



Polarimetry

Dave McConnell, ATNF 8th Synthesis Imaging School, Narrabri 2 October 2008

Polarimetry has bad press; unfortunate because its fascinating and essential to understanding the Universe. It is a topic with many simple elements; but if not assembled carefully and clearly it can be confusing, or even impenetrable. Remember, if in doubt go back to the basic simple elements. This morning we will wallow in it – we have two lectures to allow time for looking at both the simple elements and the complexities.

Electro-magnetic waves are polarized



 E/M waves have direction, amplitude, frequency and polarization

 $\mathbf{S} = c/4\pi \left(\mathbf{E} \times \mathbf{H} \right)$

Our business is to understand the Universe from the radiation we receive from it, and all characteristics of that radiation are relevant.

Outline of lecture

- Polarization what is it?
- How is it described
- Origins of polarized light
- How is it important to astrophysics
- How is it measured

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This will be a gentle introduction to radio polarimetry, to be followed by Rick Perley's more mathematical treatment. I will spend some time on descriptions and visualisations of polarized light (radio waves). I will talk in general terms about the origins of polarized light, and its propagation. I will discuss why it is worth going through the effort of radio polarimetry, sketching some of the astrophysical motivations. And I will introduce the

approach to radio polarimetry.

In very many cases, matter that emits or absorbs radiation consists of aggregates of very large numbers of atoms or molecules which in general have random orientations with respect to each other varying from one instant to the next. The net result is that such aggregates can absorb radiation of any conceivable polarisation, and radiate (as is the case for a variety of celestial objects) what is commonly called unpolarized radiation. (from Rad, 1990) Polarimetry, the measurement of the polarisation state of the observed radiation, is useful whenever there is a loss of symmetry, an isotropy of some kind in the source or in the intervening medium.

References

- Radhakrishnan. Polarisation. URSI proceedings (1990) pp. 34
- Hamaker et al. Understanding radio polarimetry. I. Mathematical foundations. Astronomy and Astrophysics Supplement (1996) vol. 117 pp. 137
- Sault et al. Understanding radio polarimetry. II. Instrumental calibration of an interferometer array. Astronomy and Astrophysics Supplement (1996) vol. 117 pp. 149
- Hamaker et al. Understanding radio polarimetry. III. Interpreting the IAU/ IEEE definitions of the Stokes parameters. Astronomy and Astrophysics Supplement (1996) vol. 117 pp. 161
- Heiles et al. Mueller Matrix Parameters for Radio Telescopes and Their Observational Determination. The Publications of the Astronomical Society of the Pacific (2001) vol. 113 pp. 1274
- Born and Wolf: 'Principle of Optics', Chapters 1 and 10
- Presentations from previous Synthesis Schools (Ohja, 2003; Perley 2006

Polarized waves: linear



$$\mathbf{E} = E_x \cos(\omega t - kz) \underline{\hat{x}}$$

5

Drop the <u>H</u> vector from the diagrams. Our detectors sense the electric field <u>E</u> at some point such as the origin of these axes. Here the waves are polarized in the cardinal directions. Whenever we talk of polarized radiation we need a reference, often celestial north or up in the local frame.

Polarized waves: linear



 $\mathbf{E} = E_x \cos(\omega t - kz) \hat{x}$

 $\mathbf{E} = E_y \cos(\omega t - kz)\hat{y}$

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... at any angle ψ



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Our convention is to state the angle of polarization as measured from that reference counterclockwise viewing the wave approaching. On the sky that is sometimes described as from north through east.



or Circular - LCP, RCP Right $\mathbf{E} = E\cos(\omega t - kz)\hat{\underline{x}}$ $+E\sin(\omega t - kz)\hat{y}$

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or Circular - LCP, RCP Right $\mathbf{E} = E\cos(\omega t - kz)\hat{x}$ $+E\cos(\omega t-kz-\frac{\pi}{2})\hat{y}$

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IEEE Standard 211, 1969

right-hand polarized wave: A circularly or an elliptically polarized electromagnetic wave for which the electric field vector, when viewed with the wave approaching the observer, rotates counter-clockwise in space. *Notes:* 1. This definition is consistent with observing a clockwise rotation when the electric field vector is viewed in the direction of propagation. 2. A right-handed helical antenna radiates a right-hand polarized wave.

