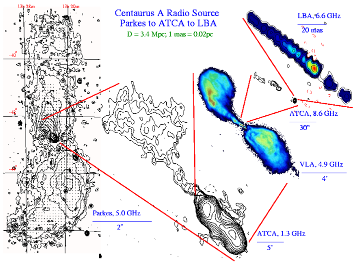
Basic imaging



SFT: Sketchy Fourier Transform

$Let: \quad f(t) \Leftrightarrow F(x) \quad$ g(t) ⇔ G(x)

$Linearity: \; af(t) + bg(t) \Leftrightarrow aF(x) + bG(x)$

$Scaling: \quad f(at) \Leftrightarrow F(x/a)$

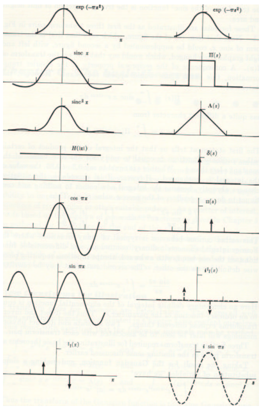
"skinny" ⇔ "wide" $\quad f(t) = 1 \Leftrightarrow \delta(x)$

discontinuities ⇔ periodicity / "ringing"

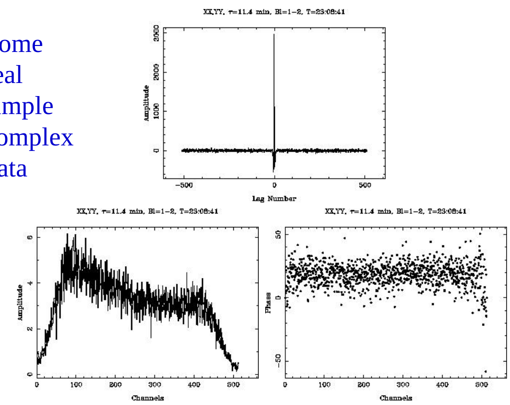
$Shift: \quad f(t+a) \Leftrightarrow F(x) \cdot e^{2\pi i a x}$

$Convolution: \; f(t) * g(t) := \int f(t')G(t-t')dt' \Leftrightarrow F(x) \cdot G(x)$

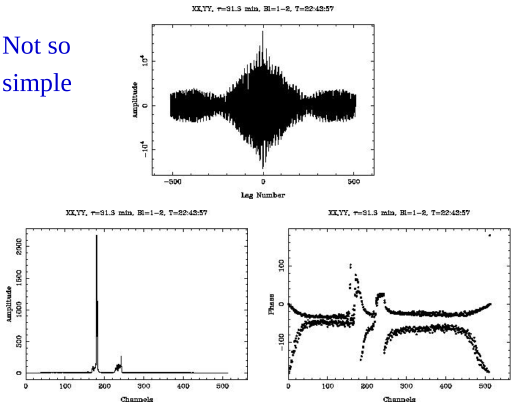
Examples…



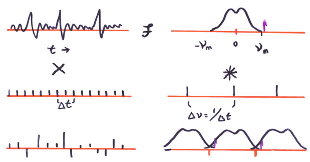
Some
real
simple
complex
data



Not so simple



Sampling theorem (aliasing)



Nyquist criterion : $\Delta t \le 1/2\upsilon_m$

In *u-v* plane : $\Delta u \le |u_{max}| = D/\lambda$

$\Delta l << 1/2|u_{max}| \approx 1/2 \cdot (\textit{beam size}) = 1/2 \cdot \theta_{synth}$

Direct Transform - DFT

$V(u,v) = \iint I(l,m) \cdot \exp(2\pi i [ul + vm])\, dldm$

V(u,v) : noisy

corrupt

incomplete : $W(u,v)$:= sampling function

$I^D(l,m) = \iint W(u,v) \cdot V(u,v) \cdot \exp\{-2\pi i [ul + vm]\, dudv$

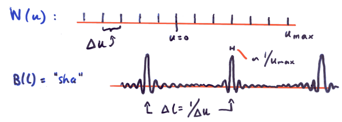
by convolution theorem :

