent is possible because of

$$(4) \quad F_{\text{tone}} \cdot dt \cdot k = 1\text{MHz} \cdot N_{\text{tone}} \cdot (1\mu\text{sec}/N_s) \cdot (\text{mod}_K(k) + n \cdot K) = \\ = F_{\text{tone}} \cdot dt \cdot \text{mod}_K(k) + \text{whole number of circles.}$$

Than (3) may be decomposed into K partial summs, each of which is accumulated at a phase $j = \text{mod}_K(k)$ inside the period of K points.

$$(5) \quad V_0 = \sum_n S(dt \cdot (n \cdot K + 0)) \cdot \exp\{-2\pi i \cdot F_{\text{offset}} \cdot dt \cdot (n \cdot K + 0)\},$$

$$V_1 = \sum_n S(dt \cdot (n \cdot K + 1)) \cdot \exp\{-2\pi i \cdot F_{\text{offset}} \cdot dt \cdot (n \cdot K + 1)\},$$

.....

$$V_j = \sum_n S(dt \cdot (n \cdot K + j)) \cdot \exp\{-2\pi i \cdot F_{\text{offset}} \cdot dt \cdot (n \cdot K + j)\},$$

.....

$$V_{K-1} = \sum_n S(dt \cdot (n \cdot K + K - 1)) \cdot \exp\{-2\pi i \cdot F_{\text{offset}} \cdot dt \cdot (n \cdot K + K - 1)\},$$

where n = whole part of k/K, $k = n \cdot K + j$.

It is significant, that partial summs V_0, \dots, V_{K-1} contain different samples and their summ

$$(6) \quad V_{\text{total}} = \sum_j V_j$$

contains all samples processed. It is also remarkable that partial summs V_j do not contain phase factors of the tone frequencies F_{tone} .

It is obvious that summ V_{total} is nothing else but accumulated voltage of the first harmonic of frequency $F = F_{\text{offset}} = 10\text{KHz}$, exactly the same what we can have using (1) at $N_{\text{tone}} = 0, F_{\text{tone}} = 0$.

To calculate a response for any other harmonic we need just to do spectral transform of the small set of already accumulated data $V_0, V_1, \dots, V_j, \dots, V_{K-1}$ as follows:

$$\begin{aligned}
 (7) \quad V_{\text{tone}} &= \sum_j V_j * \exp\{-2\pi i * F_{\text{tone}} * dt * j\} = \\
 &= \sum_j \sum_n S(dt * (n * K + j)) * \exp\{-2\pi i * F_{\text{offset}} * dt * (n * K + j)\} * \exp\{-2\pi i * F_{\text{tone}} * dt * j\} \\
 &= \sum_k S(dt * k) * \exp\{-2\pi i * (F_{\text{tone}} + F_{\text{offset}}) * dt * k\}.
 \end{aligned}$$

The result of (7) for any phase cal harmonic F_{tone} is exactly the same, as obtained by the traditional approach (1).

An advantage of the proposed algorithm is:

to calculate the partial sums V_j we need to do exactly the same number of operations as with the traditional algorithm (1) for one selected harmonic, but using accumulated partial sums V_j we can then calculate responses for all harmonics in the base-band processed. The secondary calculation takes very small number of operations with respect to the primary one.

The sensitivity and all normalizations for each harmonic obtained with the parallel algorithm are exactly the same as in the case of traditional single tone extraction algorithm.

References:

1. D.S.Bagri. Using Pulsecal Amplitudes to Determine System Temperatures. VLBI Acq. Memo 137, 1993 Mar 05.
2. S.V.Pogrebenko. Proposal for a Multiband & Multitone Phase Calibration Signal Extraction for the EVN Upgrade Project. EVN Doc #2, 1993 Jan 18.