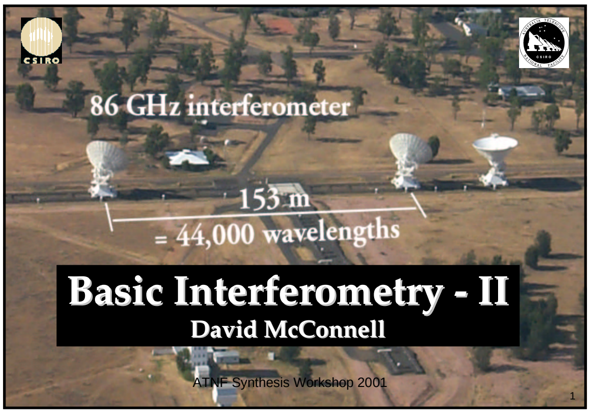
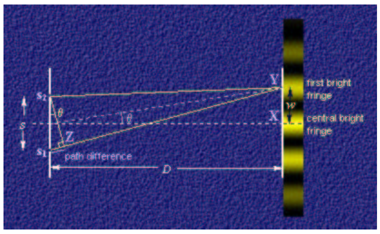


Interference

Young's double slit experiment (1801)



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Basic Interferometry II

- Coherence in Radio Astronomy
  - » follows closely Chapter 1* by Barry G Clark
- What does it mean?
- Outline of a Practical Interferometer
- Review the Simplifying Assumptions

* Synthesis Imaging in Radio Astronomy II
Edited by G.B.Taylor, C.L.Carilli, and R.A Perley

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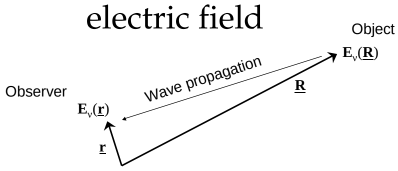
Form of the observed electric field



$$\mathbf{E}_\nu(\mathbf{r}) = \iiint P_\nu(\mathbf{R},\mathbf{r})\mathbf{E}_\nu(\mathbf{R})\,dxdydz$$

Superposition allowed by linearity of Maxwell's equations

The *propagator*

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Form of the observed electric field

$$\mathbf{E}_\nu(\mathbf{r}) = \iiint P_\nu(\mathbf{R},\mathbf{r})\mathbf{E}_\nu(\mathbf{R})\,dxdydz$$

Assumption 1: Treat the electric field as a scalar - ignore polarisation

Assumption 2: Immense distance to source; ignore depth dimension; measure "surface brightness": $\mathcal{E}_\nu(\mathbf{R})$ is electric field distribution on celestial sphere

Assumption 3: Space is empty; simple propagator

$$E_\nu(\mathbf{r}) = \int \mathcal{E}_\nu(\mathbf{R}) \frac{e^{2\pi i\nu|\mathbf{R}-\mathbf{r}|/c}}{|\mathbf{R}-\mathbf{r}|}\,dS$$

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Spatial coherence function of the field

Define the **correlation** of the field at points $\mathbf{r}_1$ and $\mathbf{r}_2$ as:

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \left\langle E_\nu(\mathbf{r}_1)E_\nu^*(\mathbf{r}_2)\right\rangle$$

where

$$E_\nu(\mathbf{r}) = \int \mathcal{E}_\nu(\mathbf{R}) \frac{e^{2\pi i\nu|\mathbf{R}-\mathbf{r}|/c}}{|\mathbf{R}-\mathbf{r}|}\,dS$$

so

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \left\langle \iint \mathcal{E}_\nu(\mathbf{R}_1)\mathcal{E}_\nu^*(\mathbf{R}_2) \frac{e^{2\pi i\nu|\mathbf{R}_1-\mathbf{r}_1|/c}}{|\mathbf{R}_1-\mathbf{r}_1|} \frac{e^{-2\pi i\nu|\mathbf{R}_2-\mathbf{r}_2|/c}}{|\mathbf{R}_2-\mathbf{r}_2|}\,dS_1 dS_2\right\rangle$$

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Spatial coherence function of the field

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \left\langle \iint \mathcal{E}_\nu(\mathbf{R}_1)\mathcal{E}_\nu^*(\mathbf{R}_2) \frac{e^{2\pi i\nu|\mathbf{R}_1-\mathbf{r}_1|/c}}{|\mathbf{R}_1-\mathbf{r}_1|} \frac{e^{-2\pi i\nu|\mathbf{R}_2-\mathbf{r}_2|/c}}{|\mathbf{R}_2-\mathbf{r}_2|}\,dS_1 dS_2\right\rangle$$