IAU resolution, 1973

Commission 40 (Radio Astronomy/Radioastronomie)

8. POLARIZATION DEFINITIONS

A working Group chaired by Westerhout was convened to discuss the definition of polarization brightness temperatures used in the description of polarized extended objects and the galactic background. The following resolution was adopted by Commissions 25 and 40: 'RESOLVED, that the frame of reference for the Stokes parameters is that of Right Ascension and Declination with the position angle of electric-vector maximum, θ , starting from North and increasing through East. Elliptical polarization is defined in conformity with the definitions of the Institute of Electrical and Electronics Engineers (IEEE Standard 211, 1969). This means that the polarization of incoming radiation, for which the position angle, θ , of the electric vector, measured at a fixed point in space, increases with time, is described as right-handed and positive.'

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Linear as sum of circulars

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A linearly polarized wave can be decomposed into two opposite handed circular waves. The position angle of the linear determines the phase difference between the two circulars. Note that if the relative phase of the circular waves changes by δ , the linear position angle changes by $\delta/2$.

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Other combinations

- The sum of two circular waves of unequal amplitude will have elliptical polarization.
- The sum of two orthogonal linears with phase difference $0 < \varphi < \pi/2$ will also have elliptical polarization.

Polarization ellipse



- $tan \alpha = E_y / E_x$
- $tan 2\psi = tan 2\alpha \cos \delta$
- ψ is the position angle
- χ is the ellipticity

Stokes description

- Defined by George Stokes in 1852
- Adopted for astronomy by Chandrasehkar (1949) in the solution of radiative transfer problems.





The Stokes parameters are a mathematically convenient way of describing the polarization state of the radiation. Why? come to that later.

Stokes parameters

$$I = E_x^2 + E_y^2$$
$$Q = E_x^2 - E_y^2$$
$$U = 2E_x E_y \cos(\delta)$$
$$V = 2E_x E_y \sin(\delta)$$

- For monochromatic waves
- I:total intensity
- Q:linear
- U : linear
- V:circular

•
$$|^2 = Q^2 + U^2 + V^2$$

Stokes parameters

$$I = E_R^2 + E_L^2$$
$$V = E_R^2 - E_L^2$$
$$Q = 2E_R E_L \cos(\delta)$$
$$U = 2E_R E_L \sin(\delta)$$

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$$I^2 = Q^2 + U^2 + V^2$$

Linear: Q and U





Mars at 22GHz



This image of Mars (credit to Steve Meyers) nicely shows the nature of Q and U.

Mars at 22GHz



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The Poincaré sphere

The Poincaré sphere





Partial polarization

- Two monochromatic signals summed:
 - different frequencies
 - different polarization ellipses

Up until now we have been talking about the polarization of monochromatic radiation. Monochromatic radiation, that is radiation with zero bandwidth, is an idealisation useful for thinking about the radiation's elementary properties. And we know that we can describe a real radiation field as a Fourier synthesis of a set of monochromatic waves. Monochromatic radiation is fully, 100%, polarized. So what is "Partial polarization"? Consider the sum of two monochromatic waves, each necessarily 100% polarized. Suppose they have different polarization ellipses, and different frequencies. Their sum has a polarization state that is no longer obvious from simple inspection.

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Pancharatnam's extension



- Radius represents I
- S = (Q,U,V)
- |S| < I
- Unpolarized radiation (I S) at centre .

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Within any time interval less than the inverse bandwidth ($\tau < 1/\Delta f$) the radiation is fully polarized and has a position on the surface of the sphere. The tip of vector S will be on the sphere. Some intervals τ later it will have moved to a different position. Watch this over a long time and form the vector average of all the positions; if the radiation is truly "unpolarized" it will average to the sphere centre. If there is any dominant polarization, the

average vector will have a finite length and will represent the state of the partial polarization.

Stokes parameters

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$$Q = \langle E_x^2 \rangle - \langle E_y^2 \rangle$$
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- I : total intensity
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- $I^2 \leq Q^2 + U^2 + V^2$

$$P = \sqrt{Q^2 + U^2 + V^2}$$
$$\tan(2\psi) = \frac{U}{Q}$$

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So now we must perform time-averaging in the definition of the Stokes parameters.

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Polarized light gives the clue to some loss of symmetry in the radiation source, or the propagation path to the observer. Something is anisotropic. Deal with the propagation first. Reflected light is well known to be partially polarized; the rainbow is an example.

• Loss of symmetry

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- Loss of symmetry
- Polarized radiation
- Polarization-dependent propagation
 - reflection
 - scattering
 - birefringence



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Scattering



- Light from the day time sky is sun light "Rayleigh scattered" by molecules in the atmosphere.
- Sky light is polarized, maximally at 90 degrees to the sun.
- The CMB is expected to be partially polarized because of Thompson scattering

What is anisotropic? The direction of solar photons: all coming from one direction.

Birefringence

Birefringence occurs when light passes through anisotropic material whose refractive index differs for the two polarization modes.