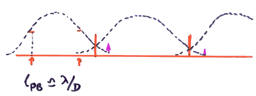
$I^D(l,m) = I^D(l,m) * B^D(l,m)$ $B^D(l,m)$: "dirty beam"

finite extent $\rightarrow$ resolution limit $\theta_{synth} \approx 1/u_{max}$

sparse/discrete $\rightarrow$ aliasing, ambiguities

Sampling in the u-v plane: How much is enough?





Ideally, $l_{pb} \le \Delta l \Rightarrow \Delta u \le D / \lambda$

For ATCA, $\lambda\Delta u = 15m$, $D = 22m$

Grating lobes

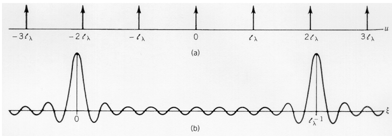
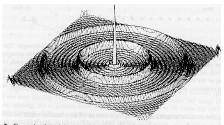
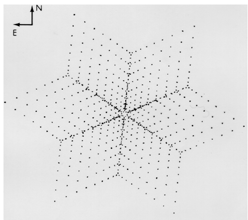
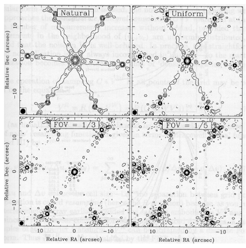


Figure 5.14 Part of a series of δ functions representing the instantaneous distribution of spacings for a uniformly spaced linear array with equal weight for each spacing. (b) Part of the corresponding series of fan beams that constitute the instantaneous response. Parts (a) and (b) represent the left- and right-hand sides of Eq.(5.19), respectively.



u-v weighting

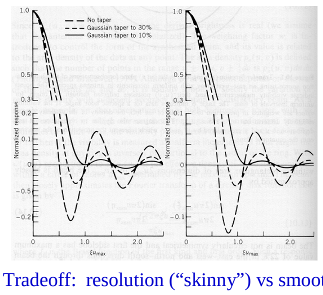




Natural – best sensitivity (point source)

Uniform – better resolution, beamshape

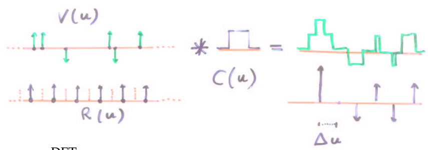
Weighting/tapering



Tradeoff: resolution ("skinny") vs smooth

FFT - regridding

Usually a necessity (CPU time) - but introduces aliasing



DFT: $I^{DFT} = \Im(W \cdot V)$

FFT: $I^{FFT} = \Im(((W.V) * C) \cdot R)\ /\ \Im(C)$

$= \Im((W \cdot V) * C)) * \Im(R)$

$= (I^{DFT} \cdot \Im(C)) * \Im(R)\ /\ \Im(C)$

Image aliasing I

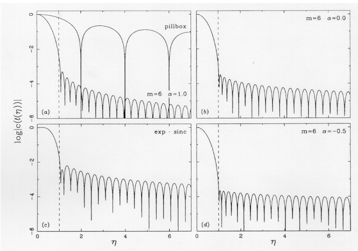


Figure 7–6. For some typical gridding convolution functions C, plots of the absolute value of the Fourier transform of C. **(a)** The spheroidal function ψ_{10}, for $m = 6$, compared with the pillbox function ($m = 1$); **(b)** the 'prolate spheroidal wave function' ψ_{00}, $m = 6$; **(c)** an optimized Gaussian-tapered sinc function, $m = 6$; **(d)** the spheroidal function $\psi_{-\frac{1}{2},0}$, $m = 6$. Panel **(a)** is comparing the function most commonly used at the VLA with the simple but particularly poor choice of a pillbox. Adapted from Schwab (1984).

Image plane aliasing