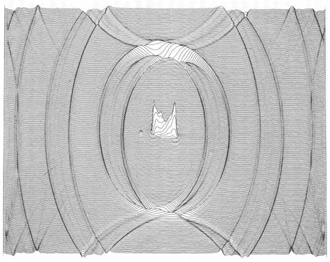
The big picture



Complexity….

1. Deformations of Galois representations

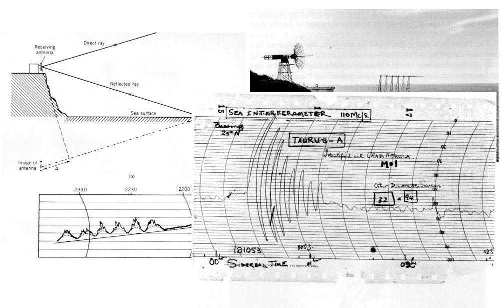
Let p be an odd prime. Let Σ be a finite set of primes including p and let $\mathbf{Q}_\Sigma$ be the maximal extension of $\mathbf{Q}$ unramified outside this set and ∞. Throughout we fix an embedding of $\overline{\mathbf{Q}}$, and so also of $\mathbf{Q}_\Sigma$, in $\mathbf{C}$. We will also fix a choice of decomposition group D_q for all primes q in $\mathbf{Z}$. Suppose that k is a finite field of characteristic p and that

$$\rho_0\colon \operatorname{Gal}(\mathbf{Q}_\Sigma/\mathbf{Q}) \to \mathrm{GL}_2(k) \tag{1.1}$$

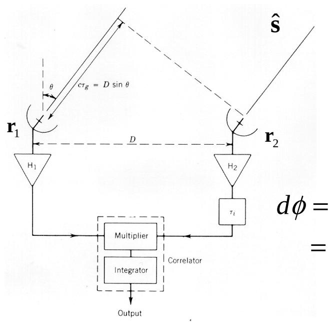
is an irreducible representation. In contrast to the introduction we will assume in the rest of the paper that ρ_0 comes with its field of definition k. Suppose further that $\det \rho_0$ is odd. In particular this implies that the smallest field of definition for ρ_0 is given by the field k_0 generated by the traces but we will not assume that $k = k_0$. It also implies that ρ_0 is absolutely irreducible. We consider the deformations $[\rho]$ to $\mathrm{GL}_2(A)$ of ρ_0 in the sense of Mazur [Ma1]. Thus if $W(k)$ is the ring of Witt vectors of k, A is to be a complete Noetherian local $W(k)$-algebra with residue field k and maximal ideal m, and a deformation $[\rho]$ is just a strict equivalence class of homomorphisms $\rho\colon \operatorname{Gal}(\mathbf{Q}_\Sigma/\mathbf{Q}) \to \mathrm{GL}_2(A)$ such that $\rho \bmod m = \rho_0$, two such homomorphisms being called strictly equivalent if one can be brought to the other by conjugation by an element of $\ker : \mathrm{GL}_2(A) \to \mathrm{GL}_2(k)$. We often simply write ρ instead of $[\rho]$ for the equivalence class.

The first page of Wiles's published proof, which goes on for over a hundred pages.

One-element interferometry



Basic interferometer geometry:



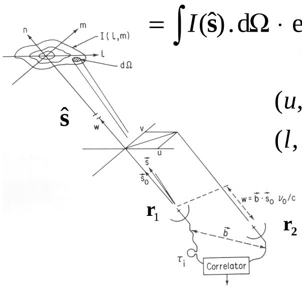
$$d\phi = 2\pi \cdot D \sin\theta / \lambda$$
$$= 2\pi\ \hat{\mathbf{s}} \bullet (\mathbf{r}_1 - \mathbf{r}_2) / \lambda$$

Coherence function:

$V(\mathbf{r}_1,\mathbf{r}_2) \equiv \Gamma_{12}$

$$V(\mathbf{r}_1,\mathbf{r}_2) \;=\; \langle E(\mathbf{r}_1)\cdot E^*(\mathbf{r}_2)\rangle$$

$$= \int I(\hat{\mathbf{s}}).\mathrm{d}\Omega \cdot \exp\{-2\pi i \cdot \hat{\mathbf{s}}\bullet(\mathbf{r}_1-\mathbf{r}_2)/\lambda\ \}$$



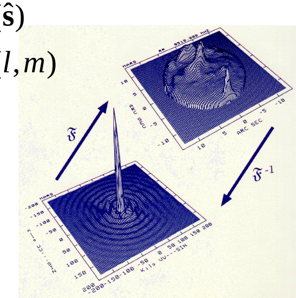
$$(u,v) \leftrightarrow \mathbf{u} = (\mathbf{r}_1-\mathbf{r}_2)/\lambda$$

$$(l,m) \leftrightarrow \hat{\mathbf{s}}$$

Fourier transform relation:

$$V(\mathbf{r}_1-\mathbf{r}_2) \leftrightarrow I(\hat{\mathbf{s}})$$

$$V(u,v) \quad \leftrightarrow \quad I(l,m)$$



Key simplifications

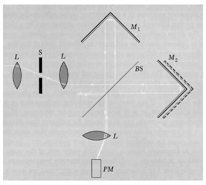
- Spatial incoherence
- Stationarity $(\partial\langle E_1 E_2^*\rangle/\partial t \rightarrow 0)$
- “Far field” (D/d >> 1)
- Propagation *in vacuo*
- Small-field *(l,m* << 1)

- Quasi-monochromatic (d*ν*/*ν* <<1)
- Polarization

Practical matters

- Antennae (primary beam)
- Receivers, filters (bandpasses)
- Sampling
- Correlators
- Computation: approximations
- Deconvolution
- Noise, Error recognition

Fourier transform spectrometer



$$V(\tau) = \langle I(t)\, I^*(t-\tau) \rangle$$

$$I(\upsilon) = \int V(\tau) \exp\{-2\pi i\, \upsilon\tau\}$$

Spectroscopic synthesis:

$$V(\mathbf{r}_1, \mathbf{r}_2) = \langle E(\mathbf{r}_1) E^*(\mathbf{r}_2) \rangle$$

$$V(\mathbf{r}_1, \mathbf{r}_2) = \langle E(\mathbf{r}_1, t) E^*(\mathbf{r}_2, t - t_0) \rangle$$

Quasi-monochromatic: $t_0 \approx |\mathbf{r}_1 - \mathbf{r}_2| / c \ll 1/d\nu$

$$V(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle E(\mathbf{r}_1, t) E^*(\mathbf{r}_2, t - \tau) \rangle \quad \equiv \Gamma_{12}(\tau)$$

$$V(\mathbf{r}_1, \mathbf{r}_2, \upsilon) = \int V(\mathbf{r}_1, \mathbf{r}_2, \tau) \exp\{-2\pi i\, \upsilon\tau\}$$