Assumption 4: Radiation from astronomical sources is **not** spatially coherent

$$\left\langle \mathcal{E}_\nu(\mathbf{R}_1)\mathcal{E}_\nu^*(\mathbf{R}_2)\right\rangle = 0 \quad \text{for } \mathbf{R}_1 \neq \mathbf{R}_2$$

After exchanging the expectation operator and integrals becomes:

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \int \left\langle |\mathcal{E}_\nu(\mathbf{R})|^2\right\rangle |\mathbf{R}|^2 \frac{e^{2\pi i\nu|\mathbf{R}-\mathbf{r}_1|/c}}{|\mathbf{R}-\mathbf{r}_1|} \frac{e^{-2\pi i\nu|\mathbf{R}-\mathbf{r}_2|/c}}{|\mathbf{R}-\mathbf{r}_2|}\,dS$$

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Spatial coherence function of the field

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \int \left\langle |\mathcal{E}_\nu(\mathbf{R})|^2\right\rangle |\mathbf{R}|^2 \frac{e^{2\pi i\nu|\mathbf{R}-\mathbf{r}_1|/c}}{|\mathbf{R}-\mathbf{r}_1|} \frac{e^{-2\pi i\nu|\mathbf{R}-\mathbf{r}_2|/c}}{|\mathbf{R}-\mathbf{r}_2|}\,dS$$

Write the unit vector as: $\mathbf{s} = \dfrac{\mathbf{R}}{|\mathbf{R}|}$

Write the observed intensity as: $I_\nu(\mathbf{s}) = |\mathbf{R}|^2 \left\langle |\mathcal{E}_\nu(\mathbf{s})|^2\right\rangle$

Replace the surface element : $dS = |\mathbf{R}|^2 d\Omega$

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) \approx \int I_\nu(\mathbf{s}) e^{-2\pi i\nu \mathbf{s}\cdot(\mathbf{r}_1-\mathbf{r}_2)/c}\,d\Omega$$

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Spatial coherence function of the field

$$\mathrm{V}_\nu(\mathbf{r}_1,\mathbf{r}_2) \approx \int I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{s}\cdot(\mathbf{r}_1-\mathbf{r}_2)/c} d\Omega$$

"An interferometer is a device for measuring this spatial coherence function."

Inversion of the Coherence Function

The Coherence Function is invertible after taking one of two further simplifying assumptions:

•Assumption 5(a): vectors $(\mathbf{r}_1 - \mathbf{r}_2)$ lie in a plane

•Assumption 5(b): endpoints of vectors **s** lie in a plane

Choose coordinates (u,v,w) for the $(\mathbf{r}_1 - \mathbf{r}_2)$ vector space

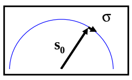
Write the components of **s** as $\left(l, m, \sqrt{1-l^2-m^2}\right)$

Then with 5(a):

$$V_\nu(u,v,w \equiv 0) = \iint I_\nu(l,m) \frac{e^{-2\pi i(ul+vm)}}{\sqrt{1-l^2-m^2}} dldm$$

Inversion of the Coherence Function

Or, taking 5(b) assuming all radiation comes from a small portion of the sky, write the vector **s** as $\mathbf{s} = \mathbf{s_0} + \sigma$ with $\mathbf{s_0}$ and σ perpendicular



Choose coordinates s.t. $\mathbf{s_0} = (0,0,1)$ then

$$\mathrm{V}_\nu(\mathbf{r}_1,\mathbf{r}_2) \approx \int I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{s}\cdot(\mathbf{r}_1-\mathbf{r}_2)/c} d\Omega$$

becomes

$$V_\nu'(u,v,w) = e^{-2\pi i w} \iint I_\nu(l,m) e^{-2\pi i(ul+vm)} dldm$$

$$V_\nu(u,v,w) = e^{2\pi i w} V_\nu'(u,v,w)$$ is independent of w

$$V_\nu(u,v) = \iint I_\nu(l,m) e^{-2\pi i(ul+vm)} dldm$$

Inversion of the Coherence Function

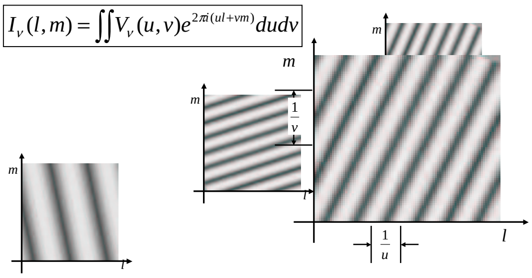
$$V_\nu(u,v) = \iint I_\nu(l,m) e^{-2\pi i(ul+vm)} dldm$$

⇕

$$I_\nu(l,m) = \iint V_\nu(u,v) e^{2\pi i(ul+vm)} dudv$$

Image analysis/synthesis

$$I_\nu(l,m) = \iint V_\nu(u,v)e^{2\pi i(ul+vm)}dudv$$



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Incomplete Sampling

$$I_\nu(l,m) = \iint V_\nu(u,v)e^{2\pi i(ul+vm)}dudv$$

Usually it is not practical to measure $V_\nu(u,v)$ for all (u,v) - our sampling of the (u,v) plane is incomplete.

