Random Media in Radio Astronomy

- Atmosphere path length ~ 6 Km
- Ionosphere path length ~100 Km

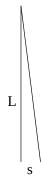
•Solar Wind (interplanetary plasma)
path length ~ 1 AU (1.5×10^8 Km)

•Interstellar Plasma
path length ~ 100-1000 pc (3×10^{16} Km)

Radio Propagation Basics

- Refractive index n phase speed v = c/n
- In air $n = 1 + \delta$ where $\delta \ll 1$ and depends on density and humidity
- In "cold" plasma $n \sim 1 - N_e r_e \lambda^2 / 2\pi = 1 - \omega_p^2 / 2\omega^2$

where N_e is electron density, λ is the wavelength
$r_e = 2.8\ 10^{-15}$ m is the classical electron radius
ω_p is called the plasma frequency

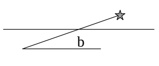


Phase is $\phi(\mathbf{s}) = 2\pi/\lambda \int n(\mathbf{s}, z)\, dz = 2\pi L/\lambda - r_e \lambda\, DM$
where $DM = \int N_e(s, z)\, dz$ is the Dispersion Measure

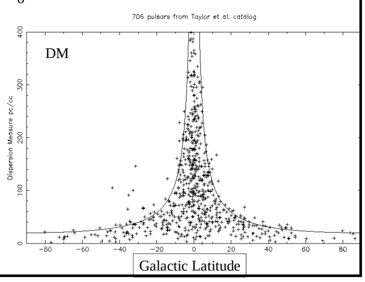
Group Delay

- Travel time for a pulse at frequency f_o

$$T_g = \frac{\partial \varphi}{\partial f} \frac{1}{2\pi} = \frac{L}{c} + \frac{r_e c DM}{f_o^{\ 2}}$$



If $L > H$ cosecb
$DM = N_e H$ cosec b
$N_e H = 20$ pc cm^{-3}

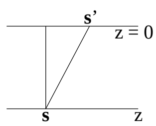


Galactic Latitude

Fresnel Diffraction Integral

Assume a plane wave incident on a phase changing layer at z=0
The emerging field is phase modulated:

$$f(\mathbf{s'}, 0) = \exp[i\phi(\mathbf{s'})]$$



The field at distance z is given by summing the contributions From position **s'** across the screen, with extra phase due to the longer slant path as approximated by the quadratic term below:

$$f(\mathbf{s}, z) = \int \exp[i\phi(\mathbf{s'}) + i\pi\, |(\mathbf{s'} - \mathbf{s})|^2 / \lambda z]\, d^2\mathbf{s'}\ (i\pi/\lambda z) \exp[i2\pi z/\lambda]$$

Interferometry with Scattering

- v_1 v_2 are voltages from antennas separated by baseline **s**
- Interferometer visibility measures

  $V = < v_1 \, v_2^* >$

- v_1 v_2 are both given by a Fresnel diffraction integral which can be combined and the average < > taken.

- The result is a remarkably simple expression

  $V = \exp[-0.5 \, D_\phi(\mathbf{s})] \; \Gamma(\mathbf{s})$ where

  $\Gamma(\mathbf{s})$ is the visibility of the source that would have resulted in the absence of the scattering medium.

  $D_\phi(\mathbf{s}) = < [\phi(\mathbf{s}') - \phi(\mathbf{s}'+\mathbf{s})]^2 >$ is called the phase structure function of the scattering medium

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- The visibility product: $V(\mathbf{s}) = e^{-0.5 D_\phi(\mathbf{s})} \; \Gamma(\mathbf{s})$ can be expressed in the image domain :
- The scattered image is the convolution of the source brightness distribution by a broadening function $P(\theta)$

- $P(\theta) =$ the Fourier Transform of $[e^{-0.5 D_\phi(\mathbf{s})}]$
- The angular width of $P(\theta)$ is θ_{scatt} called the scattering diameter which for plasma scattering varies as λ^2 (or $\lambda^{2.2}$ for scattering in a turbulent medium)

$\theta_{scatt} = \lambda / 2\pi s_o$ where s_o is the lateral scale over which there is an rms diffrence in phase of 1 radian. ie: $D_\phi(s_o) = 1$

Image Correction

- Can the loss of visibility imposed by the scattering be removed or corrected?
- Yes, sometimes…. Consider each (complex) voltage:
- $v_1 = a_1 \exp(j \, \psi_1) \, u_1$ where u_1 is the unperturbed signal
- a_1 and ψ_1 are the amplitude and phase modulations due to the scattering medium at antenna #1. Under some circumstances these can be estimated and corrected for.
- Self Calibration: With n antennas there are 2n constants to be determined. If $n(n-1)/2 > 2n$ there are more baselines than parameters and the constants may be estimated, if a suitable point source is available.

Image correction (contd)

- With $n(n-1)/2 > 2n$ and if the array is centered on a known point source, the observed visibilities can be used to estimate the 2n constants $a_n \, \psi_n$
- The commonest use is when the amplitudes do not vary, and then phase-only self-calibration estimates the ψ_n
- Such as due to atmospheric or ionospheric perturbations
- In general $a_n \, \psi_n$ vary with angular position in the sky, and thus may require a calibrator quite close to the target source. The region over which a single set of constants may apply is called the iso-planatic patch.
- Interstellar scattering has an iso-planatic patch smaller than a milliarcsecond and varies rapidly over frequency and time; consequently it cannot be corrected in practice.

Image correction (contd)

- Correction for propagation in the Earth's atmosphere is readily done by phase self-cal at frequencies of 10 GHz and lower. At higher frequencies the influence of water vapour and clouds and rain make such corrections increasingly difficult.
- Correction for propagation in the Earth's ionosphere is necessary at frequencies below about 400 MHz. As the frequency is reduced toward about 10 MHz where the ionosphere reflects radio waves, such corrections become increasingly difficult. The iso-planatic patch shrinks and requires multiple calibration sources across the field of view.

Scintillation

- As the distance from a phase screen increases, the effects of diffraction and interference turn the phase modulations into amplitude modulations. These cause the twinkling of stars at optical wavelengths and are referred to as scintillations at radio wavelengths.
- Scintillation is divided into weak and strong, according to whether the rms amplitude is less than or greater than the mean amplitude.
- Weak interstellar scintillation is typical at frequencies above ~3 GHz. $\tau = (\lambda L/2\pi)^{0.5}/V \sim 5\text{hrs}\ (f_{GHz})^{-0.5}$

  rms flux < mean flux

Strong Interstellar Scintillation (ISS)

- Below ~ 3 GHz. Interstellar Scintillation is strong, having an rms change in flux density > mean flux density
- Two Time scales:
  - Diffractive ISS $\tau_d \sim s_o / V$ rms flux ~ mean flux
  - Refractive ISS $\tau_r \sim L\,\theta_{scatt} / V$ rms flux < mean flux

For typical pulsar distance:

$\tau_d \sim$ 5-50 min ($f^{+1.2}$) $\delta\nu_d \sim$ 0.1 MHz $(f_{GHz})^{4.4}$

$\tau_r \sim$ 5-100 days ($f^{-2.2}$) $\delta\nu_r \sim f_o$