Linear modes birefringent

Circular modes birefringent

Birefringence occurs when light passes through anisotropic material whose refractive index differs for the two polarizations. In the case of certain crystals it is the linear modes that have the different refractive indices. For material permeated by a magnetic field the two circular modes have different propagation speeds, leading to the rotation of linearly polarized waves, an effect named after its discoverer, Michael Faraday.

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Now talk about polarized radiation ...

Zeeman W51C (2-b) Ba

W51C (2-b) $B_{\theta} = 2.5 \pm 0.2 \text{ mG}$



Synchrotron



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Top: non-relativisitic motion, radiation is beamed symmetrically over a broad solid angle and so is visible from most directions. The electric field vector is parallel to the acceleration and rotates, leading to circular polarized emission. Depending on the viewing angle circular or elliptical will be observed. Examples are the Jovian decametric and auroral kilometric radiation (AKR). Bottom: Relativistic case: the radiation is strongly beamed in the direction of motion. It is only visible by vieing the orbit edge on. From that perspective the radiation is linearly polarized. This is the Synchrotron radiation responsible for the Galactic non-thermal radiation. Its plane of polarization gives information on the direction of the magnetic field in the emission region.

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Faraday rotation

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Magnetised plasmas are birefringent: the two circular modes have refractive index dependent on the parallel component of the magnetic field, the electron density, and the wave frequency. Therefore, the relative phase of the two modes changes along the propagation path, and so does the position angle of the resultant linearly polarized radiation.

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$$\mathrm{RM}^{(SI)} = \frac{e^3}{8\pi^2\varepsilon_0 m^2 c^3} \int_0^d n_e B \, \mathrm{d}s = 2.62 \times 10^{-13} \int_0^d n_e B \, \mathrm{d}s,$$

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Faraday tomography

See nice presentation by Bryan Gaensler (University of Sydney) from April 2008. Just ask Bryan.

Why is it so difficult?

- Depolarization leads to weak signals
- Conceptual difficulties it is complicated
- Instrumental effects can be significant and difficult to separate from the signal

Imaging

 I, Q, U and V are equivalent for aperture synthesis imaging, but for the possibility (certainty) of negative Q, U,V.

Negative Q, U, V places constraints on some deconvolution. But, statistics: next slide

Stokes parameters

$$I = \langle E_x^2 \rangle + \langle E_y^2 \rangle$$
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- For finite bandwidth radiation
- I : total intensity
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$$P = \sqrt{Q^2 + U^2 + V^2}$$
$$\tan(2\psi) = \frac{U}{Q}$$

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IQUV have gaussian statistics (sum of many measurements). (How many is many? Provided the integration time is many times the inverse bandwidth.)

P and ψ do not. Be very careful interpreting results expressed in P or ψ or any other non-linear combination of I, Q, U, V.

Instrumental imperfections



- phase errors
- and all the usual ones!

usual ones: gain fluctuations, non-ideal primary beam, ... A gross simplification!

Jones vector



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Jones vector $\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}$

by a 2x2 matrix

Effect of the instrument described $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$

32

Jones vector

 $\left(\begin{array}{c} E_x(t)\\ E_u(t)\end{array}\right)$

Effect of the instrument described by a 2x2 matrix

 $\begin{pmatrix} E_x \\ E_y \end{pmatrix}_{c} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_{i}$

A linear polarizer

$$\begin{pmatrix} E_x \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$$

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$$\begin{pmatrix} v_x \\ v_y \end{pmatrix}_o = \begin{pmatrix} G_{xx} & G_{xy} \\ G_{yx} & G_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$$

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In general the Gs are complex. One would like the matrix to be close to "diagonal".

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix}_o = \begin{pmatrix} 1 & D_{i,x} \\ -D_{i,y} & 1 \end{pmatrix} \begin{pmatrix} G_{i,x} & 0 \\ 0 & G_{i,y} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$$

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An example treatment. See Hamaker et al (series of three papers, listed on reference page) for details.

Mueller calculus



Mueller calculus is a matrix method for manipulating Stokes vectors, which represent the polarization of incoherent light.

The ATCA is a compatively easy instrument to use for polarimetry, thanks to its excellent properties (linear feeds, stable receivers) and to the miriad doftware package that uses the Jones and Mueller calculi to calibrate data with very little effort from the user. It does pay to look carefully at the intermediate results produced by miriad (for example the solutions for leakages), to assure yourself of the reliability of your results.

Traps for players, young and old

- Missing flux (no short baselines)
- Statistics of P to bias or not to bias?
- Complexity of the instrument (off-axis, ...)

Missing flux: effect on position angle!

References

- Radhakrishnan. Polarisation. URSI proceedings (1990) pp. 34
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Polarisation a tutorial by V. Radhakrishnan