Define a sampling function: $S(u,v)$ = 1 where we have measurements, otherwise $S(u,v)$ = 0.

$$I_\nu^D(l,m) = \iint S(u,v)V_\nu(u,v)e^{2\pi i(ul+vm)}dudv$$

$I_\nu^D(l,m)$ is called the "dirty image"

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Incomplete Sampling

The convolution theorem for Fourier transforms says that the transform of the product of functions is the convolution of their transforms. So we can write:

$$I_\nu^D = I_\nu * B$$

The image formed by transforming our incomplete measurements of $V_\nu(u,v)$ is the true intensity distribution I_ν convolved with $B(u,v)$, the "synthesized beam" or "point spread function".

$$B(l,m) = \iint S(u,v)e^{2\pi i(ul+vm)}dudv$$

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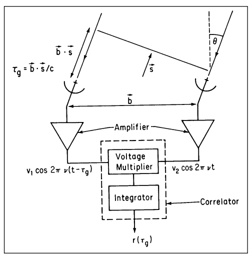
Interferometry Practice



θ and therefore τ_g change as the Earth rotates. This produces rapid changes in $r(\tau_g)$ the correlator output.

This variation can be interpreted as the "source moving through the fringe pattern".

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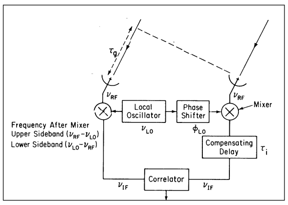
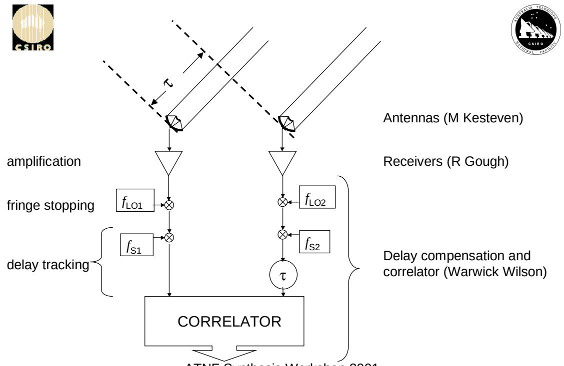
Interferometry Practice



We could variable phase reference and delay compensation to move the "fringe pattern" across the sky with the source ("fringe stopping").



Simplifying Assumptions

Assumption 1: Treat the electric field as a scalar - ignore polarisation	Polarisation is important in radioastronomy and will be addressed in a later lecture (see also Chapter 6).
Assumption 2: Immense distance to source, so ignore depth dimension and measure "surface brightness": $E_\nu(\mathbf{R})$ is electric field distribution on celestial sphere	In radioastronomy this is usually a safe assumption. Exceptions may occur in the imaging of nearby objects such as planets with very long baselines.

Simplifying Assumptions

Assumption 3: Space is empty; simple propagator	Not quite empty! The propagation medium contains magnetic fields, charged particles and atomic/molecular matter which makes it wavelength dependent. This leads to dispersion, Faraday rotation, spectral absorption, etc.
Assumption 4: Radiation from astronomical sources is **not** spatially coherent	Usually true for the sources themselves; however multi-path phenomena in the propagation medium can lead to the position dependence of the spatial coherence function.

$$V_\nu(\mathbf{r_1},\mathbf{r_2}) \approx \int I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{s}\cdot(\mathbf{r_1}-\mathbf{r_2})/c} d\Omega$$

Simplifying Assumptions

The Coherence Function is invertible after taking one of two further simplifying assumptions:

•Assumption 5(a): vectors $(\mathbf{r}_1 - \mathbf{r}_2)$ lie in a plane

•Assumption 5(b): endpoints of vectors **s** lie in a plane

5(a) violated for all but East-West arrays

5(b) violated for wide field of view

The problem is still tractable, but the inversion relation is no longer simply a 2-dimensional Fourier Transform (chapter 19).

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