Numerical Techniques for Cosmological Simulations: Initial Conditions and Groupfinding

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The Seeds of Structure - Our Universe’s Initial Conditions

Temperature Fluctuations in the Cosmic Microwave Background

Credit: NASA/WMAP Science Team (http://map.gsfc.nasa.gov)
• We define the density at location $x$ at time $t$ by
\[ \delta(x, t) = \frac{\rho(x, t) - \bar{\rho}(t)}{\bar{\rho}(t)} \]

• This can be expressed in terms of its Fourier components
\[ \delta(x, t) = \sum_k \hat{\delta}(k, t) e^{ikx} \]

• Inflation predicts that $\delta$ can be characterised as a Gaussian random field.
Gaussian Random Fields

- The properties of a Gaussian Random Field can be completely specified by the correlation function

\[ \xi(r) = \langle \delta(r) \delta(r + x) \rangle \]

or its Fourier transform, the Power Spectrum

\[ P(k) = \langle |\delta_k|^2 \rangle \]

- We find that

\[ P(k) = Ak^n T(k)^2 \]

where \( T(k) \) is the transfer function.
Generating a Gaussian Random Field

- Easiest to generate realisations in Fourier (or $k$-) space.
- Perturbations are distributed according to a Gaussian distribution, so the amplitude of a particular mode (i.e. a given $k$) is drawn from a distribution of the form,

$$f(\delta_k) = \frac{1}{2\pi P(k)} e^{-|\delta_k|^2/2P(k)}$$

- Phases are drawn from a uniform distribution.

Useful References
- See Chapter 16 of John Peacock’s “Cosmological Physics”
- See the Numerical Recipes in X (or their website www.nr.com) for more details on FFTs.
- Download the FFTW libraries from www.fftw.org.
Initial Particle Distribution

- A distribution that is stable to clumping over many expansion factors in the absence of perturbations is highly desirable.

✗ A Poisson (or random) distribution corresponds to a white noise spectrum and is inherently unstable -- not a good choice!

✔ Grids have been extremely popular -- but forces are anisotropic on scales of the mean interparticle separation.

✔ Glass files are becoming increasingly popular, and are preferable when exploring alternative cosmologies (such as WDM) which may have little power on small scales.

Glass distributions are said to be sub-random and are stable to clumping over many expansion factors (see Simon White’s 1994 Les Houches lectures; also Baugh et al 1995).
Initial Displacements and Velocities

- Still in linear regime.

- Use the Zel’dovich approximation (Zel’dovich 1970); a particle’s displacement at time $t$ from its initial (Lagrangian) position is,

$$x(t) = a(t)q + b(t)f(q)$$

where $a(t)$ is the expansion factor, $b(t)$ is the linear growth factor, $q$ is the Lagrangian coordinate of the particle and $f(q)$ is a function of the density at $q$.

Useful References

- See Klypin’s lecture notes for more details (astro-ph/0005502)
- Chapter 15 of John Peacock’s “Cosmological Physics” gives a nice treatment.
- Zel’dovich’s 1970 paper if you’re feeling brave.
Cosmological ICs in 3 Easy Steps

😊 Generate a **realisation** of Gaussian random **density field** in \( k \)-space using your power spectrum‡ of choice.

😊 Generate a **particle distribution** -- either a grid or a glass.

😊 Use the **Zel’dovich approximation** to initialise particle displacements and velocities.

Alternatively…
- Use the freely available Ed Bertschinger’s COSMICS (ascl.net/cosmics.html) and GRAFIC2 (arcturus.mit.edu/grafic2) packages.

‡ Use either LINGERS (part of COSMICS and GRAFIC2 packages) or CMBFAST (www.cmbfast.org) to generate a power spectrum for your chosen cosmology.
Groups in Cosmological Simulations

• The matter distribution in the Universe at the time of recombination (~300,000 years after the Big Bang) was smooth.

• Linear perturbation theory provides an accurate description of structure, but require simulations to describe subsequent non-linear evolution.

• How do we identify nonlinear structure in an automated manner?
The Friends-Of-Friends (FOF) Algorithm

- A **conceptually simple** approach - a particle belongs to a friends-of-friends group if it lies within some **linking length** \(b\) of any other particle in the group.
- Linking length chosen to be some fraction of the mean interparticle separation within the simulation volume -- \(b=0.2\) standard form.
- Example of a **percolation algorithm** -- widely used in identifying groups in galaxy redshift surveys (e.g. 2dF; Eke et al. 2004). See Corey Putkunz or Sarah Brough for particular applications.

**References**
- Davis et al (1985) -- the standard reference in cosmological N-body studies.
- Cole & Lacey (1996) -- systematic survey of properties of FOF haloes.

**Code**
What do FOF Groups Correspond to?
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- Compute virial mass for each group -- for LCDM cosmology, use an overdensity criterion of $\Delta \approx 97$
  
  i.e.
  
  $$M_{\text{vir}} = \frac{4\pi}{3} \Delta \rho_{\text{crit}} r_{\text{vir}}^3$$

- Good agreement between virial mass and FOF mass
What about Substructure?

- High resolution simulations reveal that dark matter haloes (and CDM haloes in particular) contain a wealth of substructure.
- How can we identify this substructure in an automated way?
- Seek gravitationally bound groups of particles that are overdense relative to the background density of the host halo.
Numerical Considerations

- We expect the amount of substructure resolved in a simulation to be sensitive to the mass resolution of the simulation.
- Efficient (parallel) algorithms becoming increasingly important.
- Still very much work in progress!
Case Study: Gill & Knebe’s AHF
References & Codes

References

• Springel et al (2001) describes the SubFind algorithm in detail, and is useful for more general considerations of how one should identify substructure for the purpose of designing an algorithm.

Codes

• Springel’s SubFind and Stadel & Diemand’s AFOF are efficient, parallel groupfinders capable of identifying substructure -- but neither are public.
• The University of Washington code SKID has been widely used;  
• Gill & Knebe’s AHF is an efficient serial code that compares favourably to SubFind and SKID (see Gill, Knebe & Gibson 2004 for details)  
  – http://www.aip.de/People/Aknebe/AMIGA